from asymptotic freedom to asymptotic safety

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standard model & beyond

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- local QFT for fundamental interactions
  - strong nuclear force
  - weak force
  - electromagnetic force
  - perturbatively renormalisable & predictive

standard model & beyond

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local QFT for fundamental interactions

- **strong** nuclear force
- **weak** force
- **electromagnetic** force
- perturbatively renormalisable & **predictive**

key feature

asymptotic freedom

couplings achieve **non-interacting**

Gaussian fixed point in the UV

standard model & beyond

local QFT for fundamental interactions

- strong nuclear force
- weak force
- electromagnetic force
- perturbatively renormalisable & predictive

key feature

asymptotic freedom
couplings achieve non-interacting
Gaussian fixed point in the UV

fundamental definition of QFT ⟷ UV fixed point

Wilson '71

standard model & beyond

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local QFT for fundamental interactions

strong nuclear force

weak force

electromagnetic force

perturbatively renormalisable & predictive

~~asymptotic freedom~~

~~couplings achieve non-interacting~~

~~Gaussian fixed point in the UV~~

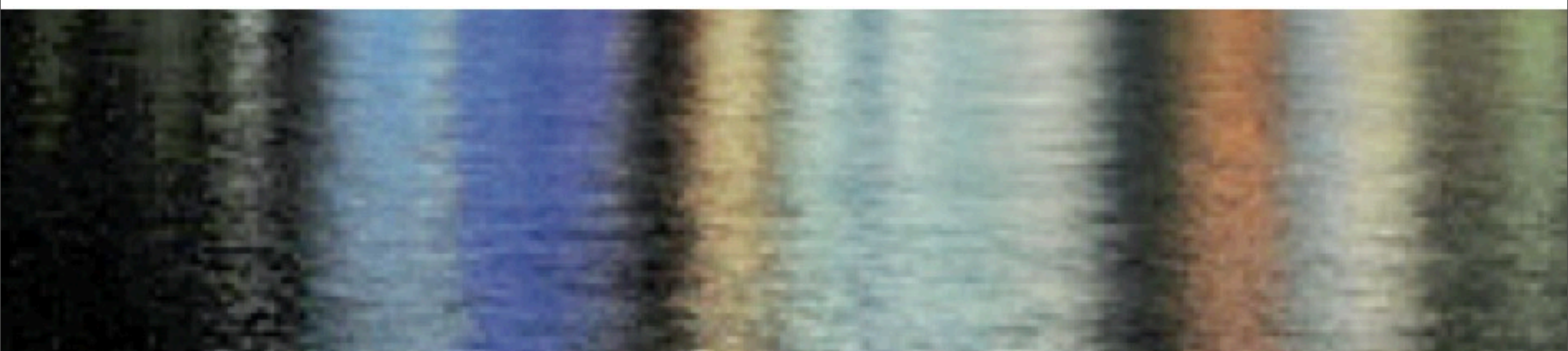
today:

asymptotic safety

couplings achieve interacting

non-Gaussian fixed point in the UV

asymptotic safety from perturbation theory



interacting fixed point

theory with coupling α **:**

$t = \ln \mu / \Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

perturbative non-renormalisability: $A > 0$

interacting fixed point

theory with coupling α**:**

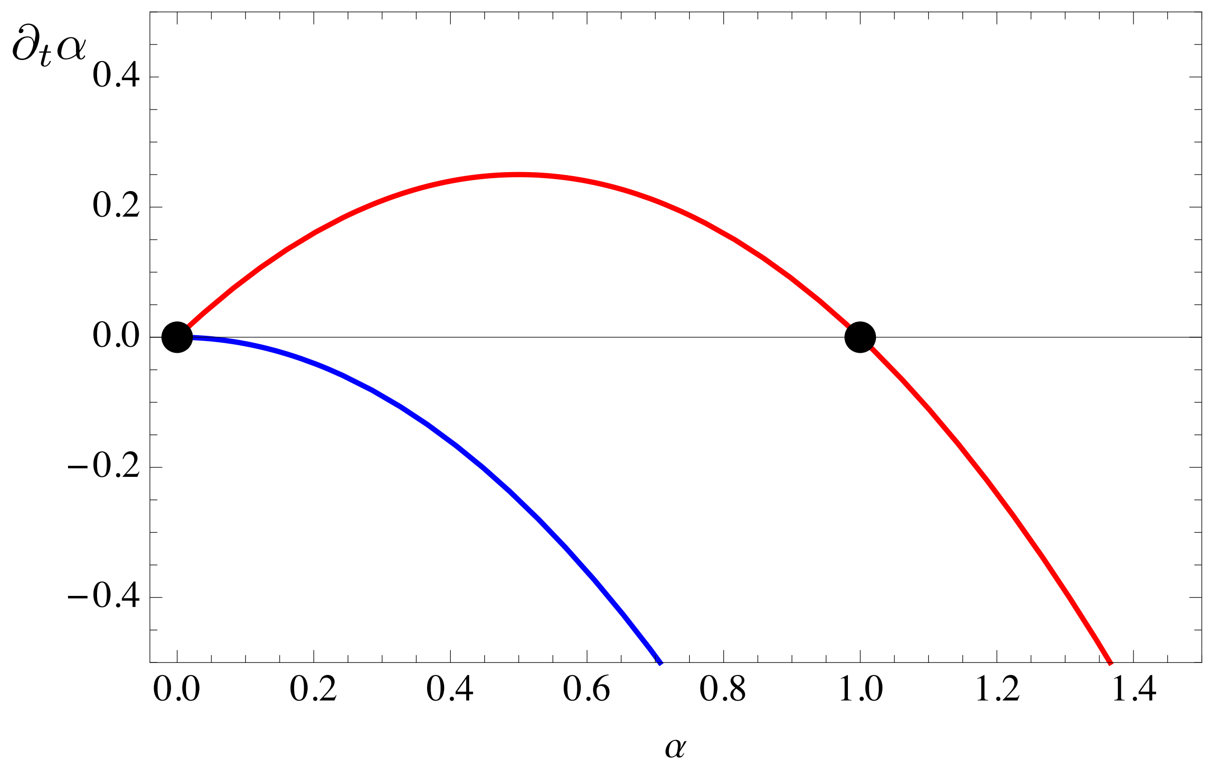
$t = \ln \mu/\Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

$\alpha_* \ll 1$

fixed points $\alpha_* = 0$ $\alpha_* = A/B$

if $A > 0\,,\ B > 0$**:** **IR** **UV**



interacting fixed point

theory with coupling α:

$t = \ln \mu / \Lambda$

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$\alpha_* \ll 1$

fixed points $\alpha_* = 0$ $\alpha_* = A/B$

epsilon expansion: $\epsilon = D - D_c$

large-N expansion: many fields

perturbation theory

theory with coupling α:

$t = \ln \mu/\Lambda$

$$\partial_t \alpha = A\,\alpha - B\,\alpha^2$$

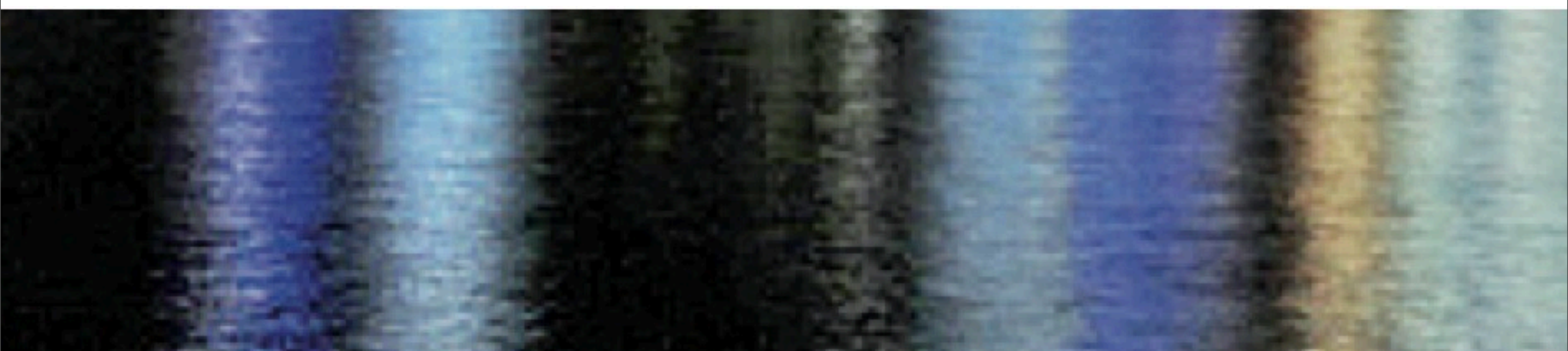
$\alpha_* \ll 1$

gravitons	$D = 2+\epsilon: \quad \alpha = G_N(\mu)\mu^{D-2}$	Gastmans et al '78 Weinberg '79 Kawai et al '90
fermions	$D = 2+\epsilon: \quad \alpha = g_{\rm GN}(\mu)\mu^{2-D}$	Gawedzki, Kupiainen '85 de Calan et al '91
gluons	$D = 4+\epsilon: \quad \alpha = g^2_{\rm YM}(\mu)\mu^{4-D}$	Peskin '80 Morris '04
scalars	$D = 2+\epsilon: \quad \alpha = g_{NL}(\mu)\mu^{D-2}$	Brezin, Zinn-Justin '76 Bardeen, Lee, Shrock '76

non-perturbative renormalisability

$$A = \epsilon \ll 1\,, \quad B = \mathcal{O}(1) > 0$$

asymptotic safety cookbook



DFL, F Sannino, 1406.2337
and work in prep.

gauge theory with fermions

SU(NC) YM with NF fermions: $$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\, \alpha_g^2$$

$\alpha_* \ll 1$

gauge theory with fermions

SU(NC) YM with NF fermions: $\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2$$

$\alpha_* \ll 1$

$B > 0$ **: asymptotic freedom**

UV fixed point $\alpha_* = 0$

$B < 0$ **: no asymptotic freedom**

UV fixed point? $\alpha_* \neq 0$

gauge theory with fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu / \Lambda$

$$\partial_t \alpha_g = -B\, \alpha_g^2 + C \alpha_g^3$$

$\alpha_* \ll 1$

↑

2-loop

gauge theory with fermions

SU(NC) YM with NF fermions: $\alpha_g = \dfrac{g^2\, N_c}{(4\pi)^2}$

$t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

$\alpha_* = 0$ $\alpha_g^* = B/C$

gauge theory with fermions

SU(NC) YM with NF fermions: $\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$ $t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

large-NF,NC (Veneziano) limit:

ϵ **continuous**

$$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

Veneziano '79

we consider $0 < -B \equiv -B\,(\epsilon) \ll 1$

gauge theory with fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$

$\alpha_* \ll 1$

$$\alpha_g^* = B/C$$

however:

no perturbative UV fixed point

in gauge theories with fermionic matter $(C > 0)$ Caswell '74

gauge theory with fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2} \qquad t = \ln \mu/\Lambda$$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3 \qquad \alpha_* \ll 1$$

$$\alpha_g^* = B/C$$

scalar fields & Yukawa couplings required

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$t = \ln \mu / \Lambda$

$\alpha_* \ll 1$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$t = \ln \mu / \Lambda$

$\alpha_* \ll 1$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

$$\partial_t \alpha_y = E\,\alpha_y^2 - F\,\alpha_g\,\alpha_y$$

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

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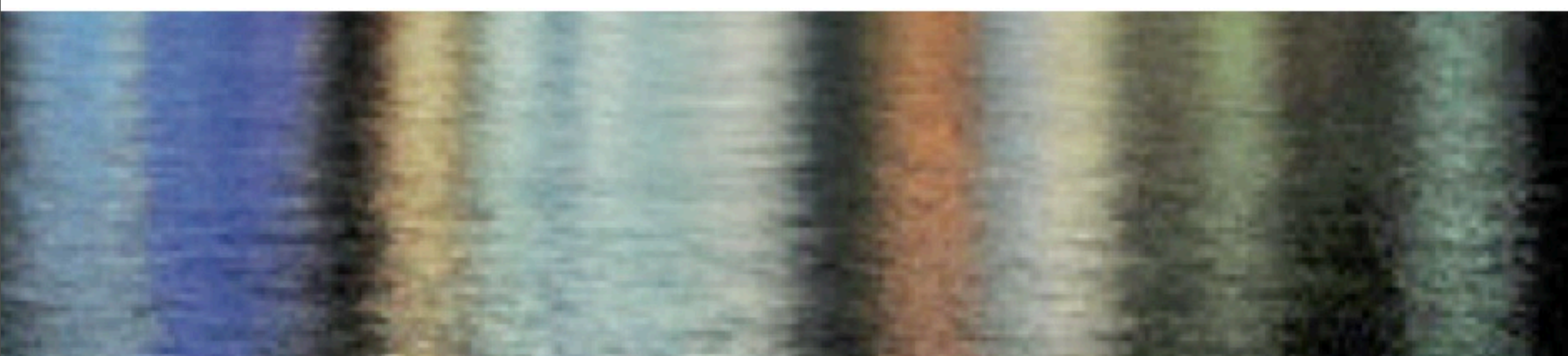
sensible interacting UV fixed point

