

# Dark matter direct detection with dielectrics

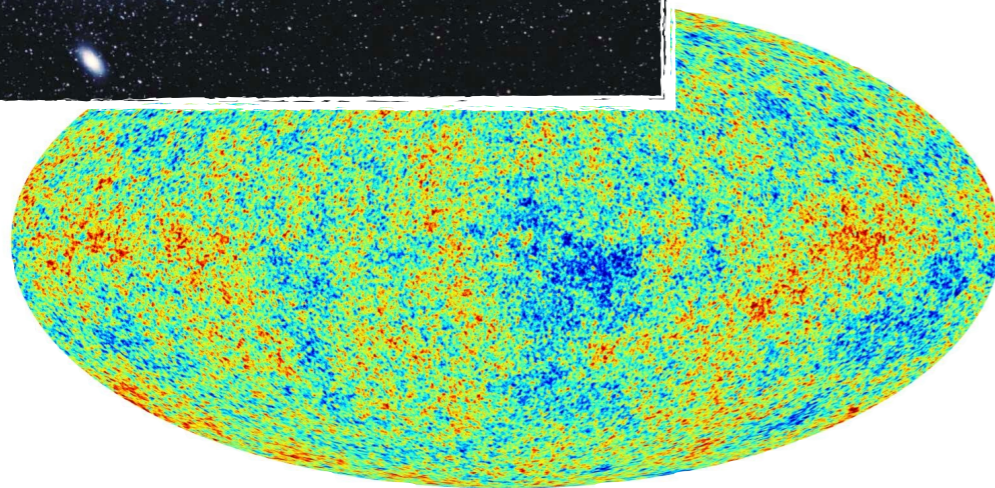
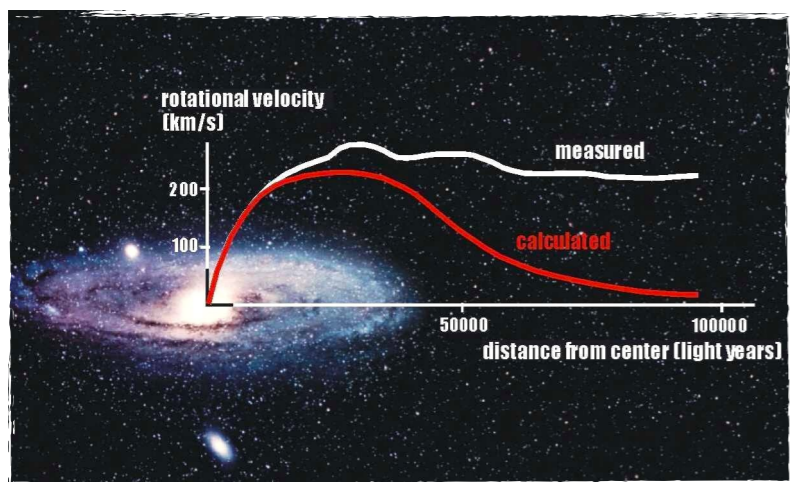
Tongyan Lin  
UCSD

January 24, 2022  
Heidelberg

Based on work with Simon Knapen and Jonathan Kozaczuk  
2003.12077, 2011.09496, 2101.08275, 2104.12786

# Dark matter: evidence and searches

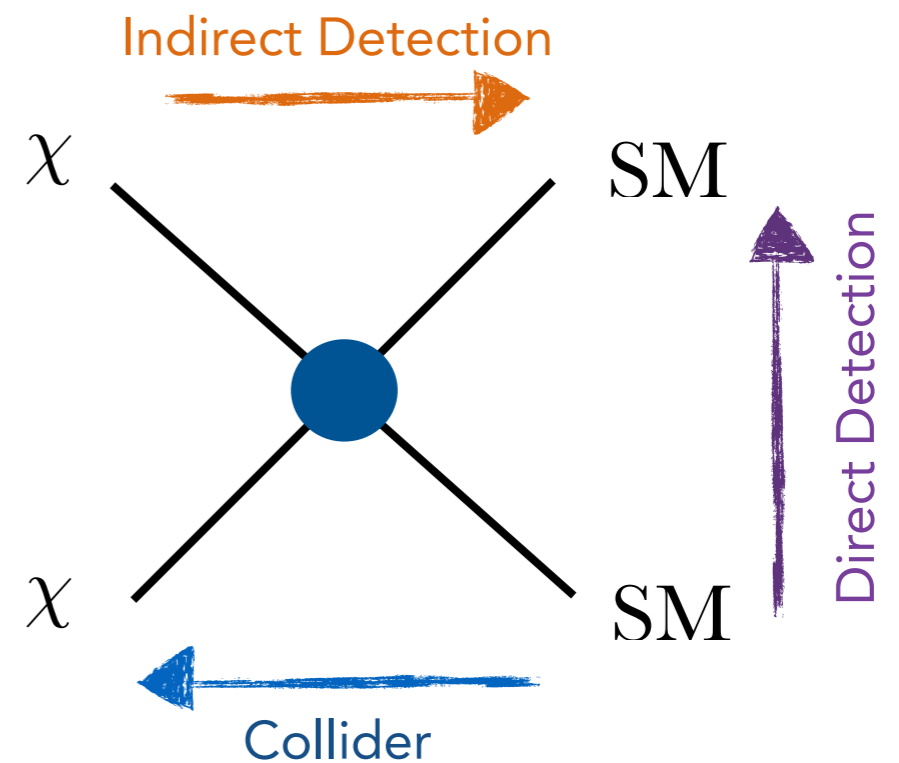
## Gravitational evidence



Cosmic Microwave Background

## Particle searches

- ★ Self-interactions
- ★ Many cosmo/astro probes



- ★ Electron scattering
- ★ Condensed mat systems

- ★ High Luminosity
- ★ Accelerator experiments

# Motivation

Traditional approach to direct detection of dark matter:  
DM-nucleus scattering

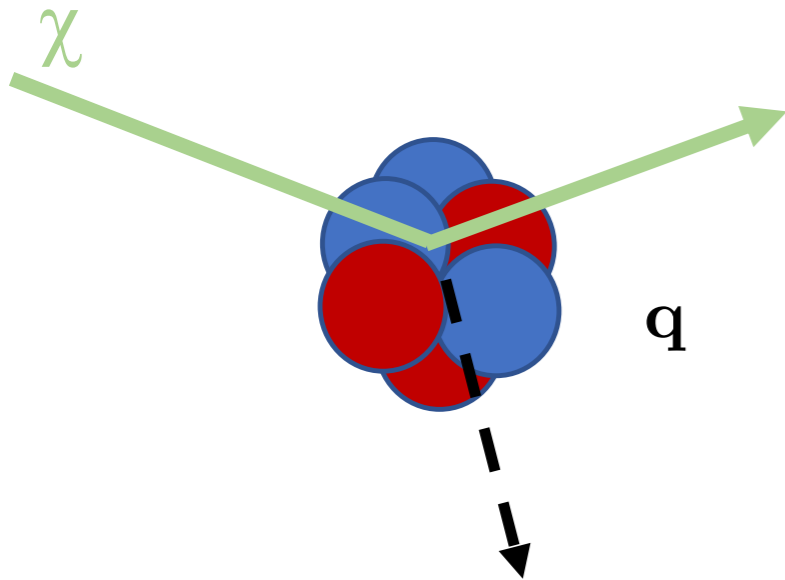
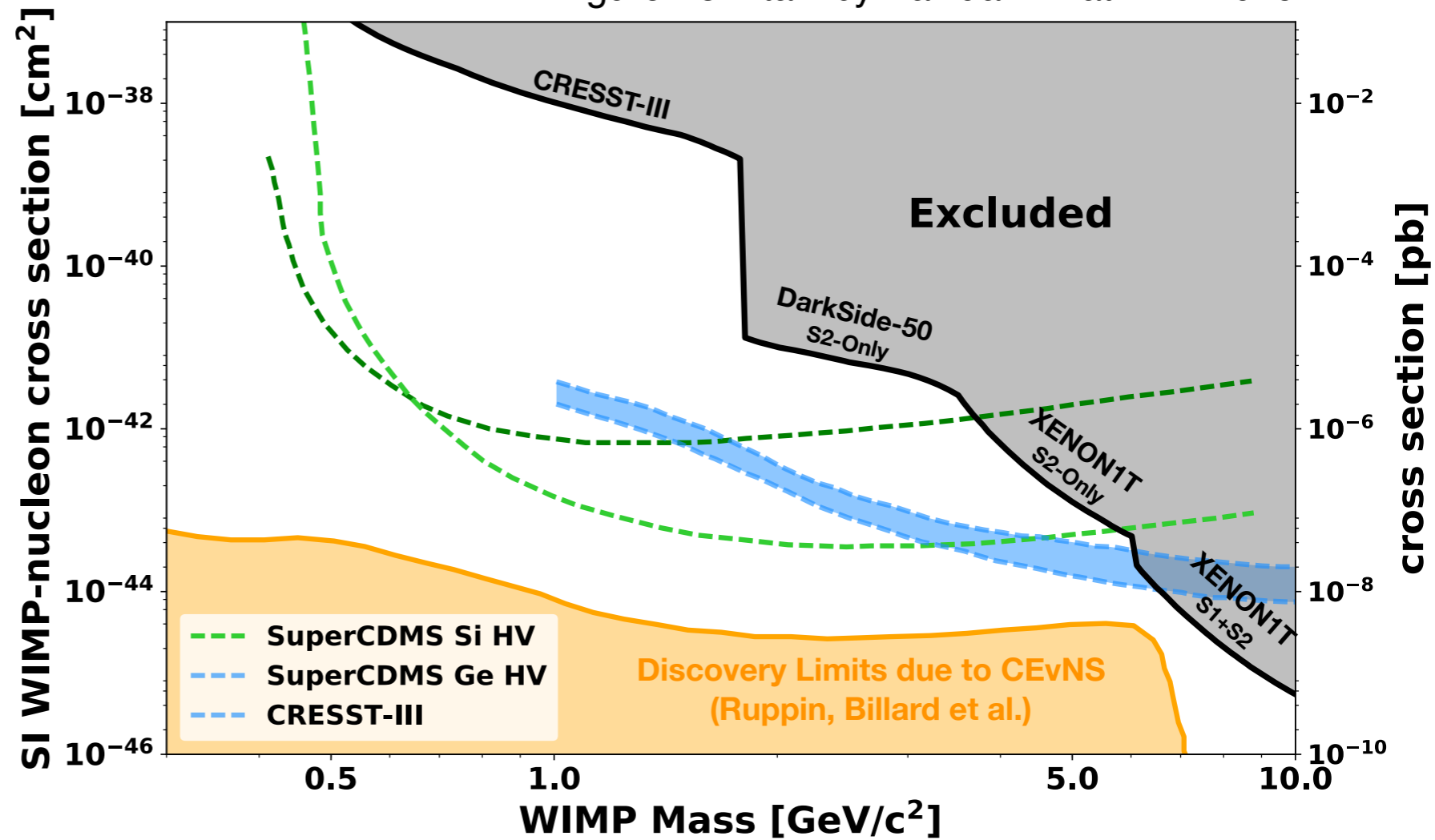


