The $B \to \pi K$ Puzzle: 2021 Status Report

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Monday, May 31, 2021

Talk based in part on work done in collaboration with N. Boisvert Beaudry, A. Datta, A. Rashed and J.-S. Roux, [arXiv:1709.07142 [hep-ph]].

Back to the 1990s

CP violation (CPV) observed only in Kaon system, in K^0-$\bar{K}^0$ mixing ($\epsilon_K \sim \mathrm{Im}\langle \bar{K}^0|\mathcal{H}|K^0\rangle$). SM explanation: CKM matrix.

Charged current: $\bar{U}_L V_{CKM} \gamma^\mu D_L W^+_\mu + h.c.$:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} .$$

V_{CKM} is unitary, parametrized by 3 angles and 1 phase.

Experiment: CKM matrix elements obey a hierarchy:

$$V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} ,$$

where $\lambda = \sin\theta_C \simeq 0.22$ (θ_C is Cabibbo angle).

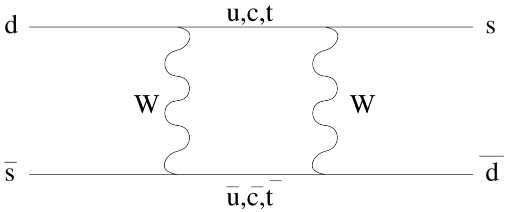
Wolfenstein parametrization: 3 angles → 3 real parameters,
1 phase → 1 complex parameter:

$$V_{CKM} \simeq \begin{array}{c} \\ u \\ c \\ t \end{array} \begin{array}{c} \begin{array}{ccc} d & s & b \end{array} \\ \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \end{array} .$$

V_{CKM} is unitary to $O(\lambda^3)$:

Note: at this order, the complex phase appears only in the corner elements V_{ub} and V_{td}.

K^0-$\bar{K}^0$ mixing:



Involves $V_{td} \Longrightarrow \mathrm{Im}\langle\bar{K}^0|\mathcal{H}|K^0\rangle$ is nonzero.

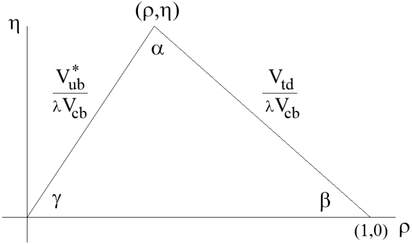
However, $\exists$ problem. $|\epsilon_K|$ measured (very precisely) at meson level, but diagram calculated at quark level $\Longrightarrow$ sizeable hadronic uncertainty in relating $|\epsilon_K|$ to the parameters of the CKM matrix.

How can we test this explanation?

Write $V_{ub} = |V_{ub}| \exp(-i\gamma)$, $V_{td} = |V_{td}| \exp(-i\beta)$.
Orthogonality of first and third columns implies

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 = |V_{ud}||V_{ub}| e^{i\gamma} + |V_{cd}||V_{cb}^*| + |V_{td}||V_{tb}| e^{-i\beta} .$$

This is a triangle relation in the complex plane $\Longrightarrow$ unitarity triangle:



Interior angles α, β and γ all proportional to $\eta \Longrightarrow$ a nonzero value of any of these angles implies CPV. The angles are not independent: $\alpha + \beta + \gamma = \pi$.

Key point: α, β and γ can all be measured in B-meson decays.

CP Violation

Suppose the decay $B \to f$ has two contributing amplitudes, X and Y:

$$A(B \to f) \equiv A = X + Y = |X| e^{i\phi_X} e^{i\delta_X} + |Y| e^{i\phi_Y} e^{i\delta_Y} ~,$$

where $\phi_{X,Y}$ and $\delta_{X,Y}$ are weak (CP-odd) and strong (CP-even) phases, respectively. The (direct) CP asymmetry is

$$\begin{aligned} A_{\rm CP}^{\rm dir} &= \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \\ &= \frac{2|X||Y|\sin(\phi_X - \phi_Y)\sin(\delta_X - \delta_Y)}{|X|^2 + |Y|^2 + 2|X||Y|\cos(\phi_X - \phi_Y)\cos(\delta_X - \delta_Y)} ~. \end{aligned}$$

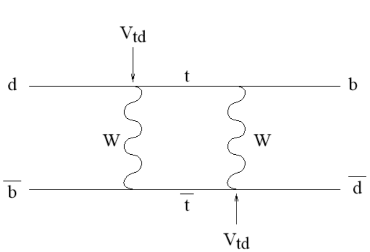
Point: a nonzero $A_{\rm CP}^{\rm dir}$ requires $\phi_X - \phi_Y \neq 0$ *and* $\delta_X - \delta_Y \neq 0$.