$$D\,F - C\,E > 0$$

asymptotic safety from template gauge-Yukawa theory



DFL, F Sannino, 1406.2337
DFL, M Mojaza, F Sannino, 1501.03061

gauge-Yukawa theory

Lagrangean

$$L_{\mathrm{YM}} = -\frac{1}{2}\mathrm{Tr}\, F^{\mu\nu} F_{\mu\nu}$$

gauge — Nc colours

$$L_F = \mathrm{Tr}\left(\overline{Q}\, i\not{D}\, Q\right)$$

$$L_Y = y\, \mathrm{Tr}\left(\overline{Q}\, H\, Q\right)$$

Yukawa — Nf flavours

$$L_H = \mathrm{Tr}\left(\partial_\mu H^\dagger\, \partial^\mu H\right)$$

$$L_U = -u\, \mathrm{Tr}\left(H^\dagger H\right)^2$$

$$L_V = -v\left(\mathrm{Tr}\, H^\dagger H\right)^2 .$$

Higgs — Nf times Nf

gauge-Yukawa theory

Lagrangean

$$L_{\rm YM} = -\frac{1}{2}{\rm Tr}\, F^{\mu\nu}F_{\mu\nu}$$

$$L_F = {\rm Tr}\left(\overline{Q}\, i\not{D}\, Q\right)$$

$$L_Y = y\,{\rm Tr}\left(\overline{Q}\, H\, Q\right)$$

$$L_H = {\rm Tr}\left(\partial_\mu H^\dagger\, \partial^\mu H\right)$$

$$L_U = -u\,{\rm Tr}\,(H^\dagger H)^2$$

$$L_V = -v\,({\rm Tr}\, H^\dagger H)^2 .$$

couplings

$$\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}, \qquad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}$$

$$\alpha_h = \frac{u\, N_F}{(4\pi)^2}, \qquad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2} .$$

small parameter:

$$0 < \epsilon \ll 1 \qquad \epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

no asymptotic freedom

gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right) \alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$

gauge

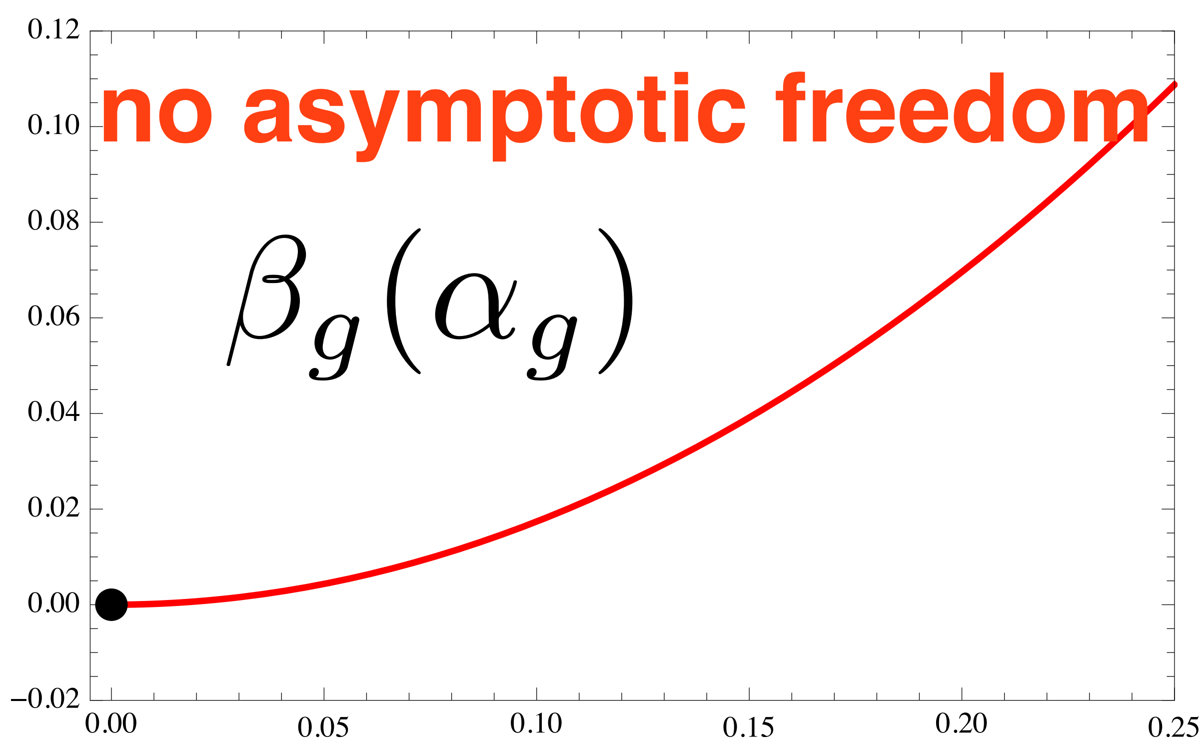
$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon)\, \alpha_y - 6\, \alpha_g \right\} .$$

Yukawa

$$\beta_h = -(11 + 2\epsilon)\, \alpha_y^2 + 4\alpha_h (\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v \left(\alpha_v + 4\alpha_h + \alpha_y\right) .$$

Higgs



gauge-Yukawa theory

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Higgs

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y) \ .$$

gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$

gauge

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Yukawa

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Higgs

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\left(\alpha_v + 4\alpha_h + \alpha_y\right) .$$

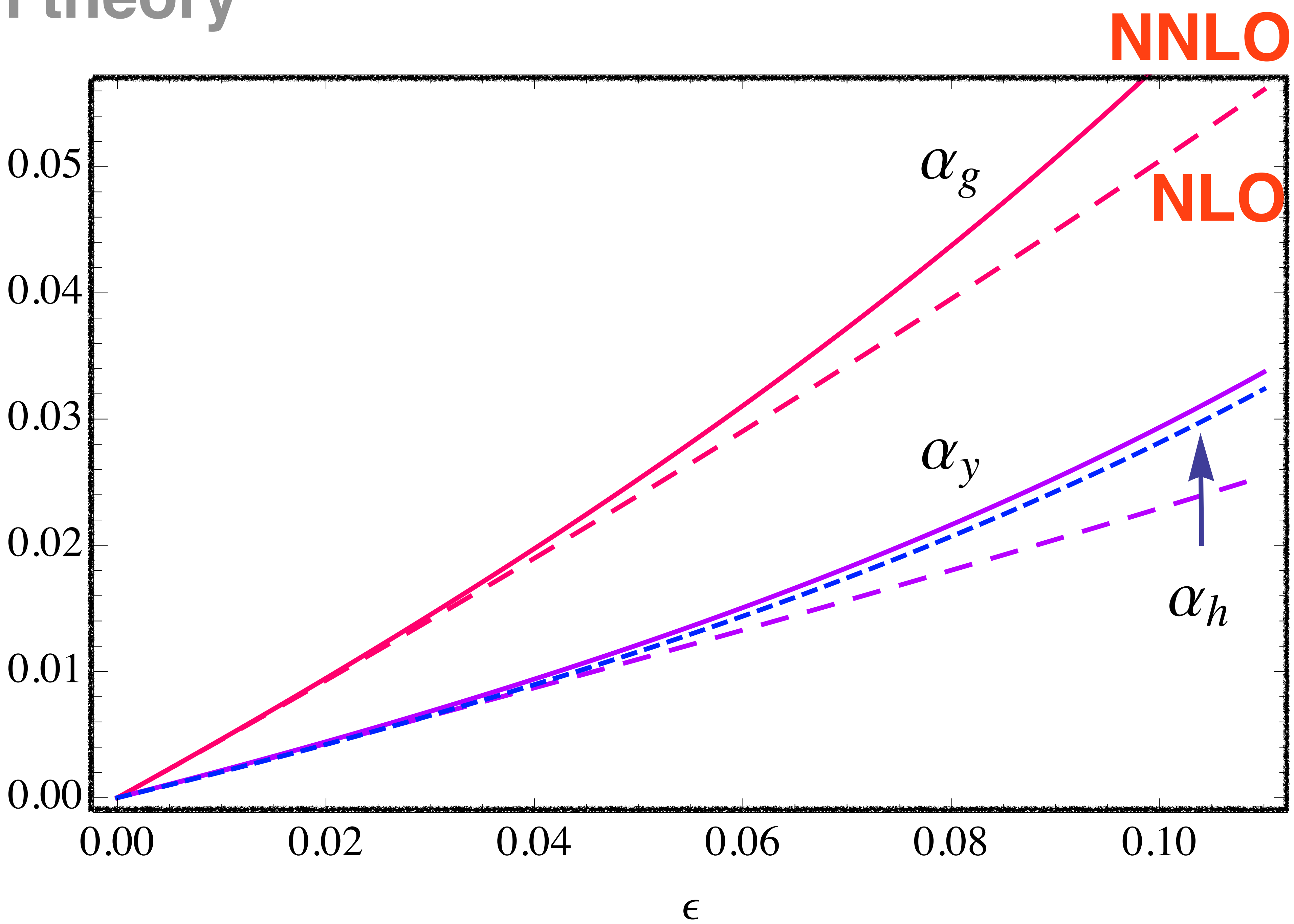
exact

UV fixed point

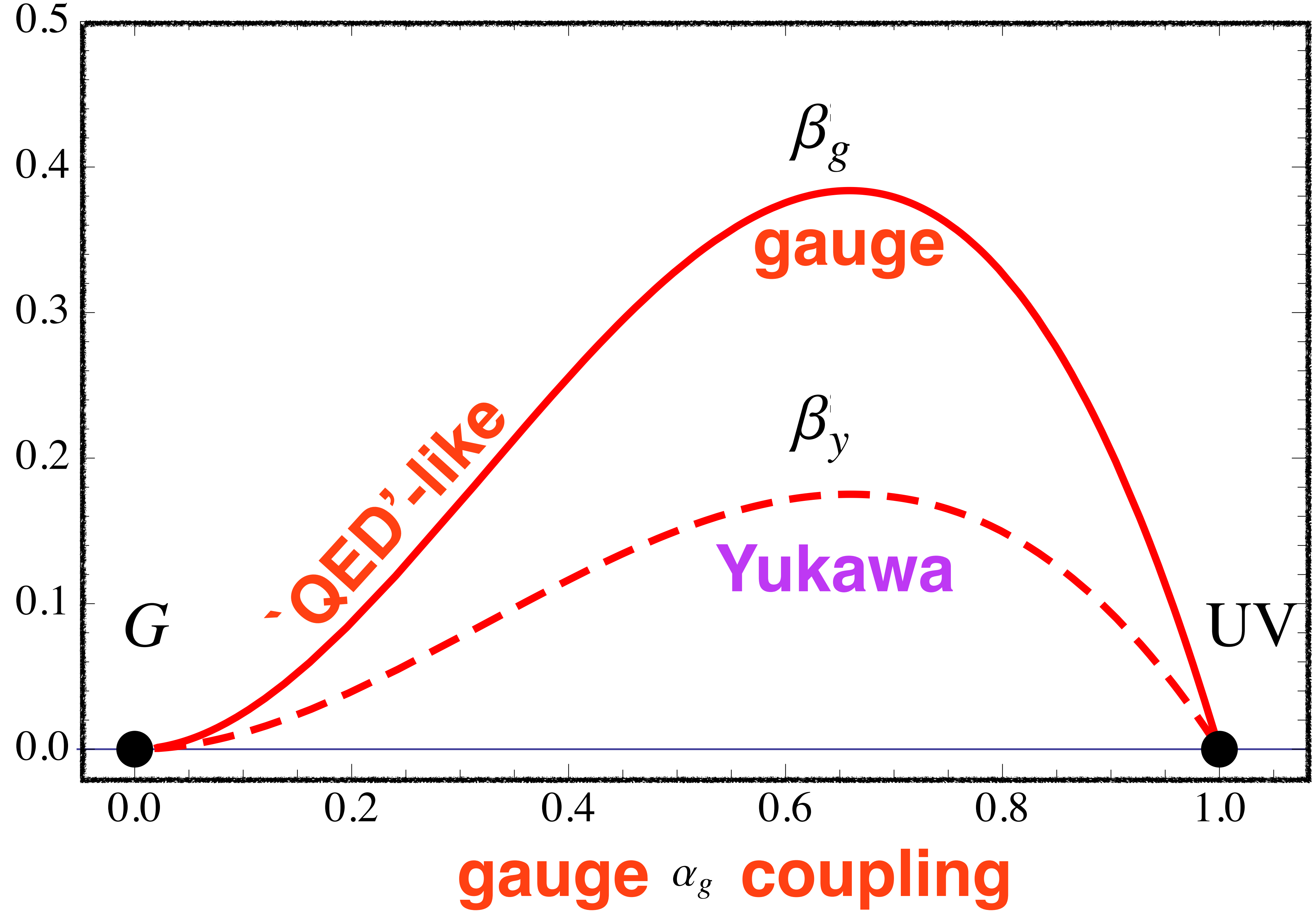
$$\begin{aligned} \alpha_g^* &= 0.4561\,\epsilon + 0.7808\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_y^* &= 0.2105\,\epsilon + 0.5082\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_h^* &= 0.1998\,\epsilon + 0.5042\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_v^* &= -0.1373\,\epsilon + \mathcal{O}(\epsilon^2) \end{aligned}$$