Figure from talk by Kaixuan Ni at DPF 2019



# Challenges for sub-GeV DM

Kinematics of nuclear recoils from light dark matter

$$E_R = \frac{|\mathbf{q}|^2}{2m_N} \leq \frac{2\mu_{\chi N}^2 v^2}{m_N}$$

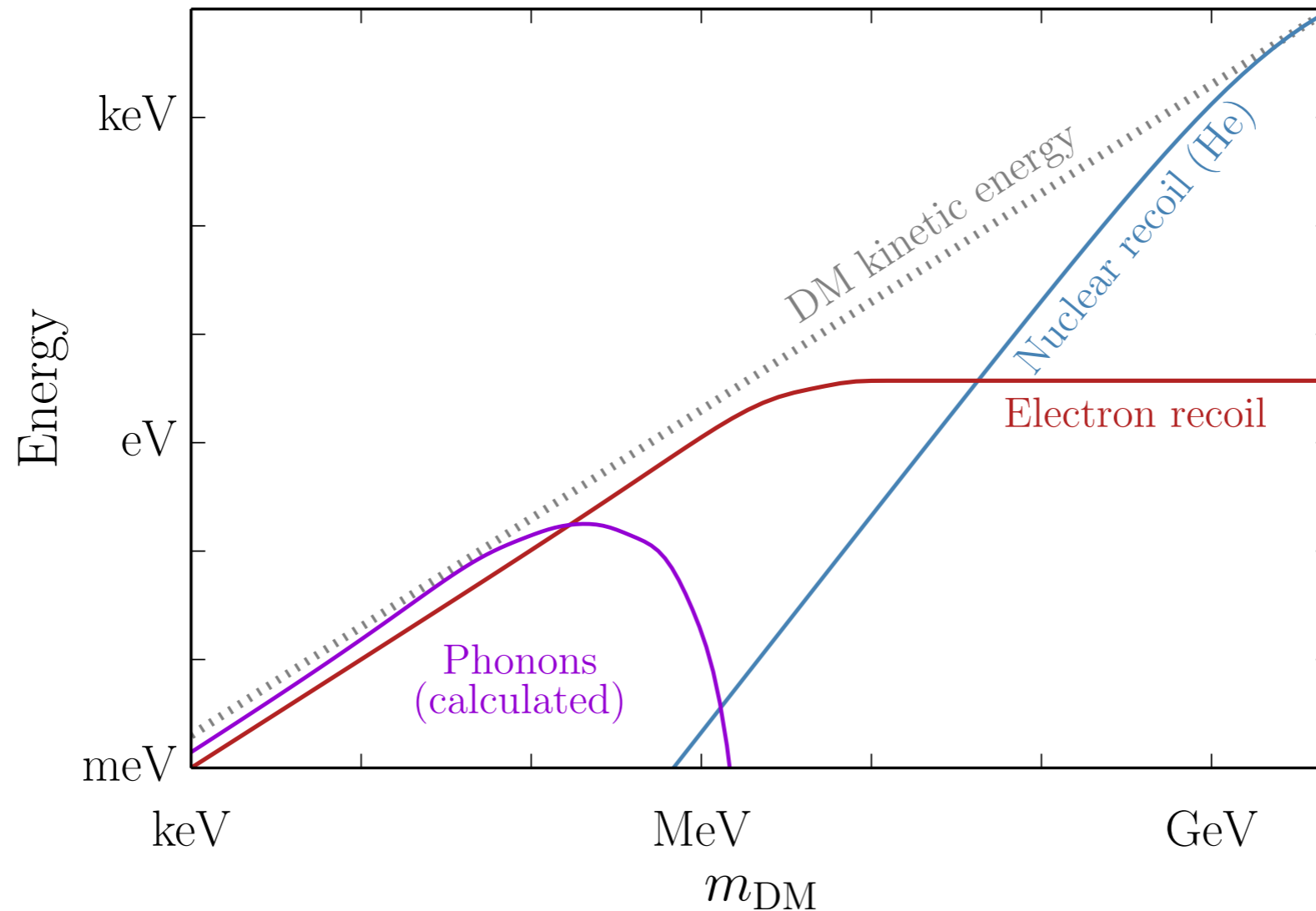
Drops quickly below  $m_\chi \sim 10$  GeV

Best nuclear recoil threshold is currently  $E_R > 30$  eV

(CRESST-III) with DM reach of  $m_\chi > 160$  MeV.

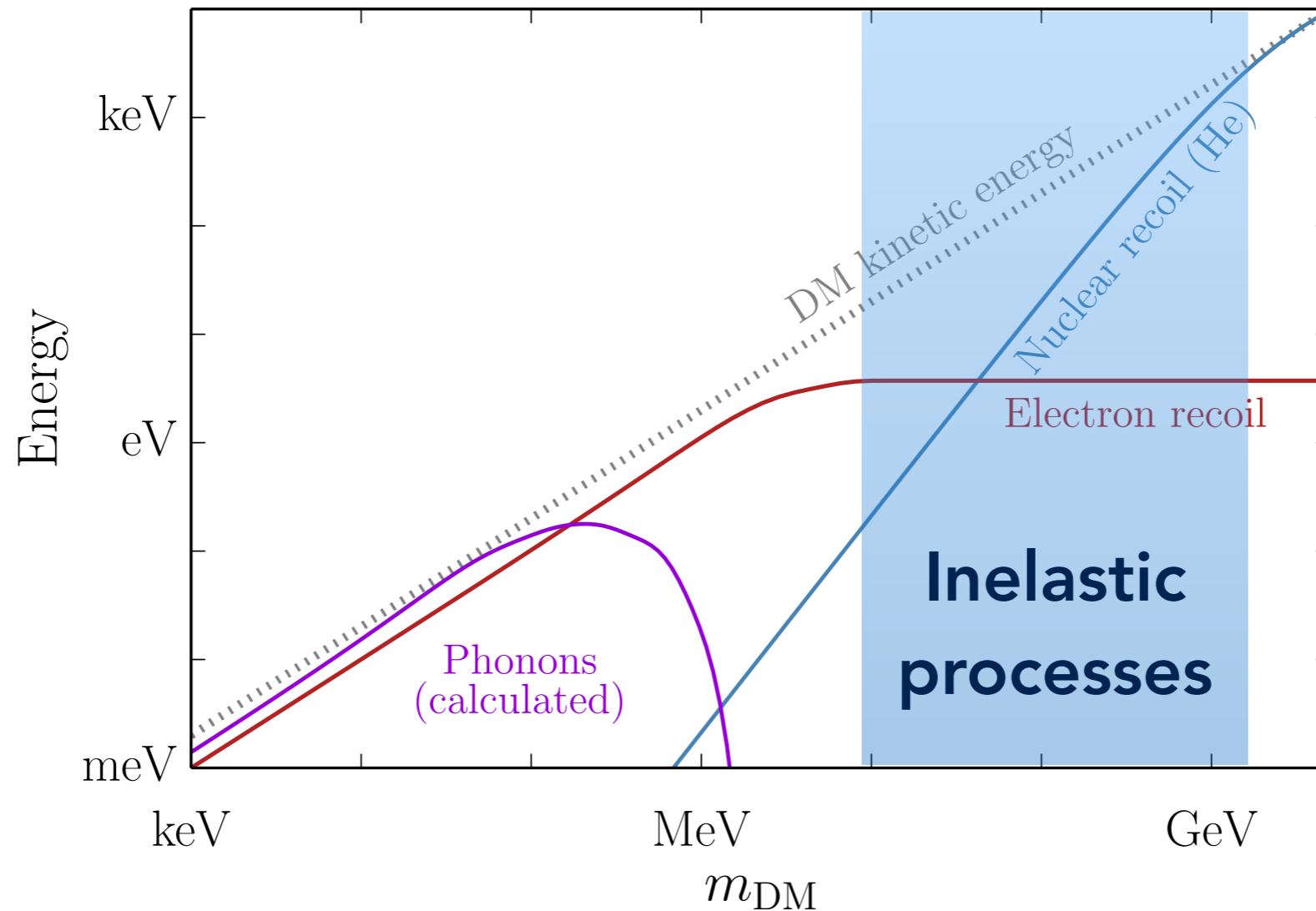
The kinematics of DM scattering against **free** nuclei is inefficient, and it does not describe target response accurately at low energies.

# Material response to DM



Nuclear response is phonon-dominated at low energies.  
Electronic response depends on details of band structure/eigenstates.

# Material response to DM



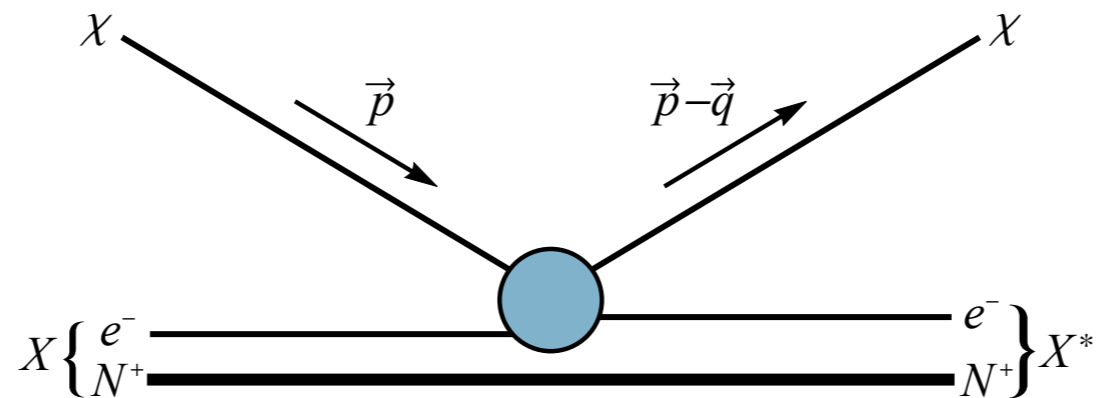
Inelastic nuclear recoils or  $2 \rightarrow 3$  processes can extract more DM kinetic energy, and give charge signals from nuclear recoils.

# Outline

Describing DM-electron scattering in terms of dielectric response  $\epsilon(\omega, \mathbf{k})$

Inelastic processes: describing the Migdal effect in terms of  $\epsilon(\omega, \mathbf{k})$

# Electron recoils



**e- in materials are not free or isolated particles**

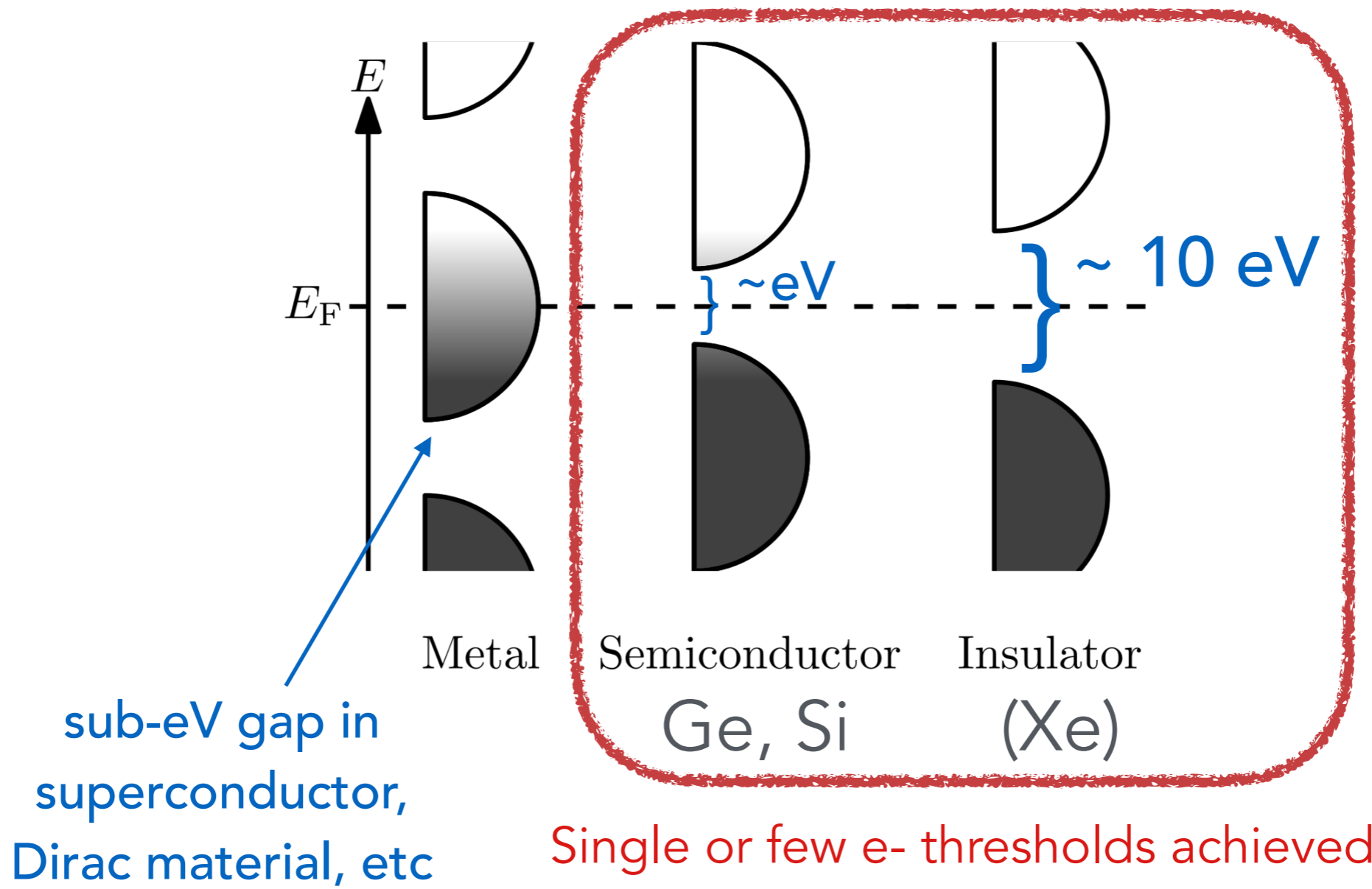
Opportunity: constrained by available energy eigenstates rather than free-particle kinematics.

