Primordial Black Holes

as

Dark Matter

Florian Kühnel

Talk at *Particle and Astroparticle Theory Seminar*
Max Planck Institute for Nuclear Physics
Heidelberg, November 20th, 2017

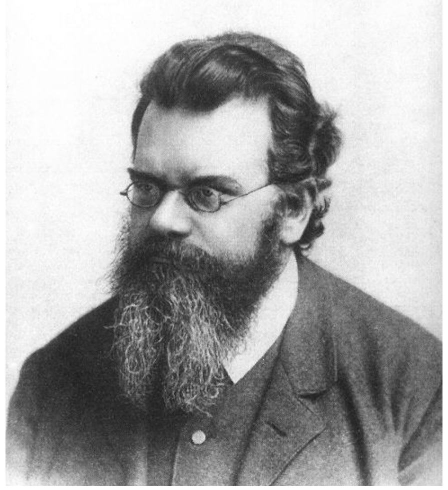
work in particular with
Bernard Carr
Katherine Freese
Pavel Naselsky
Tommy Ohlsson
Glenn Starkman

PBH Generalities

- Black-hole (BH) formation for $R < R_S$.
  - Astrophysical: From $10^9\, M_\odot$ down to $M_\odot$ but not lower.
  - Have a look at the density $$\rho_S = 10^{18} \left(\frac{M}{M_\odot} \right)^{-2} \frac{\mathrm{g}}{\mathrm{cm}^3}$$
    - → To form smaller black holes we need higher density
    - → Compare to cosmological density $$\rho_C = 10^{6} \left(\frac{t}{\mathrm{s}} \right)^{-2} \frac{\mathrm{g}}{\mathrm{cm}^3}$$
    - → Formation at early times; primordial black holes (PBHs)
  - Masses of primordial black holes:

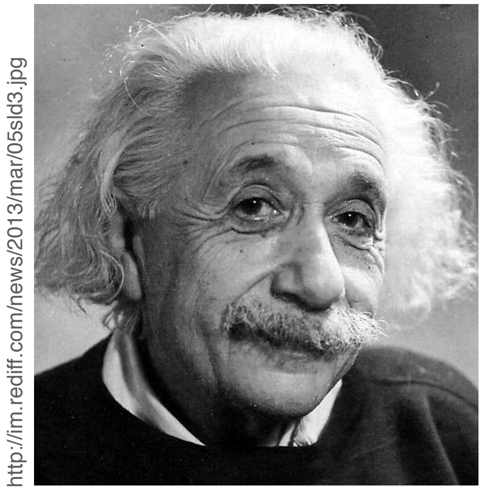
$$\boxed{M(t = 10^{-23}\,\mathrm{s}) = 10^{15}\,\mathrm{g}}, \qquad M(t = 10^{-6}\,\mathrm{s}) = M_\odot$$

Evaporation



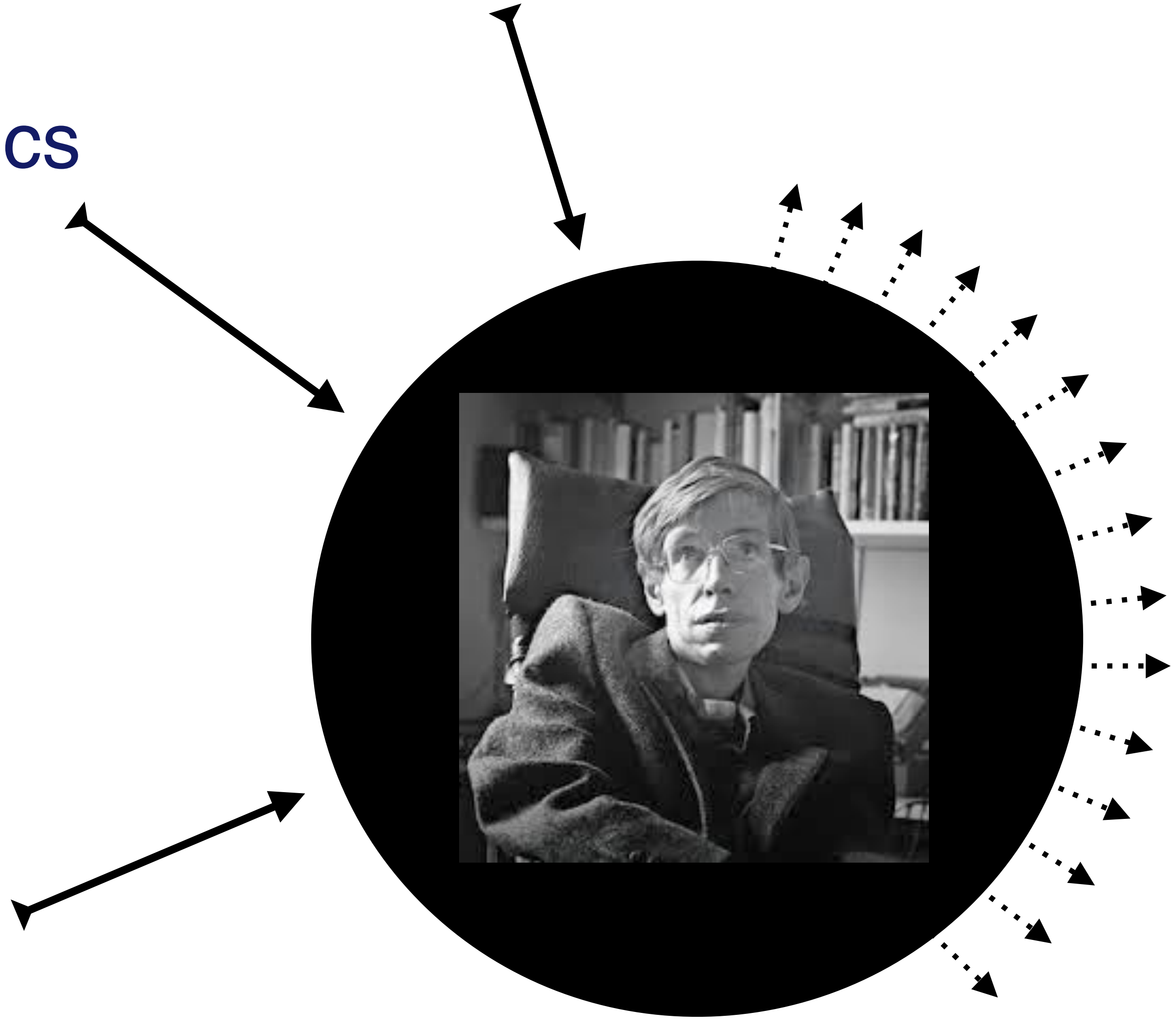
Thermodynamics

Quantum Mechanics



http://im.rediff.com/news/2013/mar/05sld3.jpg

General Relativity



★ Black-hole **radiation**

[Hawking 1974]

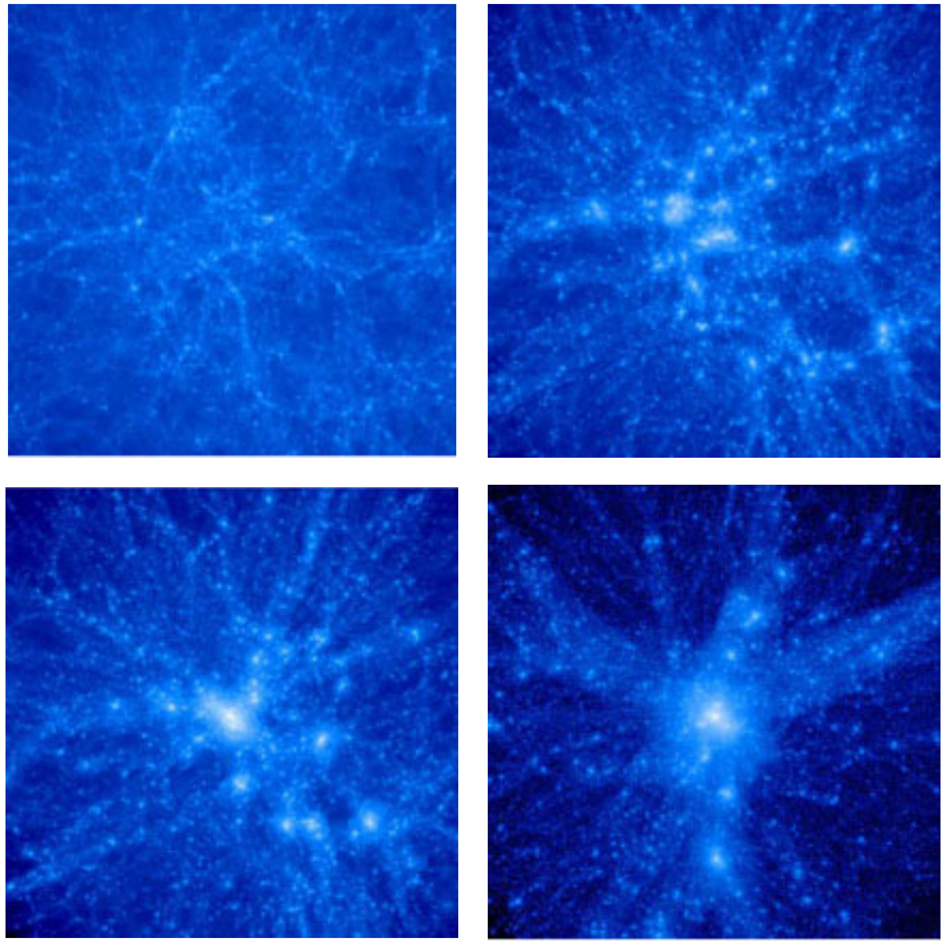
$$t_{\text{evaporation}}[\mathrm{s}] = 10^{71}\left(\frac{M}{M_\odot}\right)^3$$

$$T_{BH}[\mathrm{K}] = 10^{-7}\,\frac{M_\odot}{M}$$

$$\frac{\mathrm{d}M}{\mathrm{d}t} \sim -M^{-2}$$

PBH Formation Mechanisms

- Formation of primordial black holes by
  - Cosmic string loops
  - Bubble collisions
  - Pressure reduction
  - Large density perturbations



https://ned.ipac.caltech.edu/level5/Sept12/Kravtsov/Kravtsov3.html

→ Simple estimate:

[Carr 1975]

$$R > R_J \quad \Rightarrow \quad \boxed{\delta_H > w} \; , \quad \text{for} \quad p = w\,\rho$$

scale of the over density

Jeans length

PBH — Probes of Scales

- ★ Probe a huge range of scales:

$M \sim 10^{-5}\mathrm{g}$	Quantum Gravity:	Planck relics, Extra dimensions and higher-dimensional black holes, …
$M \lesssim 10^{15}\mathrm{g}$	Early Universe:	Baryogenesis, Nucleosynthesis, Reionisation, …
$M \sim 10^{15}\mathrm{g}$	High-Energy Physics:	Cosmological and galactic gamma-rays, …
$M \gtrsim 10^{15}\mathrm{g}$	Gravity:	Critical phenomena, Cold dark matter, Dynamical effects, Lensing effects, Gravitational waves, Black holes in galactic nuclei, …

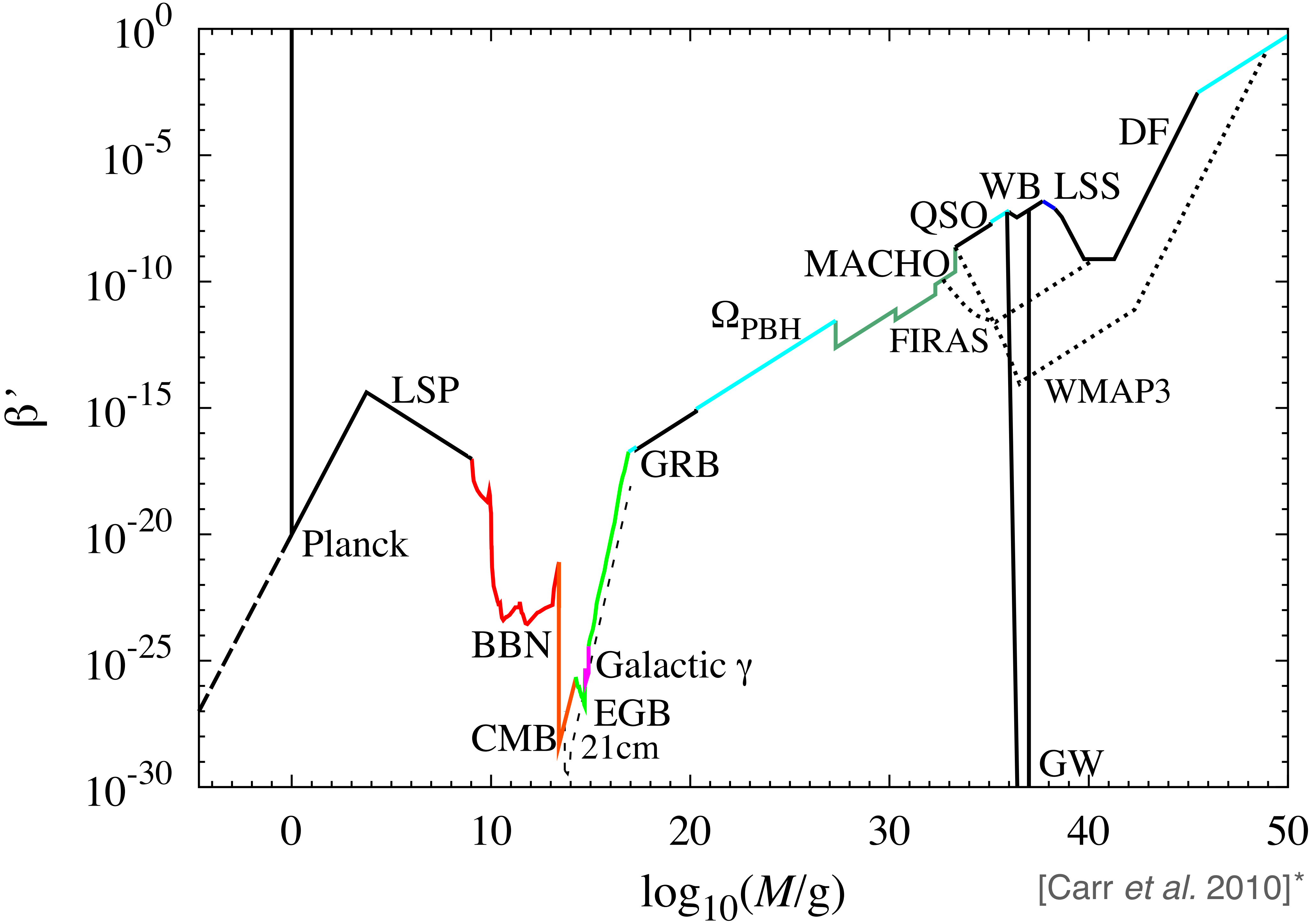
PBH — Some Numbers

- Consider an example of primordial black holes constituting all of the dark matter:

  - Mass range: $10^{20}\,\mathrm{g}$
  - Size: $10^{-8}\,\mathrm{cm}$
  - Number in our Galaxy: 10^{25}
  - Distance: $10\,\mathrm{AU}$

PBH Constraints at Formation

$$\propto \Omega_{\rm PBH}\Big|_{\rm form}$$



[Carr *et al.* 2010]*

★ Note that

$$\rho_{\rm rad} \propto a^{-4}$$

$$\rho_{\rm PBH} \propto a^{-3}$$

and hence

$$\boxed{\Omega_{\rm PBH} \propto a}$$

*(more recent) constraints for the presently allowed dark-matter fraction later…

More on PBH Formation

- Given the density distribution $P(\delta)$, derive the PBH density parameter:

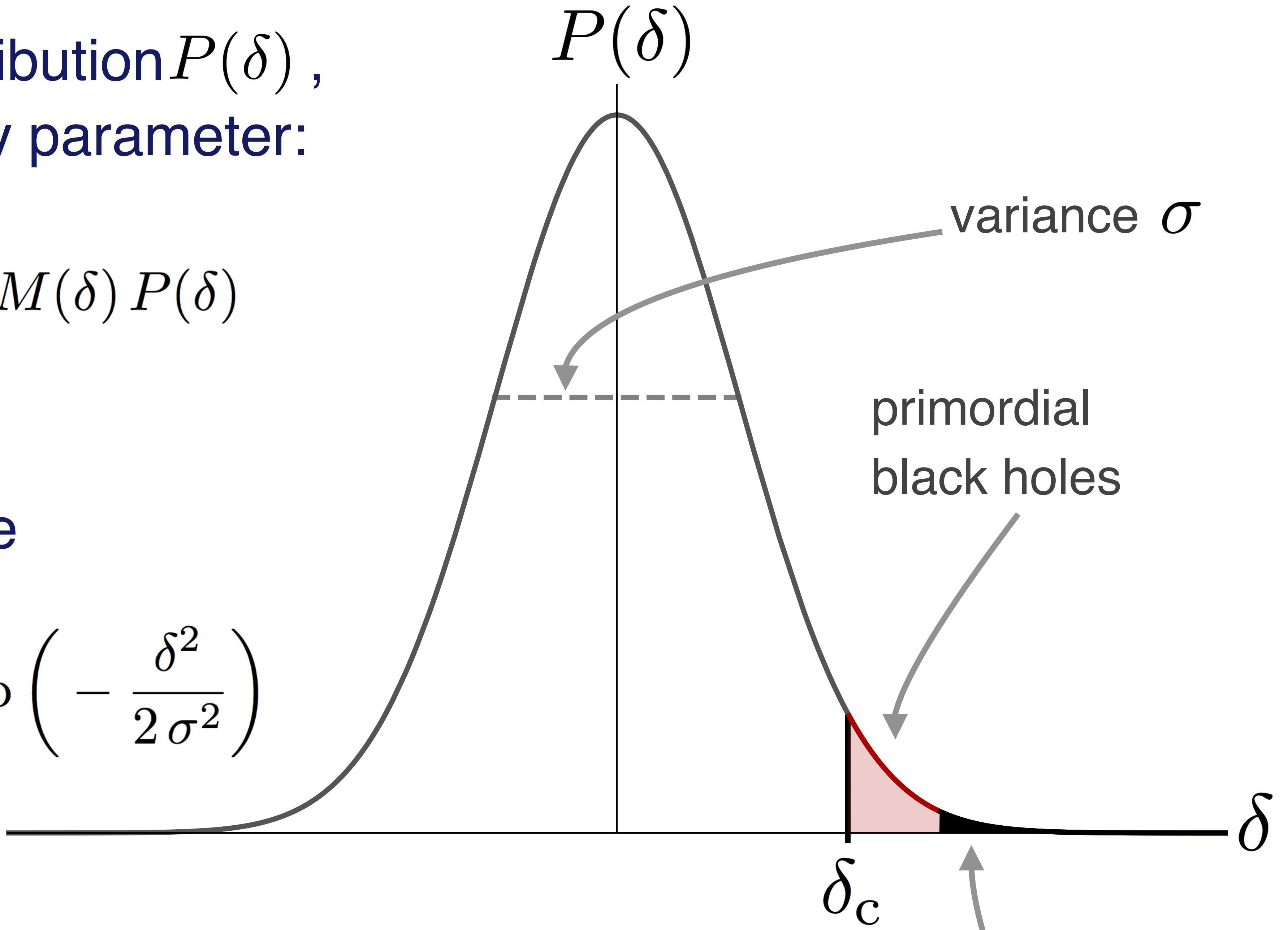
$$\Omega_{\mathrm{PBH}} = M_H^{-1} \int_{\delta_c} \mathrm{d}\delta \; M(\delta)\, P(\delta)$$

- In the Gaußian case

$$P(\delta) = \frac{1}{\sqrt{2\,\pi\,\sigma^2}} \exp\left(- \frac{\delta^2}{2\,\sigma^2} \right)$$

- If furthermore $M \sim M_H$ we find that the fraction of collapsed universes becomes

$$\beta \sim \mathrm{erfc}\left(\frac{\delta_c}{2\,\sigma} \right)$$

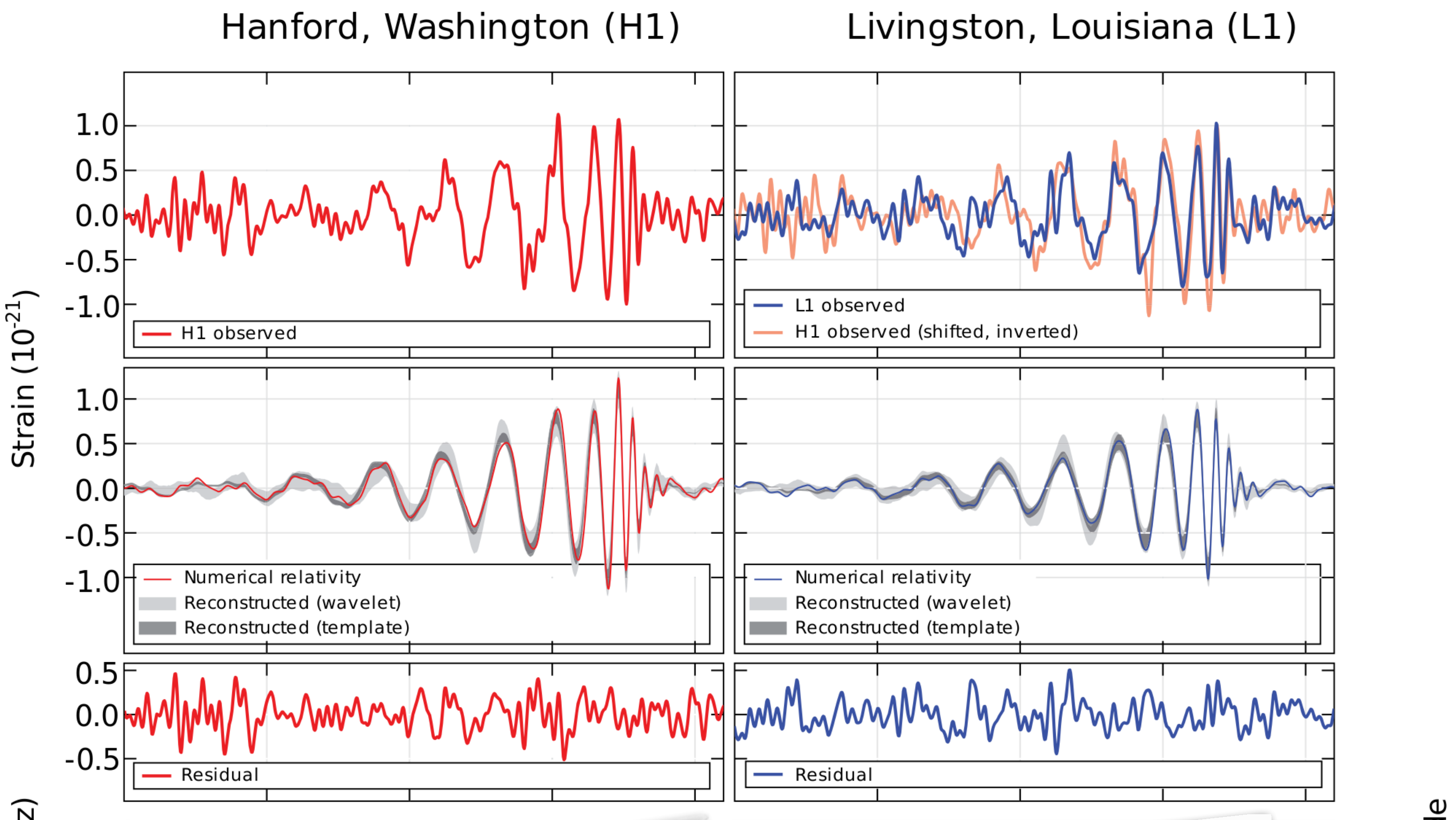


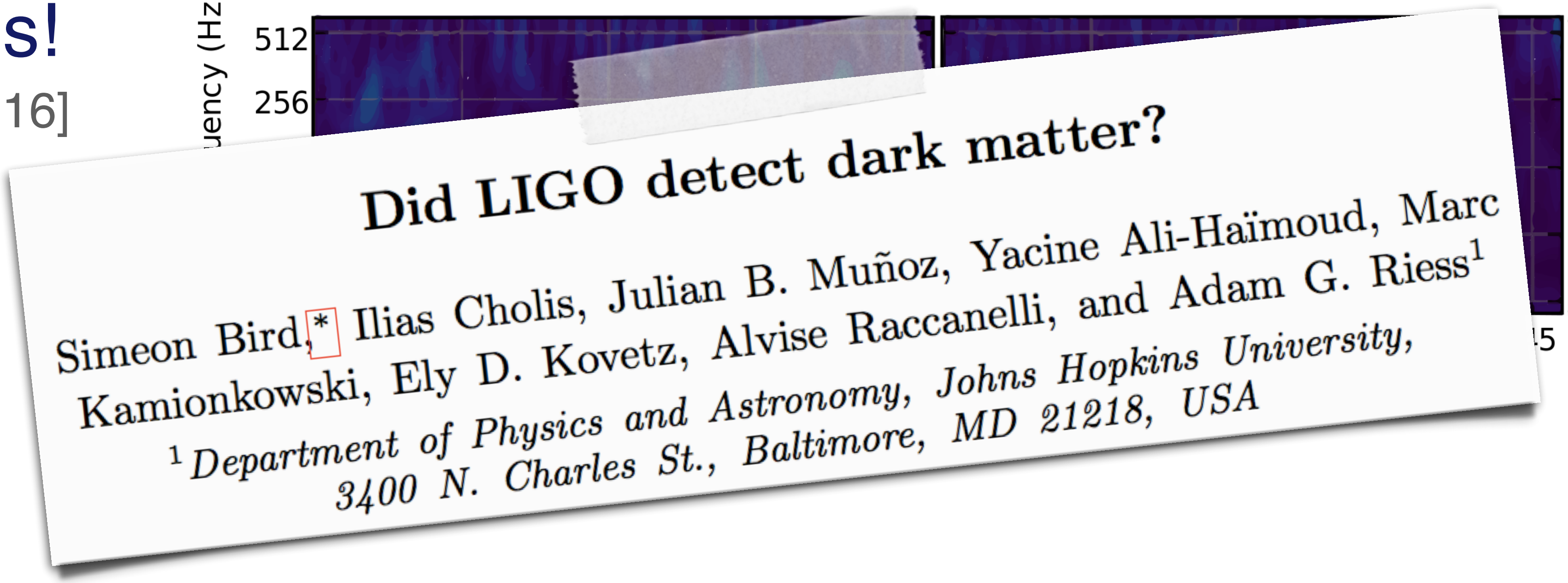
Primordial Black Holes — Observed?

★ Milestone detection of gravitational waves by LIGO

- ★ Confirmation of two merging black holes (GW150914, GW151226, …)
- ★ Masses for all BHs: $\mathcal{O}(10)\ M_\odot$
- ➔ Could be PBHs!

[Bird *et al.* 2016]





Did LIGO detect dark matter?

Simeon Bird,* Ilias Cholis, Julian B. Muñoz, Yacine Ali-Haïmoud, Marc Kamionkowski, Ely D. Kovetz, Alvise Raccanelli, and Adam G. Riess[1]

[1] *Department of Physics and Astronomy, Johns Hopkins University, 3400 N. Charles St., Baltimore, MD 21218, USA*

Critical Collapse

- Usually: Assume

$$M_{BH} \propto M_H$$

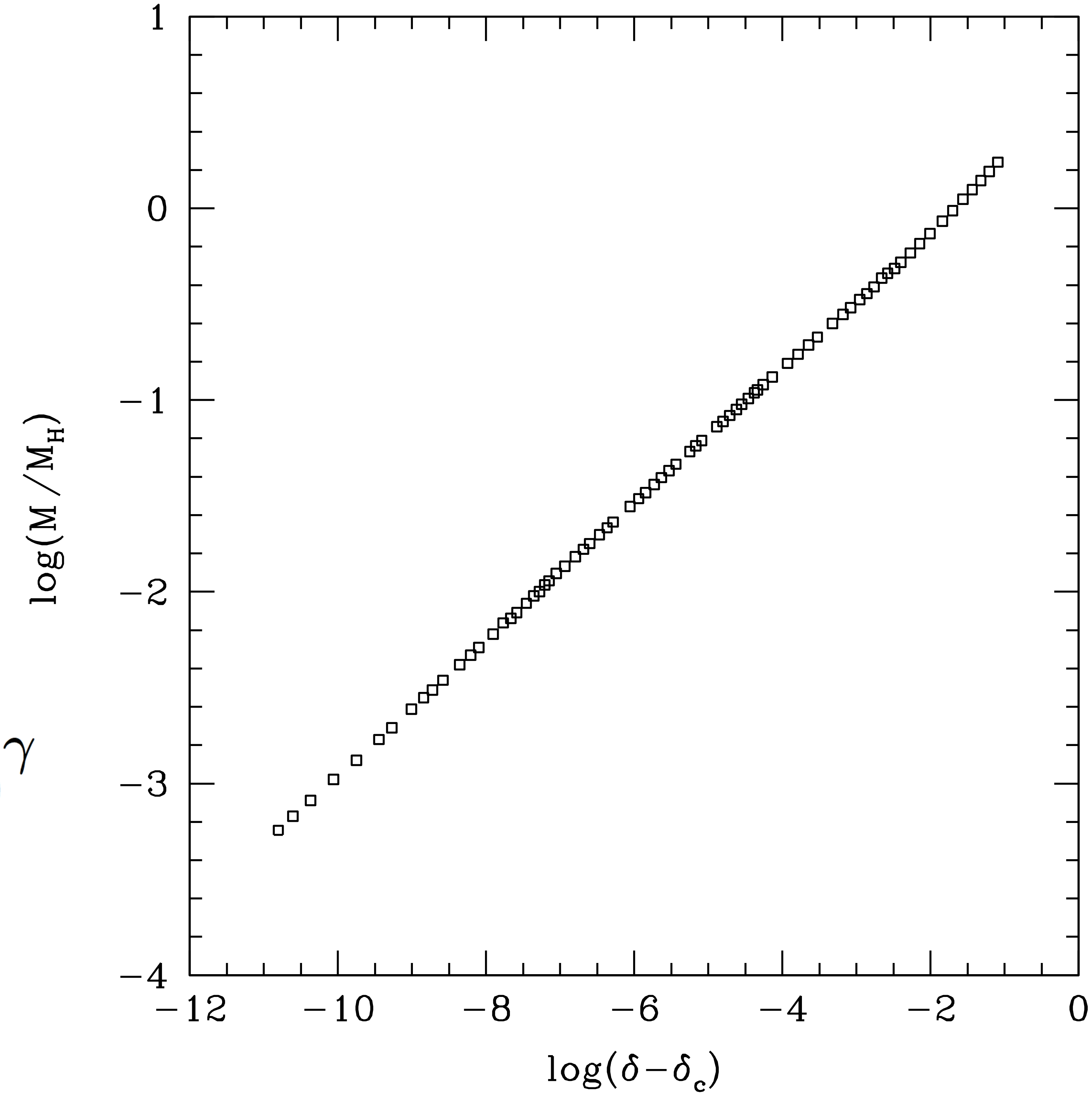
horizon mass

- Critical scaling:

