

# TwInflation

## Addressing the $\eta$ Problem with a $Z_2$ symmetry

Soubhik Kumar  
UC Berkeley and LBL

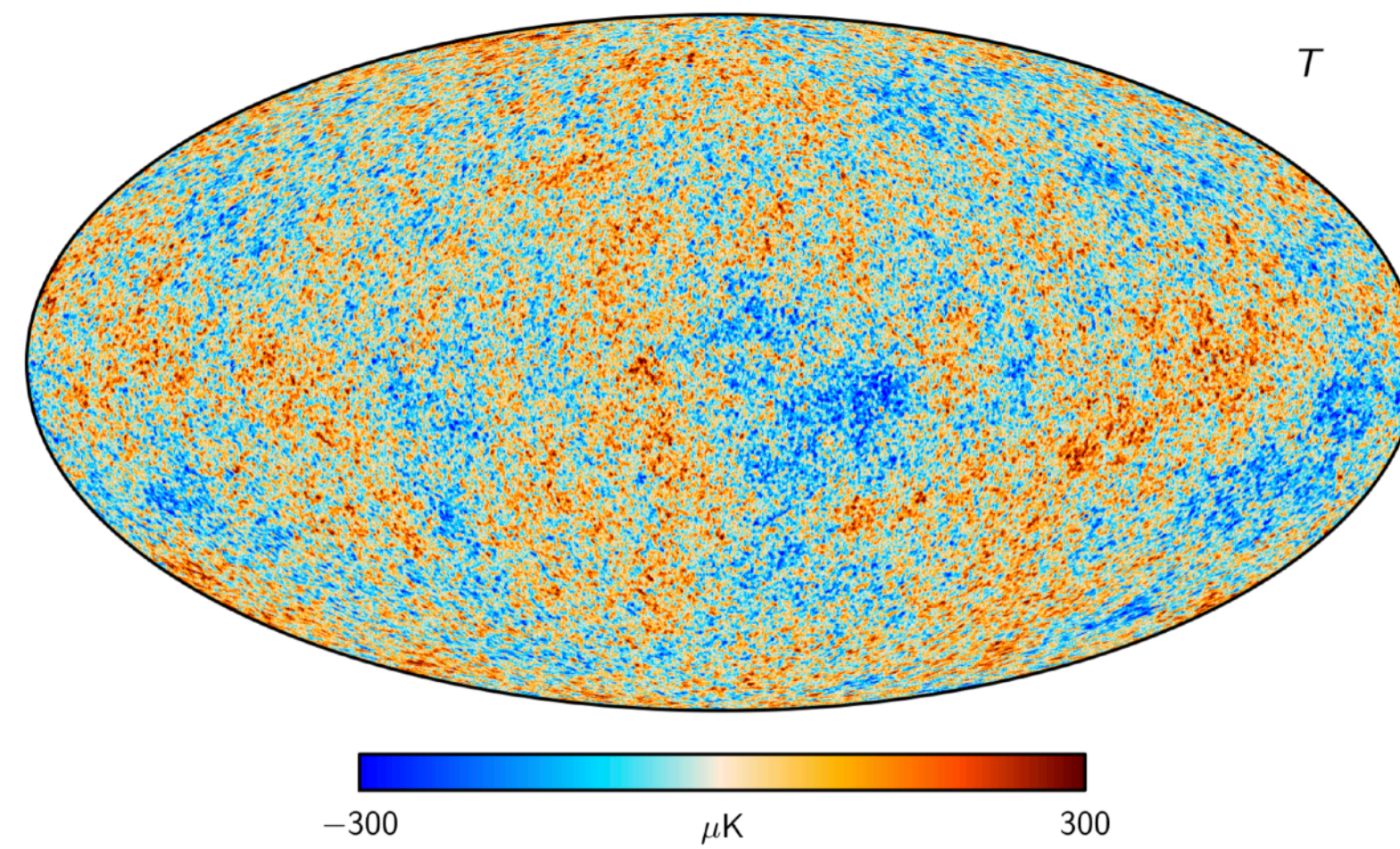
with Kaustubh Deshpande and Raman Sundrum  
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**Particle and Astroparticle Theory  
Seminar, MPIK Heidelberg**

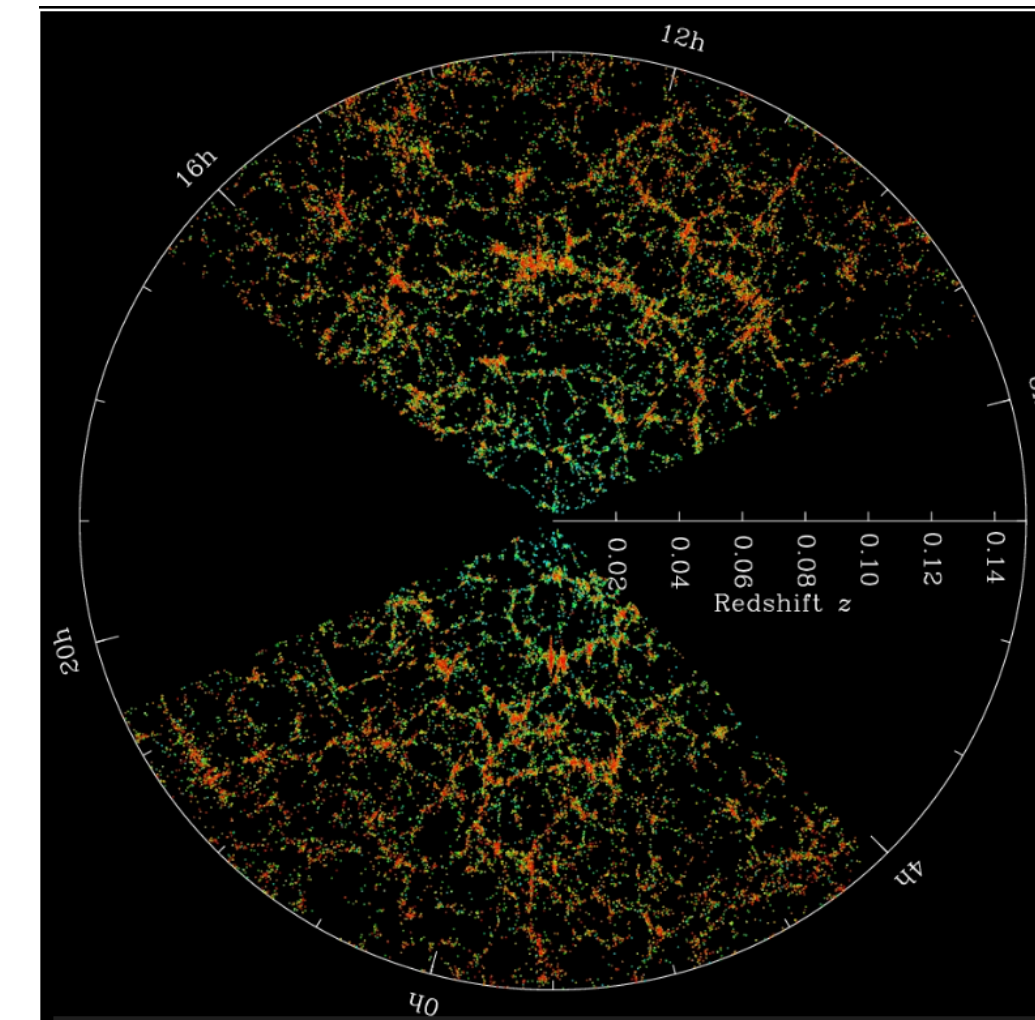


- The observed Universe is homogeneous and isotropic on very large scales

- However, we also see fluctuations at smaller scales...