results

UV fixed point from perturbation theory



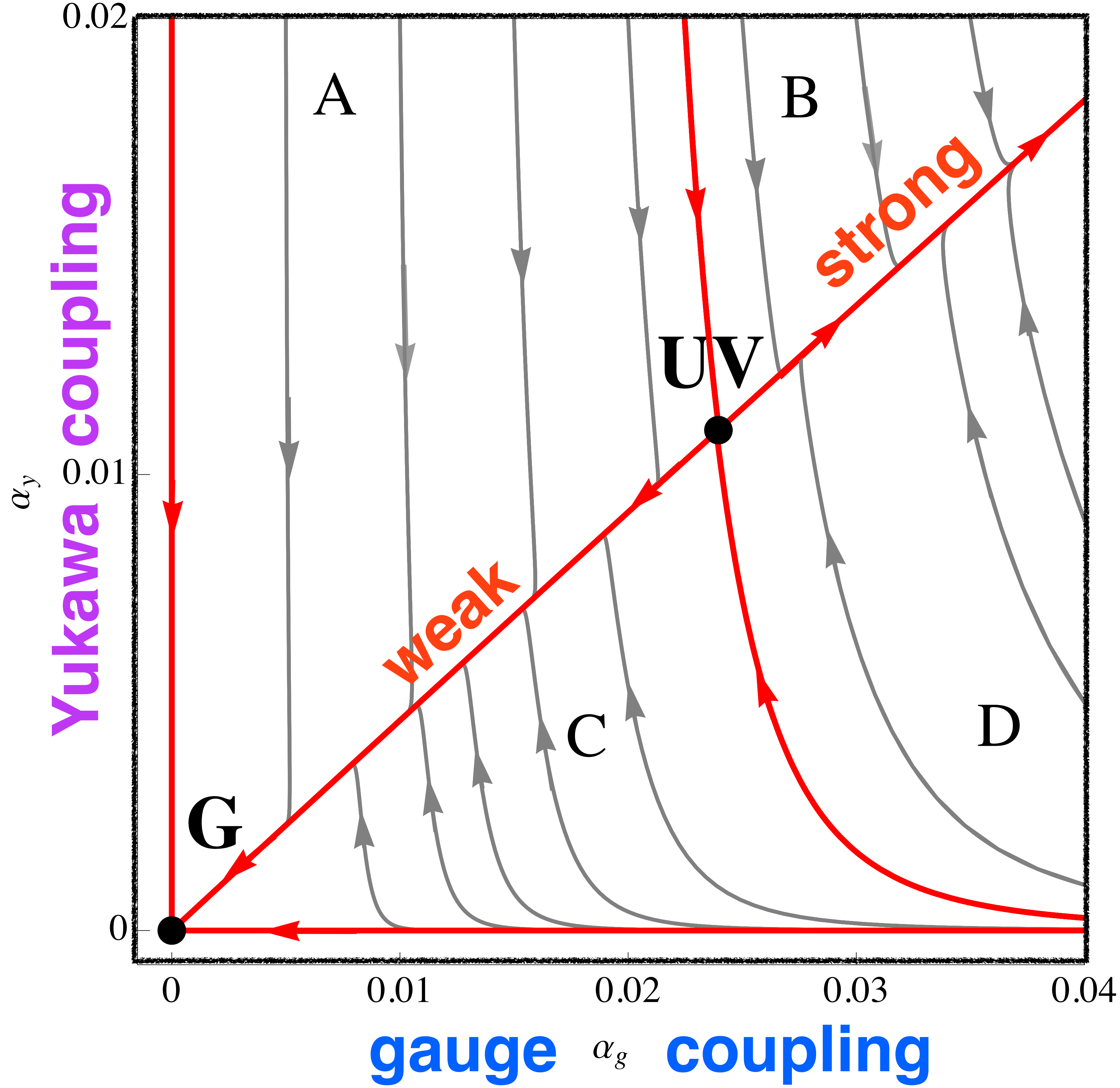
results



interacting UV fixed point
entirely due to `fluctuations'

results

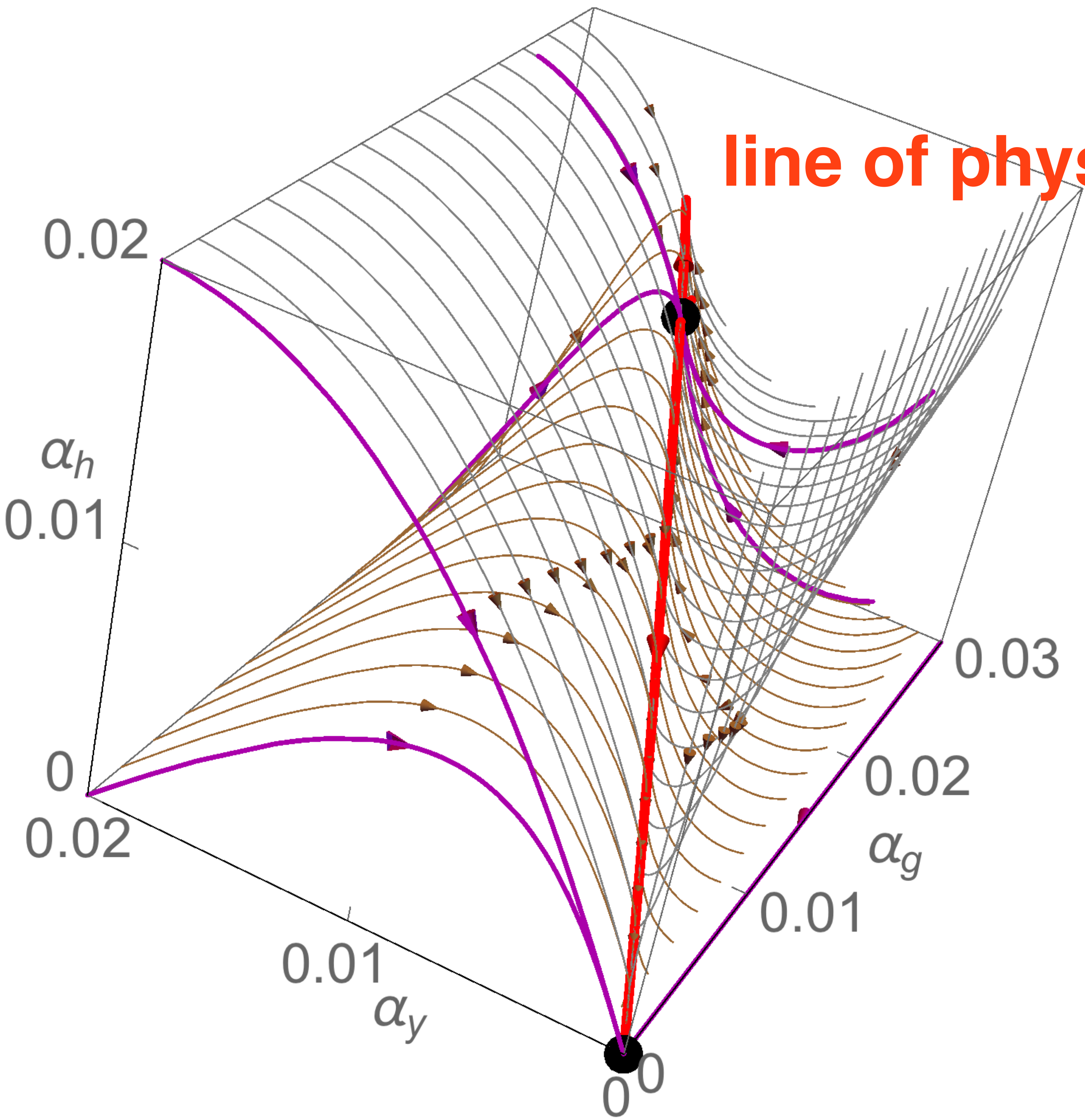
phase diagram

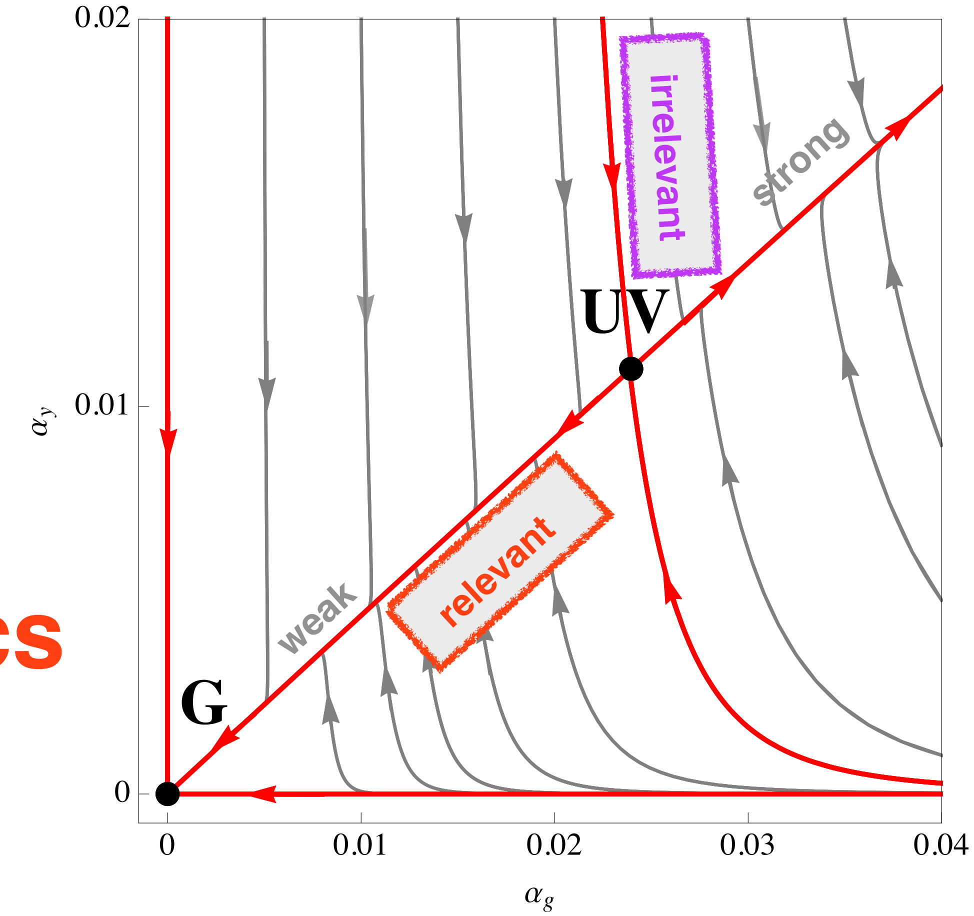


exact UV FP

strict perturbative control

phase diagram





leading order

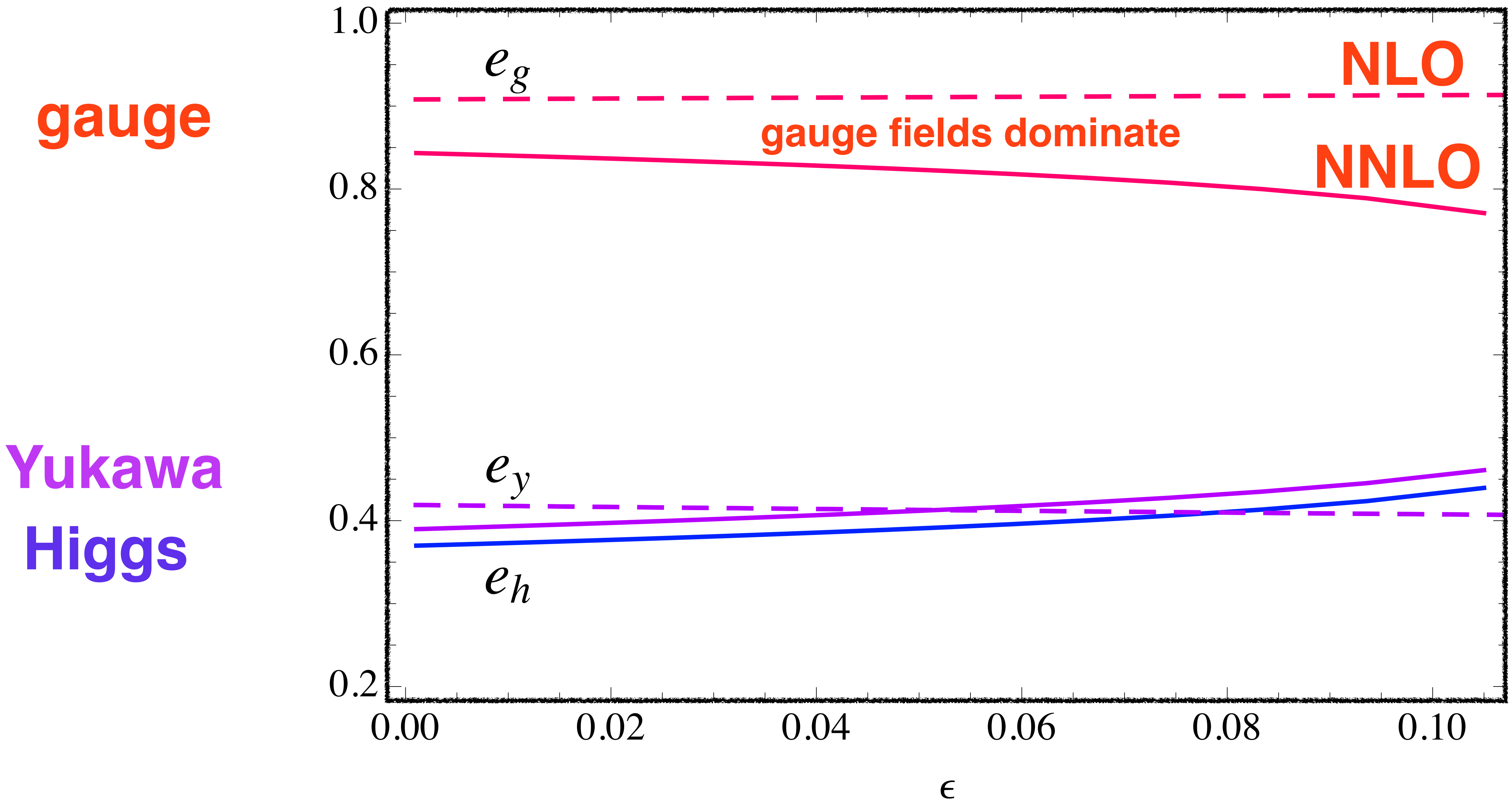
$$\alpha(\mu) = \frac{\alpha_*}{1 + W(\mu)}$$

$$W(\mu) = W_{\rm Lambert}[z(\mu)]$$

$$z = \left(\frac{\mu_0}{\mu}\right)^{-B\cdot\alpha_*} \left(\frac{\alpha_*}{\alpha_0} - 1\right) \exp\left(\frac{\alpha_*}{\alpha_0} - 1\right) .$$