[Choptuik '93]

$$M_{BH} = k\, M_H (\delta - \delta_c)^\gamma$$

density contrast



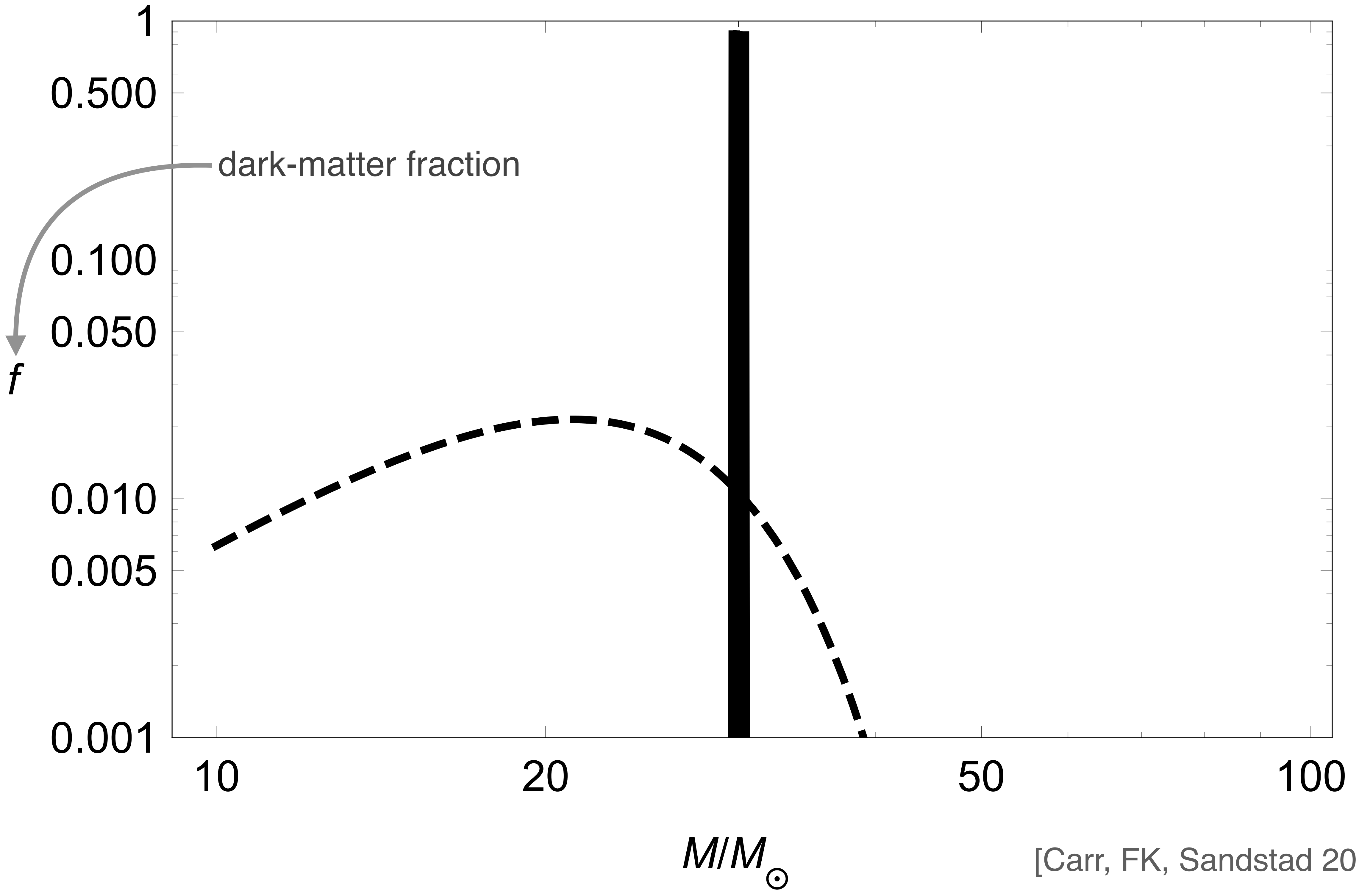
[Miller *et al.* 2004]

- Radiation domination:

$$k \approx 3.3\,, \quad \delta_c \approx 0.45\,, \quad \gamma \approx 0.36$$

Critical Collapse

- ★ How would this look for monochromatic mass function?



[Carr, FK, Sandstad 2016]

Critical Collapse

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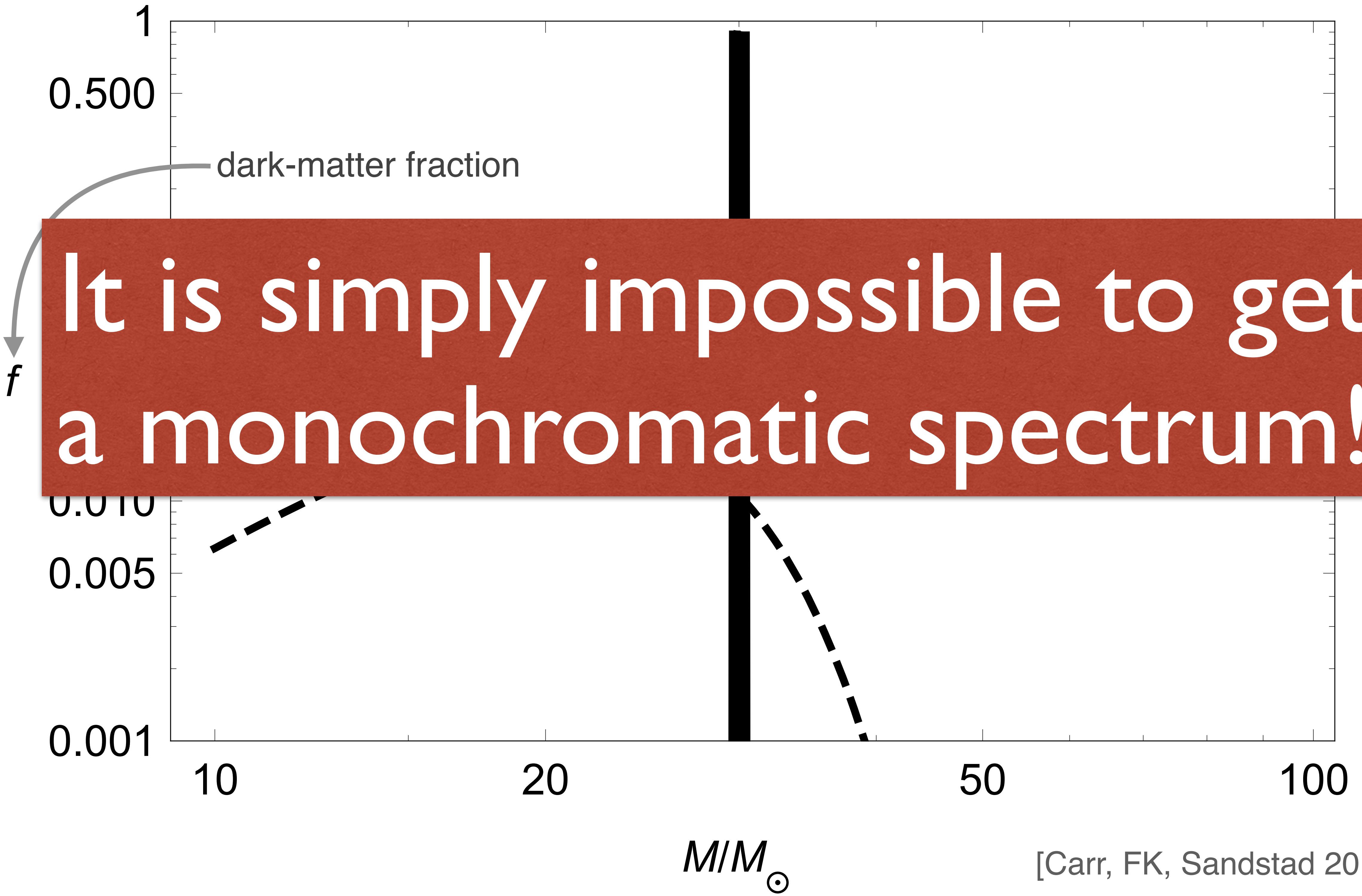
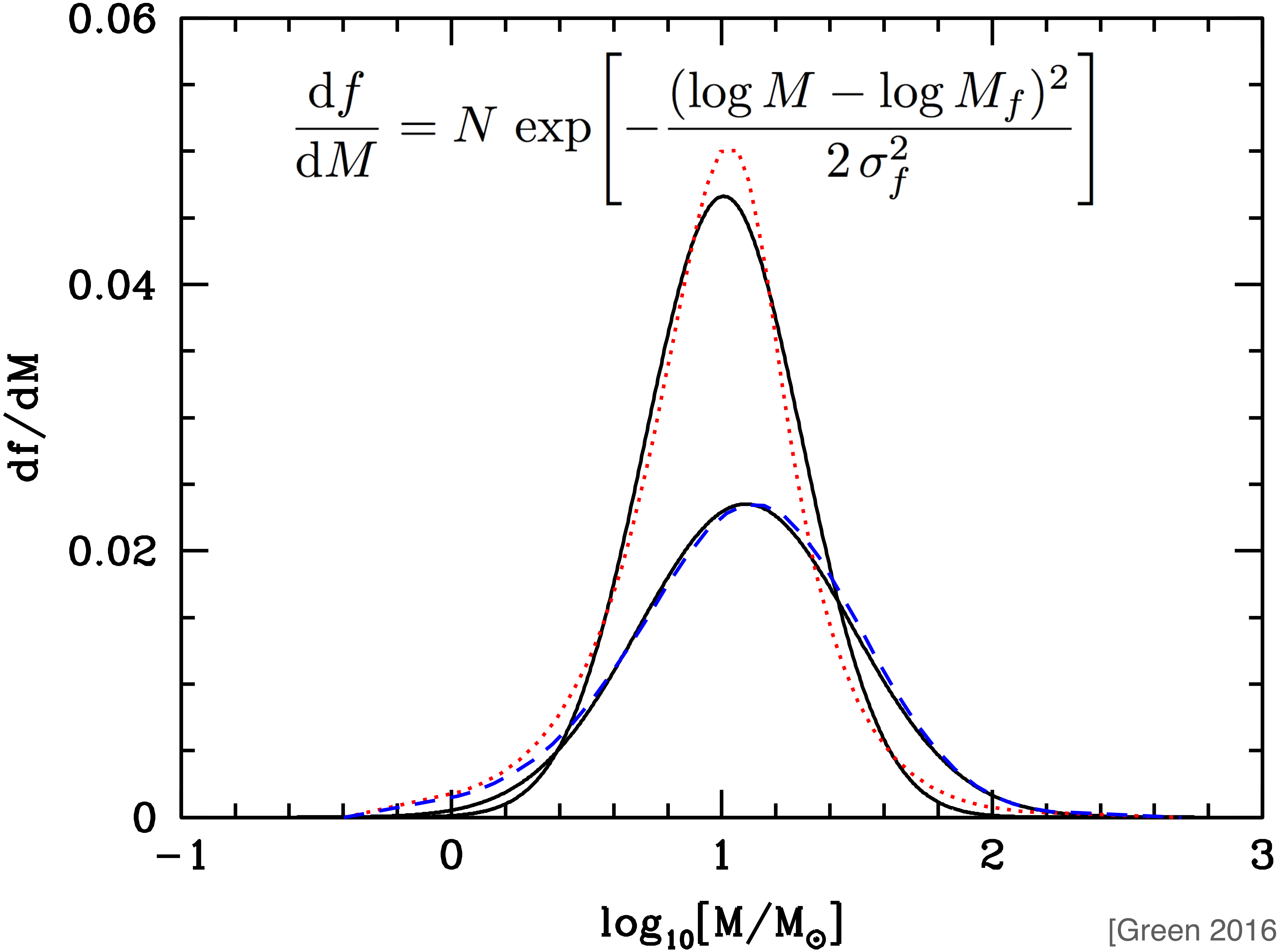


It is simply impossible to get a monochromatic spectrum!

[Carr, FK, Sandstad 2016]

More Systematic Study

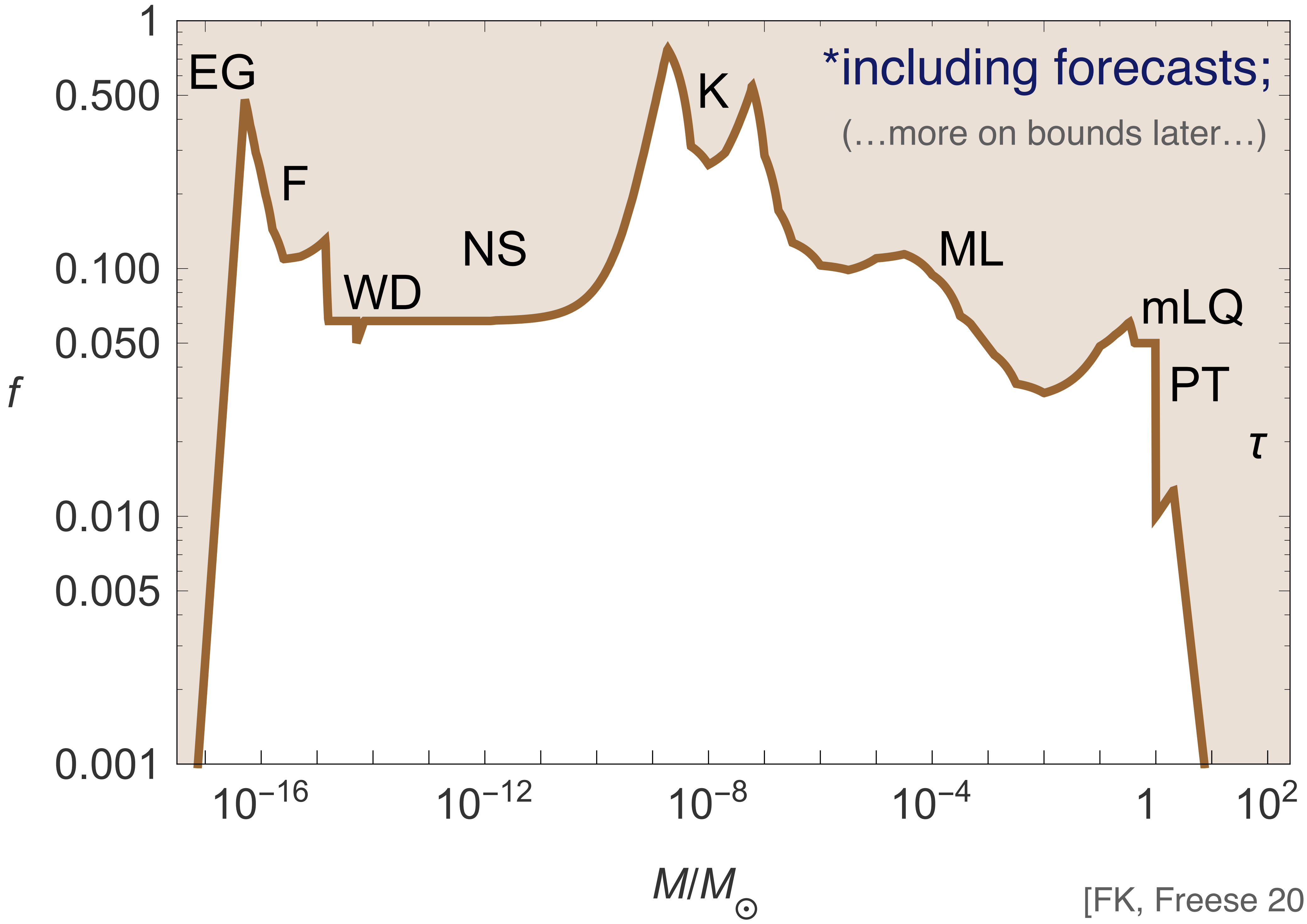


$$\frac{\mathrm{d}f}{\mathrm{d}M} = N \exp\left[-\frac{(\log M - \log M_f)^2}{2\,\sigma_f^2}\right]$$

[Green 2016]