This is problematic. The strong phases are unknown $\Longrightarrow$ cannot extract weak-phase information without significant hadronic uncertainty.

There is an alternative. $\exists$ B^0-$\bar{B}^0$ mixing $\Longrightarrow$ a particle "born" as a B^0 will become in time a mixture of B^0 and $\bar{B}^0$: $B^0(t)$.

The $B^0(t)$ can decay as a B^0 or a $\bar{B}^0$. If we consider a final state f to which both B^0 and $\bar{B}^0$ can decay, the decay $B^0(t) \to f$ has 2 amplitudes: $B^0 \to f$ and $\bar{B}^0 \to f$. These can interfere, resulting in (indirect) CPV.

The corner CKM matrix elements that have phases ($V_{ub} = |V_{ub}| \exp(-i\gamma)$, $V_{td} = |V_{td}| \exp(-i\beta)$) appear in some B mixing and decay amplitudes:



Find $|A(B^0(t) \to f)|^2$ contains two time-dependent CPV pieces:

$$A_{\rm CP}^{\rm dir} \cos(\Delta M t) + A_{\rm CP}^{\rm indir} \sin(\Delta M t) ~,$$
$$A_{\rm CP}^{\rm dir} = \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} ~~,\qquad A_{\rm CP}^{\rm indir} = {\rm Im}\left[e^{-2i\phi_M} \frac{\bar{A}}{A} \right] ~,$$

where ϕ_M is the phase of B^0-$\bar{B}^0$ mixing ($\phi_M = \beta$ (B^0), $\phi_M \simeq 0$ (B_s^0)).

Suppose $A(B \to f)$ has only one contributing amplitude:

$$A = |X| e^{i\phi_X} e^{i\delta_X} ~~\Longrightarrow~~ \bar{A} = |X| e^{-i\phi_X} e^{i\delta_X} ~.$$

This implies that

$$A_{\rm CP}^{\rm dir} = 0 ~~,\qquad A_{\rm CP}^{\rm indir} = -\sin(2\phi_M + 2\phi_X) ~.$$

Key point: strong phase cancels in $A_{\rm CP}^{\rm indir} \Longrightarrow$ weak-phase information extracted with no hadronic uncertainties!

$\exists$ 4 possibilities:

1. B^0 with $b \to u$: phase $= \beta$ (mixing) $+ \gamma$ (decay).
2. B^0 with $b \to c$: phase $= \beta$ (mixing) $+ 0$ (decay).
3. B_s^0 with $b \to u$: phase $= 0$ (mixing) $+ \gamma$ (decay).
4. B_s^0 with $b \to c$: phase $= 0$ (mixing) $+ 0$ (decay).

By considering different final states f, all three CP angles can be extracted from measurements of CPV in $B^0(t) \to f$:

- α: $B^0(t) \to \pi\pi$, $\rho\pi$, $\rho\rho$, etc.
- β: $B^0(t) \to J/\psi K_S$, ϕK_S, etc.
- γ: $B \to DK$, $B_s^0(t) \to D_s^{\pm} K^{\mp}$, etc.

Test the SM by measuring the sides and angles of the unitarity triangle in many different ways. If a discrepancy among the measurements is found $\Longrightarrow$ new physics. With this goal, the B-factories BaBar and Belle were built in the 1990s, took data in the 2000s.