CMB: Planck

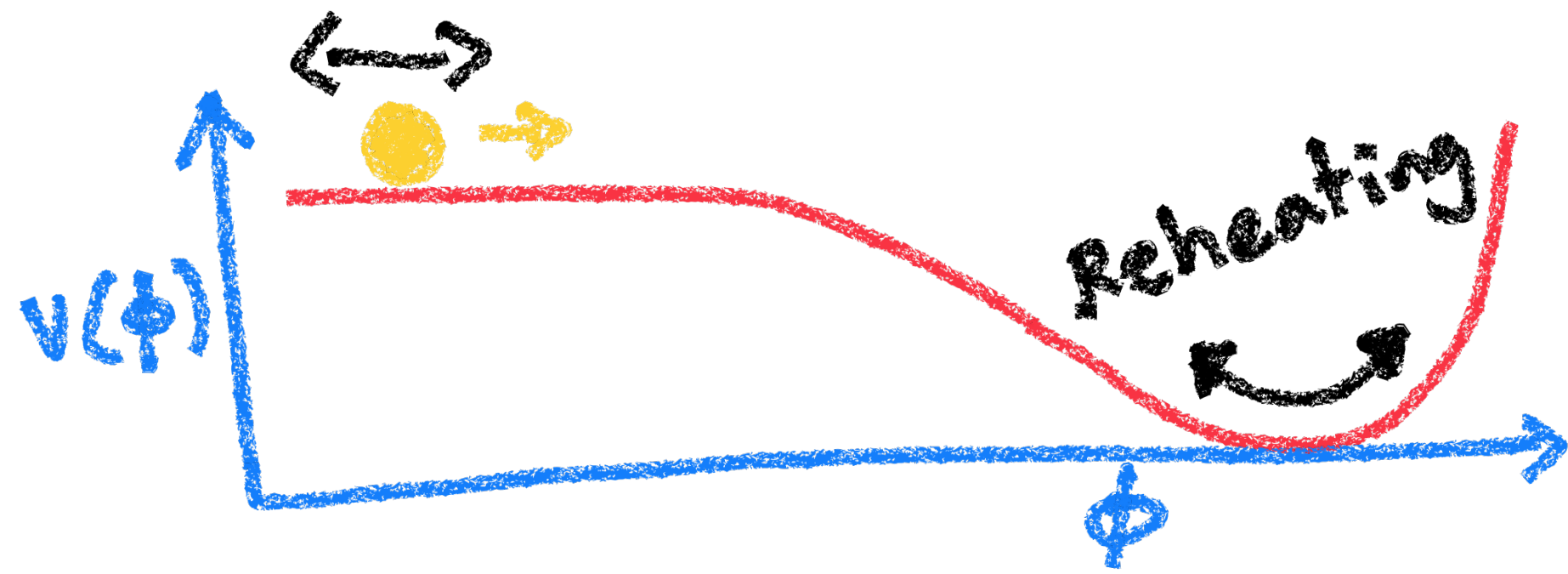


LSS: SDSS

- How to explain?



# The Inflationary Paradigm

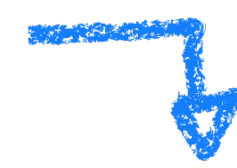


long period of slow-roll  
on a flat potential

$$a \propto \exp(Ht) \quad \text{rapid expansion}$$

Quantum fluctuations

↳ Horizon exit



Reentry and sourcing density perturbation

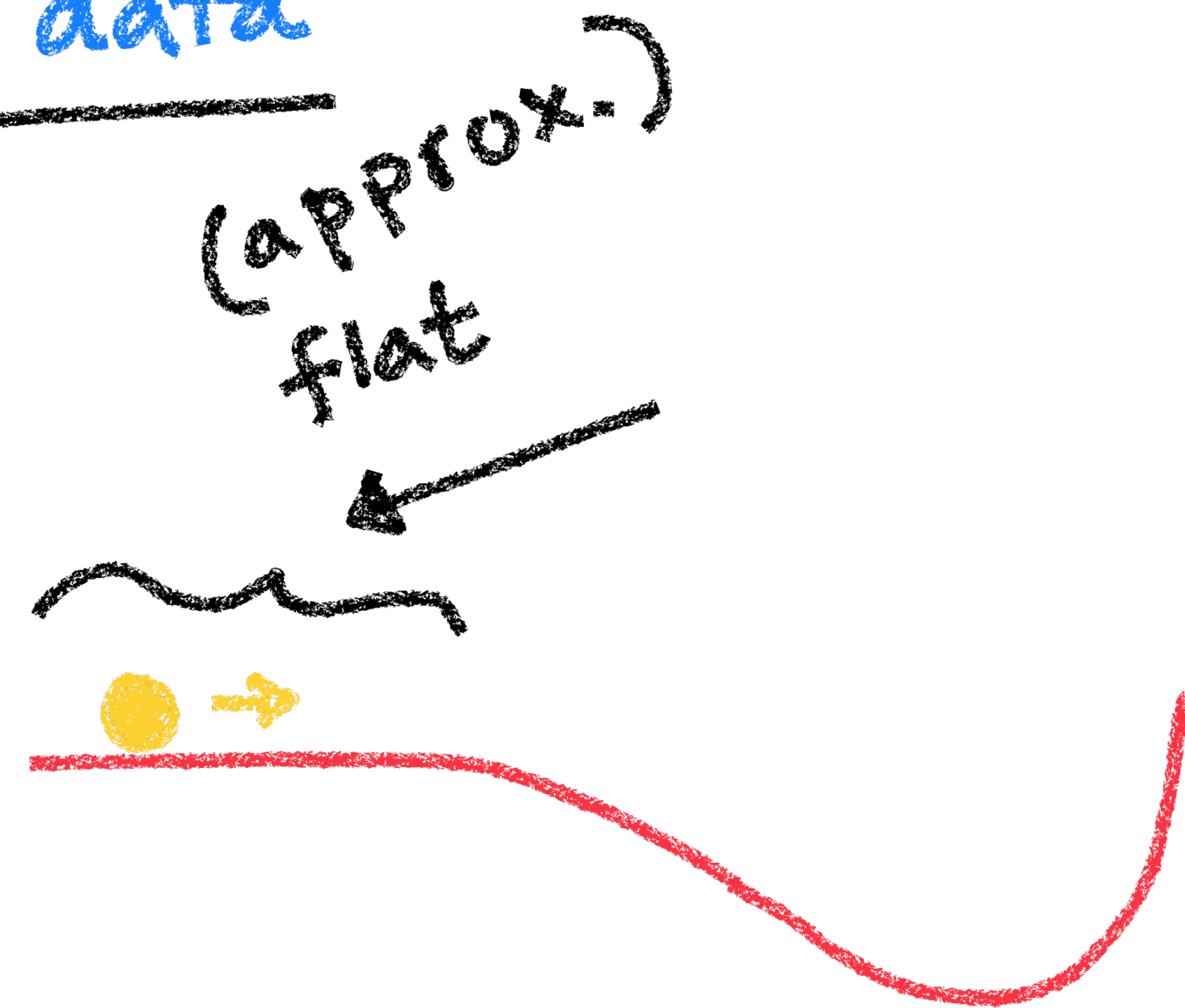


- Predictions consistent with cosmological data

- Scale invariant  
(approx.)



(approx.)  
time translation  
invariance



- Adiabatic



single field (effectively)

- Gaussian



small interactions (e.g.  $V'''/H \ll 1$ )



- Our quantitative knowledge

- know scalar power spectrum

$$\frac{k^3}{2\pi^2} P_S = A_S \left( k/k_p \right)^{n_S - 1}$$

$k_p =$  pivot scale  
(0.05/Mpc  
Planck 18)

$$A_S \approx 2.2 \times 10^{-9}$$

$$n_S \approx 0.96$$

"Scale Invariance"

- bounded tensor power spectrum

$$\frac{k^3}{2\pi^2} P_t = A_t \left( k/k_p \right)^{n_t - 1}$$

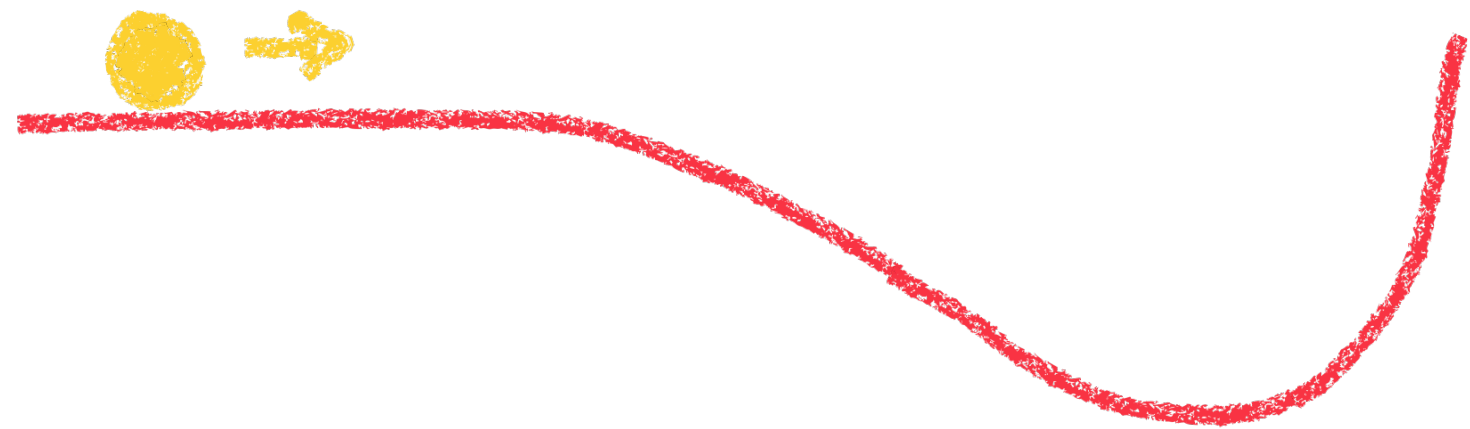
Planck '18

$$r = A_t/A_S < 0.06$$



# - Interpretation for single-field slow-roll inflation

$$\mathcal{L} \supset -\frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \quad \phi = \text{inflaton}$$



can characterize flatness via

$$\epsilon \equiv \frac{1}{2} M_{\text{P}}^2 (V'/V)^2 \quad ; \quad \eta \equiv M_{\text{P}}^2 (V''/V)$$