results

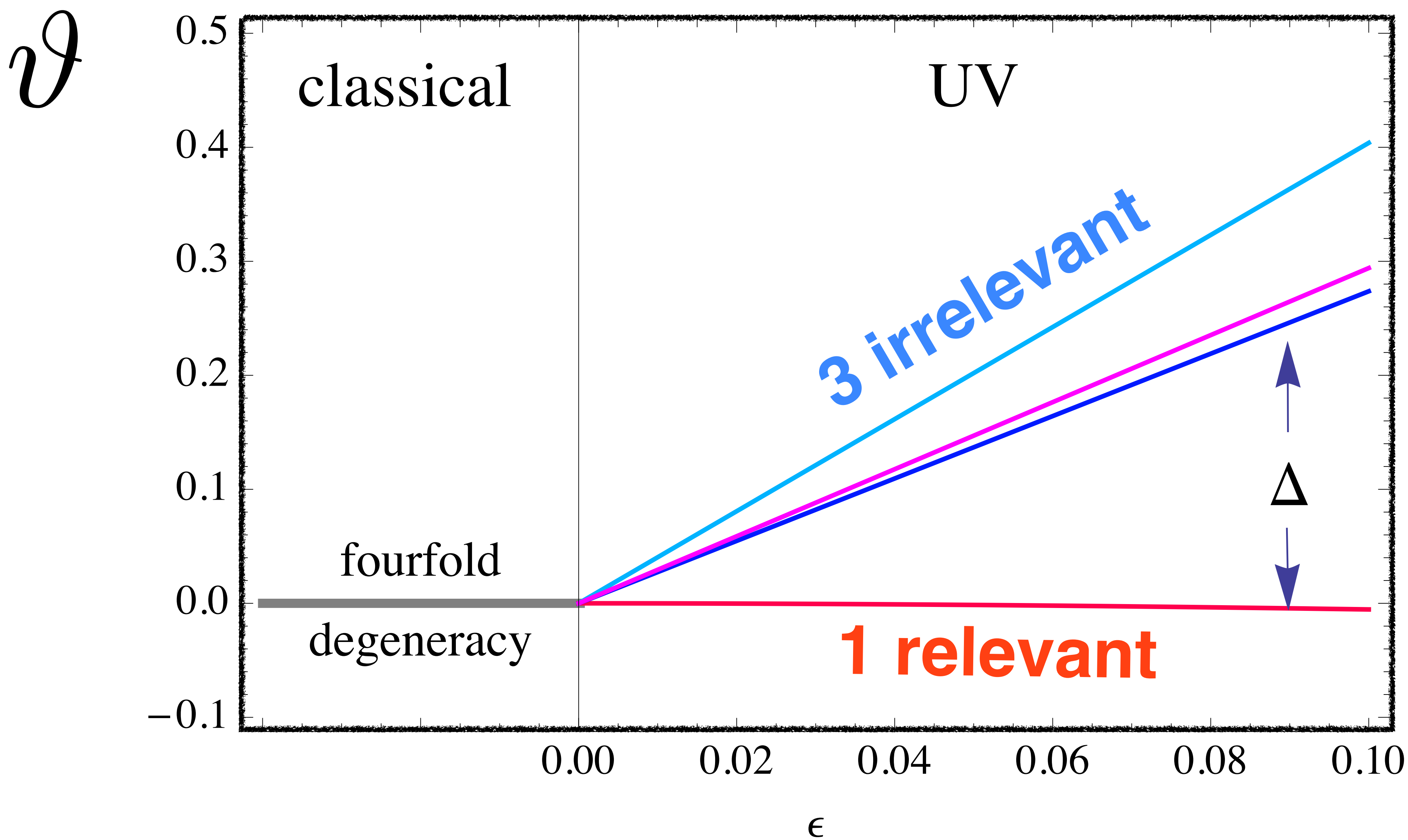
UV-**relevant** eigendirection



results

UV scaling exponents

$$\vartheta_1 < 0 < \vartheta_2 < \vartheta_4 < \vartheta_3$$



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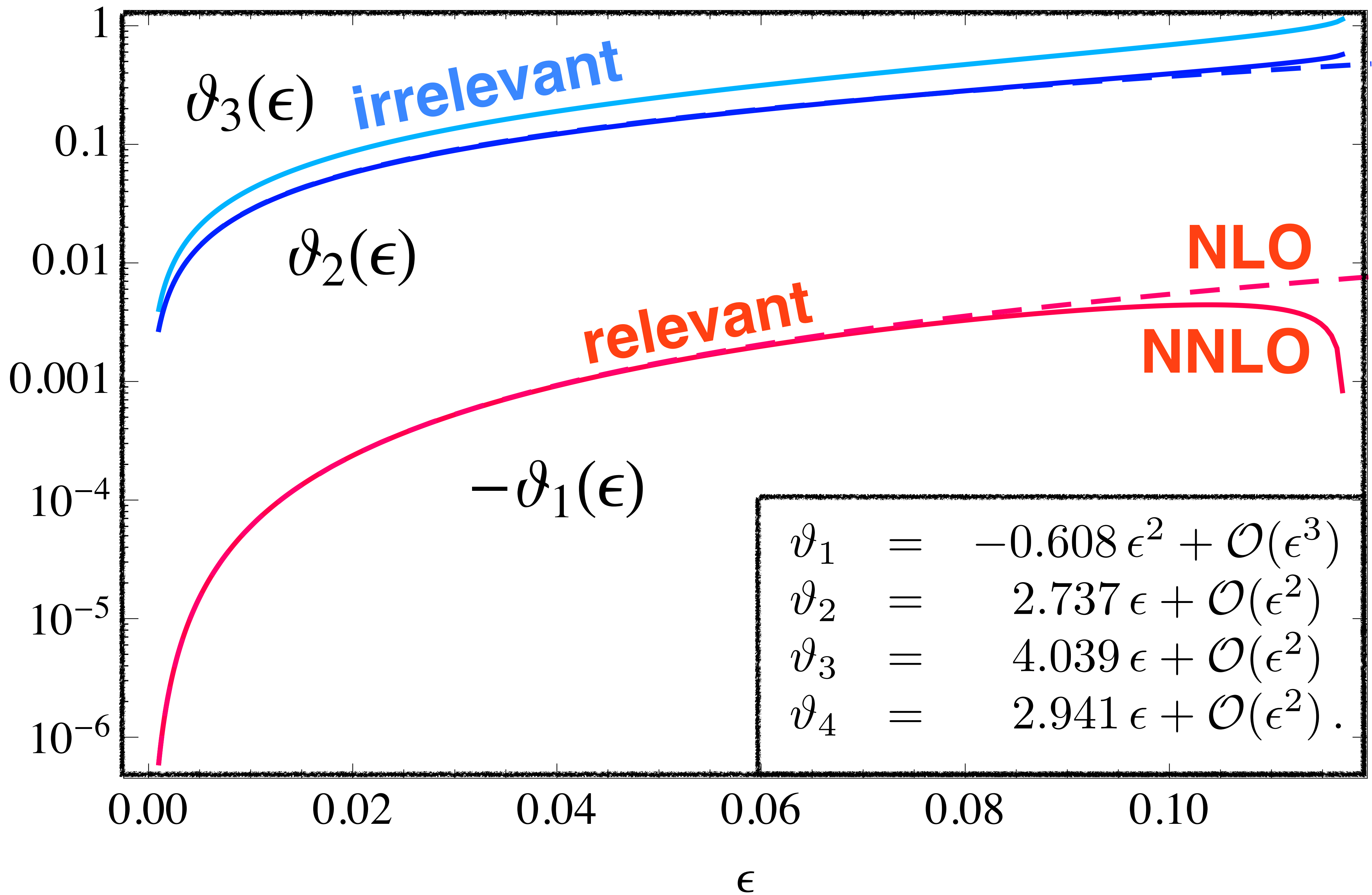


vacuum stability

vacuum must be stable classically
and quantum-mechanically

$$V \propto \alpha_v \mathrm{Tr}(H^\dagger H)^2 + \alpha_h (\mathrm{Tr} H^\dagger H)^2$$

stability

$$\alpha_h > 0 \quad \text{and} \quad \alpha_h + \alpha_v \geq 0 \qquad H_c \propto \delta_{ij}$$

$$\alpha_h < 0 \quad \text{and} \quad \alpha_h + \alpha_v / N_F \geq 0 \qquad H_c \propto \delta_{i1}$$

UV FP:

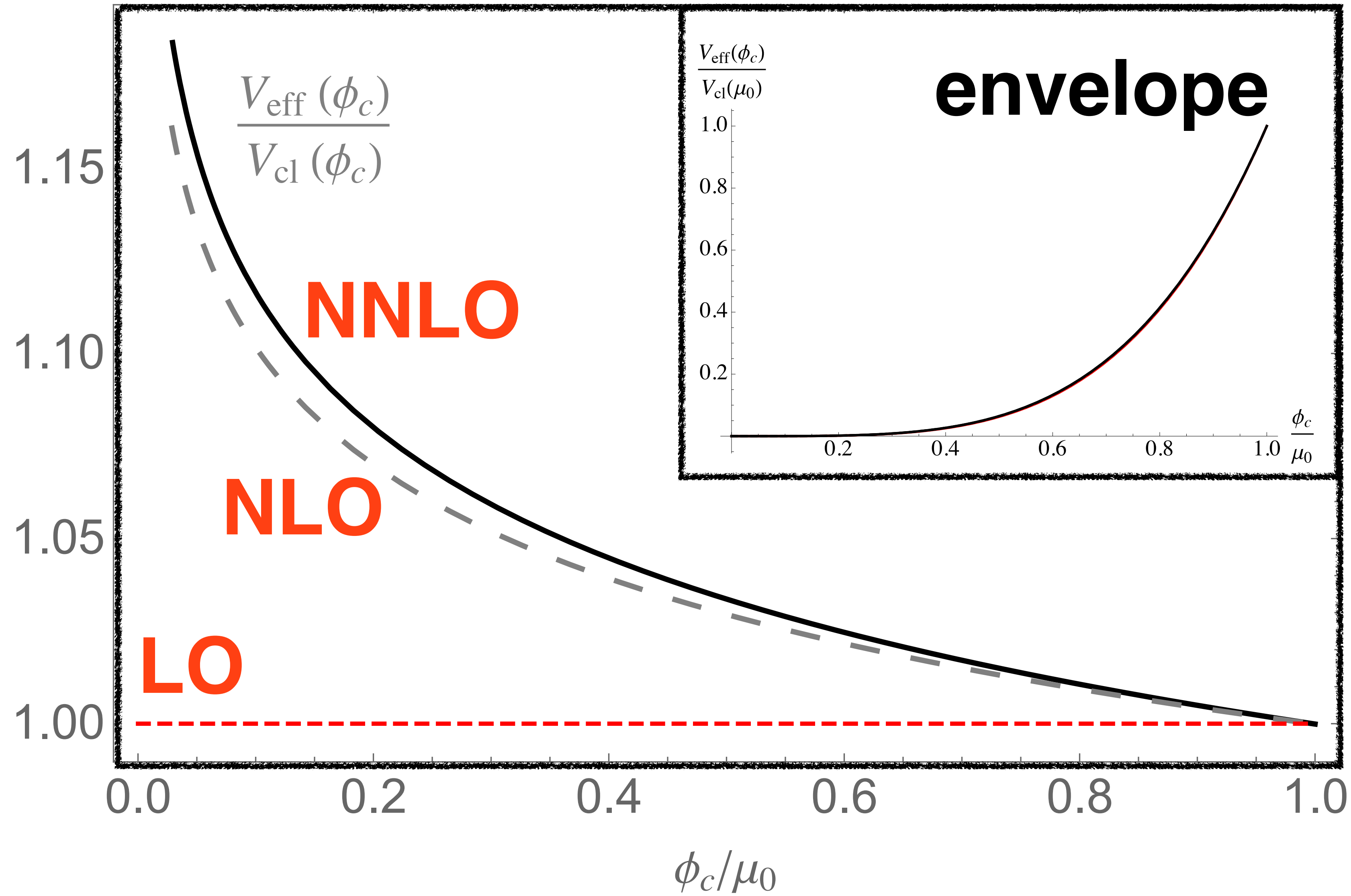
$$0 < \alpha_h^* + \alpha_v^*$$

ok

vacuum stability

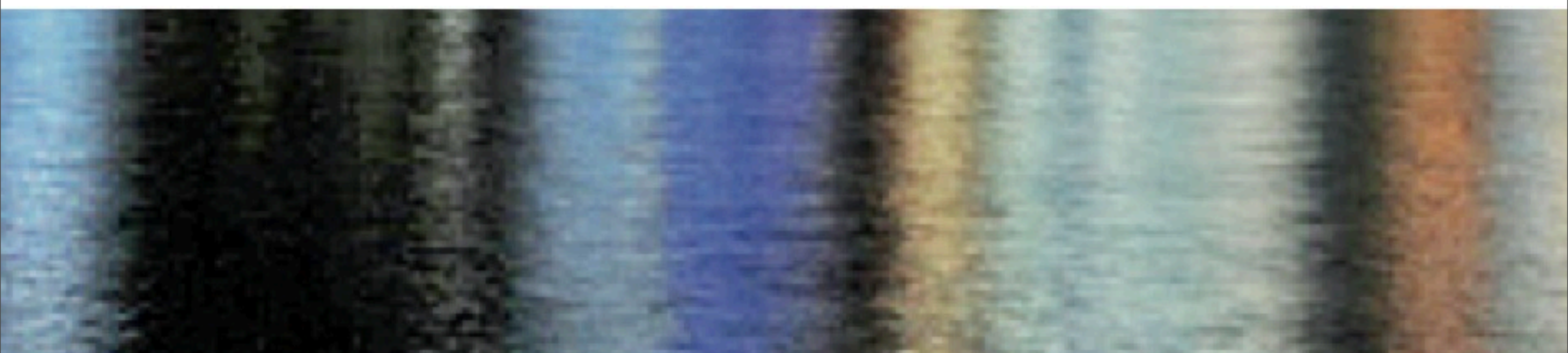
quantum stability: Coleman-Weinberg type resummation of logs

$$\left(\mu_0 \frac{\partial}{\partial \mu_0} - \gamma(\alpha_j)\,\phi_c \frac{\partial}{\partial \phi_c} + \sum_i \beta_i(\alpha_j) \frac{\partial}{\partial \alpha_i}\right) V_{\rm eff}(\phi_c, \mu_0, \alpha_j) = 0$$



effective potential well-defined for all scales

asymptotic safety beyond perturbation theory



K Falls, DL, K Nikolakopoulos & C Rahmede,   1301.4191

asymptotic freedom vs asymptotic safety

asymptotic freedom	asymptotic safety
$g_* = 0$	$g_* \neq 0$
anomalous dimensions	anomalous dimensions
$\eta_A = 0$	$\eta_N \neq 0$
canonical power counting	**non-canonical** power counting
$\{\vartheta_{\mathrm{G},n}\}$ are known	$\{\vartheta_n\}$ are **not** known
F^{256} irrelevant !	R^{256} relevant / marginal / irrelevant **?**