Results

1. SM predicts that $\alpha+\beta+\gamma=\pi$.

Latest results:

$$\begin{aligned} B^0(t)\to\pi\pi,\rho\pi,\rho\rho \quad &: \quad \alpha=\left(86.4^{+4.5}_{-4.3}\right)^\circ ~, \\ \text{charmonium} \quad &: \quad \beta=\left(22.14^{+0.69}_{-0.67}\right)^\circ ~, \\ B\to D^{(*)}K^{(*)} \quad &: \quad \gamma=\left(72.1^{+5.4}_{-5.7}\right)^\circ ~, \\ &\alpha+\beta+\gamma=\left(180.6^{+6.9}_{-7.1}\right)^\circ ~. \end{aligned}$$

2. SM predicts that $A_{\rm CP}^{\rm indir}(\text{charmonium}) = A_{\rm CP}^{\rm indir}(B^0(t)\to\phi K_S)$.

Latest results:

$$\begin{aligned} A_{CP}^{indir}(\text{charmonium}) \quad &: \quad \sin 2\beta = 0.699\pm 0.017 ~, \\ A_{CP}^{indir}(B^0(t)\to f)~(\bar{b}\to\bar{s}q\bar{q}~\text{penguin}) \quad &: \quad \sin 2\beta = 0.648\pm 0.038 ~. \end{aligned}$$

3. SM predicts phase in B_s^0-$\bar{B}_s^0$ mixing, $\varphi_s^{c\bar{c}s}$, is very small, $O(1^\circ)$.

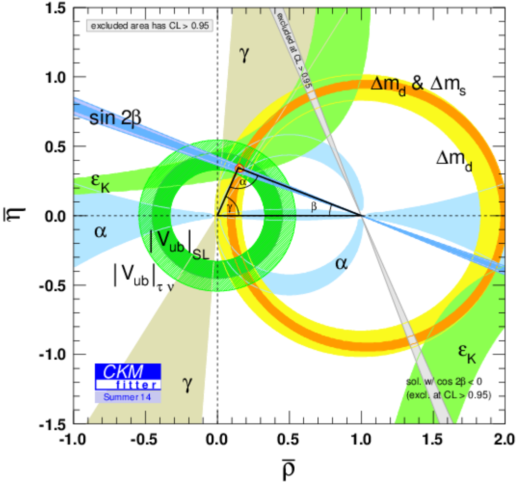
Latest results (LHCb):

$$\varphi_s^{c\bar{c}s} = (2.9 \pm 1.1)^\circ \ .$$

4. 2010: DØ measures CP asymmetry in $b\bar{b} \to \mu^\pm\mu^\pm X$:
$A_{\rm sl}^b = -(7.87 \pm 1.72 \pm 0.93) \times 10^{-3}$. Disagrees with SM prediction by 3.9σ.

Measurement not repeated (Tevatron had $p\bar{p}$ collisions, LHC has pp collisions). But result is inconsistent with other related measurements $\Longrightarrow$ looks like it was a statistical fluctuation.

As of 2018:



If new physics is present, its effects are small.

The $B \to \pi K$ Puzzle

There was a discrepancy from BaBar/Belle observed in 2003.

$\exists$ 4 $B \to \pi K$ decays: $B^+ \to \pi^+ K^0$ (designated $+0$), $B^+ \to \pi^0 K^+$ ($0+$), $B^0 \to \pi^- K^+$ ($-+$) and $B^0 \to \pi^0 K^0$ (00). Decays not independent: their amplitudes are related by isospin:

$$\sqrt{2} A^{00} + A^{-+} = \sqrt{2} A^{0+} + A^{+0} \ .$$

With these, can measure 4 branching ratios, 4 direct CP asymmetries, one indirect CP asymmetry (in $B^0(t) \to \pi^0 K^0$). When all data combined, it was found (in 2003) that there was an inconsistency among the measurements.