Extended Mass Spectra and Constraints

- ★ We applied this extended mass function to this constraint "curtain":*

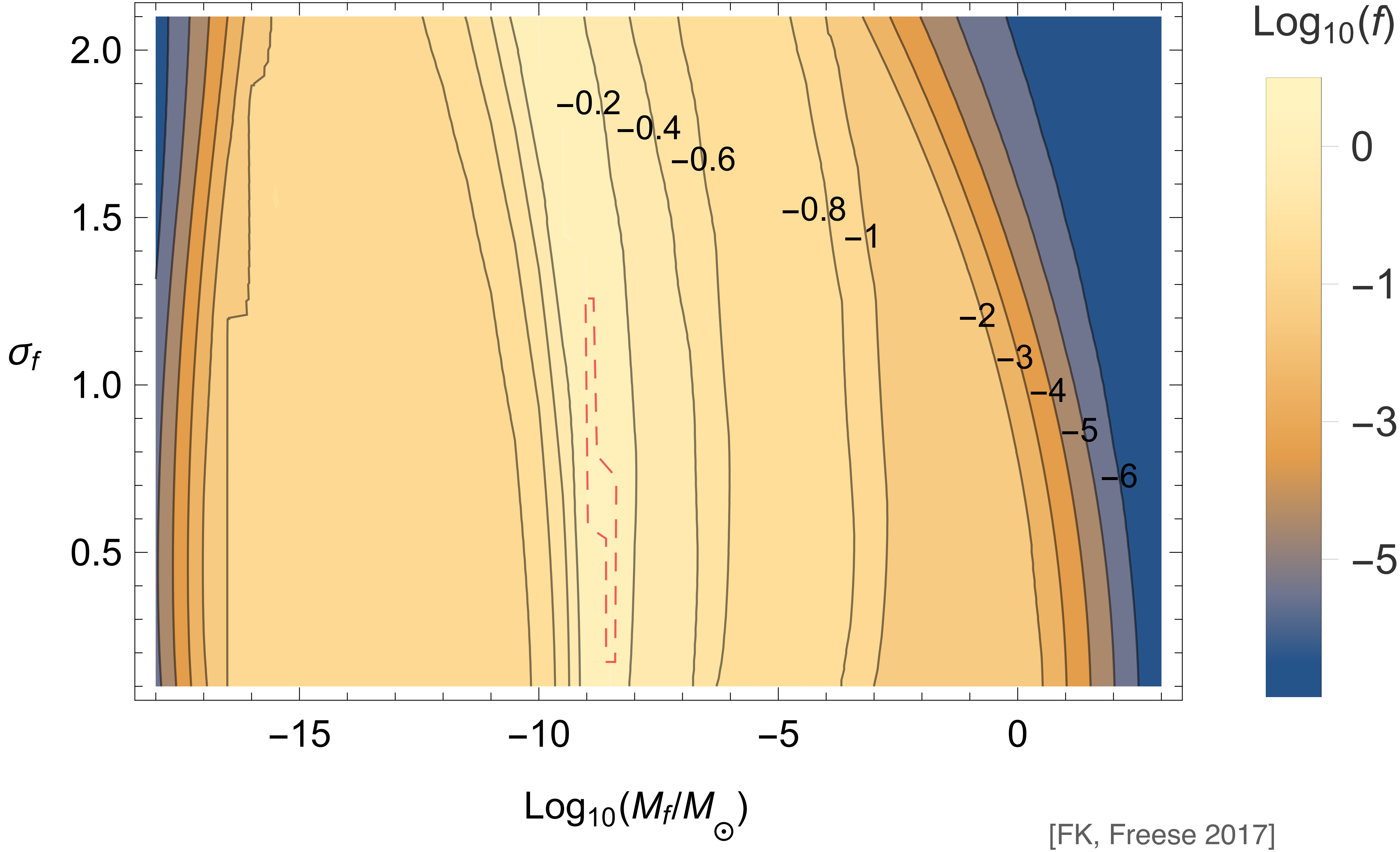


*including forecasts;

(…more on bounds later…)

[FK, Freese 2017]

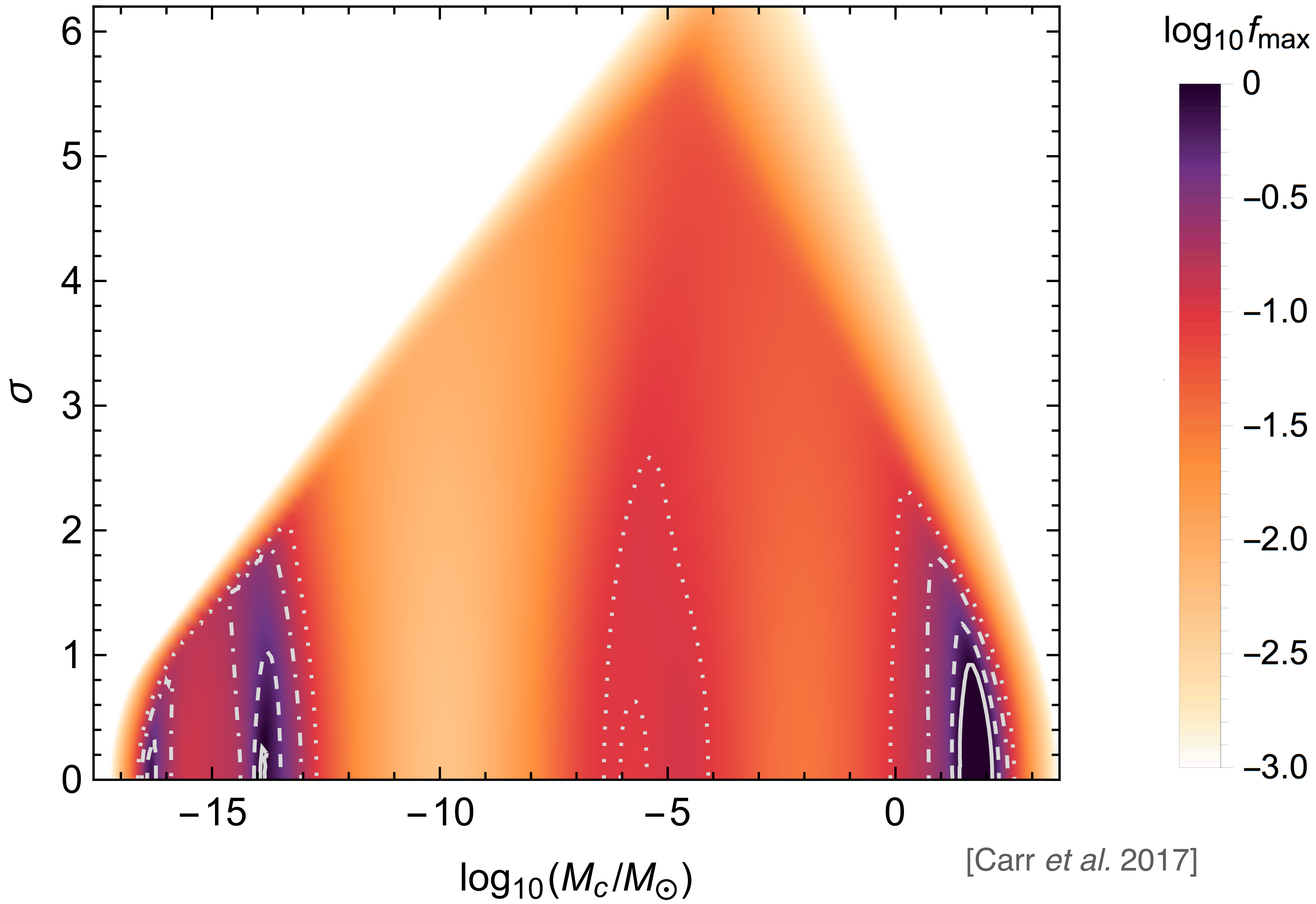
More Systematic Study — Results



[FK, Freese 2017]

More Systematic Study — Other's Results

- With partly different constraints:

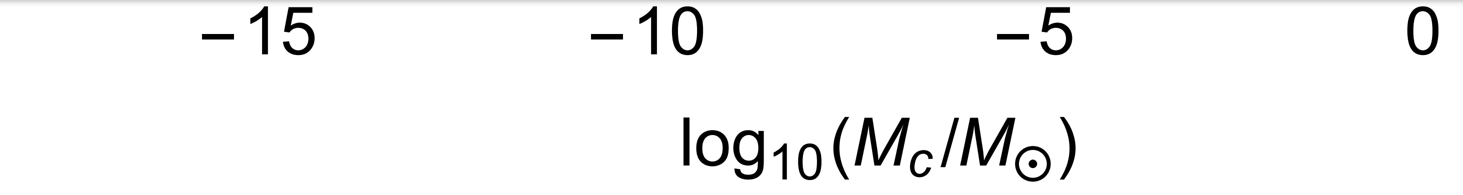


[Carr *et al.* 2017]

More Systematic Study — Other's Results

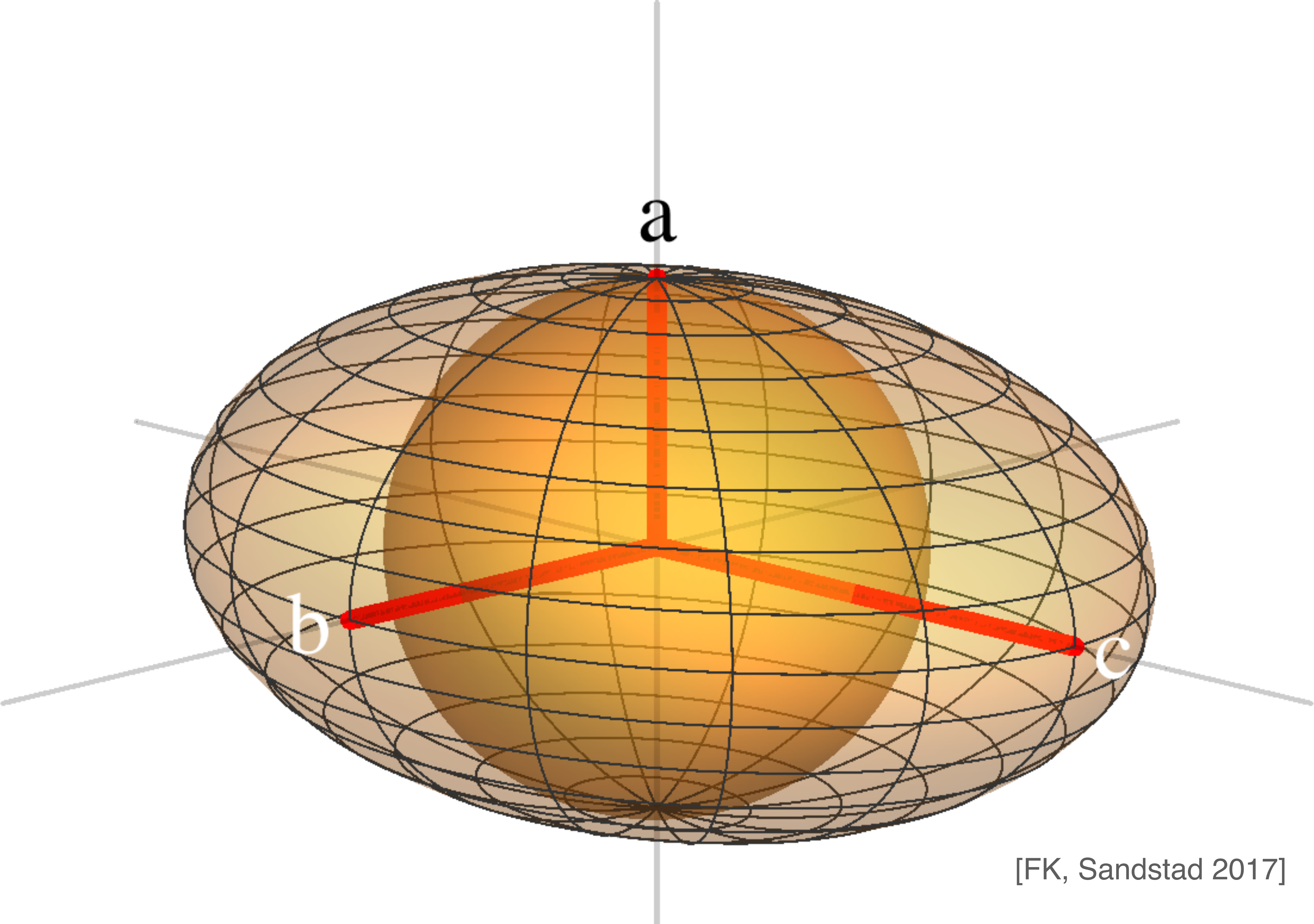
1) A good understanding of the physics of the constraints is extremely important!

2) It is crucial to re-derive the constraints for (the realistic(!) case) of extended mass functions!



[Carr *et al.* 2017]

Non-Spherical Effects



[FK, Sandstad 2017]

Non-Spherical Effects

★ Non-Sphericity

ellipsoidal threshold

$$\frac{\delta_{\rm ec}}{\delta_{\rm c}} \simeq 1 + \kappa \left(\frac{\sigma^2}{\delta_{\rm c}^2} \right)^{\gamma}$$

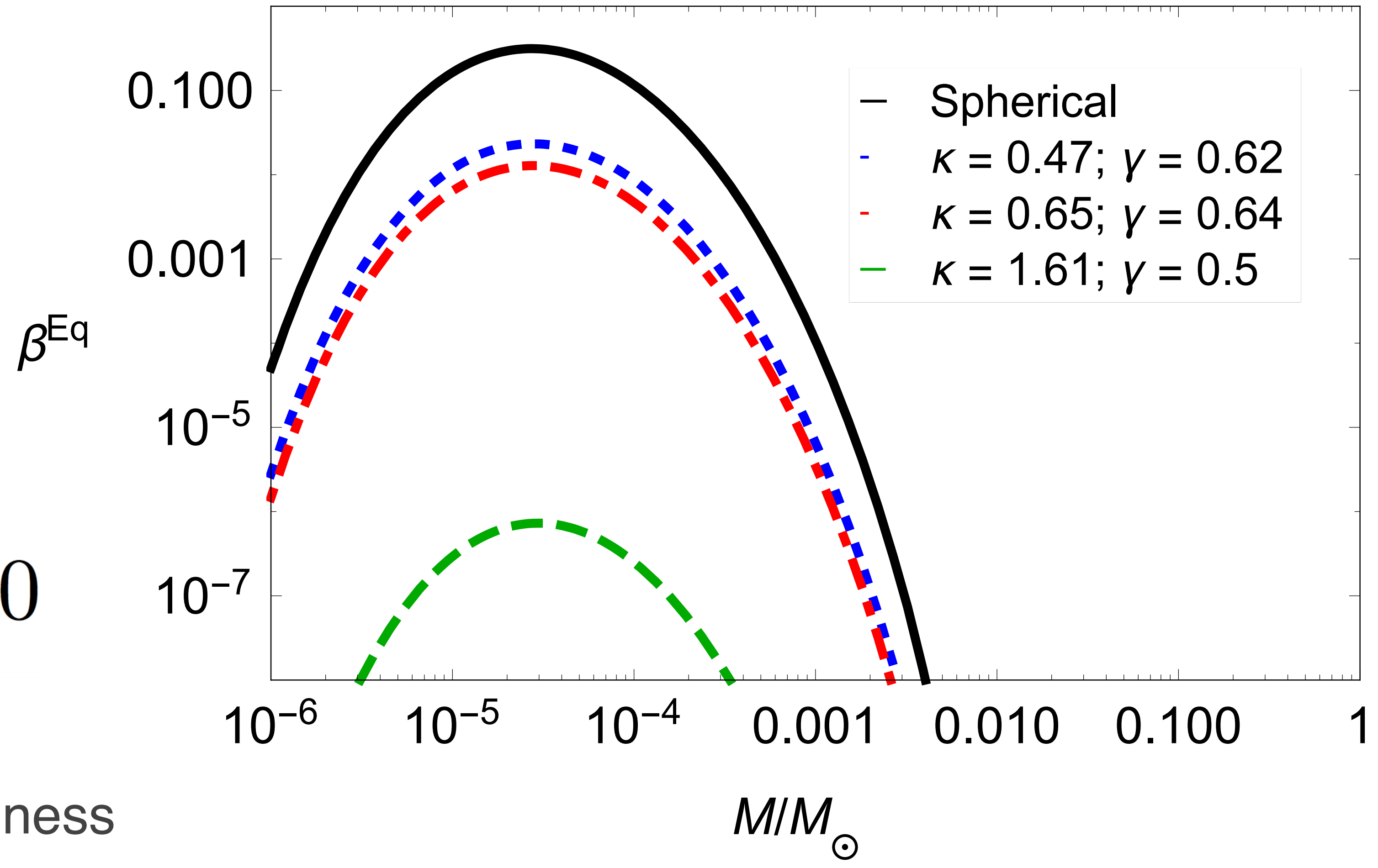
spherical threshold

$$\langle e \rangle = \frac{3\,\sigma}{\sqrt{10\pi}\;\delta}\,, \qquad \langle p \rangle = 0$$

ellipticity

prolateness

[FK, Sandstad 2016]



★ Simple estimate: As the collapse starts along shortest axis first,

→ consider collapse of largest enclosed sphere (green curve):

$$\frac{\delta_{\rm ec}}{\delta_{\rm c}} \simeq (1 + 3\,e) = 1 + \frac{9}{\sqrt{10\,\pi}} \left(\frac{\sigma^2}{\delta_{\rm c}^2} \right)^{1/2}$$