K Falls, DL, K Nikolakopoulos & C Rahmede, 1301.4191

bootstrap search strategy

hypothesis relevancy of invariants follows canonical dimension

K Falls, DL, K Nikolakopoulos & C Rahmede, 1301.4191

bootstrap search strategy

hypothesis relevancy of invariants follows canonical dimension

strategy

Step 1 retain invariants up to mass dimension D

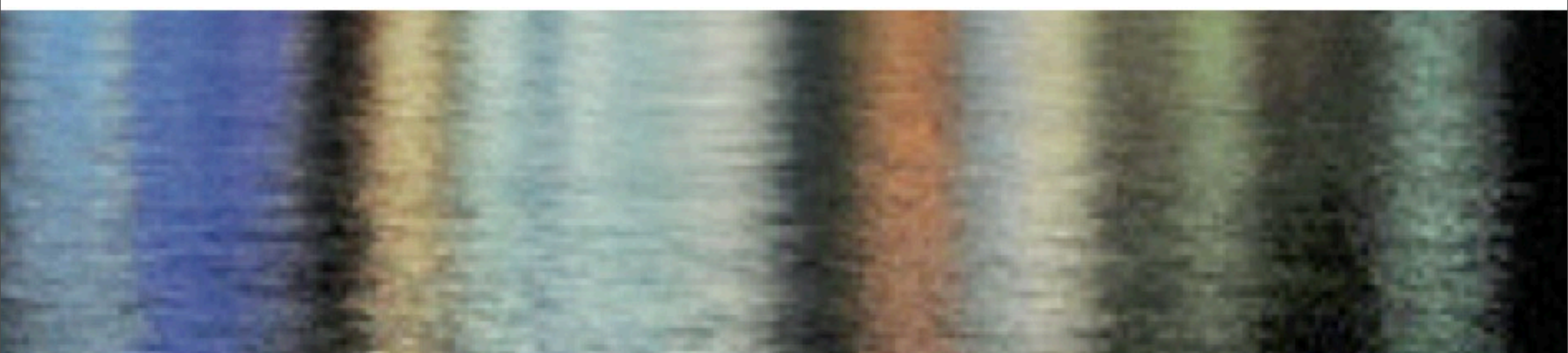
Step 2 compute $\{\vartheta_n\}$ (eg. RG, lattice, holography)

Step 3 enhance D, and iterate

convergence (no convergence) of the iteration:

hypothesis supported (refuted)

testing asymptotic safety with quantum gravity templates



K Falls, DL, K Nikolakopoulos & C Rahmede, 1301.4191
K Falls, DL, K Nikolakopoulos & C Rahmede 1410.4815
and in prep.

f(R)

$$\Gamma_k \propto f(R)$$

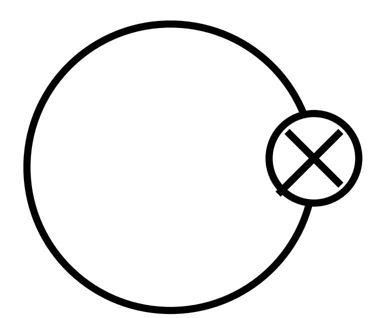
Ricci scalars

$$\Gamma_k = \sum_{n=0}^{N-1} \lambda_n \, k^{d_n} \int d^4x \sqrt{g} \, R^n$$

effective action with invariants up to mass dimension $D = 2(N-1)$

technicalities: functional renormalisation

$$k \frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2} \, \mathrm{Tr} \left[\left(\frac{\delta^2 \Gamma_k[\phi]}{\delta\phi \, \delta\phi} + R_k \right)^{-1} k \frac{\mathrm{d}R_k}{\mathrm{d}k} \right] = \frac{1}{2}$$



here:

M Reuter hep-th/9605030

DL hep-th/0103195

hep-th/0312114

Falls, DL, Nikolakopoulos, Rahmede **1301.4191.pdf**

Falls, DL, Nikolakopoulos, Rahmede 1410.4815

A Codello, R Percacci, C Rahmede 0705.1769, 0805.2909

P Machado, F Saueressig 0712.0445

identifying fixed points

$$f(R) = \sum_n \lambda_n R^n$$

polynomial expansion

generating function

$$\partial_t f + 4f - 2R\, f' = I[f]$$

$$I[f] = I_0[f] + I_1[f] \cdot \partial_t f' + I_2[f] \cdot \partial_t f''$$

recursive solution of $\beta_n \equiv \partial_t \lambda_n$ $\beta_{n-2} = 0$

family of FP candidates $\lambda_n = \lambda_n(\lambda_0, \lambda_1)$

`free' parameters (λ_0, λ_1)

interlude: Wilson-Fisher FP

$$u(\rho) = \sum_{n=0} \frac{\lambda_n}{n!}\rho^n \qquad \rho = \frac{1}{2}\phi^a\phi_a$$

polynomial expansion

generating function

$$\partial_t u' = -2u' + (d-2)\rho u'' - A\frac{u''}{(1+u')^2} - B\frac{3u''+2\rho u'''}{(1+u'+2\rho u'')^2}$$

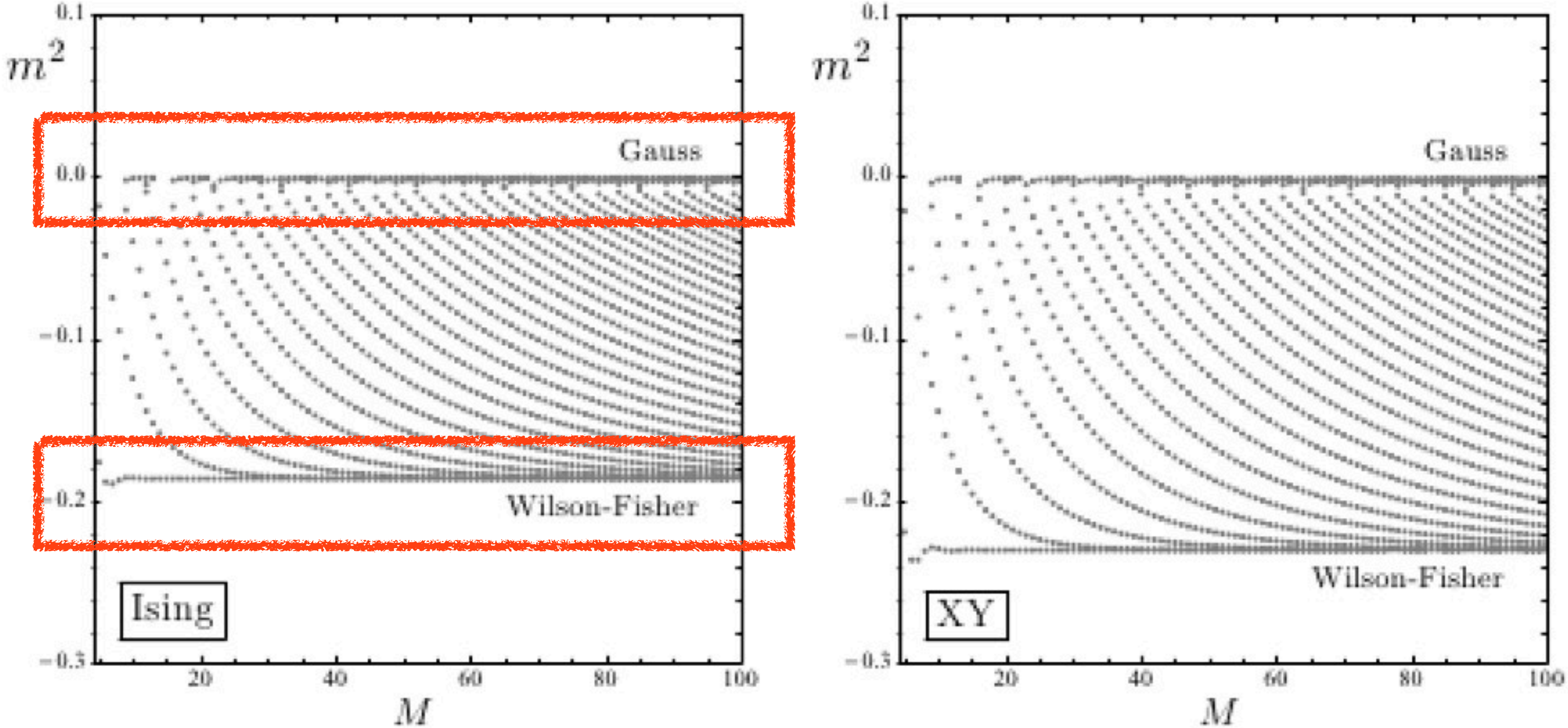
recursive solution $(\lambda_1 \equiv m^2)$

$$\lambda_n = \lambda_n(m^2)$$

Wilson-Fisher LPA

FP solutions with $\lambda_M = 0$

Juettner, DL, Marchais (in prep), arXiv:1504.00xyz



physical FP = accumulation point

Wilson-Fisher LPA

DL, hep-th/0203006

universal eigenvalues

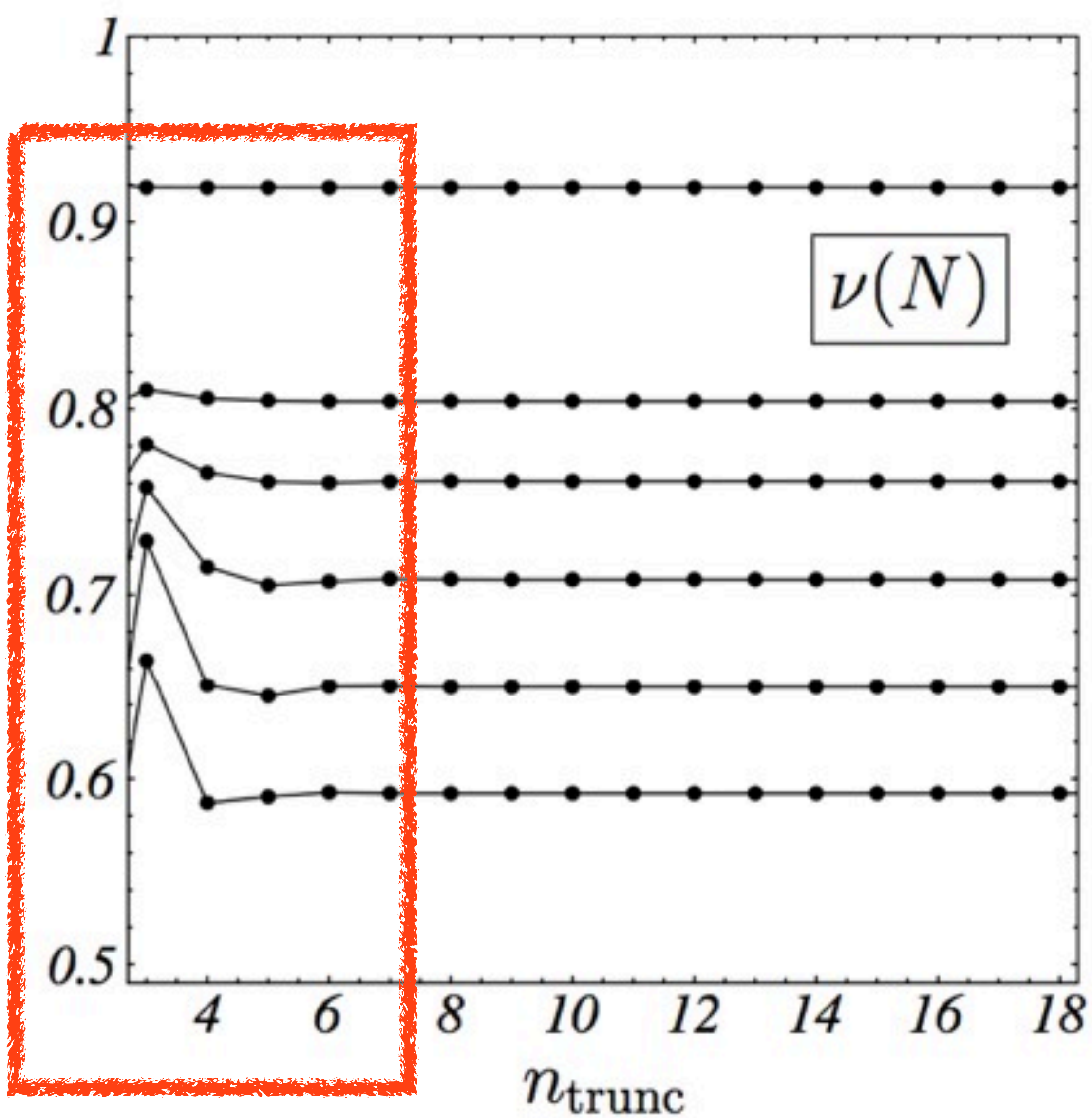


Figure 4: The exponent $\nu(N)$ as a function of N and of the order of the truncation. From top to bottom: $N = 10, 4, 3, 2, 1, 0$.