Complication: need to know eigenstates and wavefunctions in a many-body system.



Opportunity: constrained by available energy eigenstates rather than free-particle kinematics.

## Electronic band structure

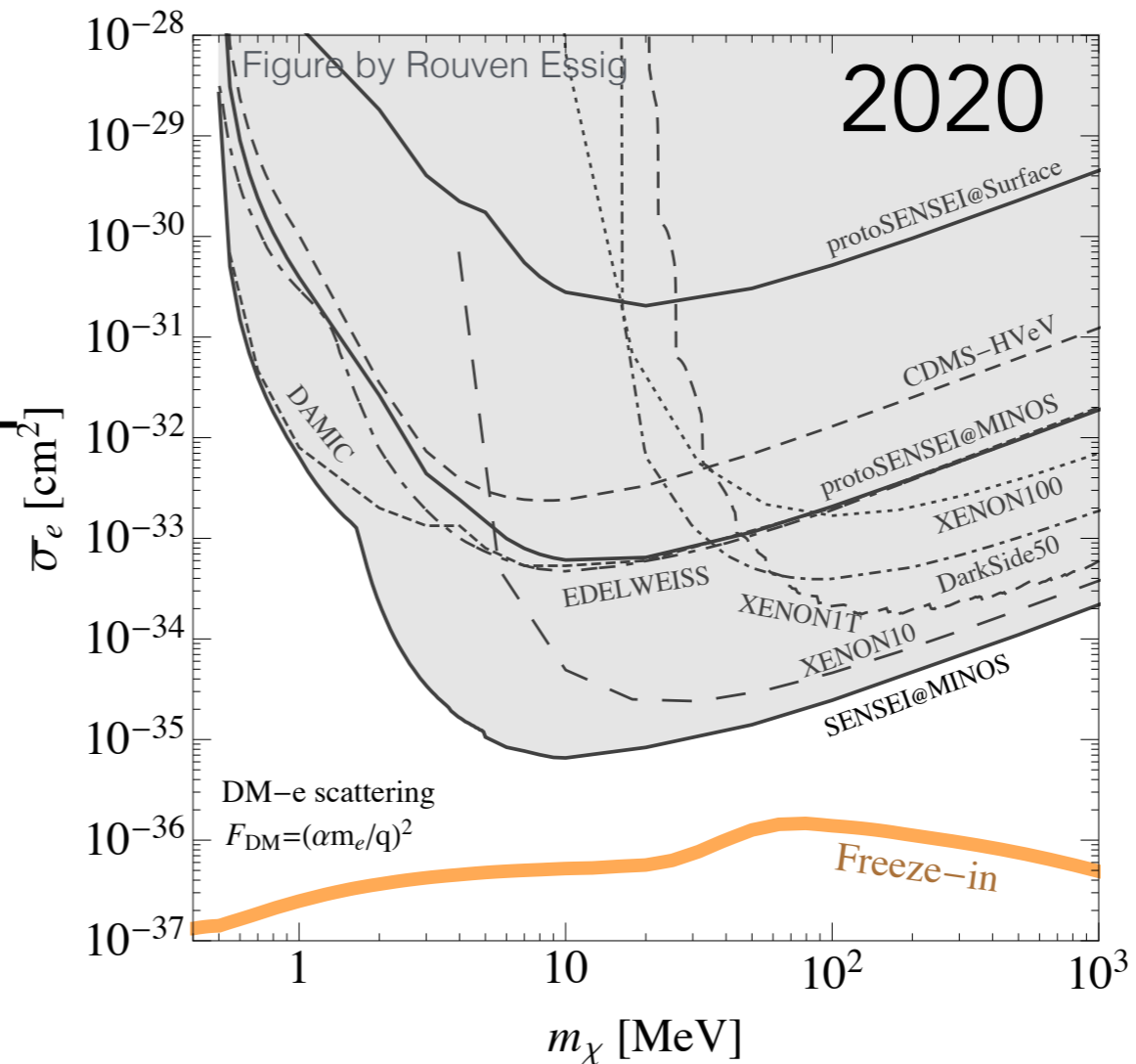
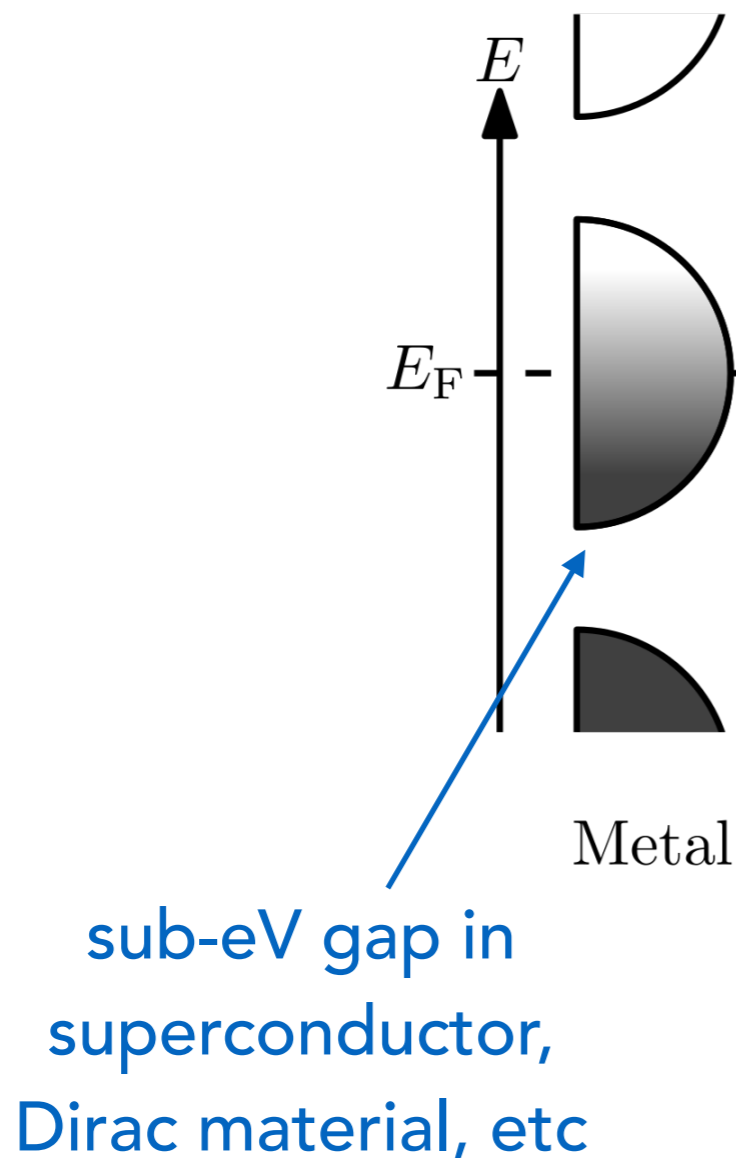


Single or few e- thresholds achieved in a number of experiments

[Hochberg, Pyle, Zhao, Zurek 2015  
Dirac: 1708.08929, 1910.02091, etc]

Opportunity: constrained by available energy eigenstates rather than free-particle kinematics.

## Electronic band structure

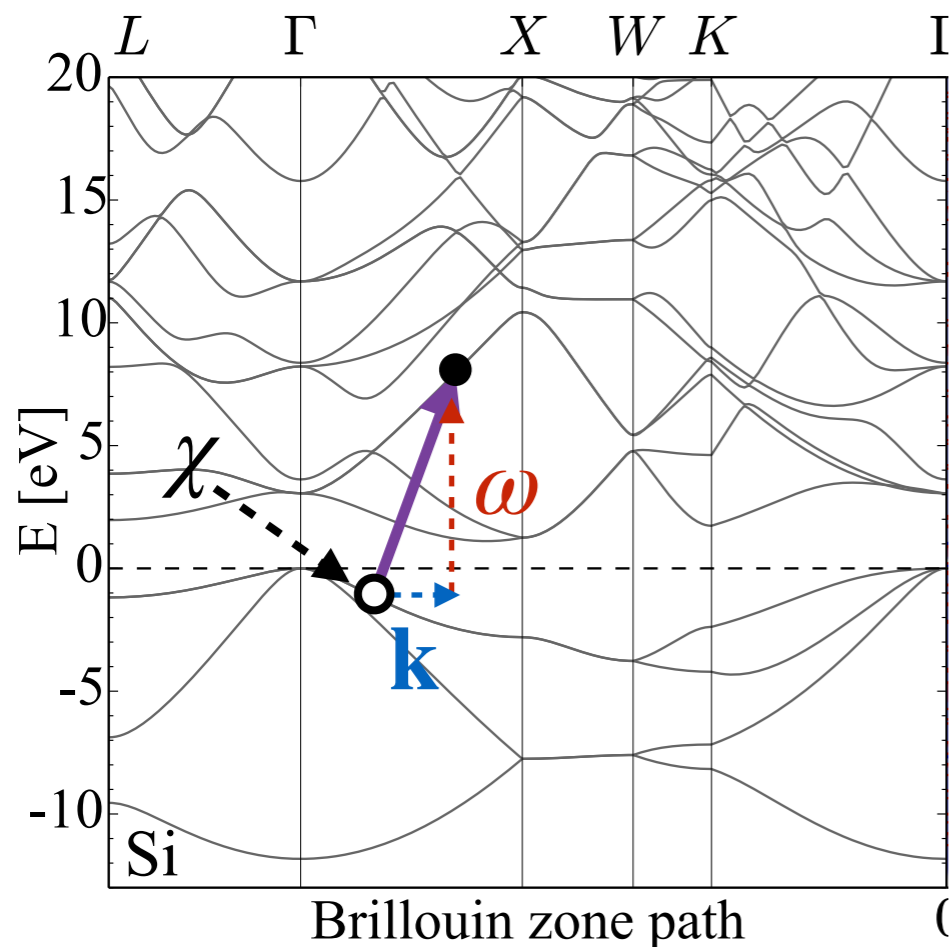


Single or few e- thresholds achieved in a number of experiments

[Hochberg, Pyle, Zhao, Zurek 2015  
 Dirac: 1708.08929, 1910.02091, etc]

Complication: need to know about excitations  
in a many-body system.