$$\frac{A_t}{A_s} = 16\epsilon < 0.06$$

$\Rightarrow$

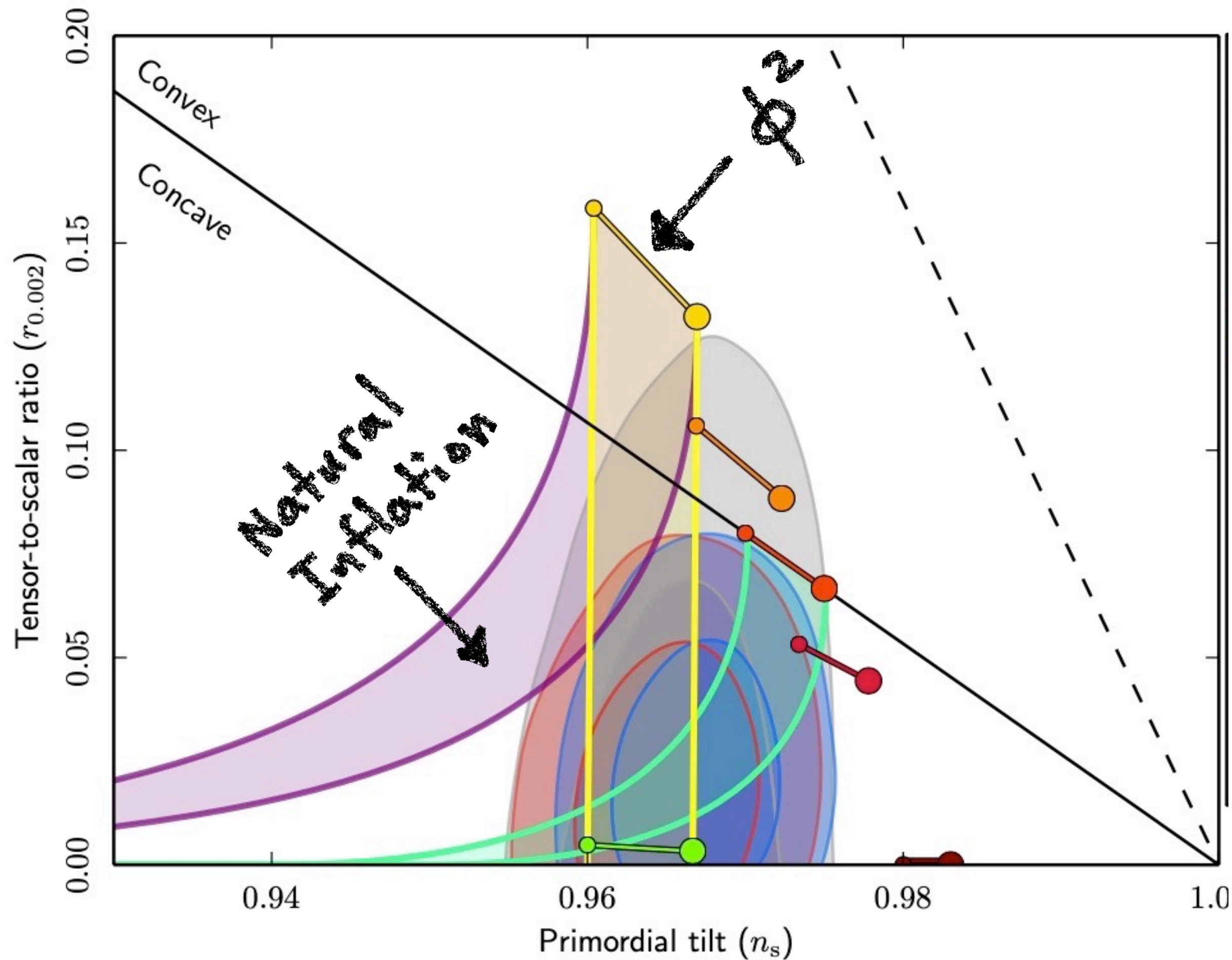
$$\eta \approx -0.02$$
$$\epsilon < 0.004$$

$$\& \quad n_s - 1 = -2\eta + 6\epsilon$$
$$= -0.04$$

Planck '18



- The  $n_s$  vs.  $r$  plot



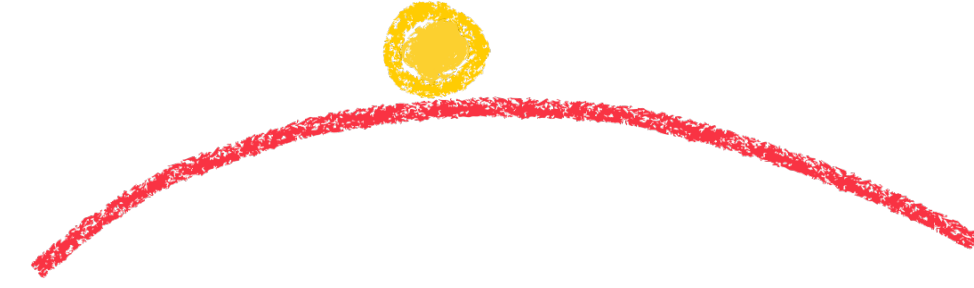
Planck '18  
+ BICEP Keck

$\propto \frac{V''}{V}$   $\propto \frac{V'''}{V}$   $\eta \approx -0.02$   
 $\epsilon < 0.004$

$|\epsilon| \ll |\eta| \ll 1$   
 $\& \eta < 0$



Initial Condition?



$m_{\phi}^2/H^2 \approx \eta$ , how to keep it light?



how to achieve  $|\epsilon| \ll |\eta|$ ?

# Outline

1. Schematic way to have  $|\epsilon| \ll |n|$

2. Hybrid Inflation and Its Issues

3. Hybrid TwInflation

├ Basic Model

└ PNGB Realization

lower bound  
on  $H$  during  
inflation

4. Domain Wall problem



? how to achieve  $|\epsilon| \ll |\eta|$ ?