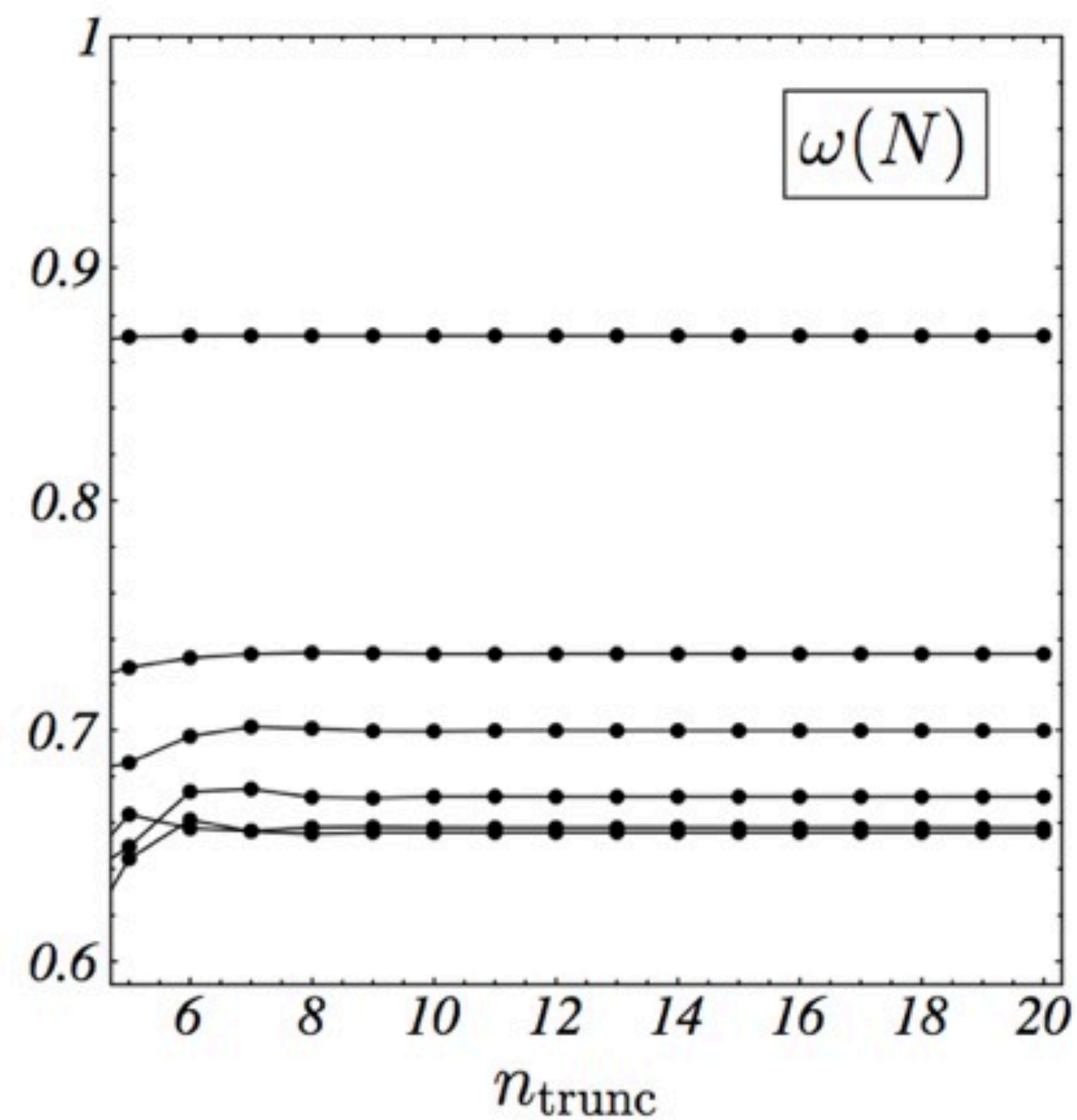


Figure 5: The eigenvalue $\omega(N)$ as a function of N and of the order of the truncation. From top to bottom: $N = 10, 4, 3, 2, 0, 1$.

f(R)

recursive solution

$$\lambda_n(\lambda_0, \lambda_1) = \frac{P_n(\lambda_0, \lambda_1)}{Q_n(\lambda_0, \lambda_1)}$$

boundary condition

$$\lambda_N = 0 \quad \& \quad \lambda_{N+1} = 0$$

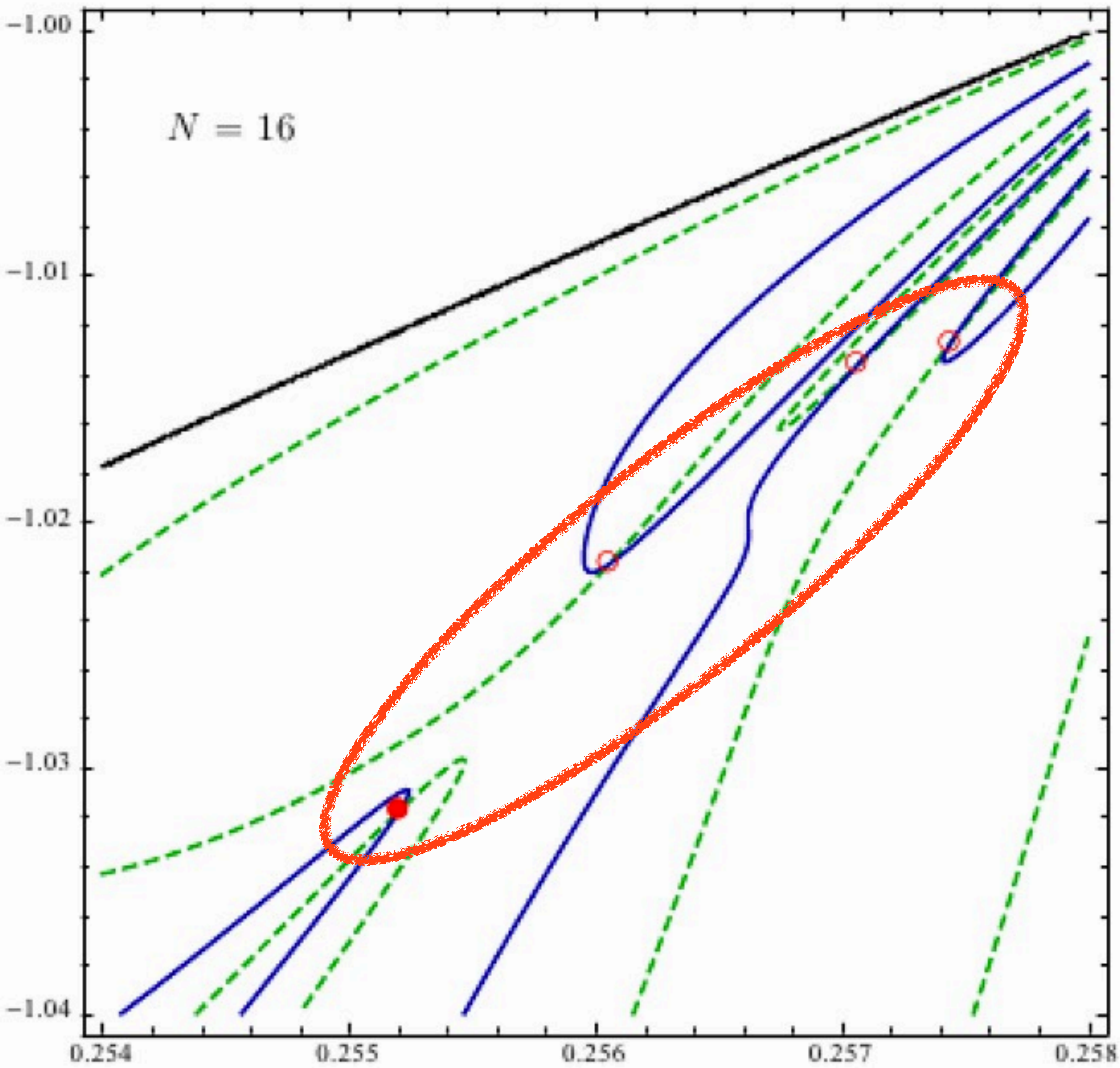
polynomials grow large, eg.

$P_{35} \approx 45.000$ terms

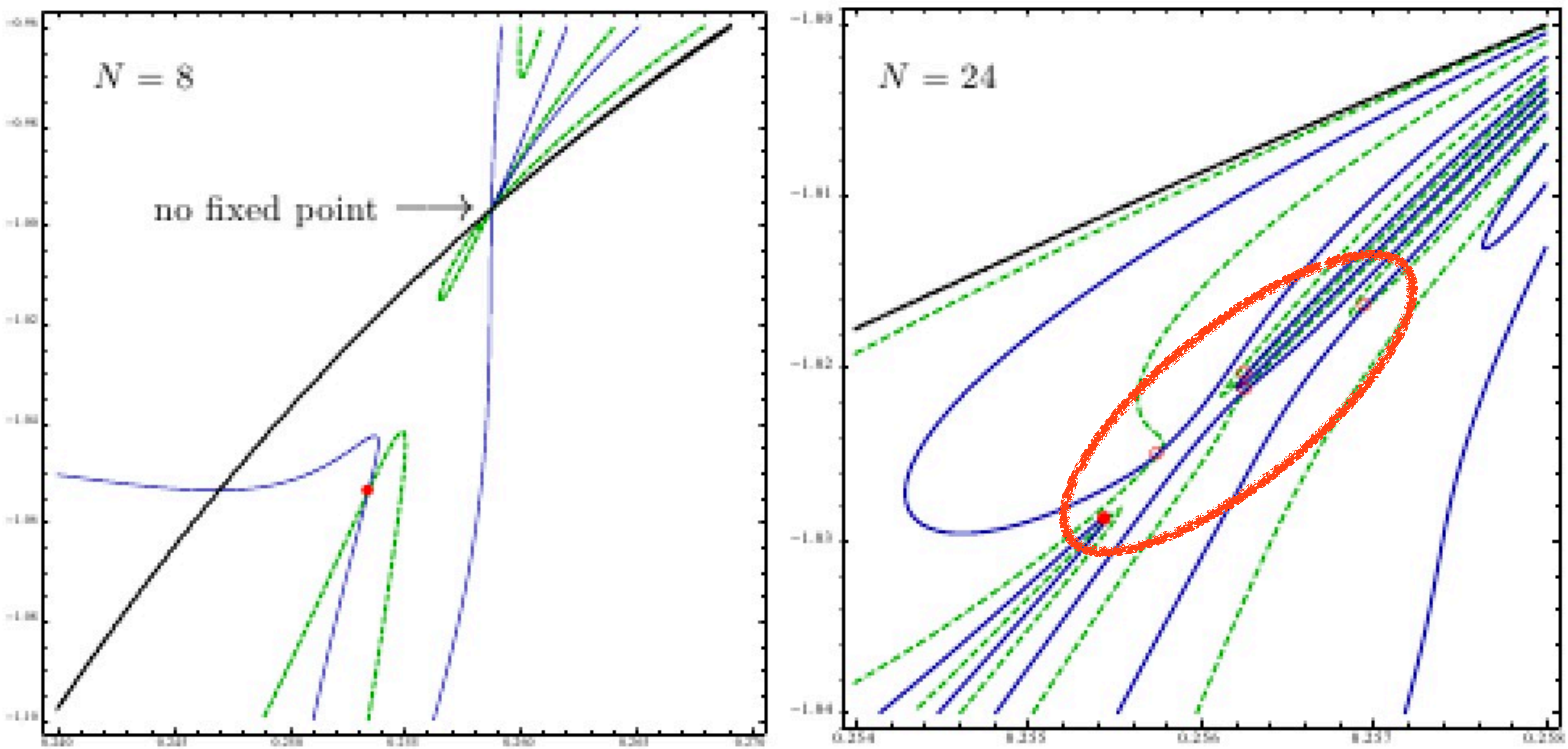
f(R)