Non-Spherical Effects

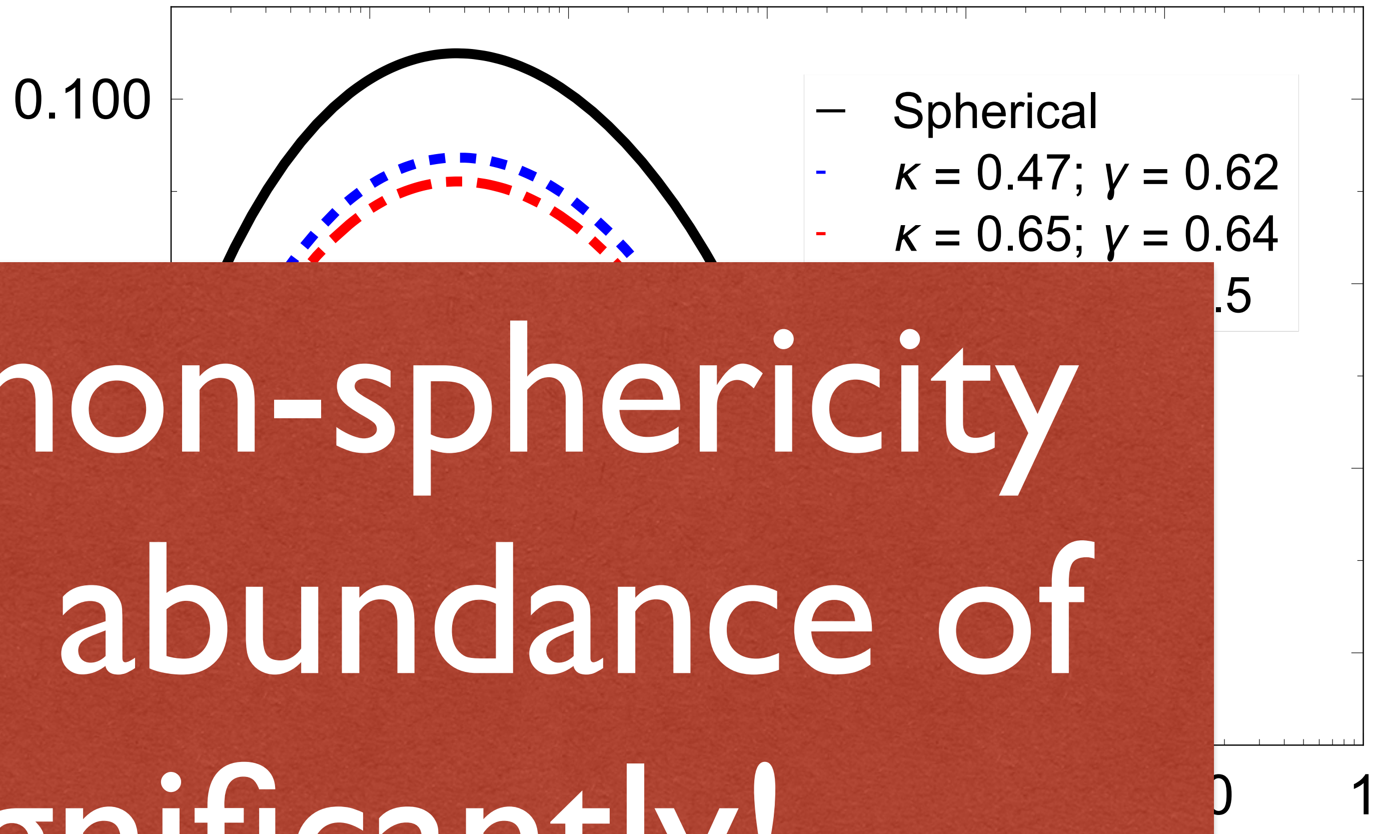
Non-Sphericity

ellipsoidal threshold

$$\frac{\delta_{\rm ec}}{\delta_{\rm c}} \sim 1+\kappa\left(\frac{\sigma^2}{\ }\right)^{\gamma}$$

$\langle e$

[FK, Sandstad 2016]



Even slight non-sphericity reduces the abundance of PBHs significantly!

- Simple estimate: As the collapse starts along shortest axis first,

→ consider collapse of largest enclosed sphere (green curve):

$$\frac{\delta_{\rm ec}}{\delta_{\rm c}} \simeq (1+3\,e) = 1+\frac{9}{\sqrt{10\,\pi}}\left(\frac{\sigma^2}{\delta_{\rm c}^2}\right)^{1/2}$$

Gravitational Waves form PBHs

- Like ordinary black holes, PBHs can emit gravitational waves. Roughly, there are two types of signals:

  1) From early formation of PBH binaries (three-body process!).

  - Starts being significant in the formation of first caustics.
  - Need *N*-body simulations!

  [FK, Mohayaee, Naselsky, von Hausegger; to appear soon]

  → Characteristic stochastic gravitational-wave background

number of events

gravitational-wave energy per event

$$\Omega_{\mathrm{gw}} \approx \frac{1}{\rho_{\mathrm{c}}\, c^2} \int \mathrm{d}z\, \frac{N(z)}{1+z} \left(\nu_{\mathrm{r}} \frac{E_{\mathrm{gw}}}{\mathrm{d}\nu_{\mathrm{r}}} \right) \Bigg|_{\nu_{\mathrm{r}}=\nu(1+z)}$$

critical density

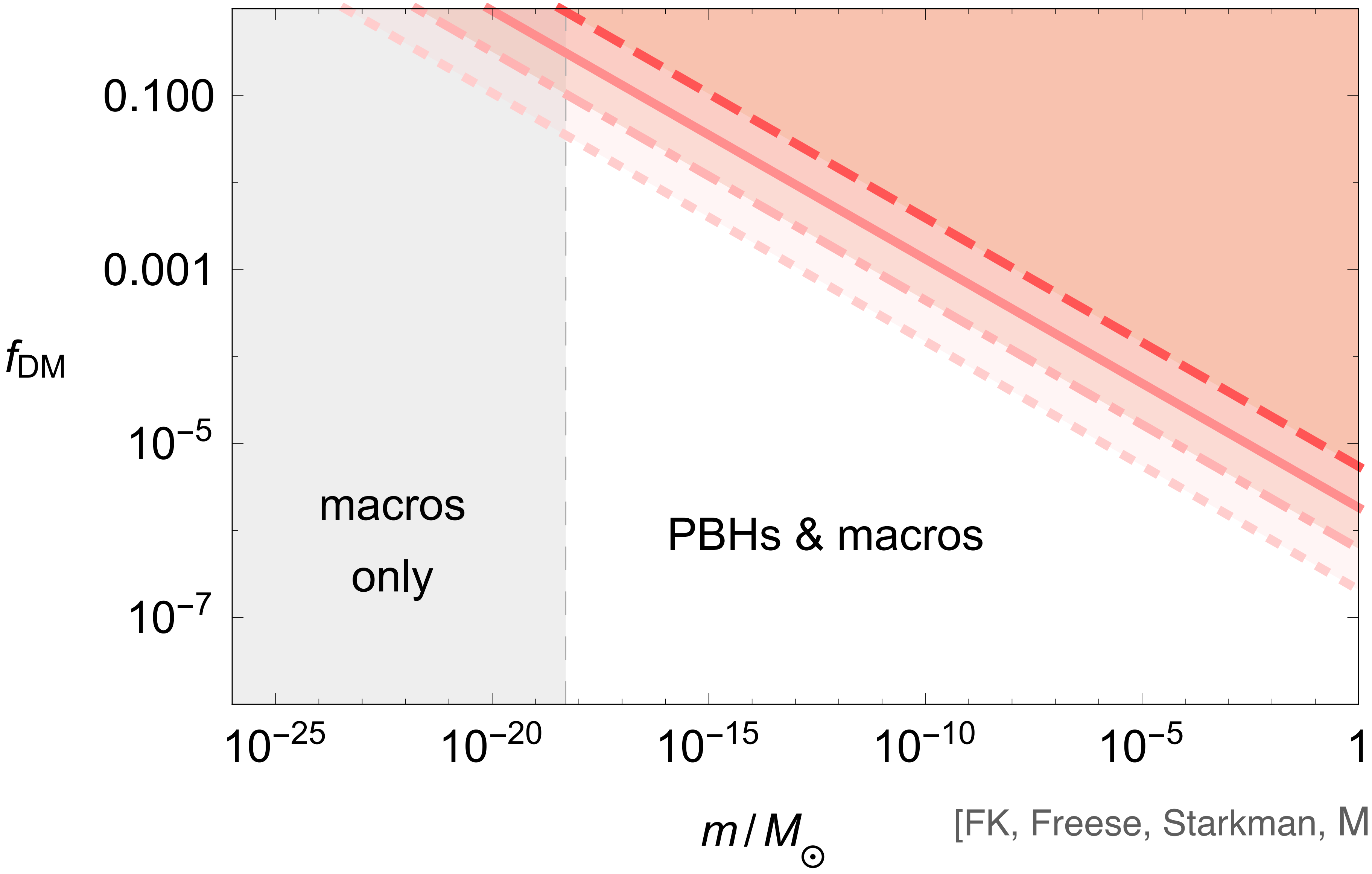
[Phinney 2001]

  2) From late formation of PBH binaries.

Gravitational Waves form PBHs

- ★ If PBHs constitute a significant fraction of the dark matter, at the center of our Galaxy one would have a very large number of PBH inspiralling into SgrA*.

→ Stochastic enhancement; Detection forecasts for LISA:



[FK, Freese, Starkman, Matas 2017]

Gravitational Waves form PBHs

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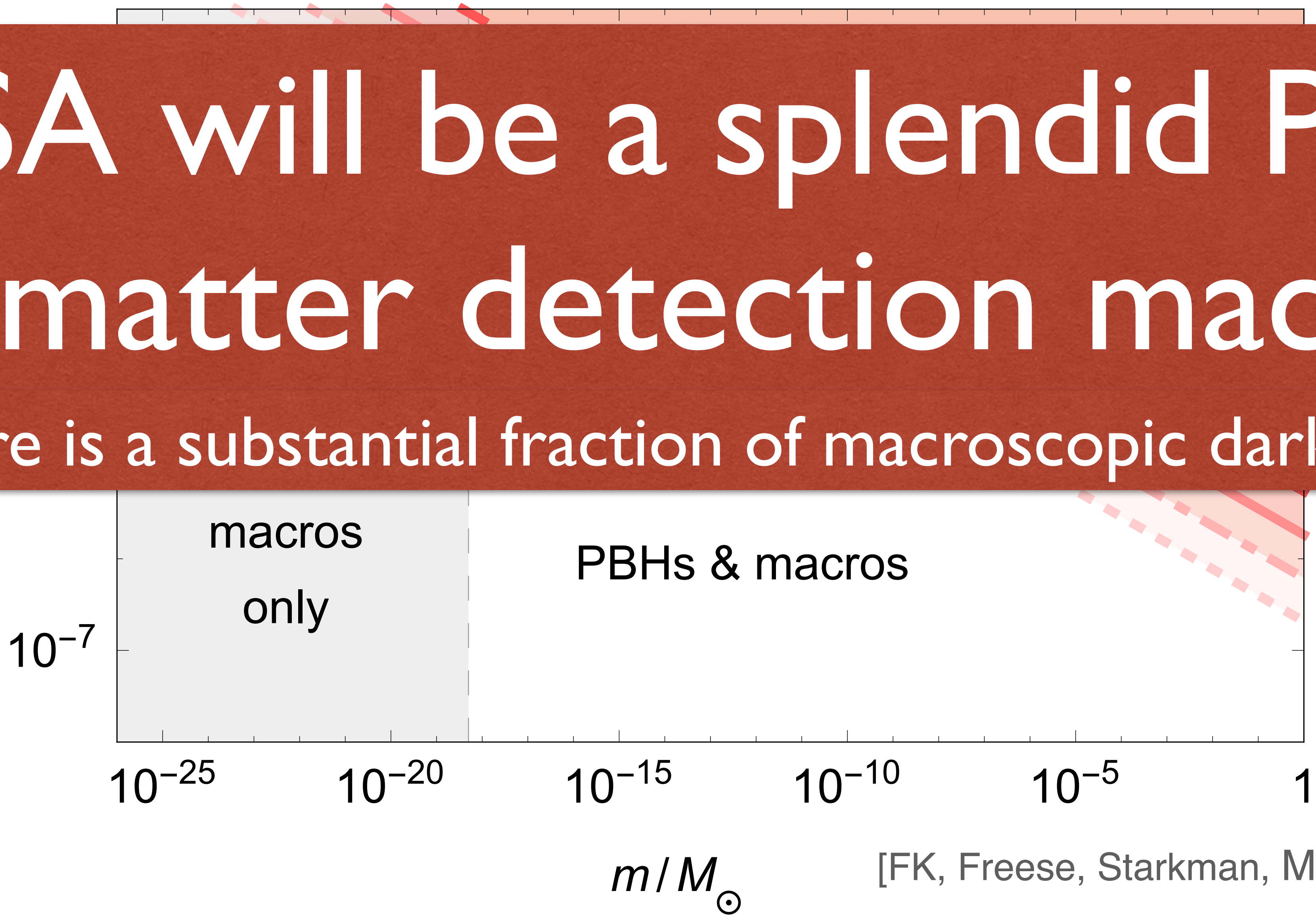
→ Stochastic enhancement; Detection forecasts for LISA:

LISA will be a splendid PBH dark-matter detection machine!*

*If there is a substantial fraction of macroscopic dark matter.



[FK, Freese, Starkman, Matas 2017]

PBH & Particle Dark Matter

- Study a combined scenario: DM = PBHs + Particles
  - The latter will be accreted by the former.
  - As an example, focus on sterile neutrinos:
    - Decays with rates:

*works also for UCMHs

$$\Gamma_{\nu_s\to 3\nu} \approx 8.7\cdot 10^{-31}\left(\frac{\sin^2(2\Theta)}{10^{-10}}\right)\left(\frac{m_{\nu_s}}{10\,\mathrm{keV}}\right)^5\ \mathrm{s}^{-1} \qquad \Gamma_{\nu_s\to\nu\gamma} \simeq \frac{1}{128}\,\Gamma_{\nu_s\to 3\nu}$$

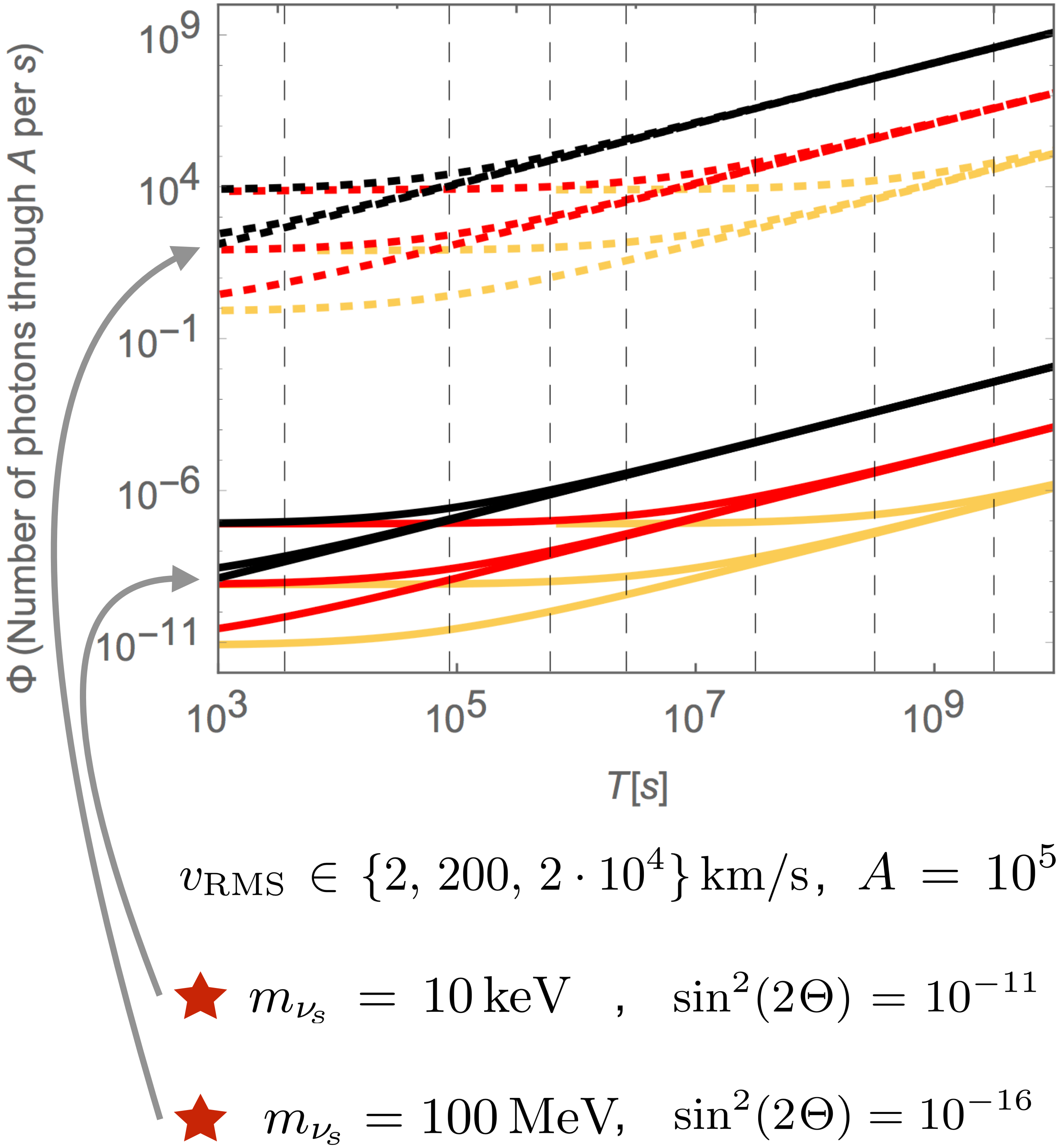
- The fluxes are quite significant with small average distances:

$M\,/\,\mathrm{g}$	N	$d\,/\,\mathrm{AU}$	$\Gamma^{\mathrm{total}}_{\nu_s\to\nu\gamma}\,/\,\mathrm{s}$	$\Gamma^{\mathrm{total}}_{\nu_s\to 3\nu}\,/\,\mathrm{s}$
10^{-3}	$6\cdot 10^{25}$	$8\cdot 10^{-7}$	$4\cdot 10^{-6}$	$5\cdot 10^{-4}$
1	$6\cdot 10^{28}$	$8\cdot 10^{-6}$	4	500
10^{3}	$6\cdot 10^{31}$	$8\cdot 10^{-5}$	$4\cdot 10^{6}$	$5\cdot 10^{8}$
10^{6}	$6\cdot 10^{34}$	$8\cdot 10^{-4}$	$4\cdot 10^{12}$	$5\cdot 10^{14}$
10^{9}	$6\cdot 10^{37}$	$8\cdot 10^{-3}$	$4\cdot 10^{18}$	$5\cdot 10^{20}$
10^{12}	$6\cdot 10^{40}$	0.08	$4\cdot 10^{24}$	$5\cdot 10^{26}$
10^{15}	$6\cdot 10^{43}$	0.8	$4\cdot 10^{30}$	$5\cdot 10^{32}$
10^{18}	$6\cdot 10^{46}$	8	$4\cdot 10^{36}$	$5\cdot 10^{38}$
10^{21}	$6\cdot 10^{49}$	80	$4\cdot 10^{42}$	$5\cdot 10^{44}$
10^{24}	$6\cdot 10^{52}$	800	$4\cdot 10^{48}$	$5\cdot 10^{50}$

[FK, Ohlsson 2017*]
(*accepted for publication in PRD)

PBH & Particle Dark Matter

- Hence, these objects possibly pass close by a detector.



$$v_{\rm RMS} \in \{2,\, 200,\, 2 \cdot 10^4\}\,{\rm km/s}, \quad A = 10^5\,{\rm cm}^2$$

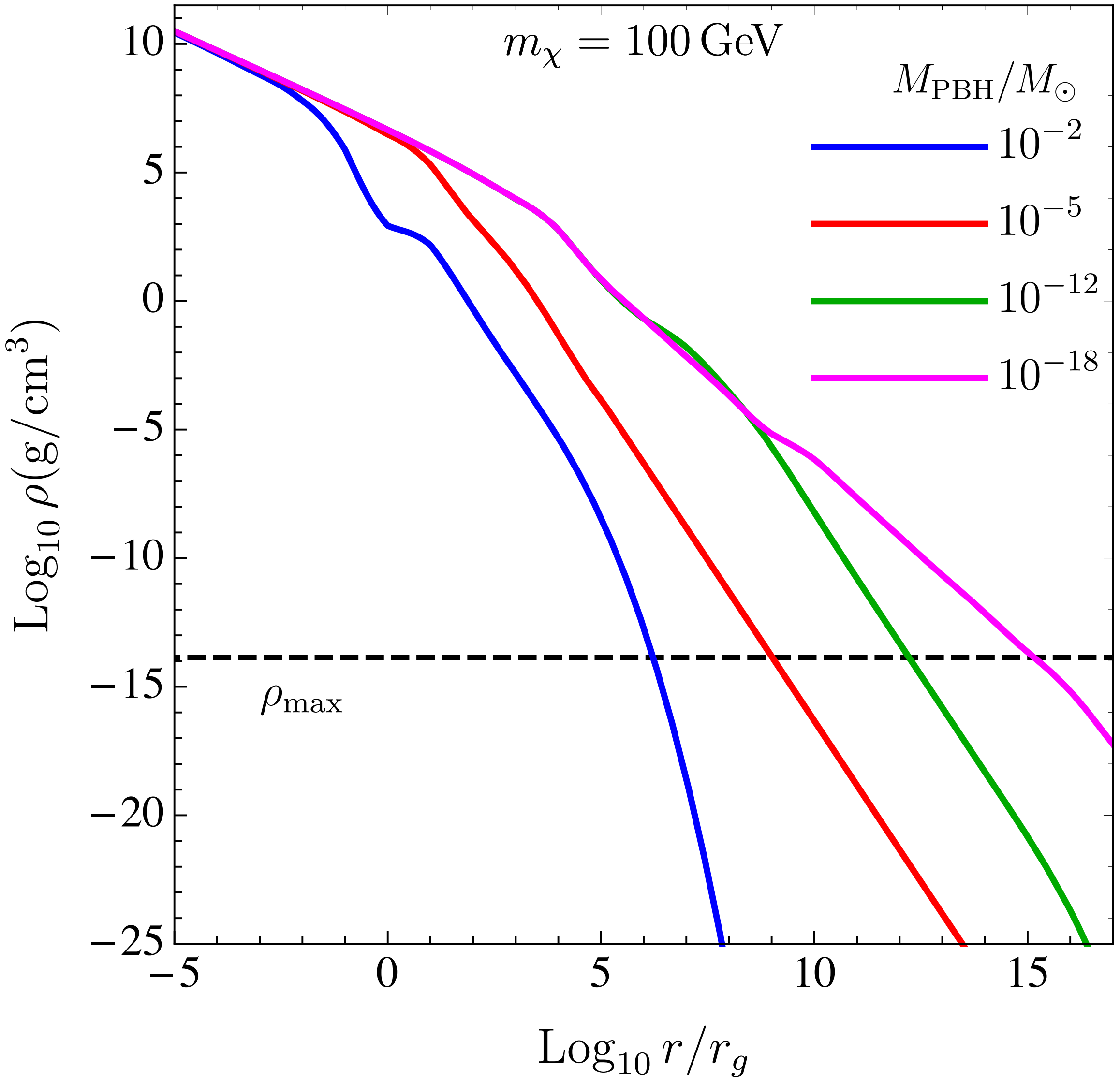
- $m_{\nu_s} = 10\,{\rm keV}\ , \quad \sin^2(2\Theta) = 10^{-11}$
- $m_{\nu_s} = 100\,{\rm MeV}, \quad \sin^2(2\Theta) = 10^{-16}$

Telescope \ Decay Rate		$\Gamma^{\rm total}/\widetilde{\Gamma}^{\rm total}$				
		0.01	0.1	1	10	100
	E / eV	Threshold masses $M_{\rm th}$/g				
Suzaku	10^3	10^{36}	10^{30}	10^{24}	10^{18}	10^{12}
NuSTAR	10^4	10^{33}	10^{27}	10^{21}	10^{15}	10^9
PoGO+	10^5	10^{39}	10^{33}	10^{27}	10^{21}	10^{15}
ACT	10^6	10^{30}	10^{24}	10^{18}	10^{12}	10^6
AdEPT	10^7	10^{33}	10^{27}	10^{21}	10^{15}	10^9
	10^8	10^{29}	10^{23}	10^{17}	10^{11}	10^5
Fermi	10^9	10^{26}	10^{20}	10^{14}	10^8	100
	10^{10}	10^{22}	10^{16}	10^{10}	10^4	0.01
	10^{11}	10^{20}	10^{14}	10^8	100	10^{-4}
	10^{12}	10^{20}	10^{14}	10^8	100	10^{-4}
IceCube	10^{13}	10^{10}	10^4	0.01	—	—
	10^{14}	10^6	1	—	—	—
	10^{15}	10^3	10^{-3}	—	—	—
	10^{16}	1	—	—	—	—

[FK, Ohlsson 2017*]
(*accepted for publication in PRD)

PBH & Particle Dark Matter

- Let us now study WIMP annihilations in PBH halos:
  - The annihilation rate $\Gamma \propto n^2$.
    - → Halo profile does matter; enhancement of Γ in density spikes.
      - 1) We derive the density profile of the captured WIMPs



[Boucenna, FK, Ohlsson, Visinelli; to appear *very* soon]

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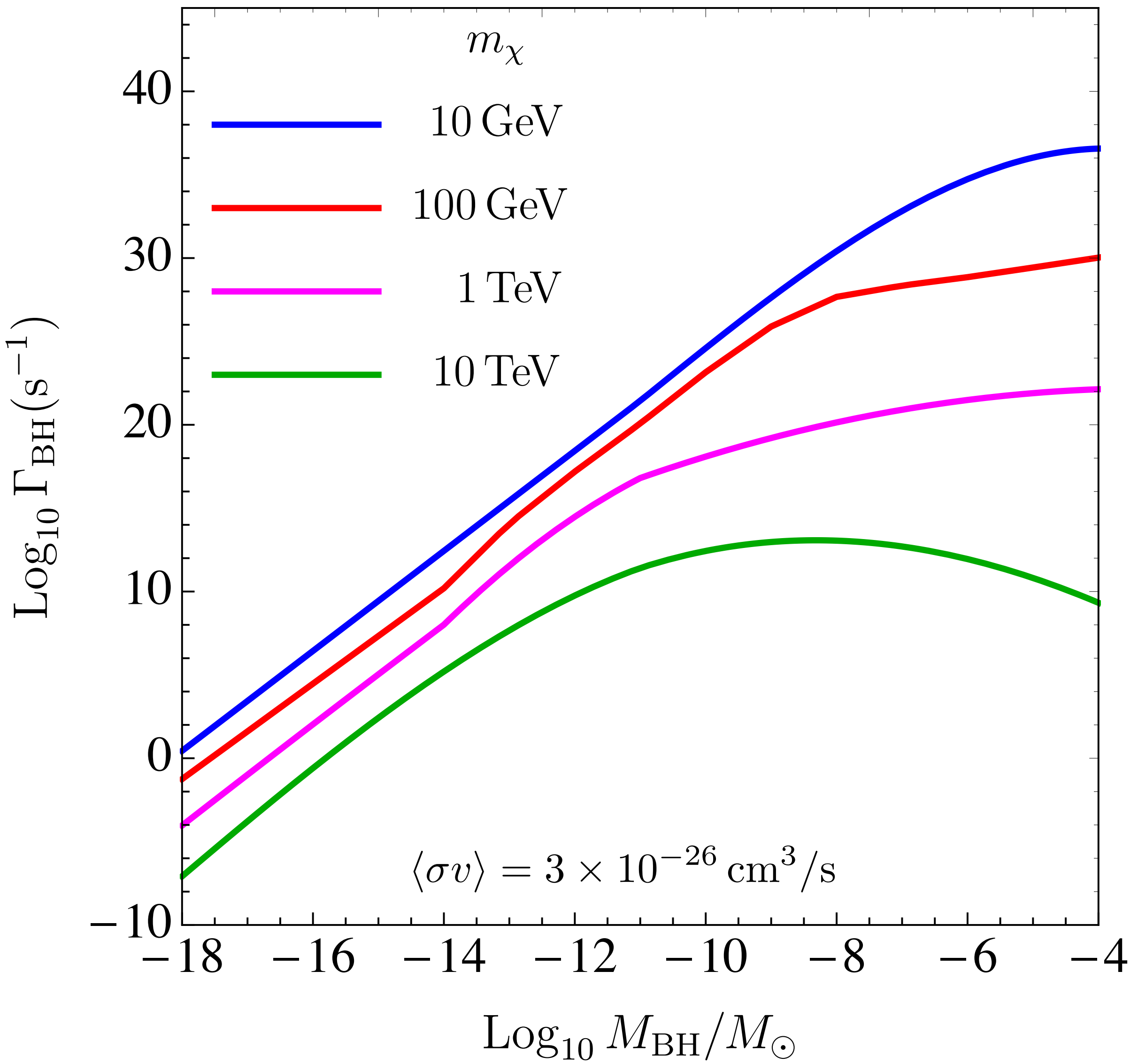

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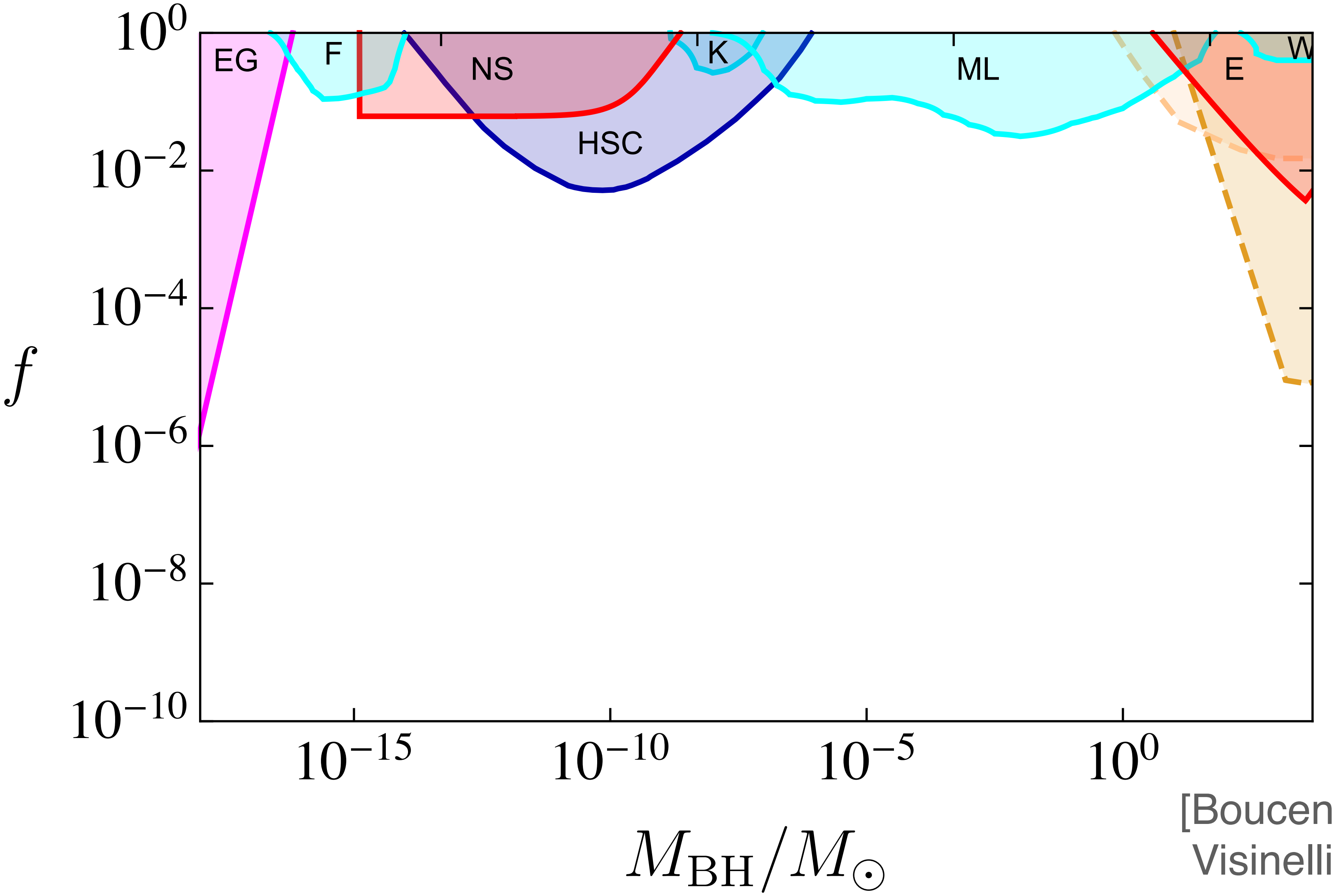