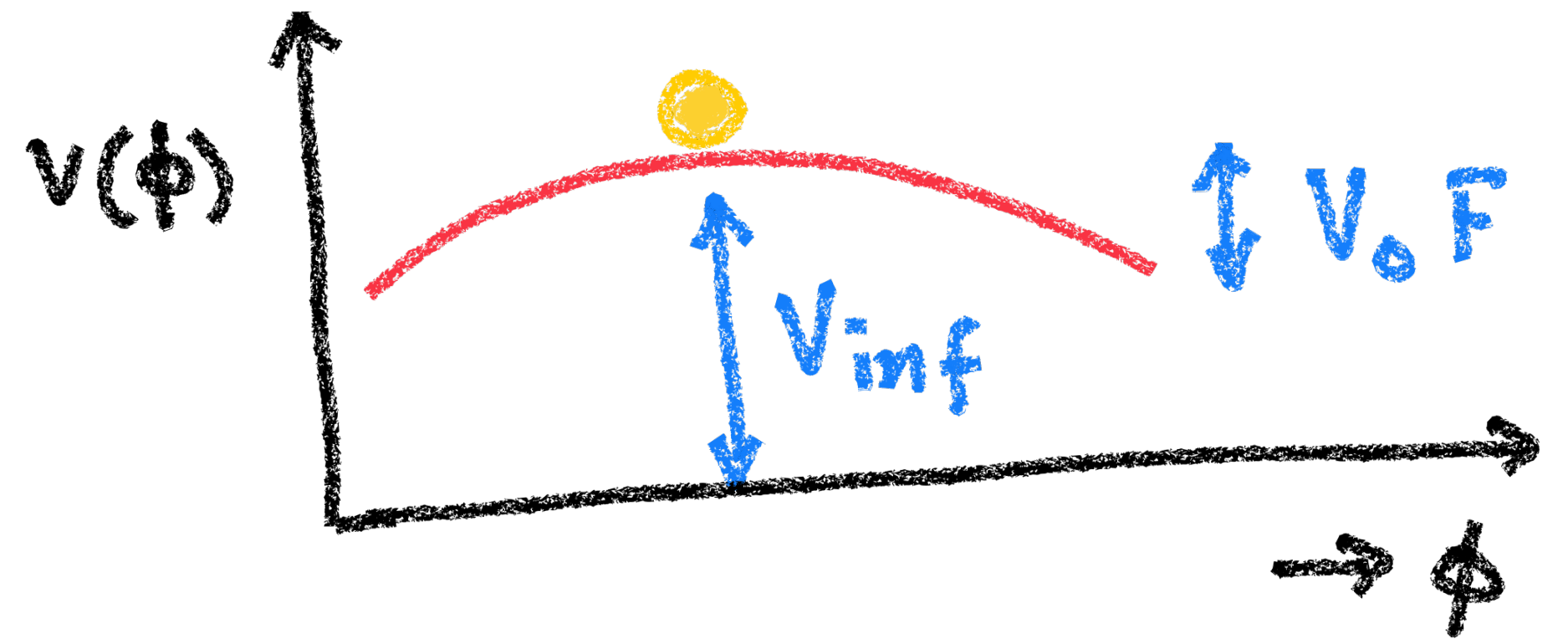
constant

$$V(\phi) = V_{\text{inf}} + \underbrace{V_0 F(\phi/f)}_{\mathcal{O}(\phi)}$$

$$\epsilon \equiv \frac{1}{2} M_{\text{P}}^2 (V'/V)^2$$

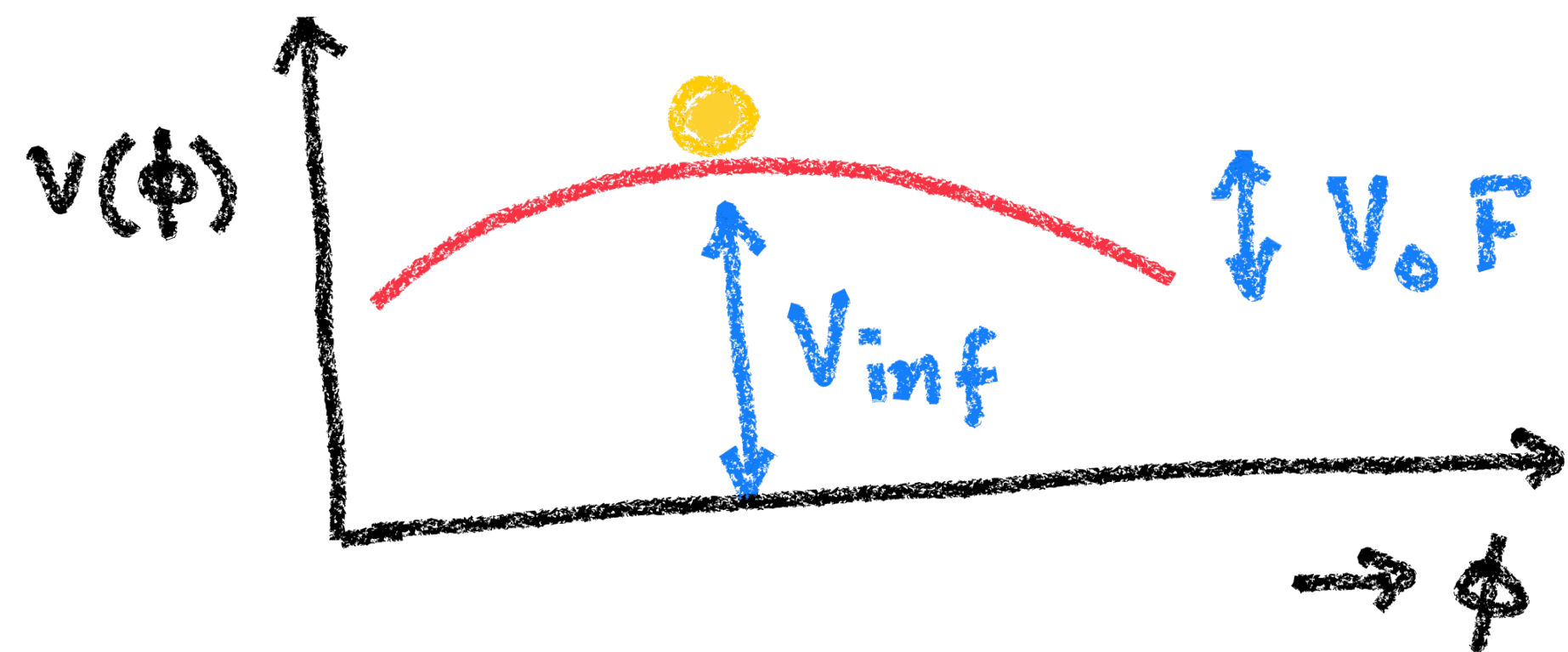
$$\eta \equiv M_{\text{P}}^2 (V''/V)$$

$$\Rightarrow \frac{\epsilon}{\eta} \sim \underbrace{\frac{V_0}{V_{\text{inf}}}}_{\mathcal{O}(1)} \cdot \frac{F'^2}{F''} \mathcal{O}(1)$$



Parametric Suppression  $\ll 1$

? how to achieve  $|\epsilon| \ll |\eta|$ ?



constant

$$V(\phi) = V_{\text{inf}} + \underbrace{V_0 F(\phi/f)}_{\mathcal{O}(\phi)}$$

$$\epsilon \equiv \frac{1}{2} M_P^2 \left( \frac{V'}{V} \right)^2$$

$$\eta \equiv M_P^2 \left( \frac{V''}{V} \right)$$

$$\Rightarrow \frac{\epsilon}{\eta} \sim \underbrace{\frac{V_0}{V_{\text{inf}}}}_{\mathcal{O}(1)} \cdot \frac{F'^2}{F''} \mathcal{O}(1)$$

Parametric Suppression  $\ll 1$

- One field for  $V_{\text{inf}}$ 
  - ↳ inflates
- Another for dynamics
  - ↳ fluctuates



# Basic Mechanism Hybrid Inflation (Linde '93)

Inflaton  $\phi$  ; Waterfall  $\sigma$

$$V(\phi, \sigma) = V_{\text{inf}} + \mathcal{V}(\phi)$$

$$+ \frac{1}{2} (M_{\sigma}^2 - g\phi^2) \sigma^2 + \frac{1}{4} \lambda_{\sigma} \sigma^4$$

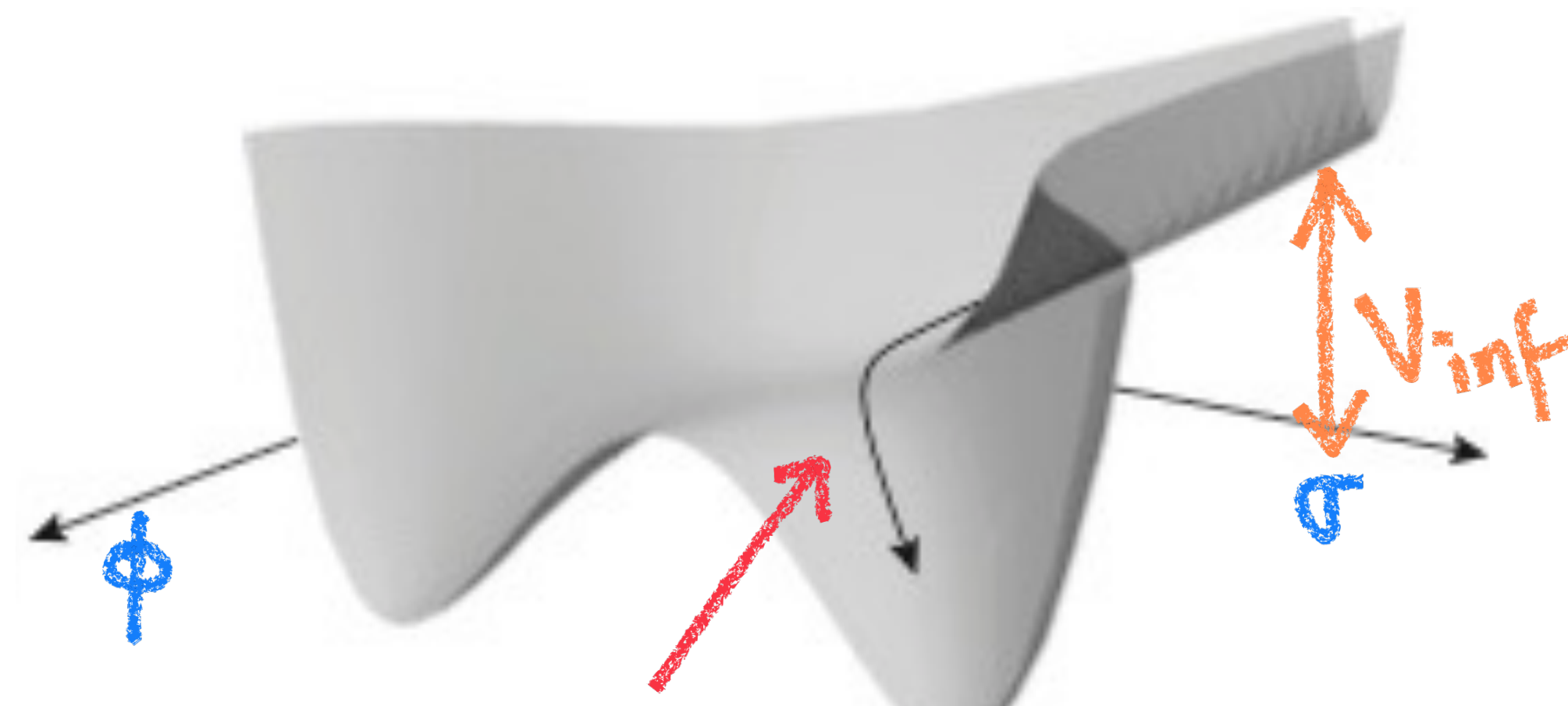
$$\phi_* = M_{\sigma} / \sqrt{g}$$

During Inflation :  $V(\phi, \sigma) \approx V_{\text{inf}} + \mathcal{V}(\phi)$

inflates  $\curvearrowright$

$\curvearrowleft$  fluctuates

$$\mathcal{V}(\phi) = -\frac{1}{2} m_{\phi}^2 \phi^2 + \frac{1}{4} \lambda_{\phi} \phi^4 \leftarrow \text{Also explains tilt}$$