## Semiconductor target



Independent particle approximation:

Wavefunction overlap

$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F_{\text{med}}^2(k) \sum_{\ell, \ell'} \sum_{\mathbf{p}, \mathbf{p}'} \overbrace{|\langle \mathbf{p}', \ell' | e^{i\mathbf{k}\cdot\mathbf{r}} | \mathbf{p}, \ell \rangle|^2}^{\text{Wavefunction overlap}} \times f^0(\omega_{\mathbf{p}, \ell}) (1 - f^0(\omega_{\mathbf{p}', \ell'})) \delta(\omega + \omega_{\mathbf{p}, \ell} - \omega_{\mathbf{p}', \ell'})$$

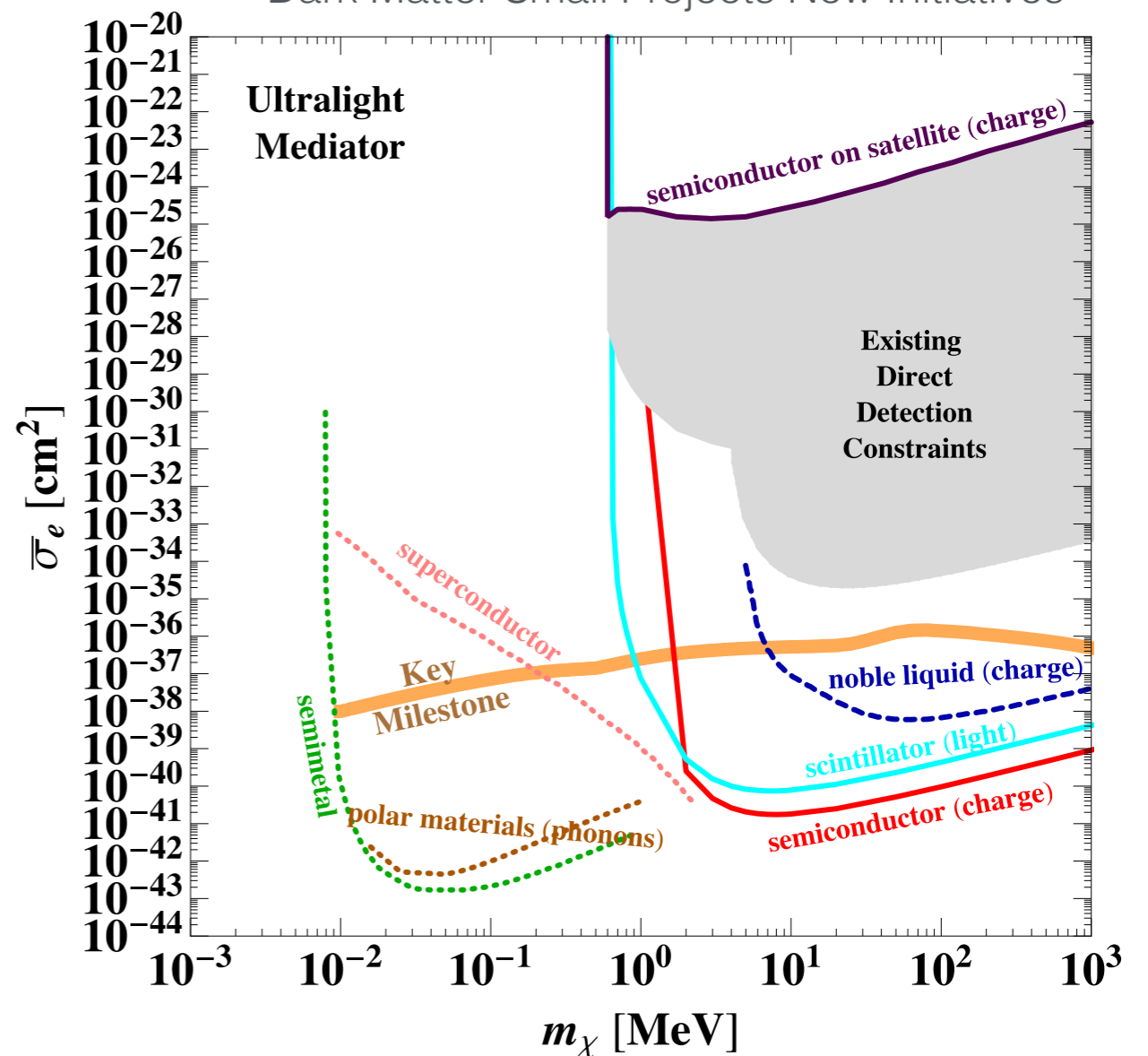
Sum over occupied bands  $\ell$  and Bloch momentum  $\mathbf{p}$  to excited state  $|\mathbf{p}', \ell'\rangle$

Does this capture all many-body effects?

Now many papers studying different targets, proposed experiments, and new experiments in development.

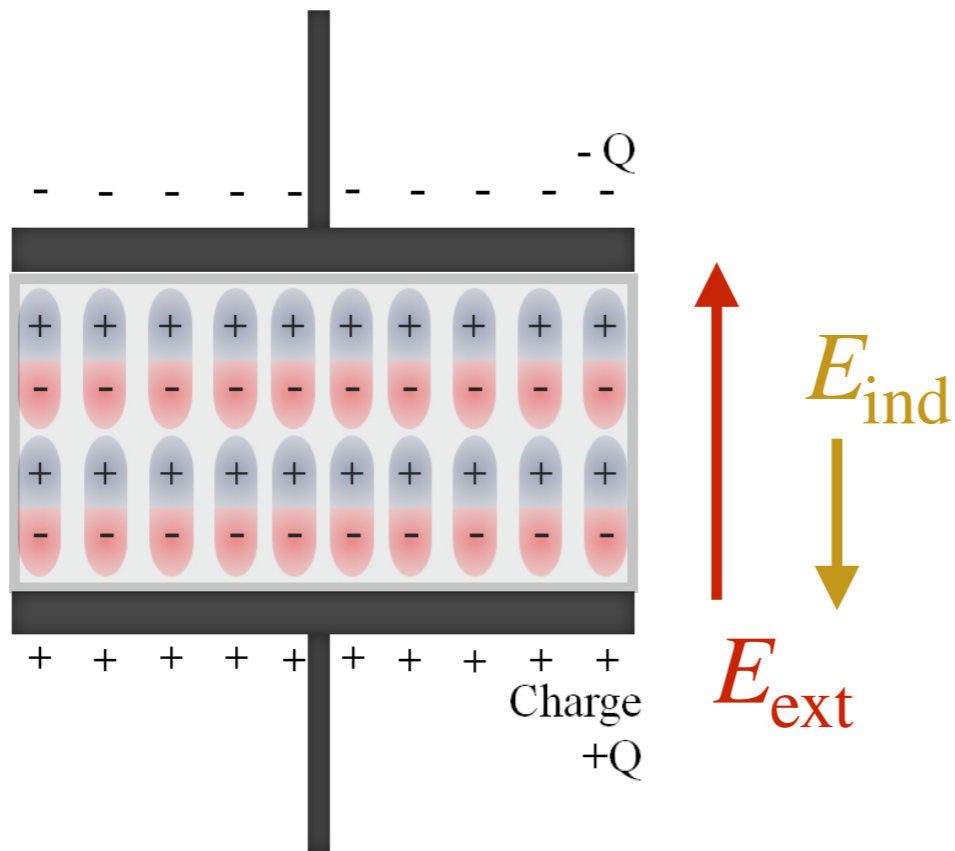
# All dielectrics

From Basic Research Needs Report:  
“Dark Matter Small Projects New Initiatives”



Today: how to describe DM-electron scattering in all these materials in terms of dielectric response function.

# Dielectric response



$$\nabla \cdot \mathbf{E} = \frac{4\pi \rho_{\text{ext}}}{\epsilon} \quad \mathbf{E} = \frac{\mathbf{E}_{\text{ext}}}{\epsilon}$$

More generally:

$$\mathbf{E}(\omega, \mathbf{k}) = \frac{\mathbf{E}_{\text{ext}}(\omega, \mathbf{k})}{\epsilon(\omega, \mathbf{k})}$$

Amount of screening is related to induced charge:

$$\frac{1}{\epsilon(\omega, \mathbf{k})} = 1 + \frac{4\pi e^2}{k^2} \chi(\omega, \mathbf{k})$$

↑  
Susceptibility, charge density response

# Energy loss function (ELF)

$$\frac{1}{\epsilon(\omega, \mathbf{k})} = 1 + \frac{4\pi e^2}{k^2} \chi(\omega, \mathbf{k})$$

External probe that couples to charge density:

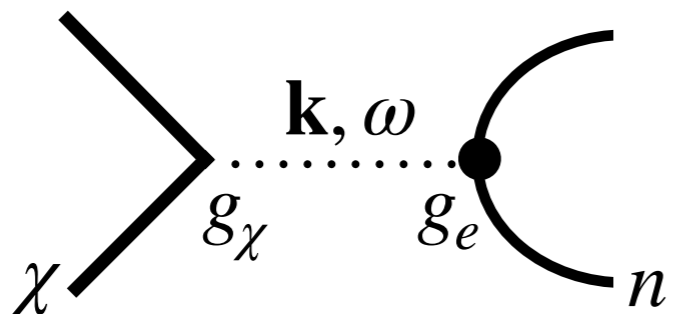
$$S(\omega, \mathbf{k}) \propto \underbrace{2 \operatorname{Im}(-\chi(\omega, \mathbf{k}))}_{\text{Dissipation}} = \frac{k^2}{2\pi\alpha_{em}} \underbrace{\operatorname{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})}\right)}_{\text{ELF}}$$

DM-electron scattering rate is determined by ELF:

$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F_{\text{med}}^2(\mathbf{k}) \operatorname{Im}\left(\frac{-1}{\epsilon(\omega, \mathbf{k})}\right)$$

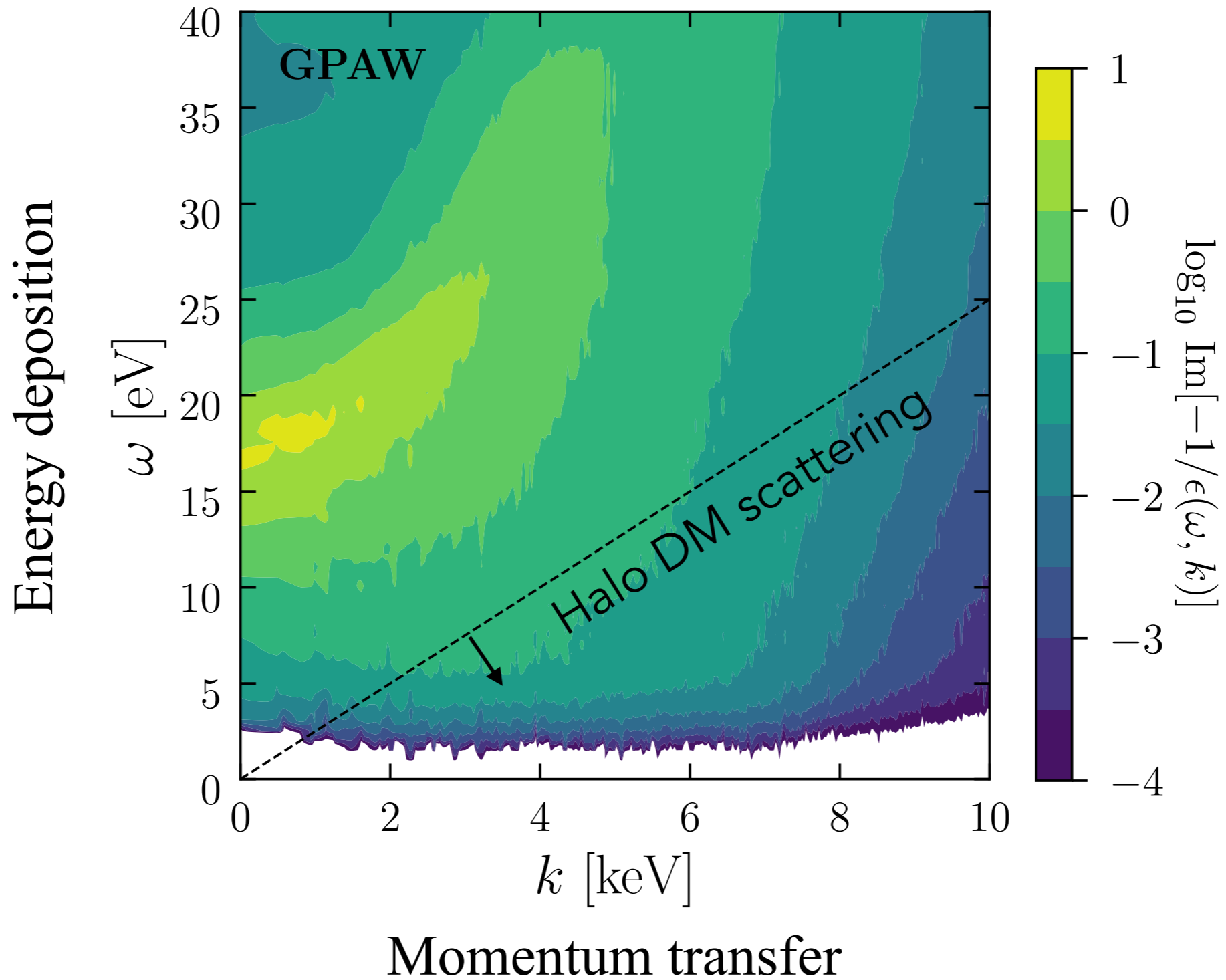
# ELF for Dark Matter

DM-electron scattering with scalar or vector mediators:


$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F_{\text{med}}^2(\mathbf{k}) \text{Im} \left( \frac{-1}{\epsilon(\omega, \mathbf{k})} \right)$$

- Packages details of material in one function
- Includes additional screening effects not captured in original approach (impact on rates)
- ELF describes response to SM probes — many existing materials science approaches

# Response of Silicon semiconductor to electron interactions





# Screening effects

$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F_{\text{med}}^2(\mathbf{k}) \text{Im} \left( \frac{-1}{\epsilon(\omega, \mathbf{k})} \right)$$

$$\propto \bar{\sigma}_e F_{\text{med}}^2(\mathbf{k}) \frac{\text{Im} \epsilon(\omega, \mathbf{k})}{|\epsilon(\omega, \mathbf{k})|^2}$$

Proportional to DM-electron scattering form factor in the independent-electron approximation (RPA)

$$\text{Im} \epsilon^{\text{RPA}}(\omega, \mathbf{k}) = \frac{4\pi^2 \alpha_{em}}{V k^2} \sum_{\ell, \ell'} \sum_{\mathbf{p}, \mathbf{p}'} |\langle \mathbf{p}', \ell' | e^{i\mathbf{k}\cdot\mathbf{r}} | \mathbf{p}, \ell \rangle|^2 \times f^0(\omega_{\mathbf{p}, \ell}) (1 - f^0(\omega_{\mathbf{p}', \ell'})) \delta(\omega + \omega_{\mathbf{p}, \ell} - \omega_{\mathbf{p}', \ell'})$$

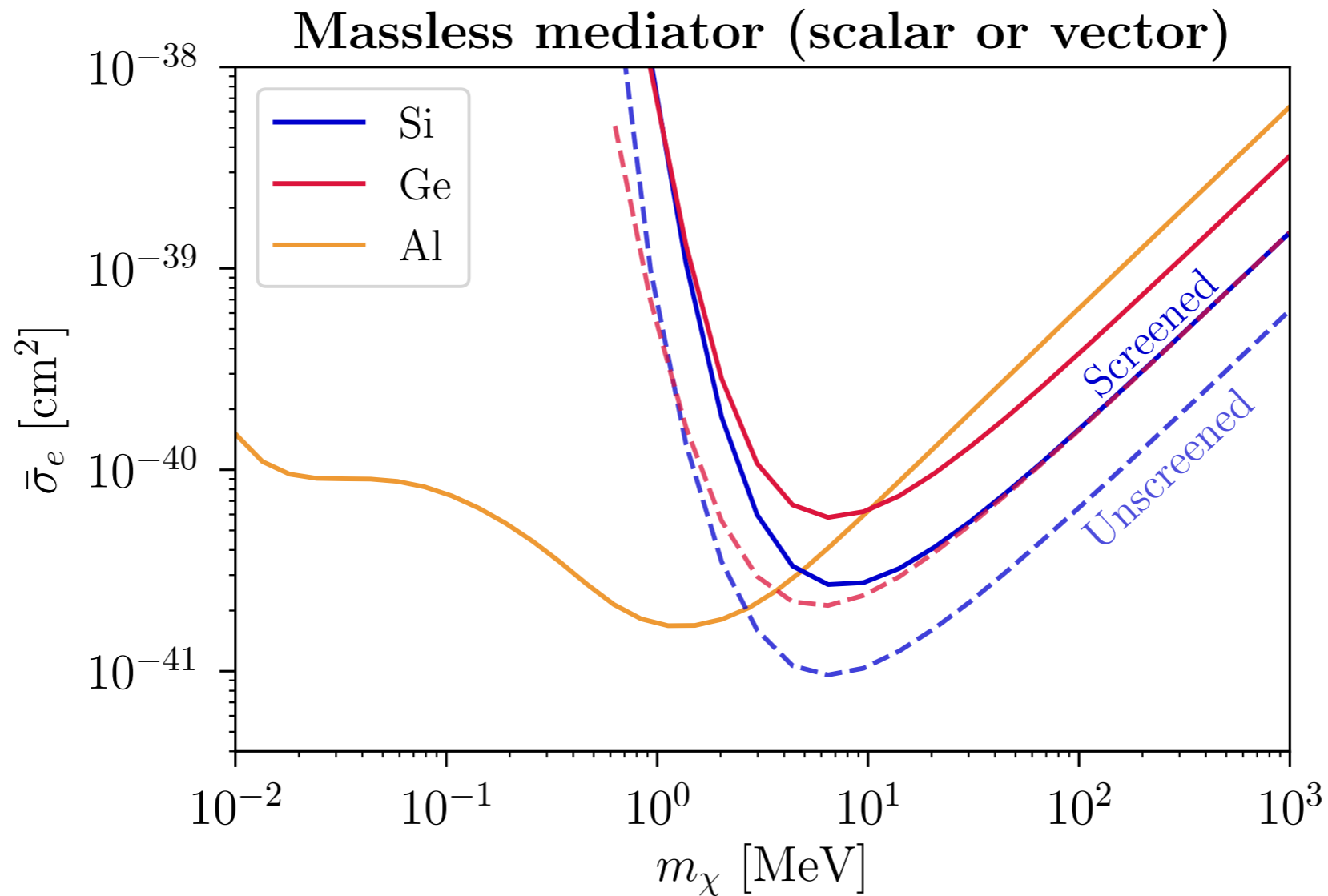
$|\epsilon(\omega, \mathbf{k})|^2$  screening for vector mediators considered in superconductors, Dirac materials.

**Not previously included in signal rates for semiconductors.**

Also not previously included for scalar mediators.

# Impact for DM-electron scattering

Using the ELF automatically incorporates screening effects:



Knapen, TL, Kozaczuk 2101.08275

Additional effects of core electrons included in Griffin, Inzani, Trickle, Zhang, Zurek 2105.05253

# The energy loss function (ELF)

$$\text{Im} \left( \frac{-1}{\epsilon(\omega, \mathbf{k})} \right)$$

## Theory

Many established approaches to  $\epsilon$

Include screening, local field effects

Include electron-electron interactions

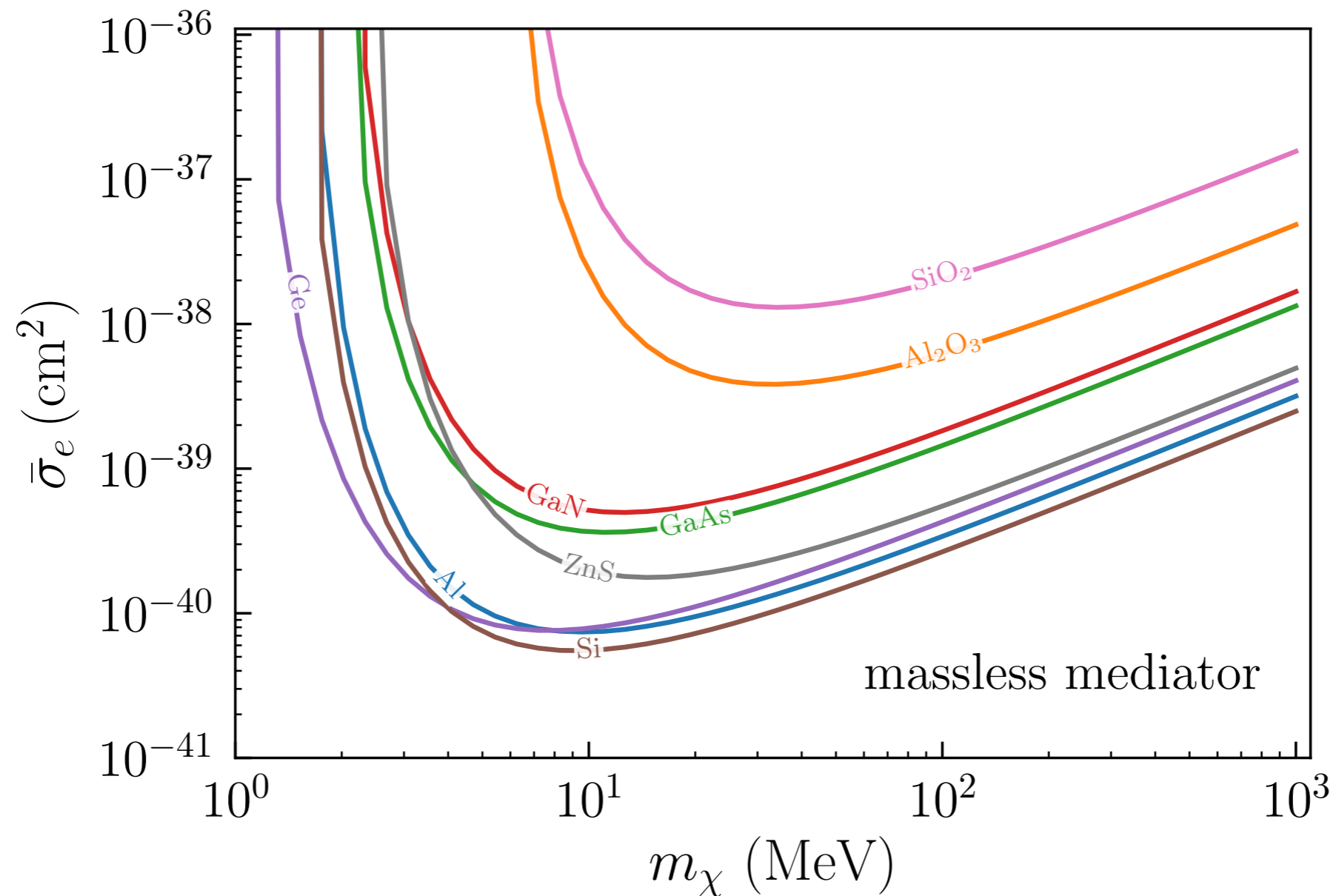
## Experiment

Optical measurements

X-ray scattering

Fast electron scattering (EELS)

# Fast target material comparison



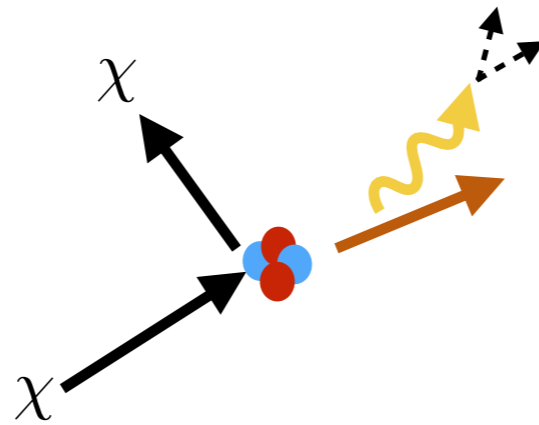
2e- threshold using data-driven Mermin method for ELF  
Si, Ge particularly good due to lower thresholds

# ELF for Dark Matter

DarkELF: python package for dark matter energy loss processes with tabulated ELF's for a variety of materials (incl. Si, Ge, GaAs)