fixed point condition

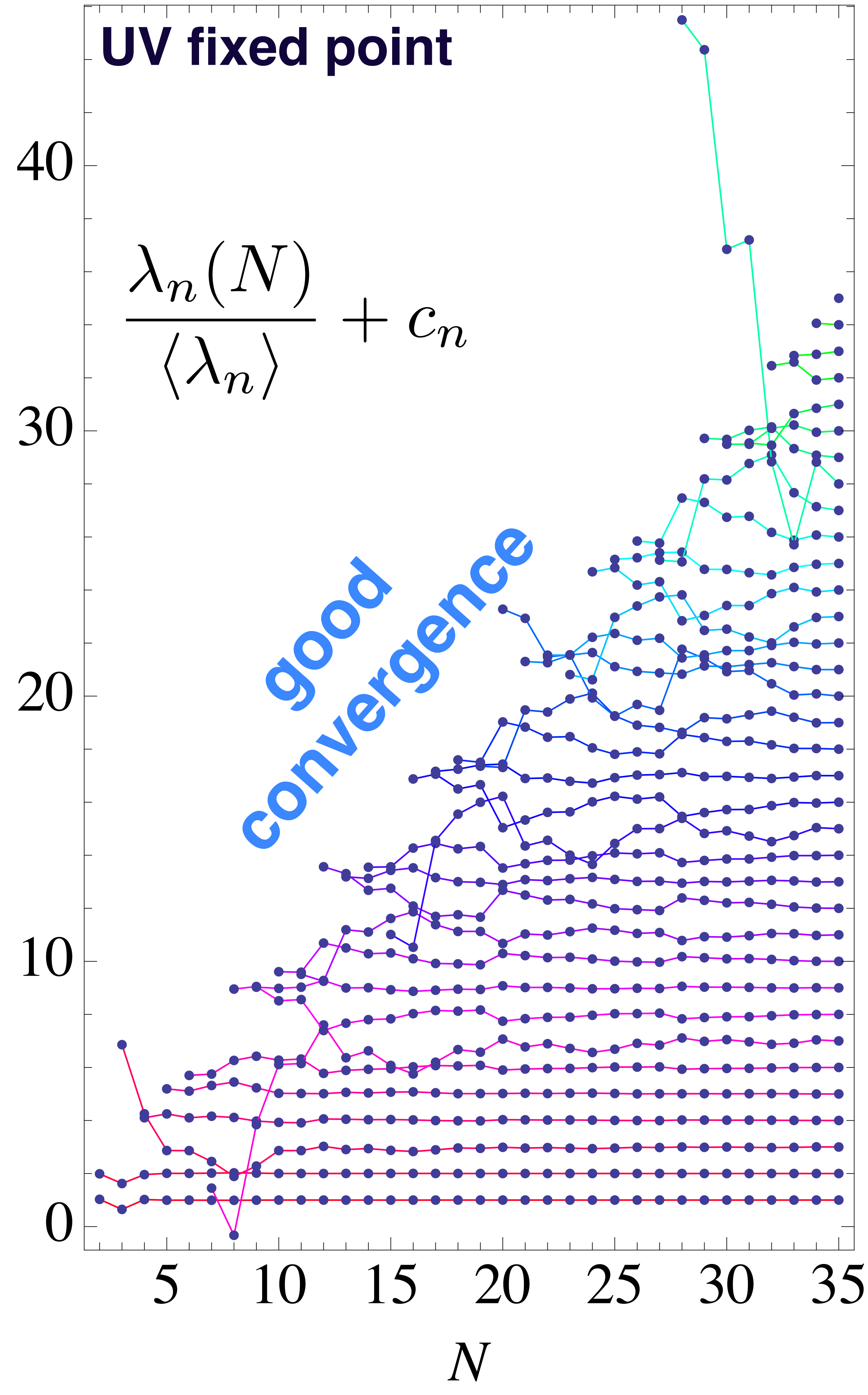
$$\lambda_N = 0$$
$$\lambda_{N+1} = 0$$



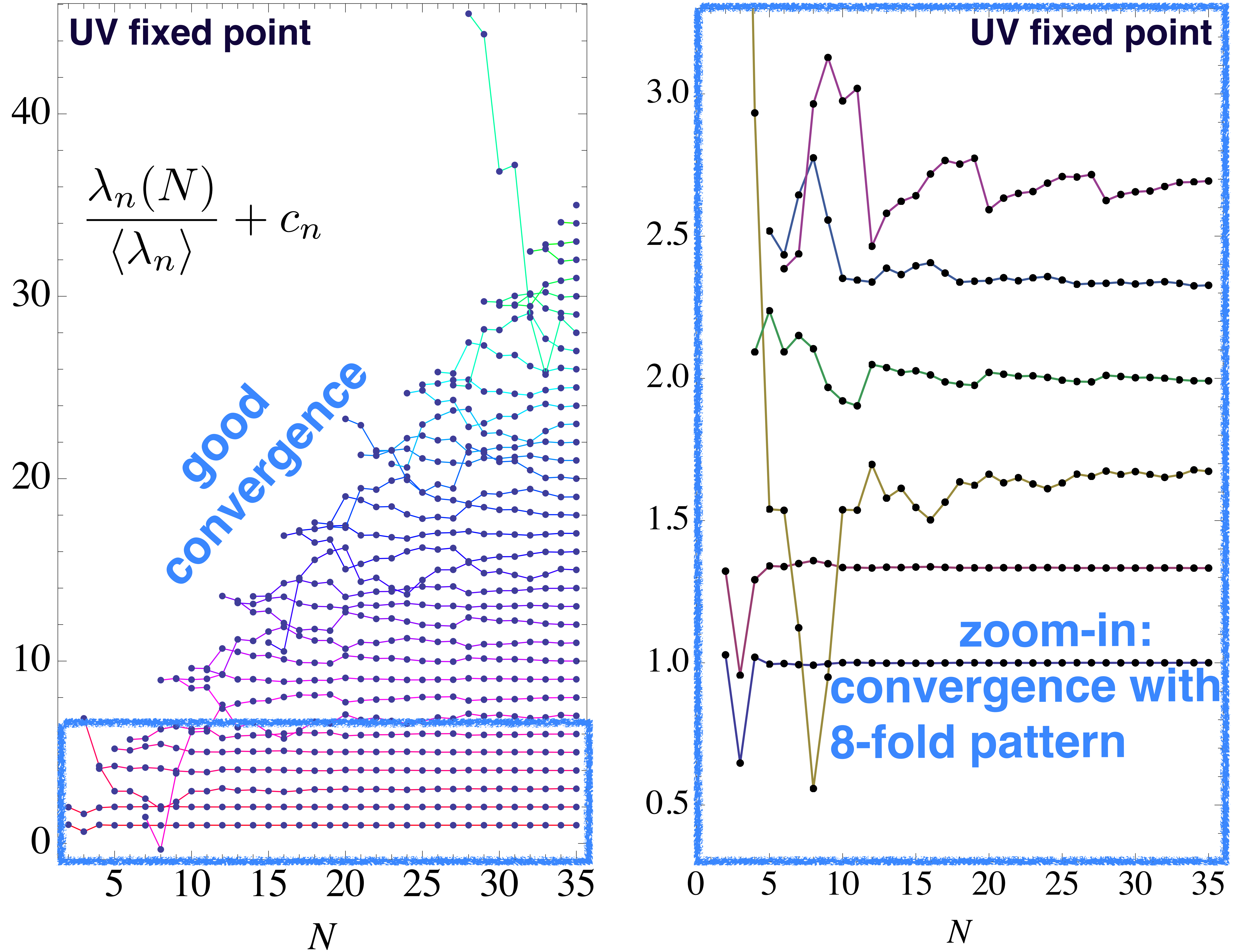
f(R)



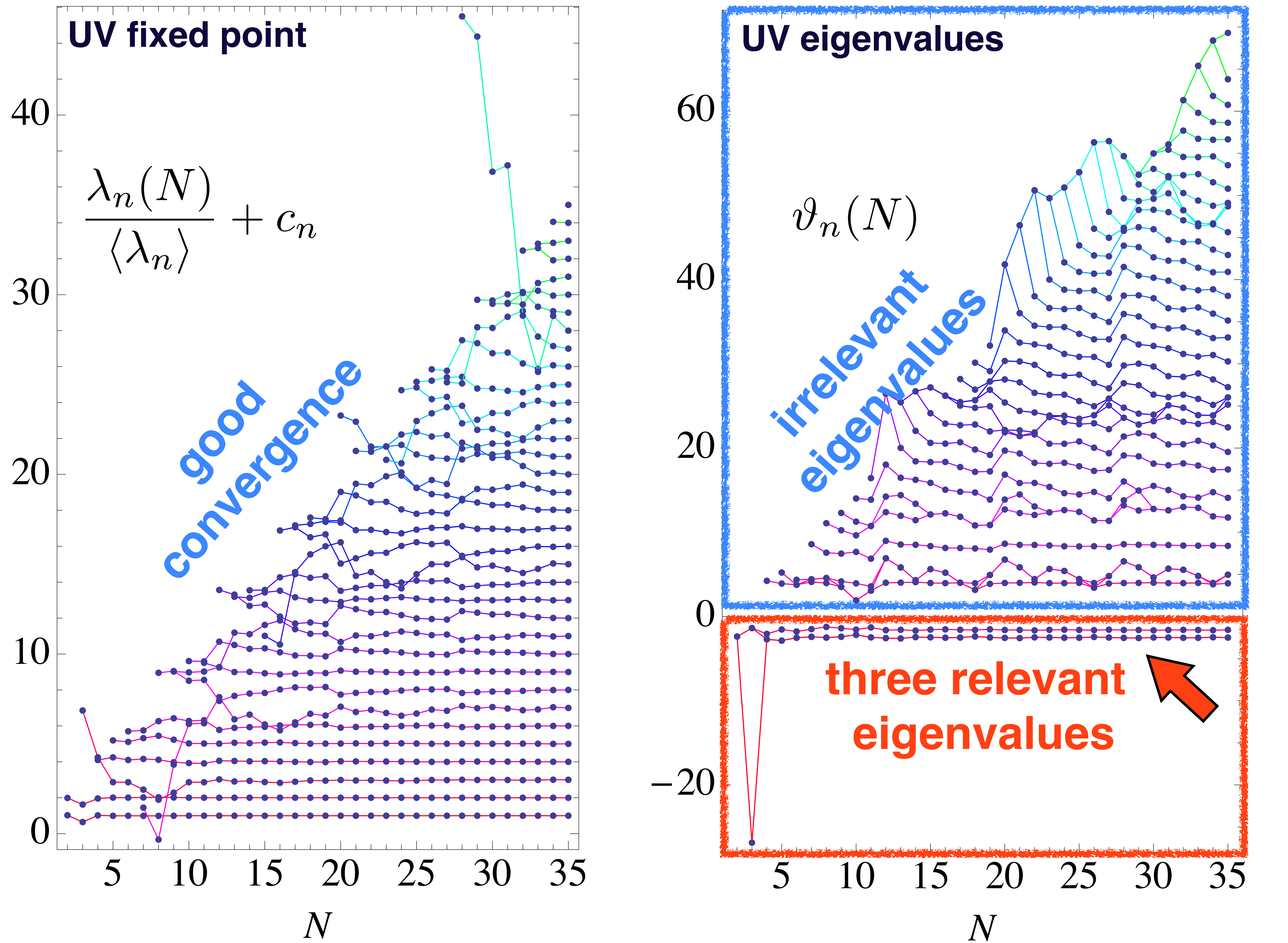
boundary condition $\lambda_N = 0 \quad \& \quad \lambda_{N+1} = 0$



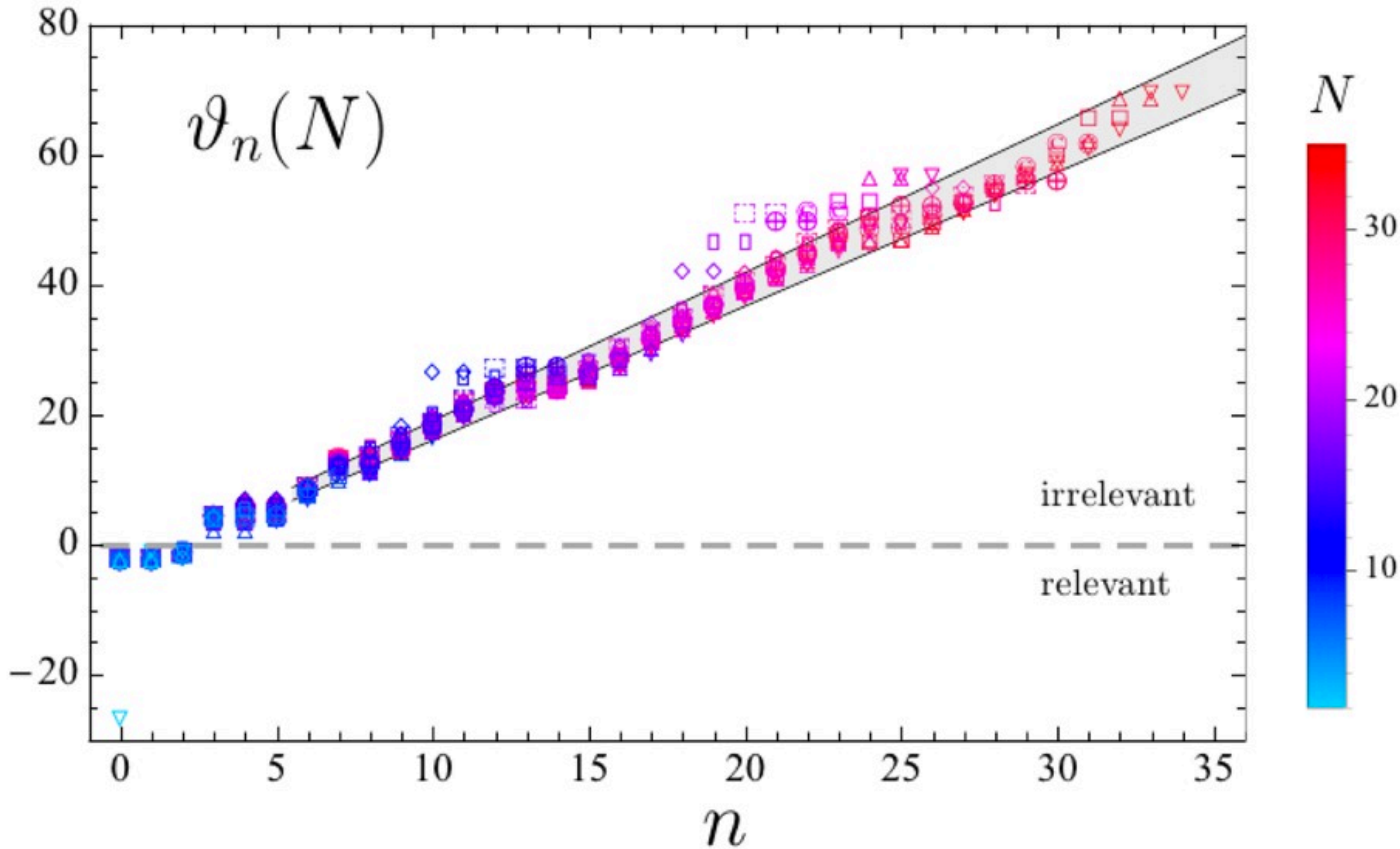
UV fixed point
$\frac{\lambda_n(N)}{\langle \lambda_n \rangle} + c_n$
good convergence
40
30
20
10
0
5
10
15
20
25
30
35
N