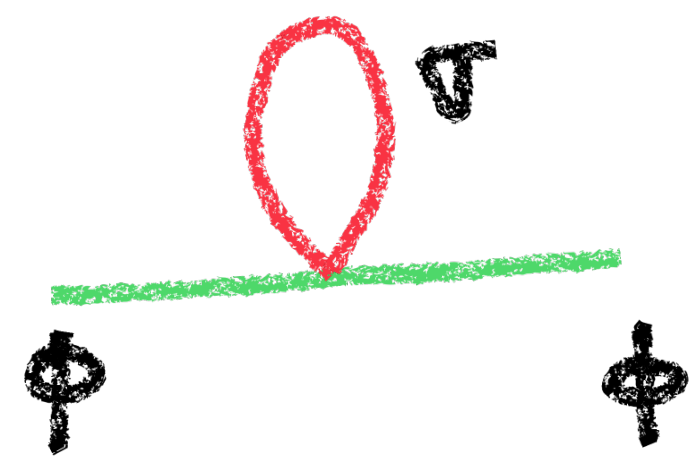


Inflation ends through tachyonic instability



## Issue

- Critical to have  $V(\phi, \sigma) \supset + \frac{1}{2} (M_\sigma^2 - g\phi^2) \sigma^2$  to have  $\phi$ -activated switching, but...



$$\delta m_\phi^2 \sim g \Lambda^2 / (16\pi^2)$$

$\Lambda$  = heavy physics scale

- $\delta m_\phi^2 < m_\phi^2$

$$\Rightarrow \underline{\Lambda^2 < (16\pi^2 g) H^2 / g}$$

- For  $\sigma$  to be static

$$\underline{M_\sigma^2 \approx g\phi_0^2 > H^2}$$

$\phi_0$  = some typical  $\phi$  value during inflation



# Issue

$$g < (16\pi^2\eta) H^2/\Lambda^2$$

↑  
Conflict



↓

$$g > H^2/\phi_0^2$$

For any  $g$  &  $\eta \approx 0.01$

$$\phi_0 \geq \Lambda$$

Sensitivity to heavy physics, uncontrolled  $(\phi/\Lambda)^n$

- Goal: Couple to waterfall field so as <sup>expansion</sup> to remove the sensitivity

## First try: Soft Breaking

- why not change  $g\phi^2\sigma^2 \rightarrow \mu\phi\sigma^2$ ?

$$V(\phi, \sigma) = V_{\text{inf}} + V(\phi) + \frac{1}{2} (M_\sigma^2 - \mu\phi) \sigma^2 + \frac{1}{4} \lambda_\sigma \sigma^4$$

$$\phi_* = M_\sigma^2 / \mu$$

- only log sensitivity

$$\delta m_\phi^2 \sim \mu^2 \ln \Lambda / (16\pi^2) \quad \text{no } \Lambda^2 \text{ sensitivity } \checkmark$$

- no large displacement

$$M_\sigma^2 \sim \mu\phi_0 > H^2 \quad \Rightarrow \quad H/\phi_0 \lesssim \mu/H \lesssim 4\pi\sqrt{\eta}$$

easy to satisfy  $\checkmark$



## Soft breaking $\rightarrow$ Large tadpole

-  $\phi \rightarrow -\phi$  is not a symmetry anymore

$$V(\phi, \sigma) \supset \frac{\mu \Lambda^2}{16\pi^2} \phi$$

- Absorb by

$$\delta\phi \sim \frac{\mu \Lambda^2}{16\pi^2 m_\phi^2} \sim \frac{\mu \Lambda^2}{H^2} \quad (\eta \approx 0.01)$$

- But this changes

$$\frac{\delta M_\sigma^2}{M_\sigma^2} \sim \frac{\delta\phi}{\phi} \sim \frac{M_\sigma^2}{H^2} \cdot \frac{\Lambda^2}{\phi_0^2} \rightarrow \frac{\delta M_\sigma^2}{M_\sigma^2} < 1$$

$\Rightarrow \phi_0/\Lambda > M_\sigma/H > 1.$

# Summary so far

## Original Coupling

$$(M_\sigma^2 - g\phi^2)\sigma^2$$

$\phi \rightarrow -\phi$  sym; no tadpole

$\delta m_\phi^2 \propto g\Lambda^2$

## Soft Coupling

$$(M_\sigma^2 - \mu\phi)\sigma^2$$

$\phi \not\rightarrow -\phi$  sym; large tadpole

$\delta m_\phi^2 \propto \mu^2 \ln \Lambda$

Want to have best of both



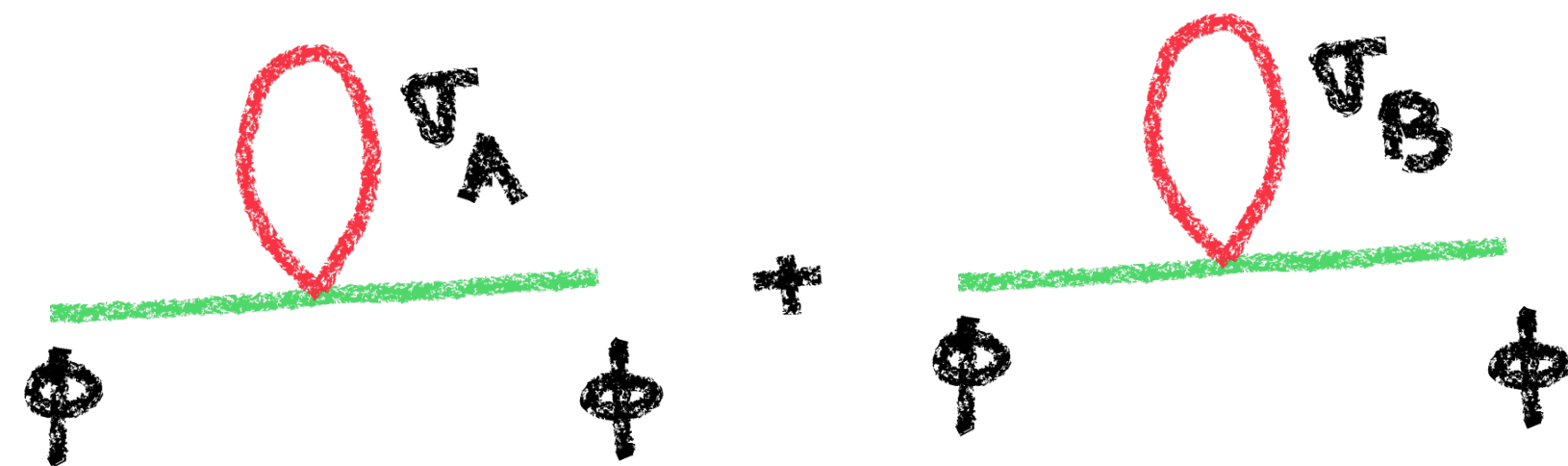
# Hybrid Twinflation

- A twin sector to remove  $\Lambda^2 = \frac{1}{2} (M_\sigma^2 - g\phi^2) \sigma^2$

Chacko, Goh  
Harnik '06

$$g\phi^2\sigma^2 \rightarrow g\phi^2(\sigma_A^2 - \sigma_B^2)$$

naively,


$$\text{Diagram 1} + \text{Diagram 2}$$

$$\rightarrow \delta m_\phi^2 \propto g\Lambda^2 - g\Lambda^2$$

but not protected by symmetry

- Hence try

$$\mu\phi\sigma^2 \rightarrow \mu\phi(\sigma_A^2 - \sigma_B^2)$$

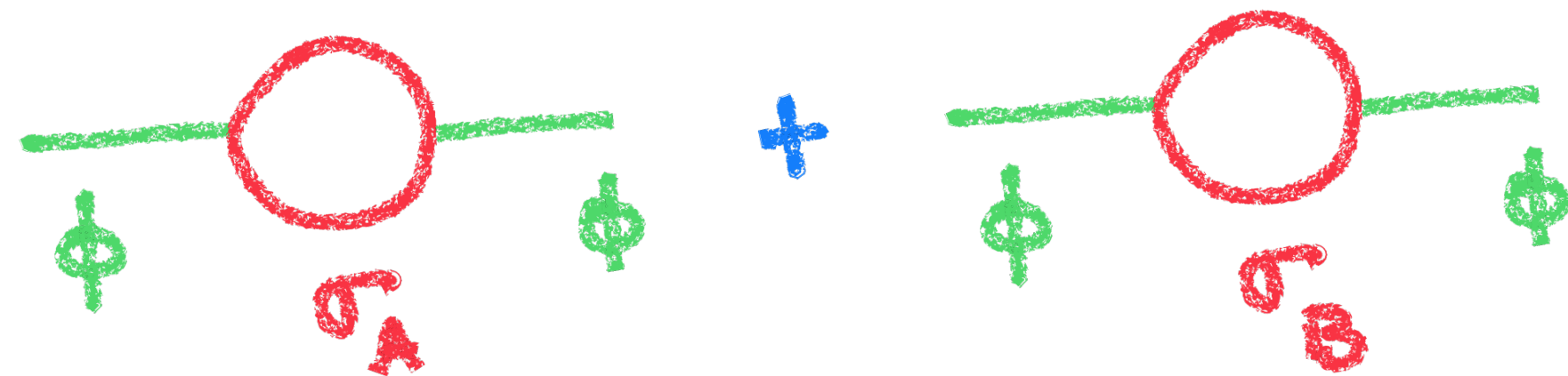
Protected via  $\{ \phi \rightarrow -\phi, \sigma_A \rightarrow \sigma_B \}$