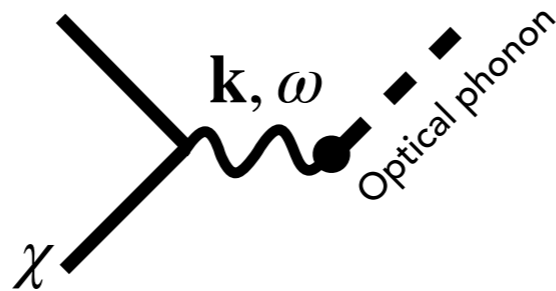
<https://github.com/tongylin/DarkELF>

**DM-nucleus scattering via Migdal effect**



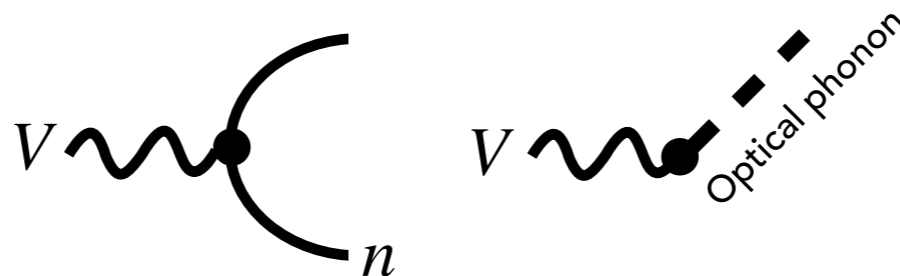
$$\frac{dP}{d^3\mathbf{k}d\omega} \propto \frac{4\pi\alpha_{em}Z_{ion}^2}{\omega^4} \frac{|\mathbf{v}_N \cdot \mathbf{k}|^2}{k^2} \text{Im} \left( \frac{-1}{\epsilon(\omega, \mathbf{k})} \right)$$

**DM-phonon scattering**



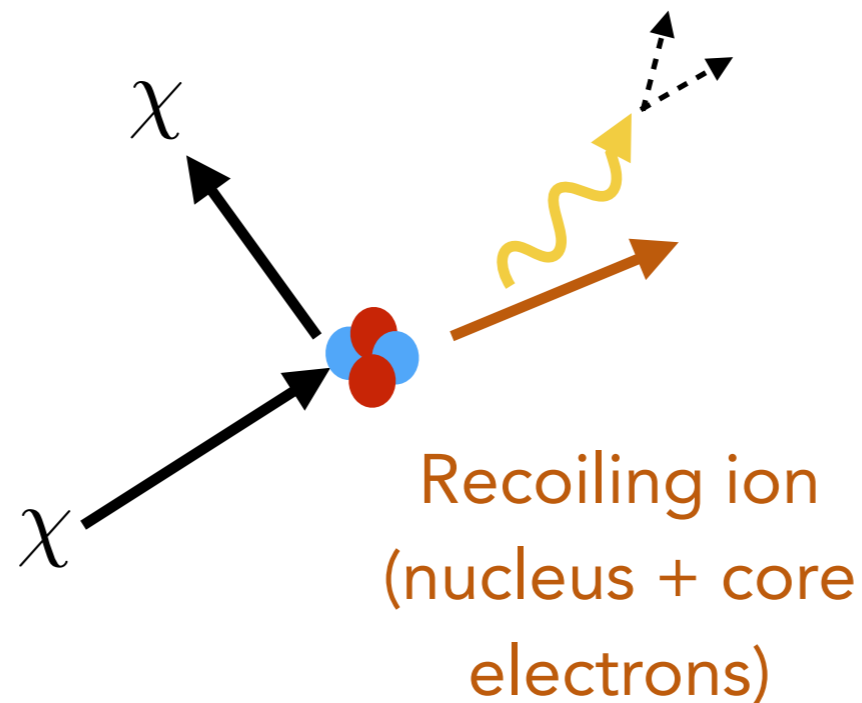
$$\frac{d\sigma}{d^3\mathbf{k}d\omega} \propto \bar{\sigma}_e F_{\text{med}}^2(\mathbf{k}) \text{Im} \left( \frac{-1}{\epsilon(\omega, \mathbf{k})} \right)$$

**Bosonic DM absorption**



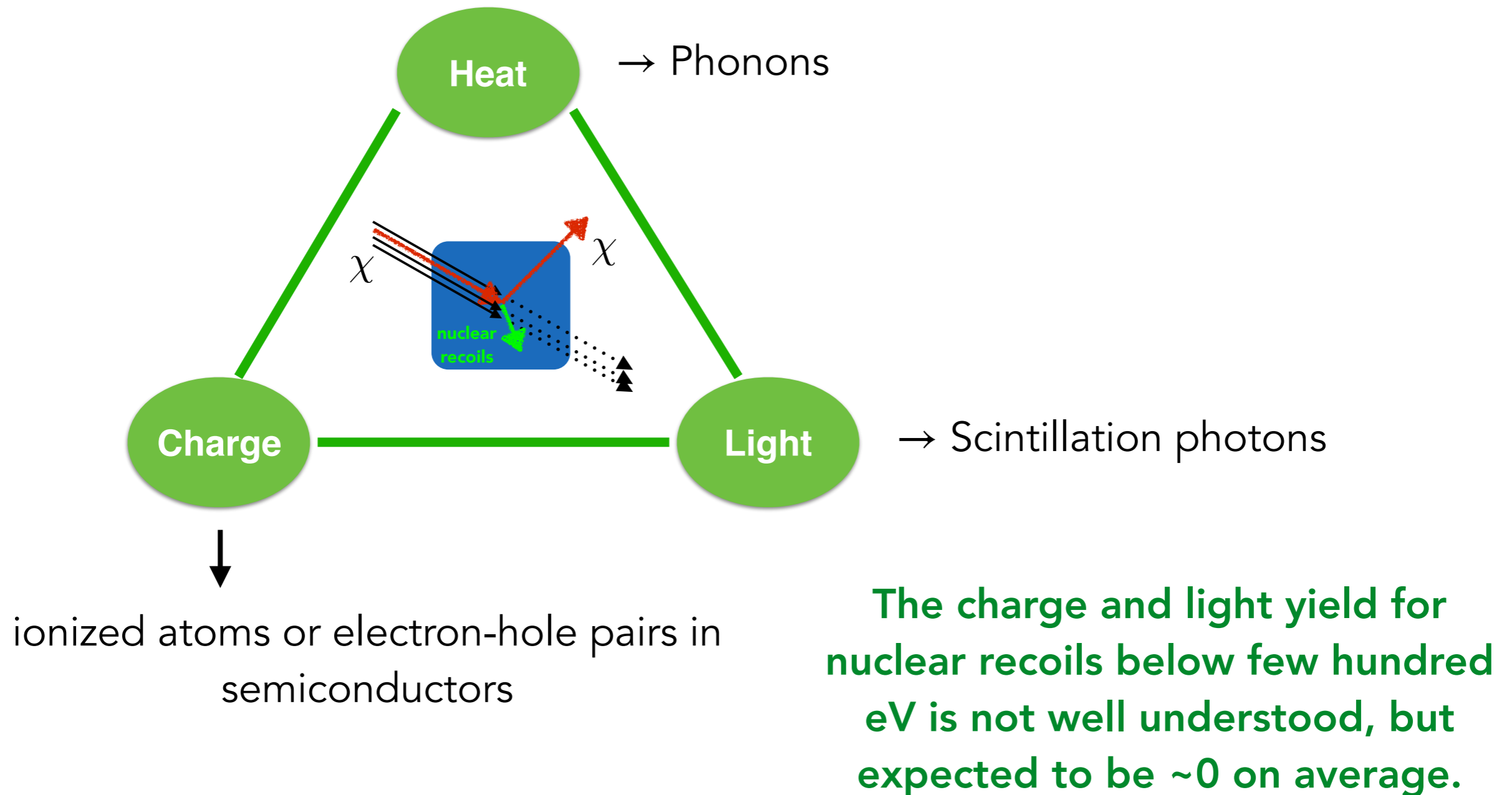
$$R = \frac{1}{\rho_T} \frac{\rho_{\text{DM}}}{m_V} k^2 m_V \text{Im} \left( \frac{-1}{\epsilon(\omega, \mathbf{k})} \right)$$

# Detecting nuclear recoils via the Migdal effect



With Jonathan Kozaczuk (2003.12077)  
and with Jonathan Kozaczuk and Simon Knapen (2011.09496)

# Challenges of low-energy nuclear recoils

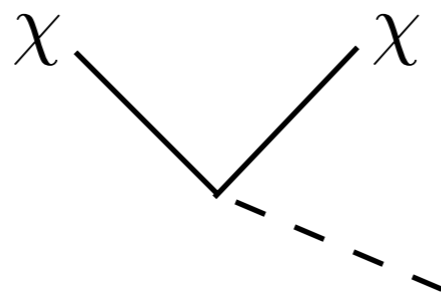


# Strategies for detecting nuclear recoils from sub-GeV DM

## 1. Decreasing the heat threshold

- Detectors in development to reach heat/phonon thresholds of  $\sim$  eV and below (e.g. SuperCDMS SNOLAB)
- Direct phonon excitations from DM scattering  
 $\omega \approx 1 - 100$  meV for acoustic and optical phonons in crystals  
(e.g. phonons: Griffin, Knapen, TL, Zurek 2018; molecules: Essig, Perez-Rios, Ramani, Slone 2019)

DM-phonon  
scattering



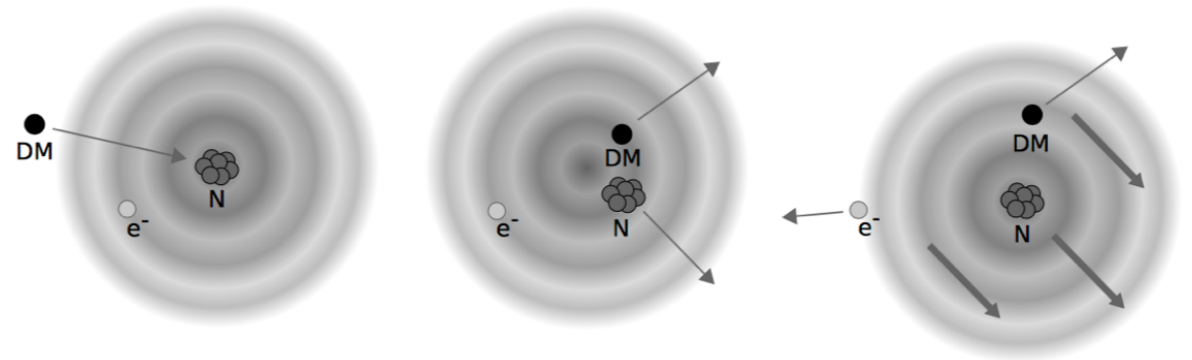
Kinematics of phonons  
relevant (and advantageous)  
for sub-MeV dark matter



# Strategies for detecting nuclear recoils from sub-GeV DM

## 2. Increasing the charge signal

- **Atomic Migdal effect**  
Ionization of electrons  
which have to 'catch up'  
to recoiling nucleus  
(e.g. Ibe, Nakano, Shoji, Suzuki 2017)