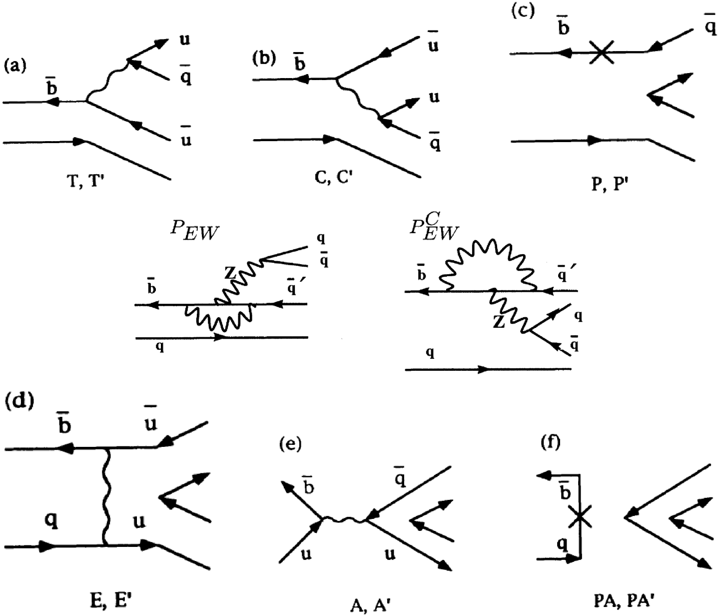
This is the $B \to \pi K$ puzzle, and it remains even today, some 20 years after its observation.

Amplitudes

Isospin: (u, d) form a doublet under $SU(2)_I \Longrightarrow (\pi^+, \pi^0, \pi^-)$ form a triplet, (K^+, K^0) form a doublet, etc. Matrix elements can be evaluated using Clebsch-Gordan coefficients and the Wigner-Eckart theorem.

Flavour symmetry: (u, d, s) form a triplet under $SU(3)_f \Longrightarrow$ $\{\pi^+, \pi^0, \pi^-, K^+, K^0, K^-, \bar{K}^0, \eta_1\}$ form an octet. Matrix elements much more difficult to calculate.

Instead, use topological diagrams. Note: these are not Feynman diagrams, but they do represent the currents involved. The set of diagrams can be mapped to the set of $SU(3)_f$ matrix elements.



(a)
b̄
u
q̄
ū
T, T'
(b)
b̄
ū
u
q̄
C, C'
(c)
b̄
q̄
P, P'
P_{EW}
b̄
Z
q
q̄
q̄ ′
q
P_{EW}^{C}
b̄
q̄ ′
Z
q
q̄
q
(d)
b̄
ū
q
u
E, E'
(e)
b̄
q̄
u
u
A, A'
(f)
b̄
q
PA, PA'

$B \to \pi K$ decays involve the transitions $\bar{b} \to \bar{s} q \bar{q}$, $q = u, d$. The amplitudes involve T', C', P', P'_{EW}, P'^C_{EW}, E', A', PA'. Observations:

- E', A', PA' suppressed by $f_B/m_B = O(1\%) \Longrightarrow$ these diagrams can be neglected to a first approximation.
- C' is colour-suppressed w.r.t. T'. Naively, this suppression is $1/3$, but more detailed theoretical estimates find $|C'/T'| \simeq 0.2$.
- P' contains t, c, u quarks in the loop. Using CKM unitarity, can write $P' = P'_{tc} + P'_{uc}$, where $|P'_{uc}/P'_{uc}| = |V^*_{ub} V_{us} / V^*_{tb} V_{ts}| \sim O(\lambda^2) = 0.04$.
- Can show that, assuming $SU(3)_f$ symmetry, to a good approximation P'_{EW} and P'^C_{EW} are proportional to T' and C', respectively:

$$P'_{EW} = \frac{3}{2} \frac{c_9}{c_1} \frac{|V^*_{tb} V_{ts}|}{|V^*_{ub} V_{us}|} \, T' \quad , \qquad P'^C_{EW} = \frac{3}{2} \frac{c_9}{c_1} \frac{|V^*_{tb} V_{ts}|}{|V^*_{ub} V_{us}|} \, C' \; .$$

$c_9 \ll c_1$, so that P'_{EW} and T' are roughly the same size, as are P'^C_{EW} and C'.

$\Longrightarrow$ the relative sizes of all the $B \to \pi K$ diagrams are roughly

$$1 : |P'_{tc}| \ , \quad \mathcal{O}(\bar{\lambda}) : |T'|, \ |P'_{EW}| \ , \quad \mathcal{O}(\bar{\lambda}^2) : |C'|, \ |P'_{uc}|, \ |P'^{C}_{EW}| \ ,$$

where $\bar{\lambda} \sim 0.2$.