# Safety from radiative corrections

$$\mu \phi \sigma^2 \rightarrow \mu \phi (\sigma_A^2 - \sigma_B^2)$$

Protected via  $\{ \phi \rightarrow -\phi, \sigma_A \rightarrow \sigma_B \}$

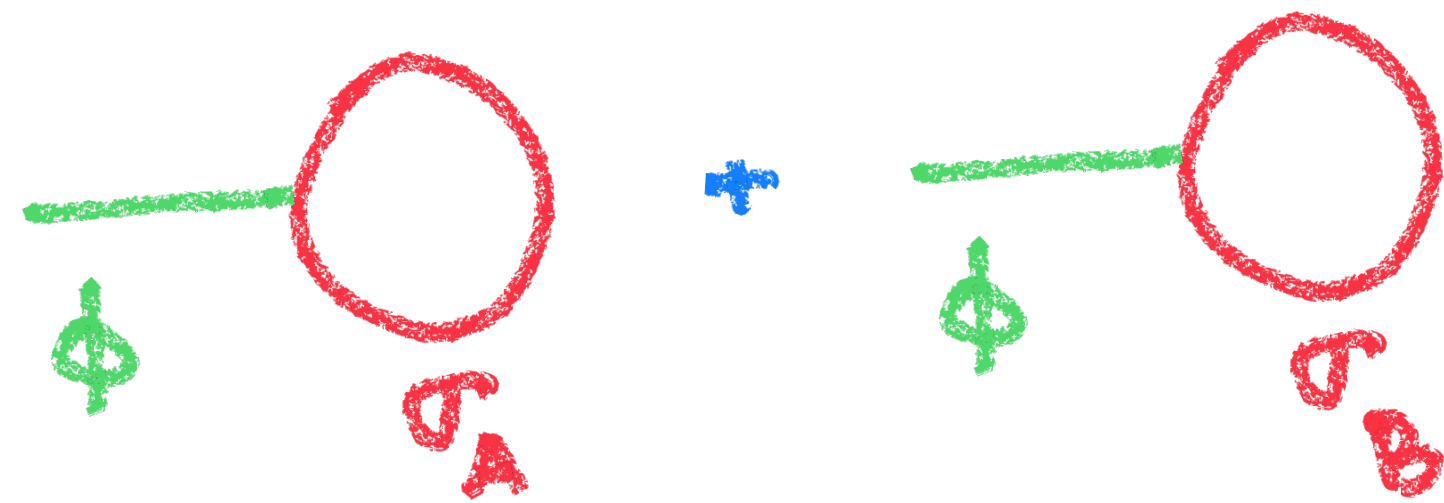
-  $\delta m_\phi^2 \rightarrow$



$$\sim \mu^2 \ln \Lambda / 16\pi^2$$

no cancellation, but  
only log sensitivity

- tadpole  $\rightarrow$



$$\sim \mu \Lambda^2 - \mu \Lambda^2$$

cancellation!

since linear in  $\phi$  violates symmetry



# Summary so far

## Original Coupling

$$(M_\sigma^2 - g\phi^2)\sigma^2$$

✓  $\phi \rightarrow -\phi$  sym; no tadpole

✗  $\delta m_\phi^2 \propto g\Lambda^2$

## Twin Soft Coupling

$$(M_{\sigma_A}^2 - \mu\phi)\sigma_A^2 + (M_{\sigma_B}^2 + \mu\phi)\sigma_B^2$$

✓  $\phi \rightarrow -\phi$  sym; <sup>NO</sup> large tadpole  
 $\sigma_A \rightarrow \sigma_B$

✓  $\delta m_\phi^2 \propto \mu^2 \ln \Lambda$

Now have best of both

# Basic Model

- Symmetry to obey  $\phi \rightarrow -\phi$  &  $\sigma_A \rightarrow \sigma_B$

$$V(\phi, \sigma_A, \sigma_B) = V_{\text{inf}} + \left( -\frac{1}{2} m_\phi^2 \phi^2 + \frac{\lambda_\phi}{4} \phi^4 + \dots \right) \\ + \left[ \left( \frac{1}{2} M_\sigma^2 \sigma_A^2 + \frac{\lambda_\sigma}{4} \sigma_A^4 \right) + (A \rightarrow B) \right] + \frac{\bar{\lambda}_\sigma}{4} \sigma_A^2 \sigma_B^2$$

$$+ \frac{1}{2} \mu \phi (\sigma_A^2 - \sigma_B^2) + \kappa \phi^2 (\sigma_A^2 + \sigma_B^2) + \dots$$

$\mu, m_\phi \ll M_\sigma$   
 $\kappa, \lambda_\phi \ll \lambda_\sigma, \bar{\lambda}_\sigma$

} Approx. shift  
symmetry for  
 $\phi$

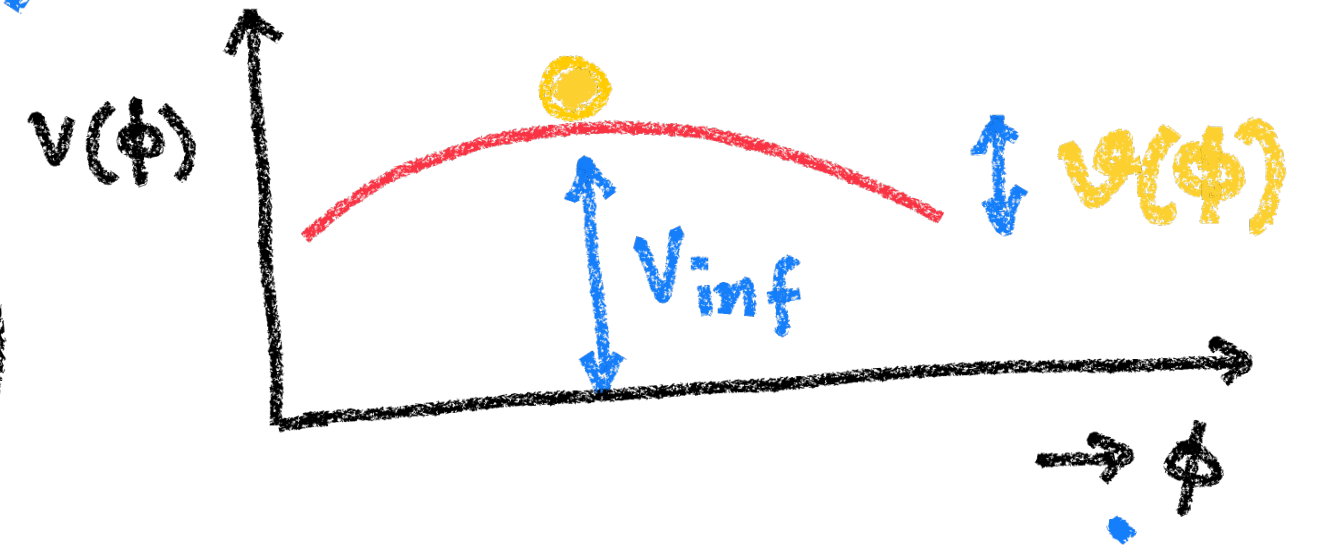
- Consider  $\mu \phi \ll M_{\sigma_{A,B}}^2$   
during inflation  
-  $\sigma_A, \sigma_B$  stabilized at  $\langle \sigma \rangle = 0$ .