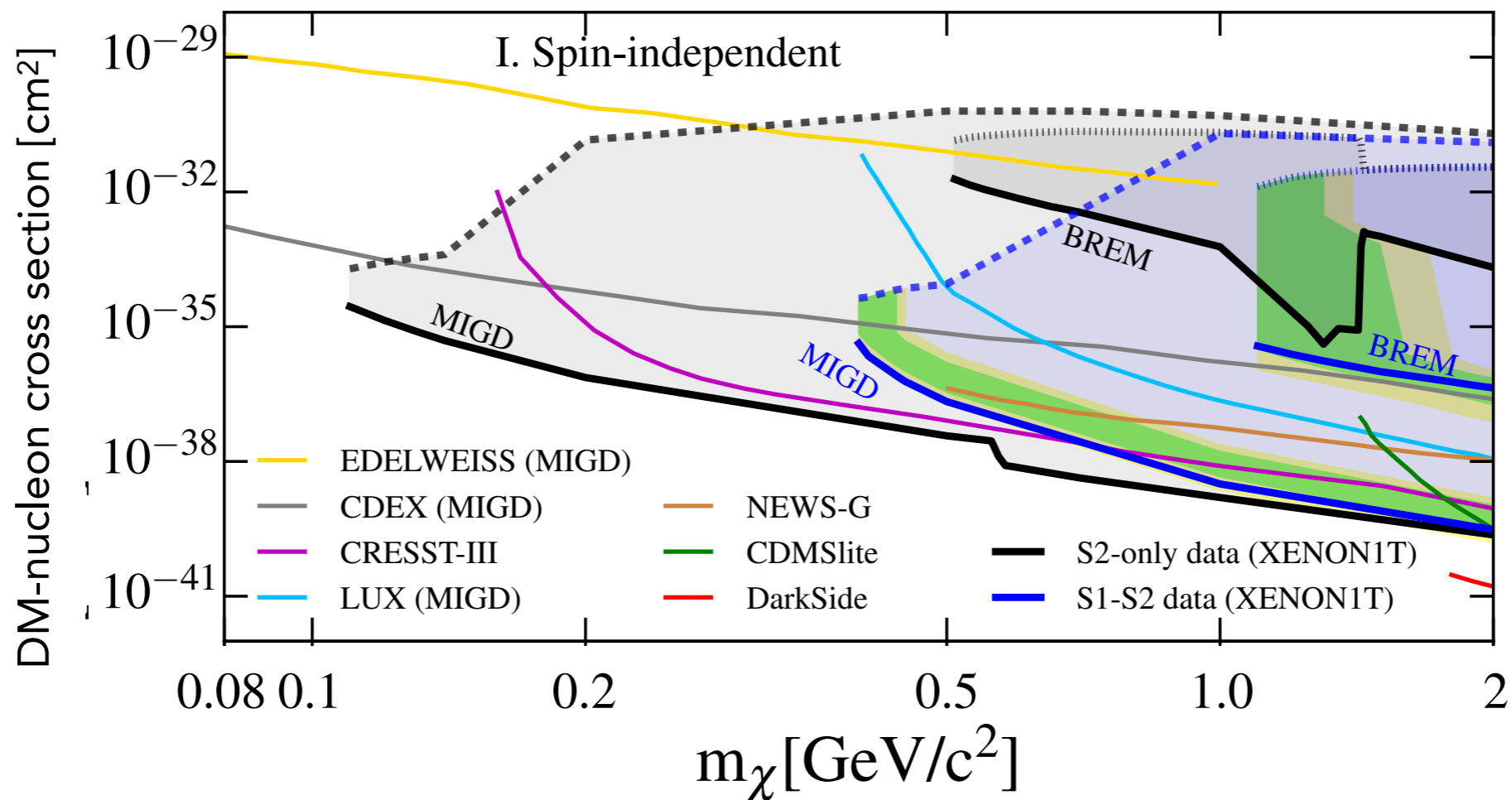


From 1711.09906 (Dolan et al.)

- **Bremsstrahlung of (transverse) photons in LXe**  
Kouvaris & Pradler 2016

# Strategies for detecting nuclear recoils from sub-GeV DM

## 2. Increasing the charge signal

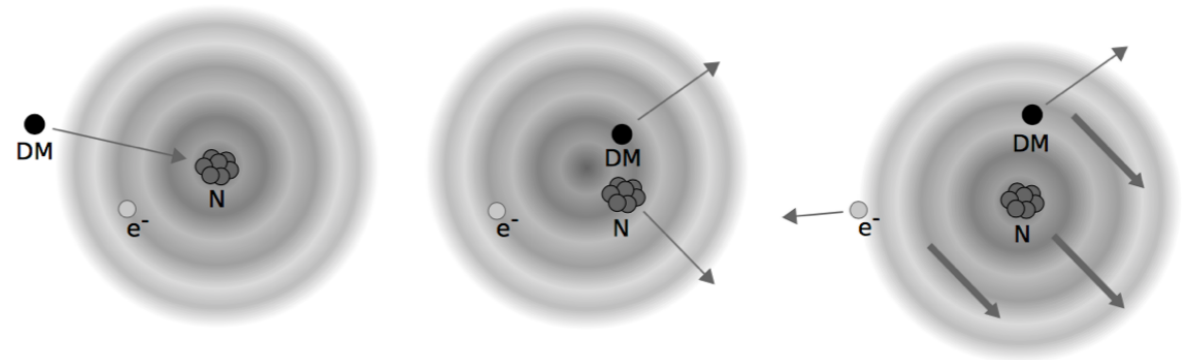


Results from XENON1T search (PRL 2019)

# Strategies for detecting nuclear recoils from sub-GeV DM

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From 1711.09906 (Dolan et al.)

- **Bremsstrahlung of (transverse) photons in LXe**  
Kouvaris & Pradler 2016
- **Migdal effect in semiconductors with lower thresholds**

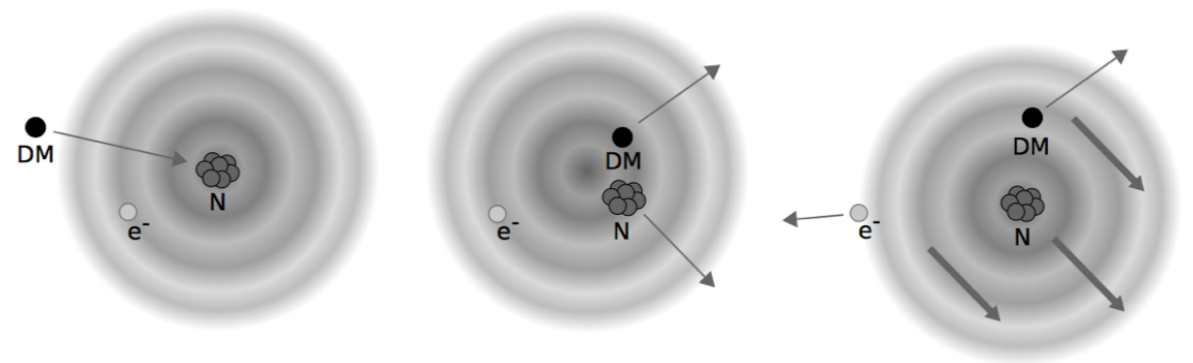
# Atomic Migdal effect

Electrons have to 'catch up' to recoiling nucleus

Boost initial state to frame of moving nucleus:

$$|i\rangle \rightarrow e^{im_e \mathbf{v}_N \cdot \sum_{\beta} \mathbf{r}_{\beta}} |i\rangle$$

From 1711.09906 (Dolan, Kahlhoefer, McCabe)



Transition probability  $|\mathcal{M}_{if}|^2$

$$\mathcal{M}_{if} = \langle f | e^{im_e \mathbf{v}_N \cdot \sum_{\beta} \mathbf{r}_{\beta}} |i\rangle \approx im_e \langle f | \mathbf{v}_N \cdot \sum_{\beta} \mathbf{r}_{\beta} |i\rangle$$

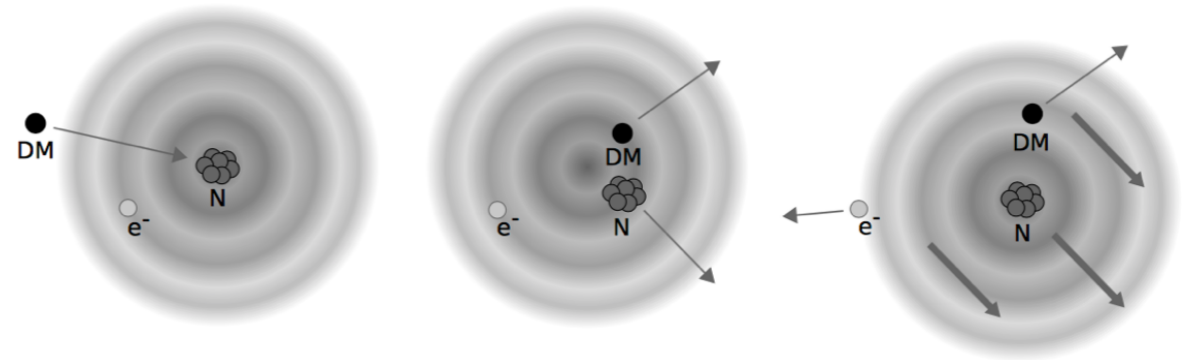
Nucleus recoils with velocity  $\mathbf{v}_N$

Small probability for "shake-off" electron, but allows low-energy nuclear recoil to be above the e- recoil threshold

# Atomic Migdal effect

Boost initial state to frame  
of moving nucleus:

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Nucleus recoils with velocity  $\mathbf{v}_N$

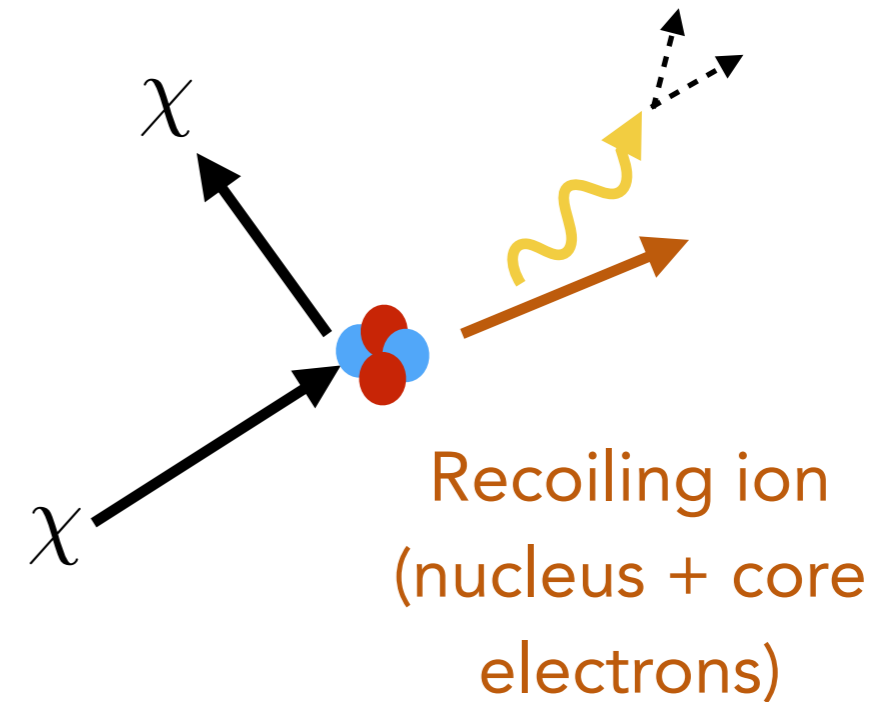
**Problem: applying this to semiconductors does not work.  
Boosting argument does not apply because of crystal lattice.**

# The Migdal effect as bremsstrahlung

Bremsstrahlung calculation



treating  $N$  as nucleus with tightly bound core electrons. Valid for  $10 \text{ MeV} \lesssim m_\chi \lesssim 1 \text{ GeV}$ .