The $B \to \pi K$ decay amplitudes are given by

$$\begin{aligned}
A^{+0} &= -P'_{tc} + P'_{uc} e^{i\gamma} - \frac{1}{3} P'^{C}_{EW} \ , \\
\sqrt{2} A^{0+} &= -T' e^{i\gamma} - C' e^{i\gamma} + P'_{tc} - \ P'_{uc} e^{i\gamma} - \ P'_{EW} - \frac{2}{3} P'^{C}_{EW} \ , \\
A^{-+} &= -T' e^{i\gamma} + P'_{tc} - P'_{uc} e^{i\gamma} - \frac{2}{3} P'^{C}_{EW} \ , \\
\sqrt{2} A^{00} &= -C' e^{i\gamma} - P'_{tc} + P'_{uc} e^{i\gamma} - P'_{EW} - \frac{1}{3} P'^{C}_{EW} \ .
\end{aligned}$$

Note: The weak-phase dependence is written explicitly; the diagrams contain both strong phases and the magnitudes of the CKM matrix elements. The amplitudes for the CP-conjugate processes are obtained by changing the sign of the weak phase γ.

The Naive $B \to \pi K$ Puzzle

Neglect $\mathcal{O}(\bar{\lambda}^2)$ diagrams in amplitudes:

$$
\begin{aligned}
A^{+0} &= -P'_{tc} \ , \\
\sqrt{2}A^{0+} &= -T' e^{i\gamma} + P'_{tc} - P'_{EW} \ , \\
A^{-+} &= -T' e^{i\gamma} + P'_{tc} \ , \\
\sqrt{2}A^{00} &= -P'_{tc} - P'_{EW} \ .
\end{aligned}
$$

With these amplitudes, $A_{\rm CP}^{\rm dir}(B^+ \to \pi^0 K^+) = A_{\rm CP}^{\rm dir}(B^0 \to \pi^- K^+)$.

2017:

Mode	$BR[10^{-6}]$	$A_{\rm CP}^{\rm dir}$	$A_{\rm CP}^{\rm indir}$
$B^+ \to \pi^+ K^0$	23.79 ± 0.75	-0.017 ± 0.016	
$B^+ \to \pi^0 K^+$	12.94 ± 0.52	0.040 ± 0.021	
$B^0 \to \pi^- K^+$	19.57 ± 0.53	-0.082 ± 0.006	
$B^0 \to \pi^0 K^0$	9.93 ± 0.49	-0.01 ± 0.10	0.57 ± 0.17

Not only are $A_{CP}(B^+ \to \pi^0 K^+)$ and $A_{CP}(B^0 \to \pi^- K^+)$ not equal, they are of opposite sign! We have $(\Delta A_{CP})_{\rm exp} = (12.2 \pm 2.2)\%$. This differs from 0 by 5.5σ. This is the naive $B \to \pi K$ puzzle.

Can quantify this. $P'_{EW} \propto T' \Longrightarrow \exists$ 5 unknown theoretical parameters: $|T'|$, $|P'_{tc}|$, one relative strong phase, and γ and β (appears in $A_{\rm CP}^{\rm indir}$). Constraints: the 9 $B \to \pi K$ observables and the independent measurements of β and γ. With more observables (11) than theoretical unknowns (5), a fit can be performed.

Terrible fit: $\chi^2_{\rm min}/{\rm d.o.f.} = 30.9/6$, corresponding to a p-value of 3.0×10^{-5}. This is the true $B \to \pi K$ puzzle.

$\chi^2_{\rm min}/{\rm d.o.f.} = 30.9/6$, p-value $= 3.0 \times 10^{-5}$	
Parameter	Best-fit value
γ	$(67.2 \pm 4.7)^\circ$
β	$(21.80 \pm 0.68)^\circ$
$\lvert T'\rvert$	7.0 ± 1.4
$\lvert P'_{tc}\rvert$	50.5 ± 0.6
$\delta_{P'_{tc}} - \delta_{T'}$	$(-15.6 \pm 3.4)^\circ$

SM Fits

1. Add small diagrams $\Longrightarrow$ with EWP-tree relations, now have 9 unknown theoretical parameters: $|T'|$, $|C'|$, $|P'_{tc}|$, $|P'_{uc}|$, three relative strong phases, and γ and β. Have 11 observables $\Longrightarrow$ can do a fit.