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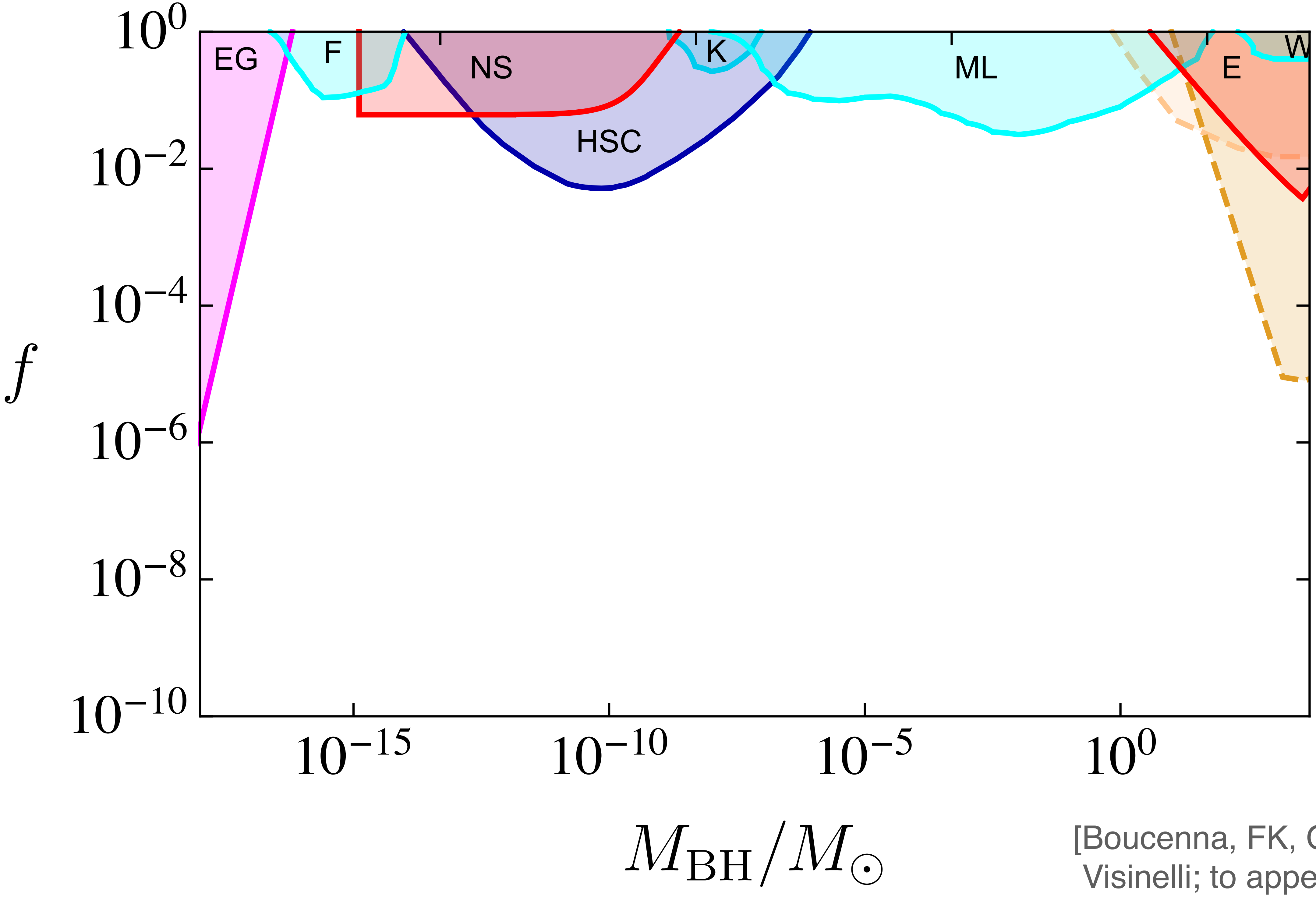

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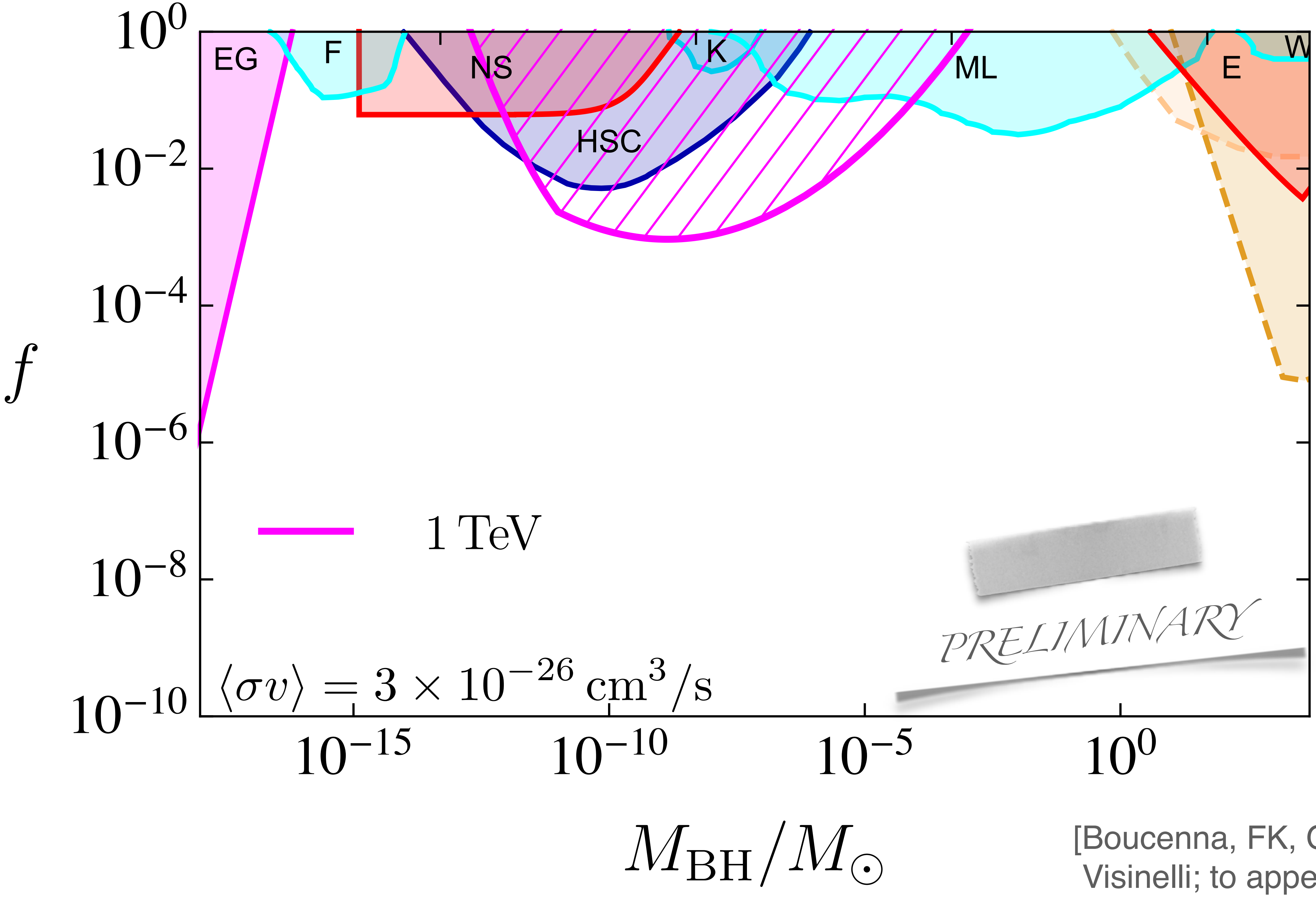

[Boucenna, FK, Ohlsson, Visinelli; to appear *very* soon]

PBH & Particle Dark Matter

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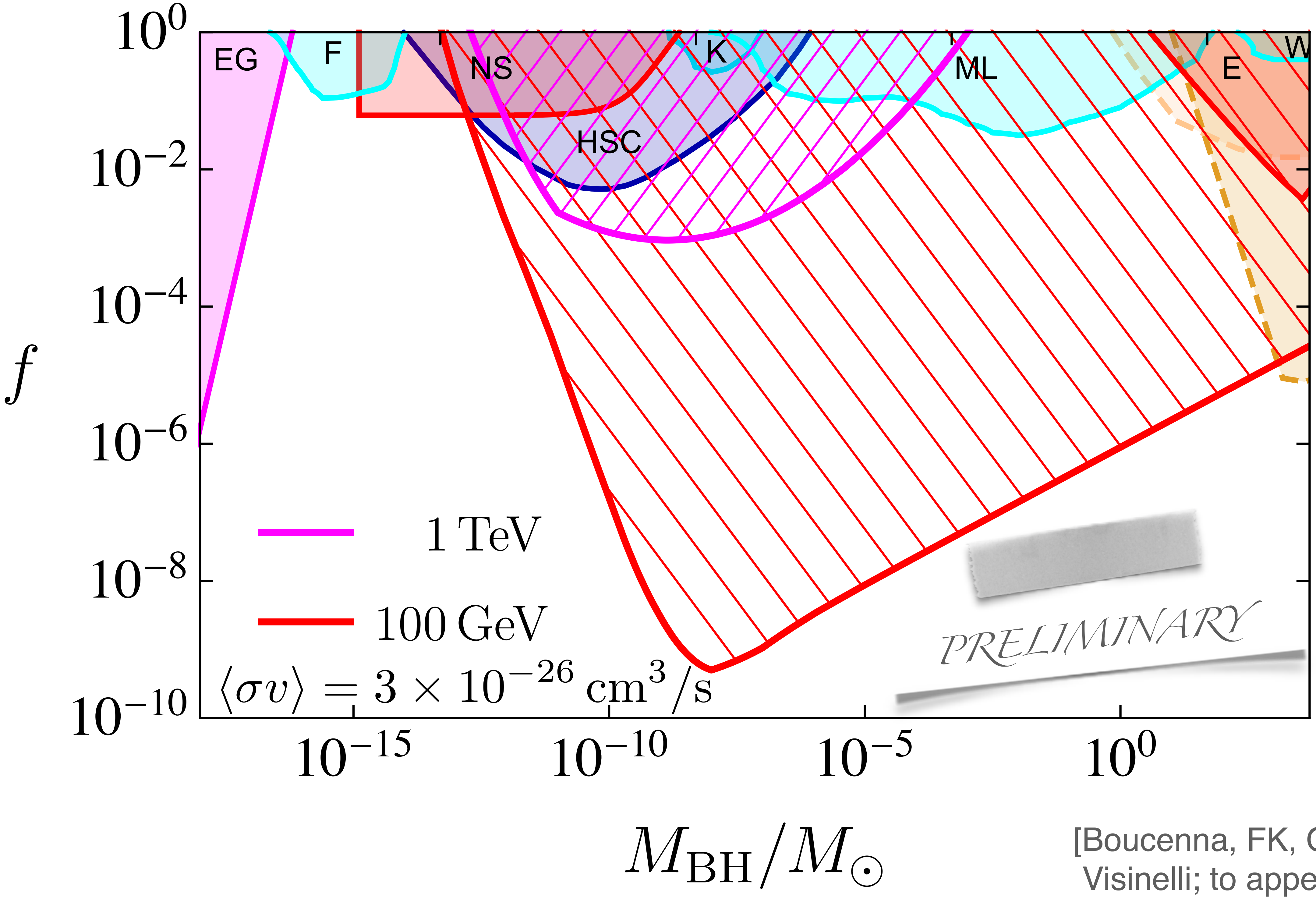


[Boucenna, FK, Ohlsson, Visinelli; to appear *very* soon]

Sad or Happy?

- ★ Are these sad prospects for a PBH-WIMP “coalition”?

[https://www.welt.de/img/politik/deutschland/mobile170761324/5932500847-ci102l-w1024/Scheitern-der-Jamaika-Sondierungen.jpg]

Sad or Happy?

- ★ … or should we be happy?

https://www.welt.de/img/politik/deutschland/mobile170766403/4302507967-ci102l-w1024/Christian-Lindner-head-of-the-Free.jpg]

Sad or Happy?

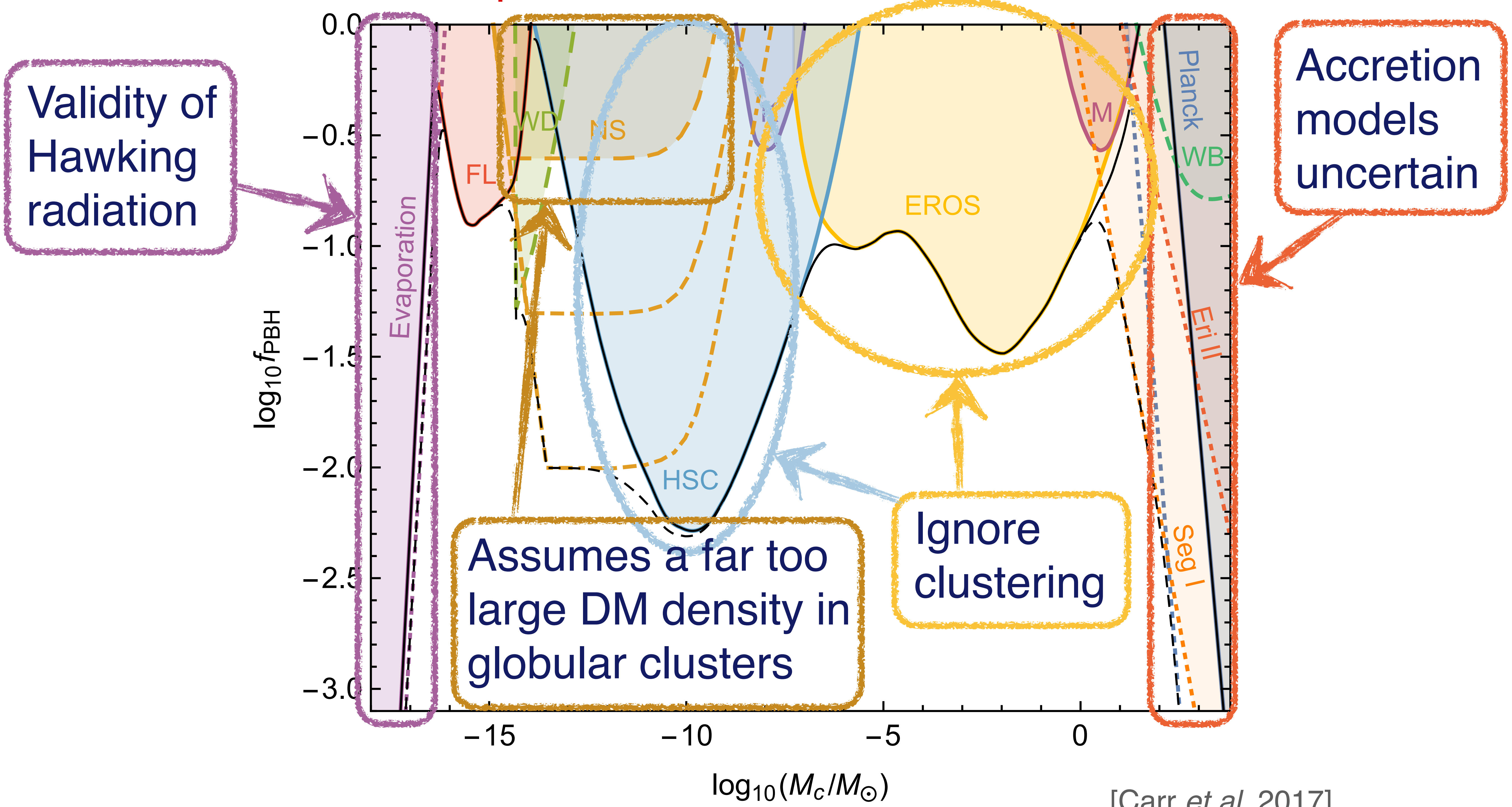
- … or should we be happy?

https://www.welt.de/img/politik/deut
3/4302507967-
ci102l-w1024/Christian-Lindner-hea

Keep smiling, by choosing well your coalition partner.