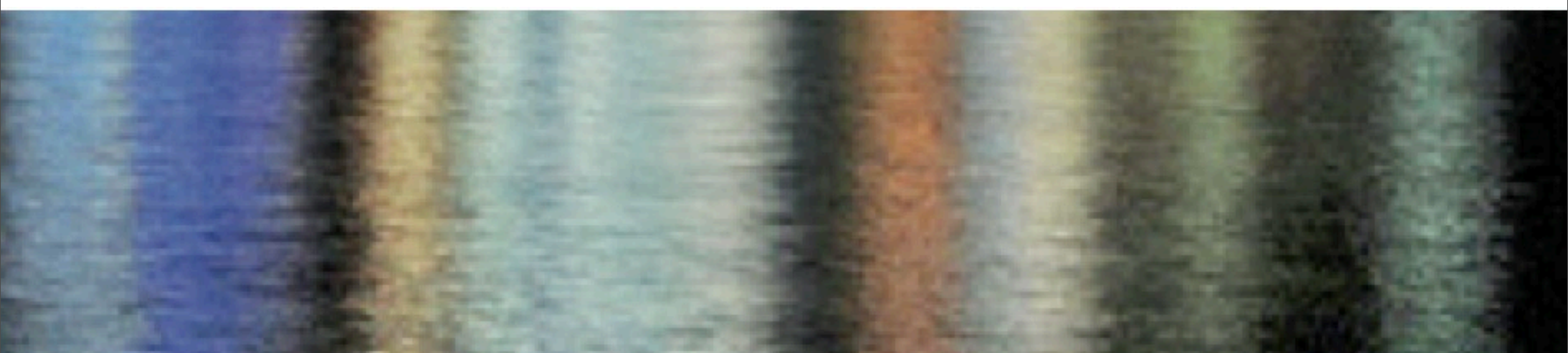
UV fixed point
$\frac{\lambda_n(N)}{\langle\lambda_n\rangle} + c_n$
good convergence
40
30
20
10
0
5 10 15 20 25 30 35
N
UV fixed point
3.0
2.5
2.0
1.5
1.0
0.5
0 5 10 15 20 25 30 35
N
zoom-in:
convergence with
8-fold pattern


UV fixed point
$\frac{\lambda_n(N)}{\langle\lambda_n\rangle} + c_n$
good convergence
UV eigenvalues
$\vartheta_n(N)$
irrelevant eigenvalues
three relevant eigenvalues
N
N

near-Gaussian



beyond Ricci scalars



f(Ricci)

$$\Gamma_k \propto \int d^d x \sqrt{g} \left[f_k(R_{\mu\nu}R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu}R^{\mu\nu}) \right]$$

f(Ricci)

$$\Gamma_k \propto \int d^dx\sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R\cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

$$\begin{aligned}\partial_t\Gamma[\bar{g},\bar{g}] =& \frac{1}{2}\mathrm{Tr}_{(2T)}\left[\frac{\partial_t\mathcal{R}_k^{h^Th^T}}{\Gamma^{(2)}_{h^Th^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\xi\xi}}{\Gamma^{(2)}_{\xi\xi}}\right] + \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\sigma\sigma}}{\Gamma^{(2)}_{\sigma\sigma}}\right] + \frac{1}{2}\mathrm{Tr}_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{hh}}{\Gamma^{(2)}_{hh}}\right]\\ &+ \mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\sigma h}}{\Gamma^{(2)}_{\sigma h}}\right] - \mathrm{Tr}_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{C}^TC^T}}{\Gamma^{(2)}_{\bar{C}^TC^T}}\right] - \mathrm{Tr}_{(0)'}\left[\frac{\partial_t\mathcal{R}_k^{\bar{\eta}\eta}}{\Gamma^{(2)}_{\bar{\eta}\eta}}\right] - \mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{\lambda}\lambda}}{\Gamma^{(2)}_{\bar{\lambda}\lambda}}\right]\\ &+ \frac{1}{2}\mathrm{Tr}''_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\omega\omega}}{\Gamma^{(2)}_{\omega\omega}}\right] - \mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{c}^Tc^T}}{\Gamma^{(2)}_{\bar{c}^Tc^T}}\right] + \frac{1}{2}\mathrm{Tr}'_{(1T)}\left[\frac{\partial_t\mathcal{R}_k^{\zeta^T\zeta^T}}{\Gamma^{(2)}_{\zeta^T\zeta^T}}\right] + \mathrm{Tr}'_{(0)}\left[\frac{\partial_t\mathcal{R}_k^{\bar{s}s}}{\Gamma^{(2)}_{\bar{s}s}}\right]\end{aligned}$$

f(Ricci)

$$\Gamma_k \propto \int d^d x \sqrt{g}\,[f_k(R_{\mu\nu}R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu}R^{\mu\nu})]$$

generating function

$$384\pi^2 \left[4f + 2\rho\, z - \rho^2\,(f' + \rho\, z') + \partial_t f + \rho\,\partial_t z\right] = I[f,z](\rho)$$

$$\begin{aligned} I[f,z](\rho) =& I_0[f,z](\rho) + \partial_t z\, I_1[f,z](\rho) + \partial_t f'\, I_2[f,z](\rho) + \partial_t z'\, I_3[f,z](\rho) \\ &+ \partial_t f''\, I_4[f,z](\rho) + \partial_t z''\, I_5[f,z](\rho)\ . \end{aligned}$$

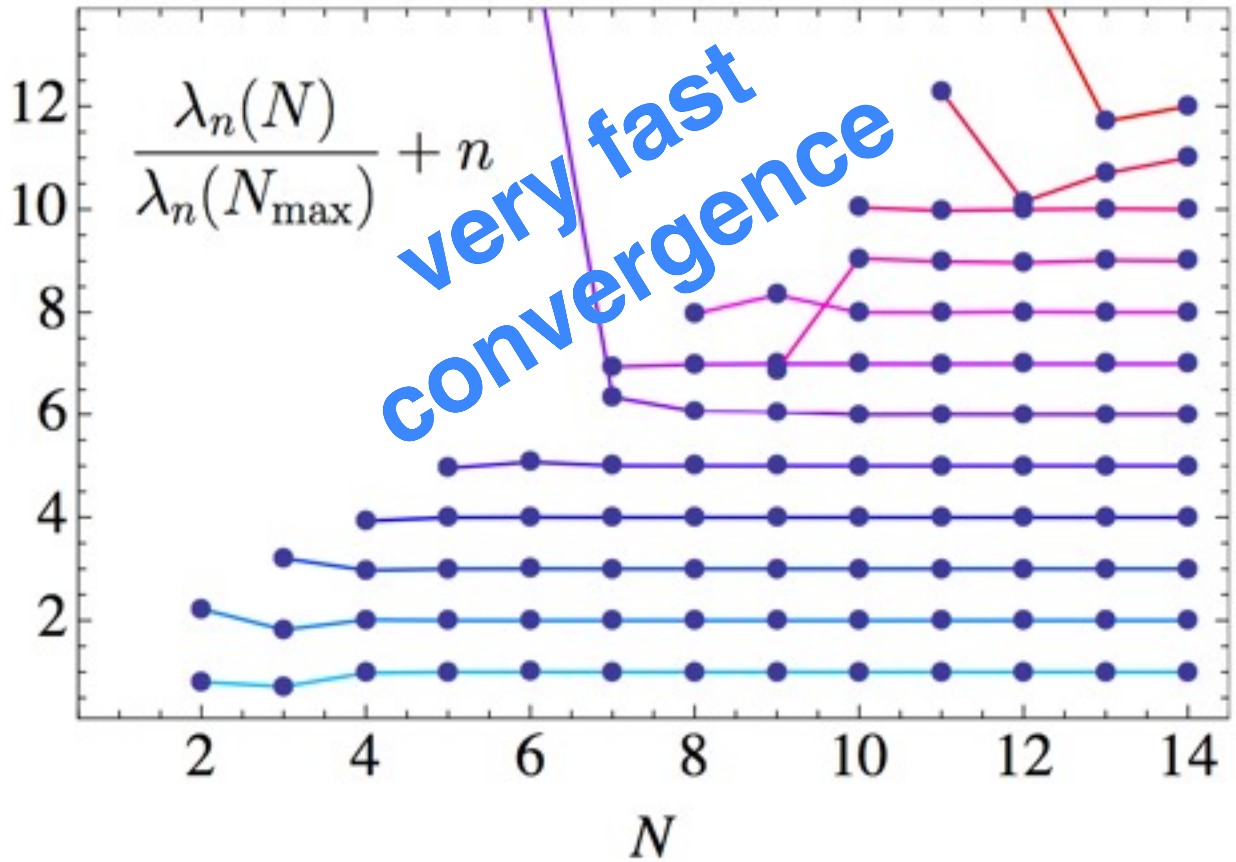
fixed points

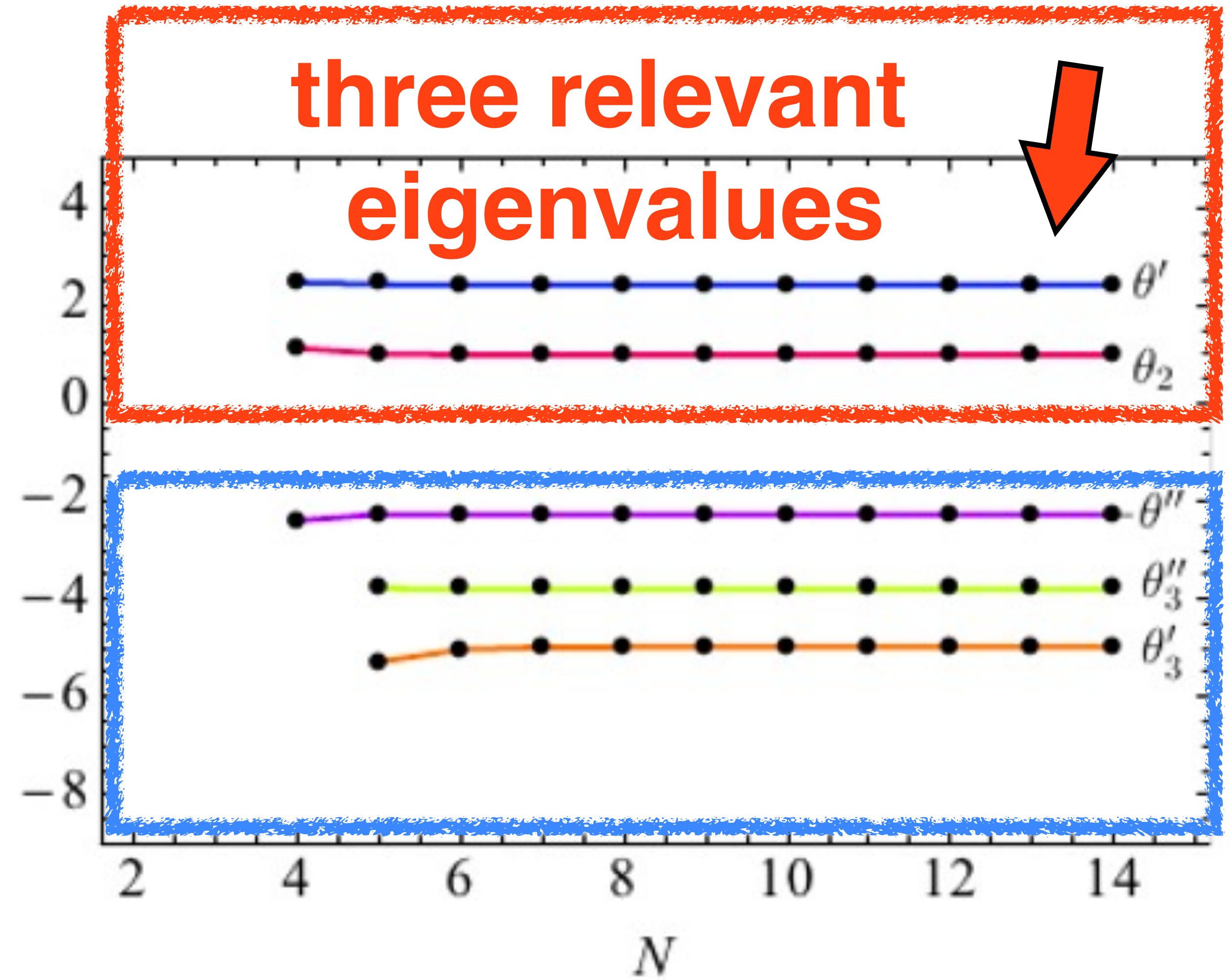
recursive solution more demanding

f(Ricci)

$$\Gamma_k \propto \int d^d x \sqrt{g} \left[f_k(R_{\mu\nu} R^{\mu\nu}) + R \cdot z_k(R_{\mu\nu} R^{\mu\nu}) \right]$$

results:





conclusions

QFTs beyond asymptotic freedom

4D matter-gauge theories

exact proof of asymptotic safety
all types of fields required
sensible UV finite theory
no additional (super-)symmetry

4D quantum gravity

systematic non-perturbative search strategies
strong hints for interacting UV fixed point
intriguing near Gaussianity