Fit is OK. However, $|C'/T'| = 0.75 \pm 0.32$, considerably larger than the estimate of $|C'/T'| = O(\bar{\lambda}) = 0.2$.

$\chi^2/\text{d.o.f.} = 3.5/2$, p-value $= 0.17$	
Parameter	Best-fit value
γ	$(72.0 \pm 5.8)^\circ$
β	$(21.85 \pm 0.68)^\circ$
$\|T'\|$	5.2 ± 1.5
$\|C'\|$	3.9 ± 1.2
$\|P'_{tc}\|$	50.7 ± 0.9
$\|P'_{uc}\|$	1.1 ± 2.4
$\delta_{C'} - \delta_{T'}$	$(209.8 \pm 21.3)^\circ$
$\delta_{P'_{tc}} - \delta_{T'}$	$(-16.2 \pm 7.3)^\circ$
$\delta_{P'_{uc}} - \delta_{T'}$	$(4.9 \pm 51.3)^\circ$

SM fit prefers a large value of $|C'/T'|$. Theory: QCD factorization: $|C'/T'| \simeq 0.2$. But pQCD: $|C'/T'|$ may be as large as 0.5 $\Longrightarrow$ fix $|C'/T'| = 0.2$ or 0.5. Also, $|P'_{uc}/P'_{tc}|$ found to be $= \mathcal{O}(\bar{\lambda}^3) \Longrightarrow$ negligible.

2. $|C'/T'| = 0.2$, $P'_{uc} = 0$, constraint on γ added.

Poor fit: $\chi^2_{\rm min}/{\rm d.o.f.} = 12.1/5$, corresponding to a p-value of 3%.

Conclusion: if $|C'/T'| = 0.2$, the $B \to \pi K$ puzzle cannot be explained by the SM.

$\chi^2_{\rm min}/{\rm d.o.f.} = 12.1/5$, p-value $= 0.03$	
Parameter	Best-fit value
γ	$(67.2 \pm 4.6)^\circ$
β	$(21.80 \pm 0.68)^\circ$
$\|T'\|$	7.9 ± 1.2
$\|P'_{tc}\|$	50.7 ± 0.6
$\delta_{P'_{tc}} - \delta_{T'}$	$(346.5 \pm 2.6)^\circ$
$\delta_{C'} - \delta_{T'}$	$(253.1 \pm 23.5)^\circ$

3. $|C'/T'| = 0.5$, $P'_{uc} = 0$, constraints on γ added

Good fit: $\chi^2_{\rm min}/{\rm d.o.f.} = 4.9/5$, for a p-value of 43%.

Conclusion: if $|C'/T'| = 0.5$, there is no $B \to \pi K$ puzzle – the data can be explained by the SM.

$\chi^2_{\rm min}/{\rm d.o.f.} = 4.9/5$, p-value $= 0.43$	
Parameter	Best-fit value
γ	$(70.6 \pm 5.3)^\circ$
β	$(21.82 \pm 0.68)^\circ$
$\|T'\|$	6.2 ± 0.9
$\|P'_{tc}\|$	50.5 ± 0.5
$\delta_{P'_{tc}} - \delta_{T'}$	$(162.4 \pm 3.5)^\circ$
$\delta_{C'} - \delta_{T'}$	$(42.8 \pm 18.1)^\circ$

Now, γ constrained by its independently-measured value. However, what happens if we treat γ as an unknown parameter? After all, if the SM explains the $B \to \pi K$ data, we would expect the extracted value of γ to be the same as that measured in tree-level decays.

4. $|C'/T'| = 0.5$, $P'_{uc} = 0$, γ free.

Reasonable fit: $\chi^2_{\rm min}/{\rm d.o.f.} = 4.3/4$, for a p-value of 36%.

However: preferred value of γ is $\gamma = (51.2 \pm 5.1)^\circ$, which deviates from its measured value of $(72.1 \pm 5.8)^\circ$ by 2.7σ.