# Dynamics during inflation

-  $\sigma_A, \sigma_B$  stable during inflation at  $\langle \sigma_{A,B} \rangle = 0$

$$V_{\text{eff}}(\phi) \approx V_{\text{inf}} + \underbrace{\left( -\frac{1}{2} m_\phi^2 \phi^2 + \frac{\lambda_\phi}{4} \phi^4 + \dots \right)}_{\psi(\phi)}$$



standard Dynamics

- Towards the end of inflation,

$$M_{\sigma_{A,B}}^2(\phi) = M_\sigma^2 \pm \mu \phi$$

remains  
at  $\langle \sigma_A \rangle = 0$

Symmetry  
breaking

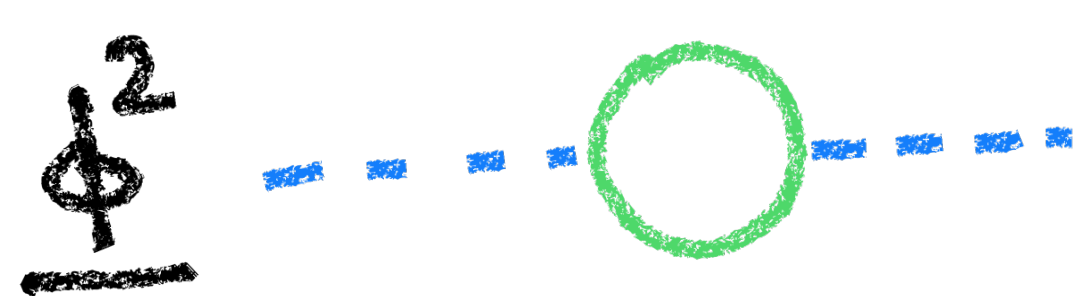
$$V_{\text{inf}} = \frac{M_\sigma^4}{4\lambda_\sigma} \left( \frac{\phi_{\text{min}}}{\phi_*} - 1 \right)^2$$

Drop in CC

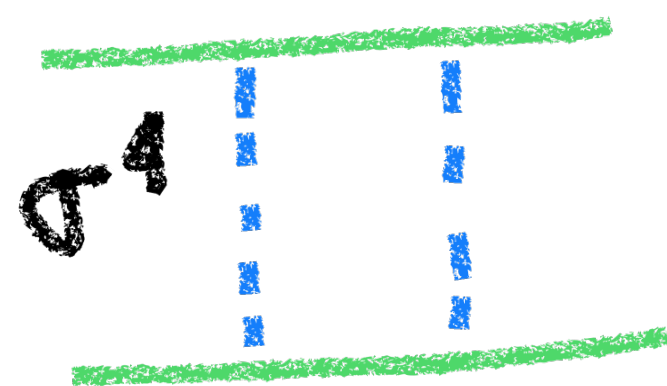
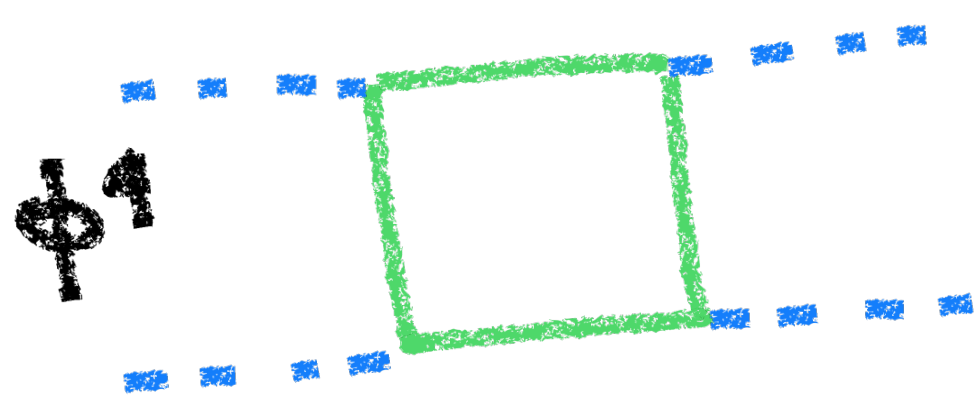


# Radiative Stability and Naturalness

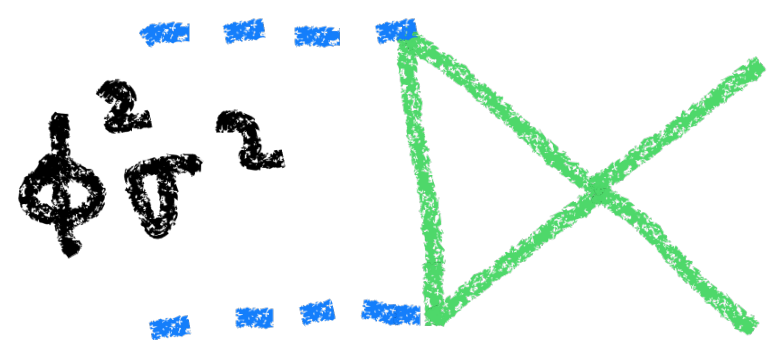
\* Source of  $\phi$ -shift symmetry breaking =  $\mu \phi \sigma_{A,B}^2$



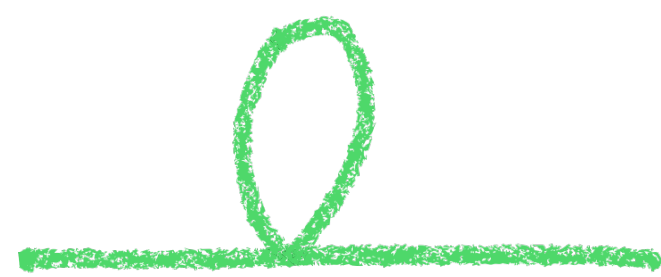
$$\delta m_\phi^2 \sim \mu^2 \ln \Lambda / 16\pi^2$$



$$\delta(\lambda_\sigma, \bar{\lambda}_\sigma, \lambda_\phi) \sim \frac{\mu^4}{16\pi^2 M_\sigma^4}$$



$$\delta \kappa \sim \frac{\lambda_\sigma \mu^2}{16\pi^2 M_\sigma^2}$$



$$\delta M_\sigma^2 \sim \frac{\lambda_\sigma \Lambda^2}{16\pi^2}$$

— These determine the natural sizes of parameters



# Getting consistent inflation

$$\mu^2 \sim 16\pi^2 \eta H^2 \Rightarrow \mu \approx H \text{ (correct } \eta \text{ parameter)}$$

$$A_s = \frac{1}{8\pi^2} \frac{H^2}{M_p^2 \epsilon} \sim \frac{10^{-2}}{\eta^2} \frac{H^2}{\phi_0^2} \Rightarrow \phi_0 / H \sim 10^6 \text{ (correct } A_s)$$

← typical inflaton value

$$\frac{M_\sigma^4}{\lambda_\sigma} \sim H^2 M_p^2 \Rightarrow \frac{\mu^2 \phi_0^2}{\lambda_\sigma} \sim H^2 M_p^2$$

$$\Rightarrow H/M_p \sim 10^{-6} \sqrt{\lambda_\sigma} \text{ (satisfies } r)$$

How small can  $H$  be?

---

$$\frac{M_\sigma^4}{\lambda_\sigma} \sim \frac{\mu^2 \phi_0^2}{\lambda_\sigma} \sim H^2 M_P^2$$

$$\Rightarrow \lambda_\sigma \sim \frac{\mu^2}{M_P^2} \cdot \left(\frac{\phi_0}{H}\right)^2$$

but also, radiative  
corrections,

$$\lambda_\sigma > \frac{\mu^2}{16\pi^2 \phi_0^2}$$

$$H \gtrsim 10^5 \text{ GeV}$$

- Lower  $T_{RH}$ , avoids overclosure due to unwanted relics
- suppresses axion DM isocurvature



# PNGB Realization

- model  $\phi$  as a PNGB of  $U(1)$  field  $\Phi \propto \langle \Phi \rangle e^{i\phi/\langle \Phi \rangle}$

$$\phi \rightarrow \phi + c \quad \Leftrightarrow \quad \Phi \rightarrow \Phi e^{ic/\langle \Phi \rangle}$$

$$\mathcal{L}_{UV} = -|\partial\Phi|^2 - V(|\Phi|^2) + \mathcal{L}[\sigma_A, \sigma_B] \quad \text{same as before}$$

$$+ \left\{ \mu \Phi (\sigma_A^2 - \sigma_B^2) + (\mu \Phi)^2 + \text{h.c.} \right\} - g |\Phi|^2 (\sigma_A^2 + \sigma_B^2) - V_{\text{inf}}$$

$$\left\{ \Phi \rightarrow -\Phi, \sigma_A \rightarrow \sigma_B \right\}$$

$\mu = U(1)$  "spurion" to denote soft  $U(1)$  breaking



# The IR Lagrangian

$$\mathcal{L}_{IR} = \mathcal{L}[\sigma_A, \sigma_B] \quad \text{same as before}$$

$$+ \frac{1}{2}(\partial\phi)^2 - \frac{\mu f}{2} \sin(\phi/f) (\sigma_A^2 - \sigma_B^2) - \mu^2 f^2 \cos(2\phi/f)$$