Usual DM-nucleus scattering

$$\frac{d\sigma}{d\omega} = \frac{2\pi^2 A^2 \sigma_n}{m_\chi^2 v_\chi} \int \frac{d^3 \mathbf{q}_N}{(2\pi)^3} \int \frac{d^3 \mathbf{p}_f}{(2\pi)^3} \delta(E_i - E_f - \omega - E_N) \times \int \frac{d^3 \mathbf{k}}{(2\pi)^3} F(\mathbf{p}_i - \mathbf{p}_f - \mathbf{q}_N - \mathbf{k})^2$$

$$\times 4\alpha_{em} Z_{\text{ion}}^2 \left[ \frac{1}{\omega - \mathbf{q}_N \cdot \mathbf{k} / m_N} - \frac{1}{\omega} \right]^2 \text{Im} \left( \frac{-1}{\epsilon(\omega, \mathbf{k})} \right)$$

Form factor accounting for multiphonon response in a harmonic crystal

Differential probability of ion to excite an electron

# Relation with atomic Migdal effect

From boosting argument:

$$\begin{aligned}
 & im_e \langle f | \mathbf{v}_N \cdot \sum_{\beta} \mathbf{r}_{\beta} | i \rangle \\
 &= \mathbf{v}_N \cdot \frac{1}{\omega} \langle f | \sum_{\beta} \mathbf{p}_{\beta} | i \rangle \\
 &= -\mathbf{v}_N \cdot \frac{1}{\omega^2} \langle f | \sum_{\beta} [\mathbf{p}_{\beta}, H_0] | i \rangle = \frac{-i}{\omega^2} \langle f | \sum_{\beta} \frac{Z_N \alpha \mathbf{v}_N \cdot \hat{\mathbf{r}}_{\beta}}{|\mathbf{r}_{\beta} - \mathbf{r}_N|^2} | i \rangle.
 \end{aligned}$$

Dipole potential from recoiling nucleus

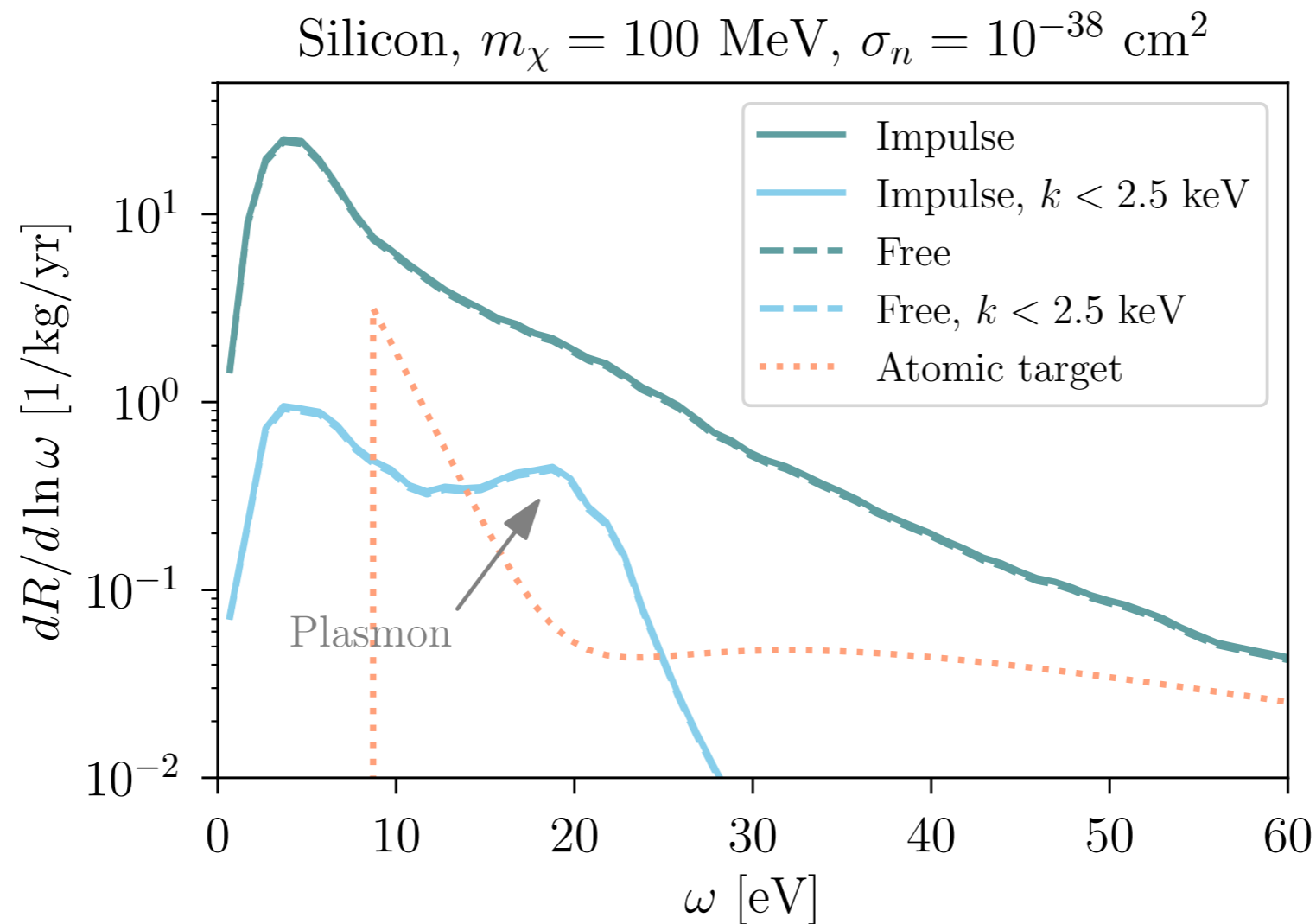
## Atomic Migdal effect

$$\frac{dP(E_N)}{d\omega} \approx \left( \frac{4\pi Z_N \alpha}{\omega^2} \right)^2 \sum_{i,f} \left| \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{\mathbf{v}_N \cdot \mathbf{k}}{k^2} \langle f | e^{i\mathbf{k} \cdot \mathbf{r}} | i \rangle \right|^2 \delta(E_i + \omega - E_f)$$

## Semiconductor Migdal effect

$$\frac{dP}{d\omega} \approx \frac{(4\pi Z_{\text{ion}} \alpha)^2}{\omega^4 V} \sum_{\mathbf{p}_e} \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{|\mathbf{v}_N \cdot \mathbf{k}|^2}{k^4} \frac{|[\mathbf{p}_e + \mathbf{k} | \mathbf{p}_e]_{\Omega}|^2}{|\epsilon(\mathbf{k}, \omega)|^2} \times (f(\mathbf{p}_e) - f(\mathbf{p}_e + \mathbf{k})) \delta(\omega_{\mathbf{p}_e + \mathbf{k}} - \omega_{\mathbf{p}_e} - \omega)$$

# Full rate in semiconductors

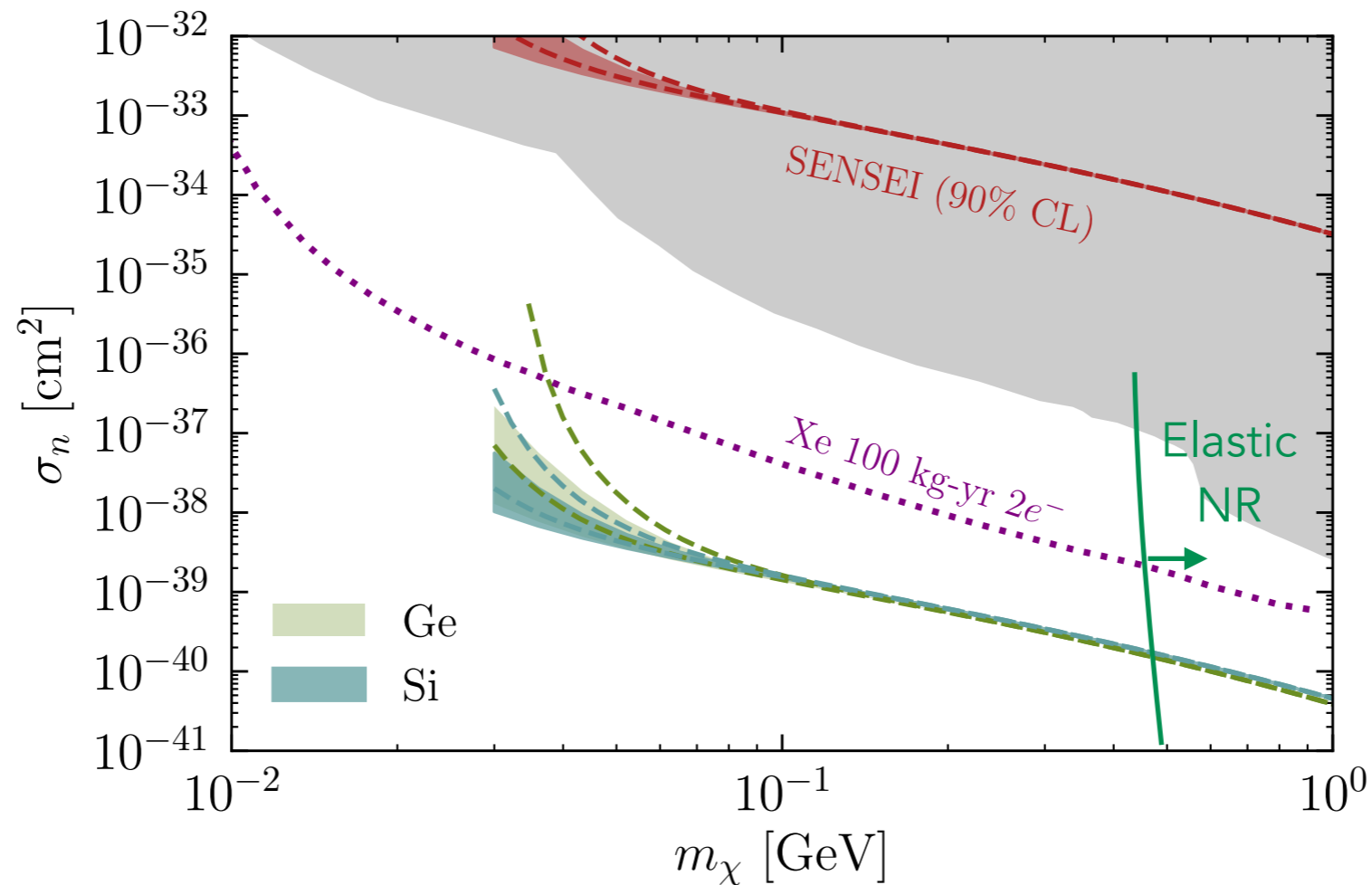


Migdal rate in semiconductors is much larger due to lower gap for excitations.



# Sensitivity in semiconductors

1 kg-year exposure, with  $Q > 2$  (similar to proposed experiments)



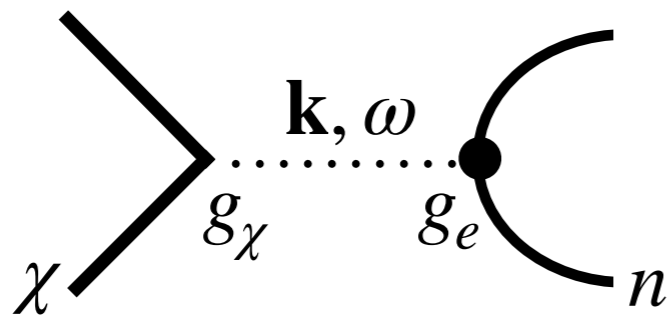
The Migdal effect in semiconductors can enhance sensitivity to nuclear recoils from sub-GeV dark matter

# Conclusions

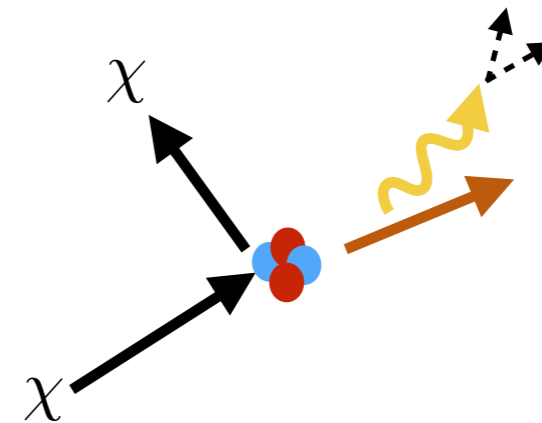
The energy loss function (ELF) in dielectric materials describes response to any electromagnetic probe (Standard Model or DM):

$$\text{Im} \left( \frac{-1}{\epsilon(\omega, \mathbf{k})} \right)$$

Appears in multiple types of DM interactions,  
applies for arbitrary target material



**DM-electron  
scattering**



**DM-nucleus scattering  
via Migdal effect**

# Conclusions

The energy loss function (ELF) in dielectric materials describes response to any electromagnetic probe (Standard Model or DM):

$$\text{Im} \left( \frac{-1}{\epsilon(\omega, \mathbf{k})} \right)$$

Appears in multiple types of DM interactions,  
applies for arbitrary target material

We welcome use of DarkELF, a python package for  
DM interactions in terms of the ELF:  
<https://github.com/tongylin/DarkELF>