Conclusion: even if $|C'/T'| = 0.5$, this is a reason not to be entirely satisfied that the SM explains the $B \to \pi K$ puzzle.

$\chi^2_{\rm min}/{\rm d.o.f.} = 4.3/4$, p-value $= 0.36$	
Parameter	Best-fit value
γ	$(51.2 \pm 5.1)^\circ$
β	$(21.78 \pm 0.68)^\circ$
$\lvert T'\rvert$	10.1 ± 3.4
$\lvert P'_{tc}\rvert$	51.8 ± 1.0
$\delta_{P'_{tc}} - \delta_{T'}$	$(168.6 \pm 4.6)^\circ$
$\delta_{C'} - \delta_{T'}$	$(131.2 \pm 24.7)^\circ$

Experimental Updates since 2017

Analysis based on 2017 data. Since then:

- 2020, LHCb: $A_{CP}^{indir}(B^+ \to \pi^0 K^+) = 0.025 \pm 0.016$ (was previously 0.040 ± 0.021), $A_{CP}^{dir}(B^0 \to \pi^- K^+) = 0.084 \pm 0.004$ (was previously 0.082 ± 0.006).

  An LHCb experimentalist repeated our analysis, found preferred value is $|C'/T'| = 0.67$ (was previously 0.75 [page 20]).
- 2021, Belle II presented measurements of the BR and A_{CP}^{dir} for $B^+ \to \pi^0 K^+$ and $B^0 \to \pi^0 K^0$. Consistent with previous data, but errors not yet competitive.

Previous analysis of the $B \to \pi K$ puzzle still valid.

But Belle II has started to make measurements of $B \to \pi K$ decays
$\Longrightarrow$ we will learn more in the coming years.

New-Physics Explanations

Need NP contribution to $\bar{b} \to \bar{s} u \bar{u}$ and/or $\bar{b} \to \bar{s} d \bar{d}$.

- Z' models with a flavour-changing $\bar{b} s Z'$ coupling. Add this NP contribution, get reasonably good fit, but only if the Z' couples to RH quarks, with $g_R^{dd} \neq g_R^{uu}$.
  B anomalies involving $\bar{b} \to \bar{s} \mu^+ \mu^-$ decays: One simple explanation: Z' boson. Many Z' models proposed, in some the Z' couples to RH $u\bar{u}$ and/or $d\bar{d}$, with $g_R^{dd} \neq g_R^{uu}$. These models can potentially also explain the $B \to \pi K$ puzzle.
- Diquarks D: contribute at tree level to $\bar{b} \to \bar{s} q \bar{q}$ ($q = u, d$) via $\bar{b} \to q D^* (\to \bar{s} \bar{q})$. The diquark that provides a reasonably-good fit transforms as $(6, 1, \frac{2}{3})$ and couples to $q_L^i q_L^j$ and $u_R^i d_R^j$.
- Axion-like particle a that mixes with the π^0 and has a mass close to the π^0 mass. a decays to γ; its addition modifies only those amplitudes involving a π^0, A^{0+} and A^{00}. Get a reasonably good fit. (B. Bhattacharya, A. Datta, D. Marfatia, S. Nandi and J. Waite, [arXiv:2104.03947 [hep-ph]])

Conclusions

Unitarity triangle constrained by many independent measurements. All consistent, lead to well-defined unitarity triangle. If NP is present, its effects are small.

One exception: one can measure 9 observables using the 4 $B \to \pi K$ decays. Problem: measurements not entirely consistent. This is the $B \to \pi K$ puzzle. It was first noticed in 2003, but it remains even today.

Caveat: not a "clean" discrepancy – $\exists$ theoretical input. In particular, if $|C'/T'| = 0.5$, data can be explained by the SM. But if $|C'/T'| = 0.2$, which is the preferred theoretical value, new physics is required.

One interesting NP solution is a Z' boson. If it couples to $\mu^+\mu^-$, might also be able to explain the $\bar{b} \to \bar{s}\mu^+\mu^-$ B anomalies.

Belle II has started to make measurements of $B \to \pi K$ decays $\Longrightarrow$ hopefully we will learn more in the coming years.