→ gives the previous structure for  $\phi \ll f$  -  $V_{inf}$

→ other dynamics same as before

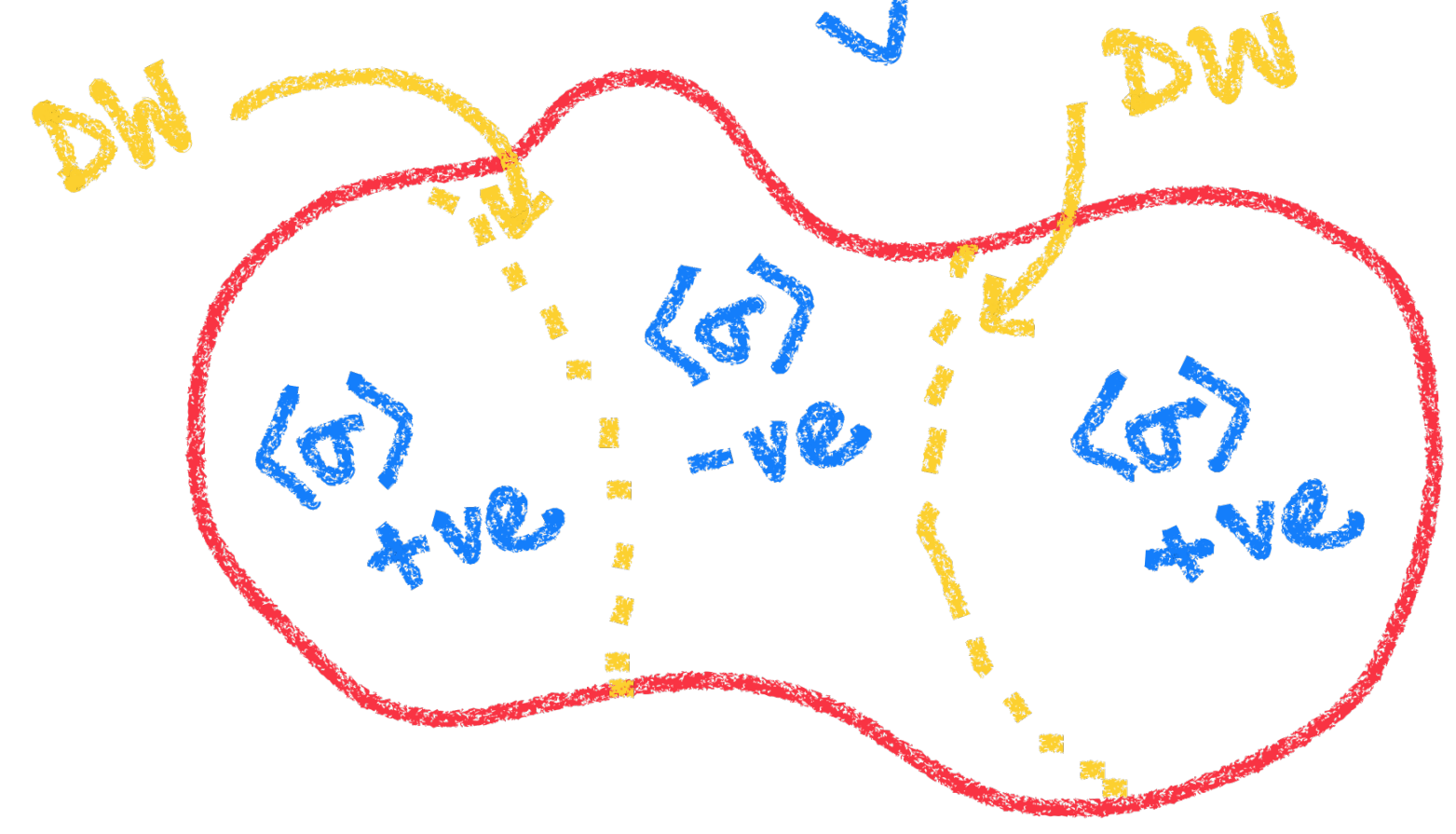
→  $\phi_{initial}/f \approx 0.1\pi$  gives 60 e-foldings

no fine tuning of initial position is needed



# Addressing the Domain Wall Problem

- Spontaneous breaking of  $\sigma \rightarrow -\sigma$  symmetry



not essential in our mechanism

- Can be avoided by breaking  $\sigma_i \rightarrow -\sigma_i$  symmetry

$$\mathcal{L} \supset M \sigma_i^3$$

biases one vacuum, annihilation of DW

- Breaking affects inflaton potential, increases  $H \gtrsim 10^7 \text{ GeV}$



## Summary

- Current data is increasing favoring low (ish) - scale, hilltop models  
 $\epsilon \ll |\eta|, \eta < 0$

- Hybrid inflation achieves  $\epsilon < |\eta|$  parametrically

- Constructed a natural, EFT controlled version with

$Z_2$  twin symmetry

- Low-scale  $M \gtrsim 10^7$  GeV [axion isocurvature, overclosure]

Gravitational Waves ?

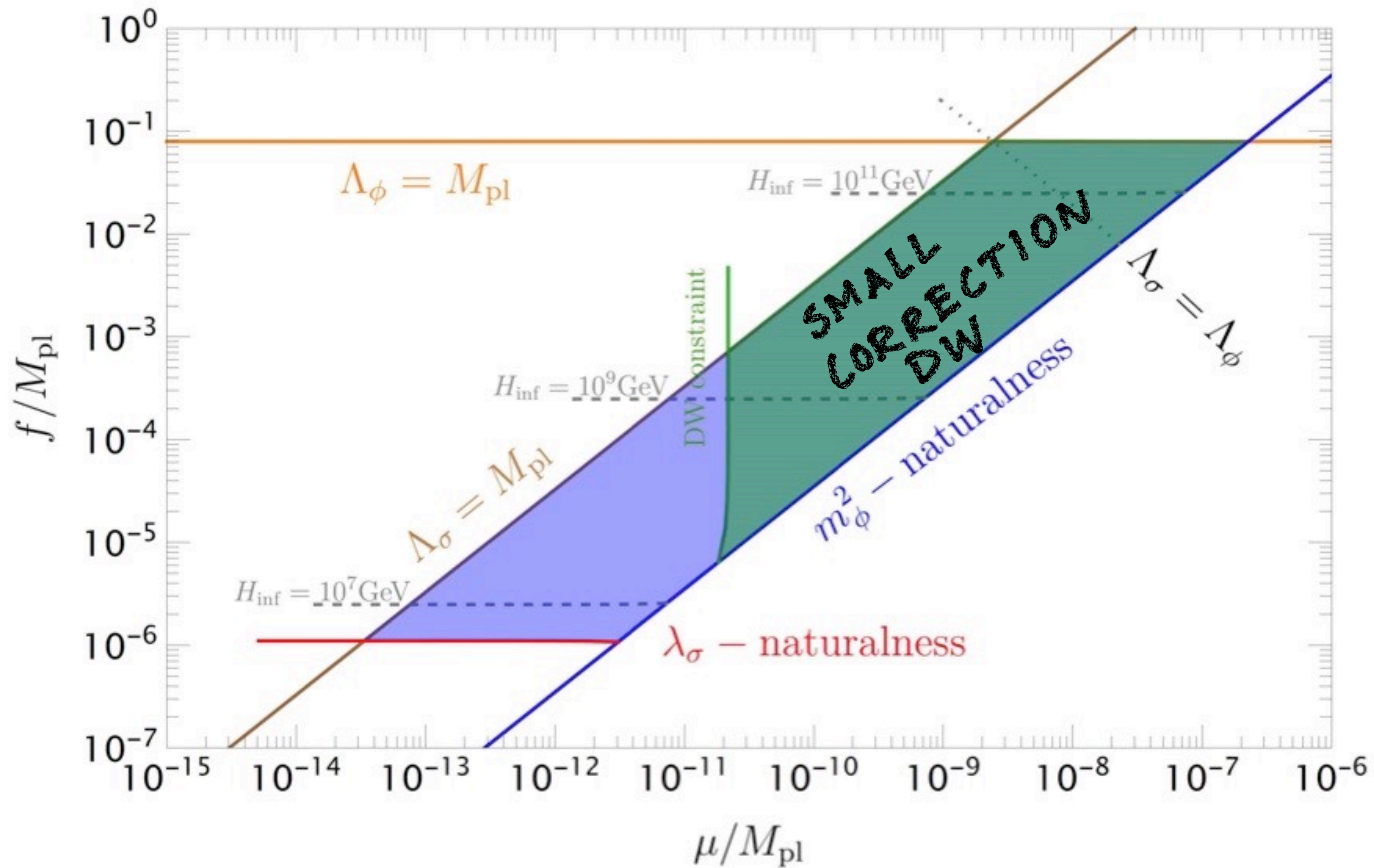


# Full Parameter Space

$$\Lambda_\sigma \approx 4\pi \frac{M_\sigma}{\sqrt{\lambda_\sigma}} \sqrt{V}$$

$$\sim 4\pi \frac{H M_\sigma}{M_{\text{pl}}}$$

$$\Lambda_\phi \approx 4\pi f$$





# Details on Domain Wall Problem

$$V(\phi, \sigma_i) \supset M \sigma_i^3 \rightarrow \Delta V_{\text{bias}} / V_{\text{inf}} \sim M / (M_\sigma \sqrt{\lambda_\sigma})$$

Annihilation before domination

$$\mathcal{O}(1) \gtrsim \Delta V_{\text{bias}} / V_{\text{inf}} \gtrsim \frac{M^2}{\lambda_\sigma M_p^2}$$

M generates a tadpole  $V \supset M \lambda_\sigma^2 \sigma_i / (16\pi^2)$

$$\delta \sigma_i \sim \frac{M M_\sigma^2}{\lambda_\sigma M_{\sigma_i}^2} \Rightarrow \delta V_{\text{eff}} \sim \frac{M M_\sigma^2}{\lambda_\sigma} \cdot \frac{M M_\sigma^2}{\lambda_\sigma M_{\sigma_i}^2}$$

Ensure  $\delta V_{\text{eff}} < V_{\text{eff}}$  itself