Constraints — Words of Caution

- May constraints rely on rather on uncertain, restrictive, simplistic or even incorrect assumptions!



Validity of Hawking radiation

Assumes a far too large DM density in globular clusters

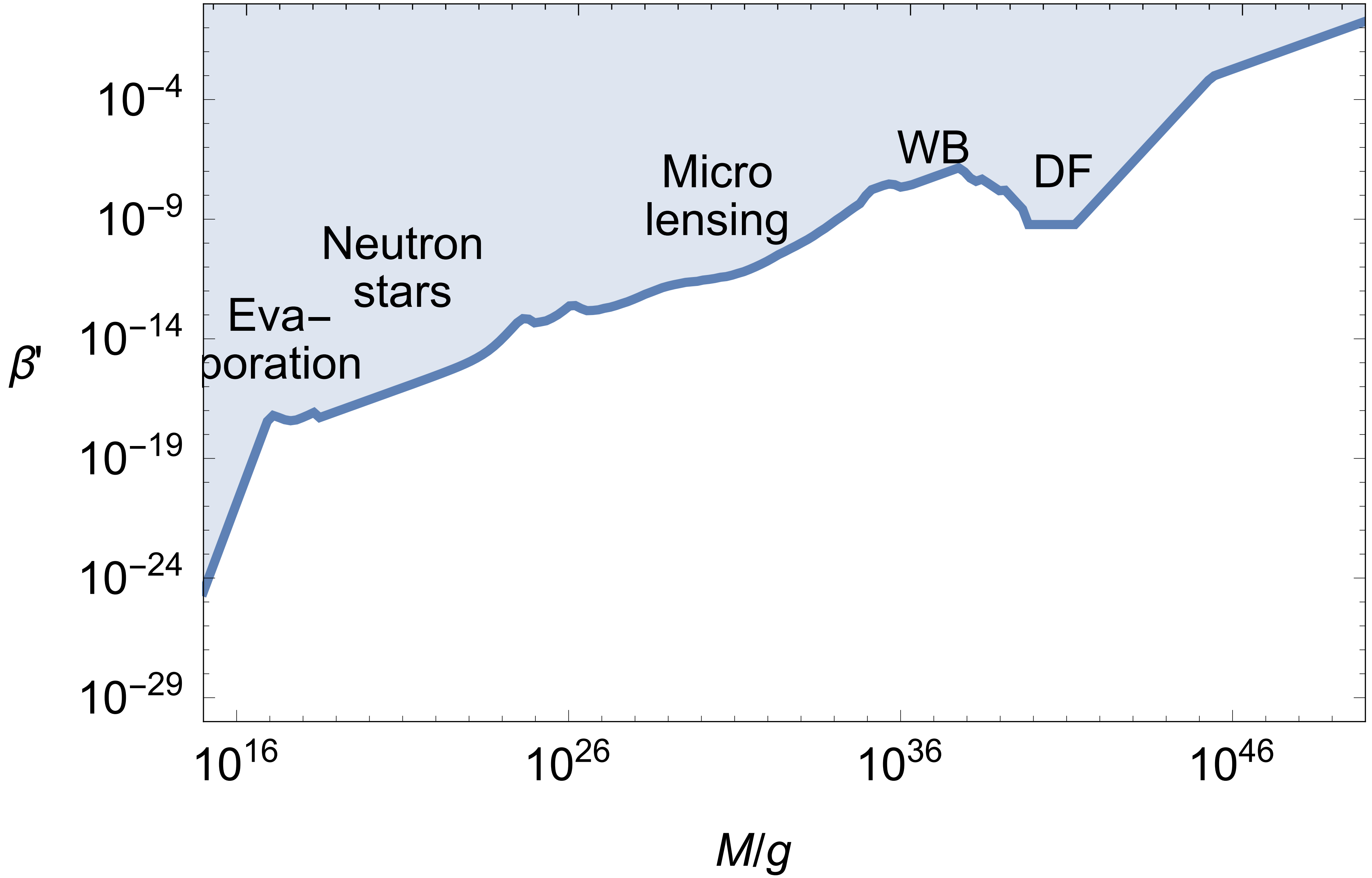
Ignore clustering

Accretion models uncertain

[Carr *et al.* 2017]

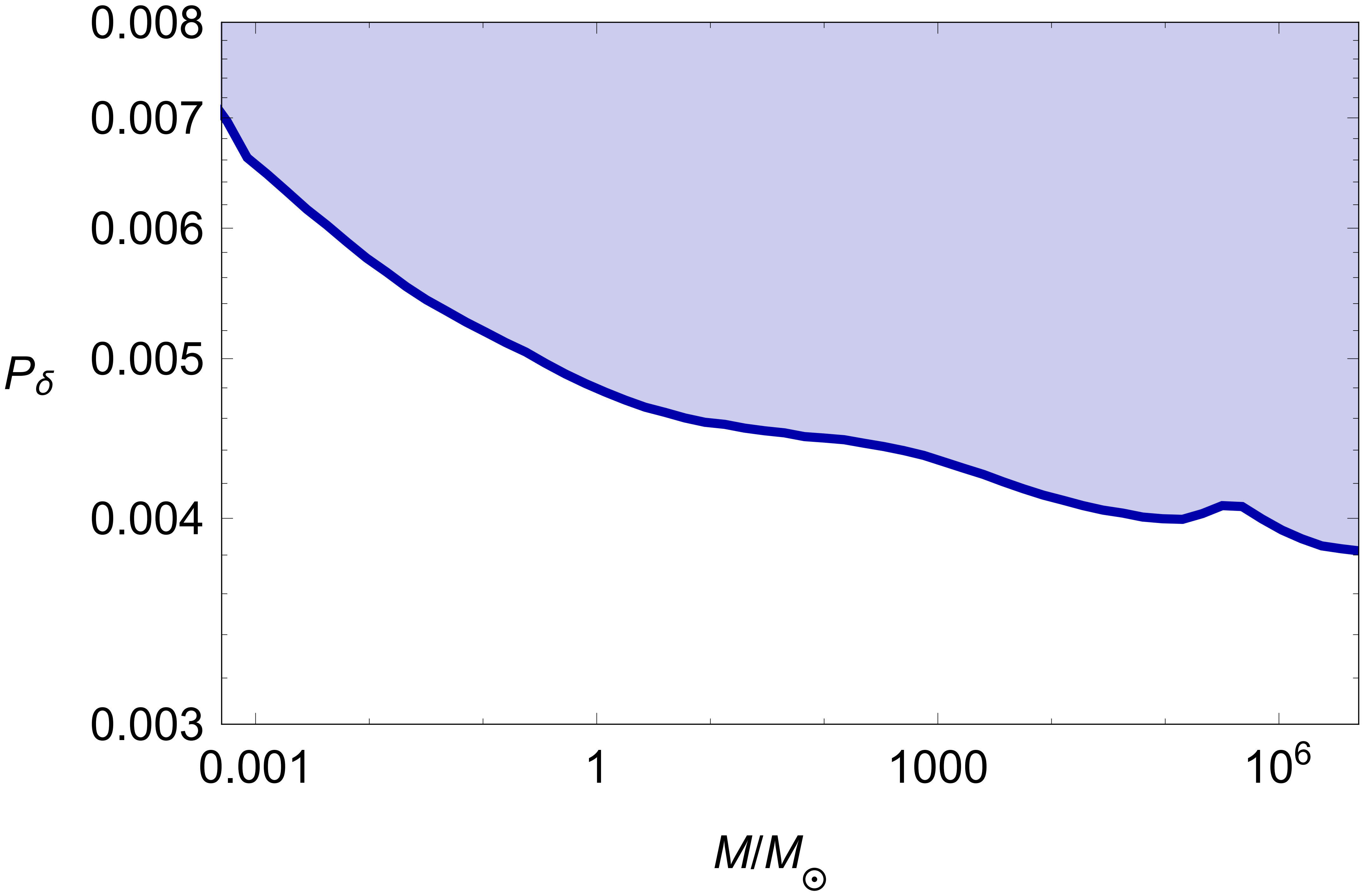
More Words of Caution

- One may wonder how the constraints on the PBH dark-matter fraction constrain the primordial power spectrum.
  - Go back to the constraints at the time of formation:



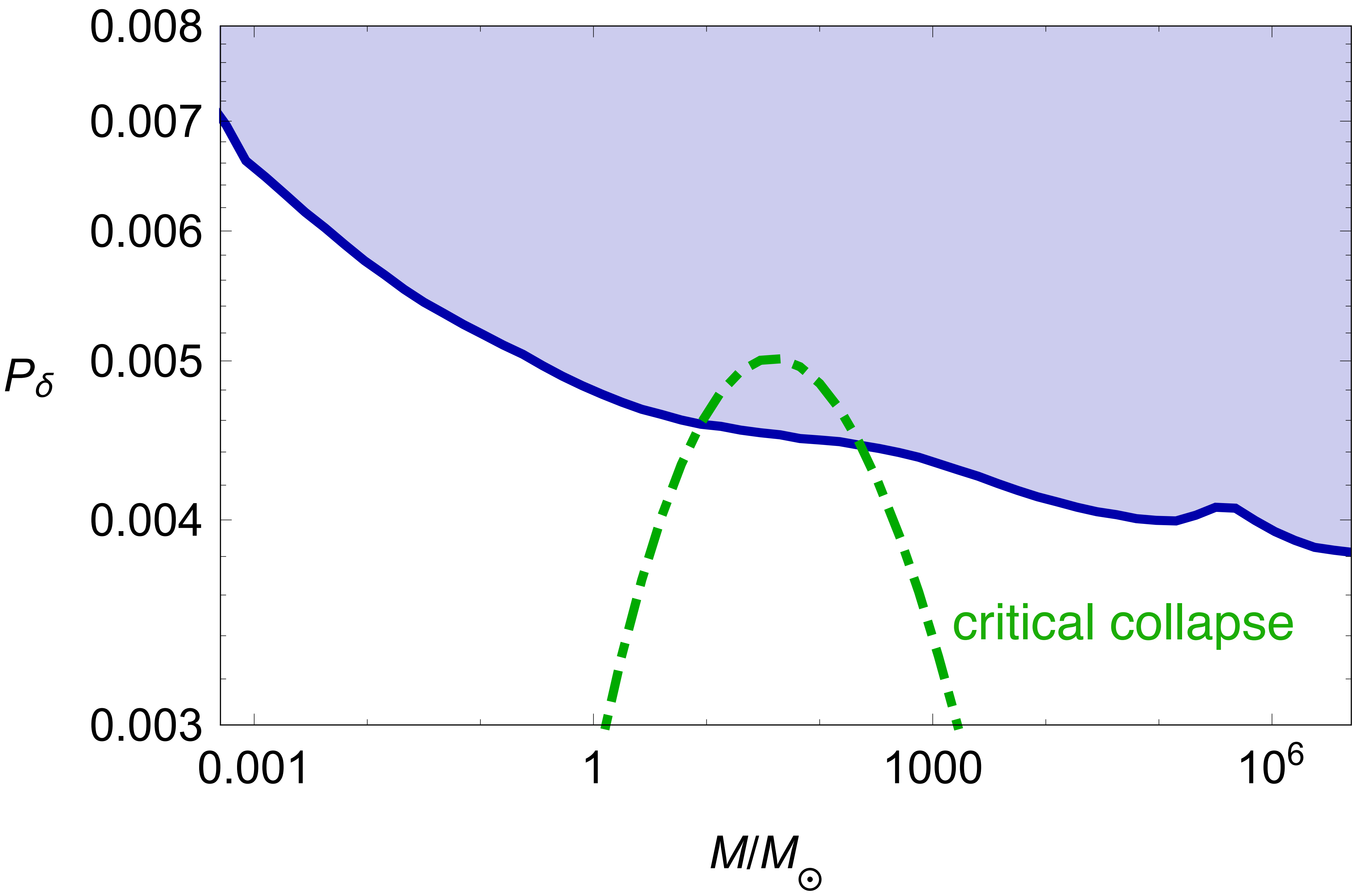
More Words of Caution

- These constraints naïvely translate to:



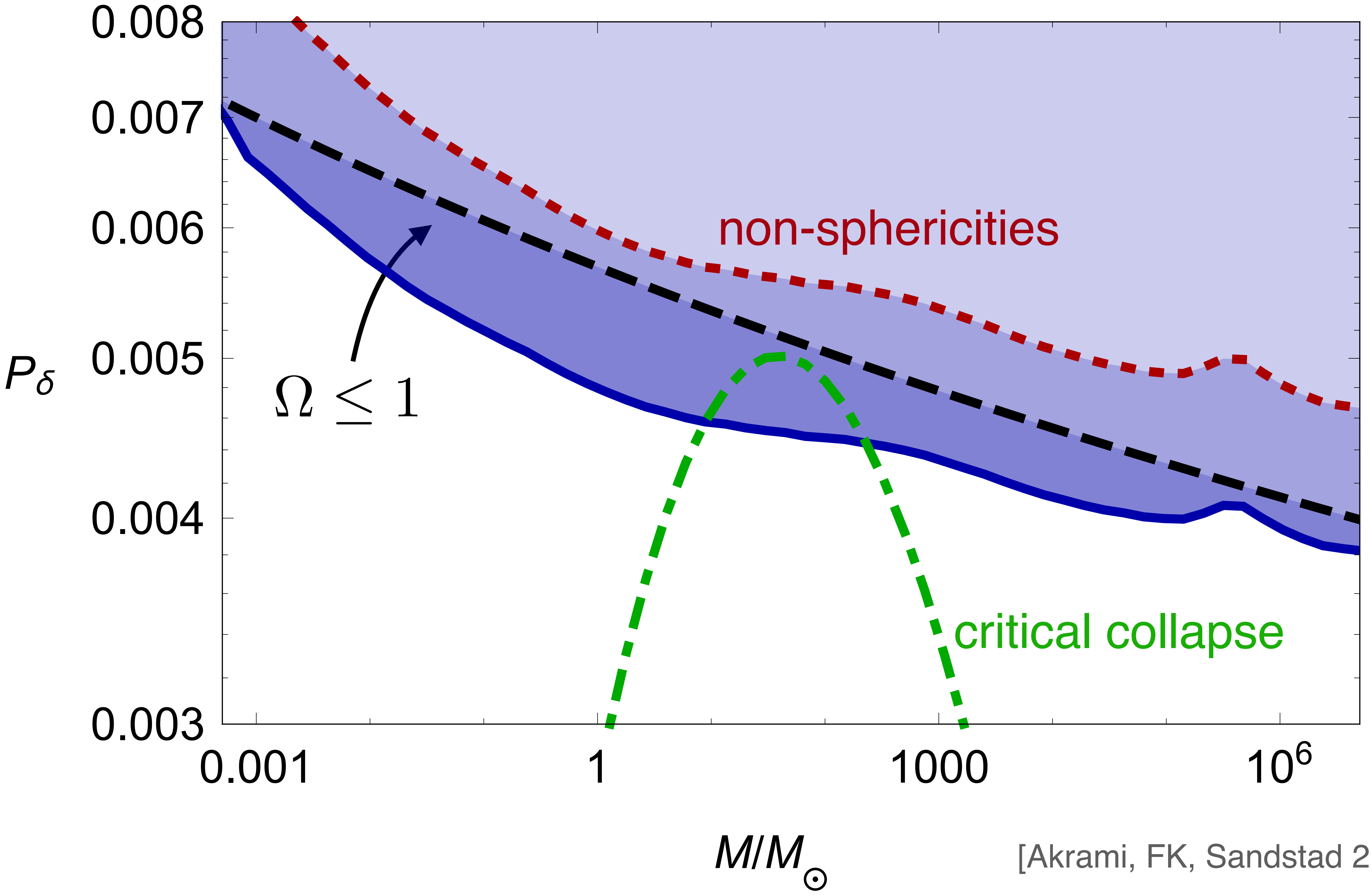
More Words of Caution

- … drawing in the power spectrum of a running-mass model, which is perfectly d’accord with the mentioned constraints:



Constraints on the Primordial Power Spectrum?

★ Moreover, take the uncertainty due to non-sphericities into account:



[Akrami, FK, Sandstad 2017]

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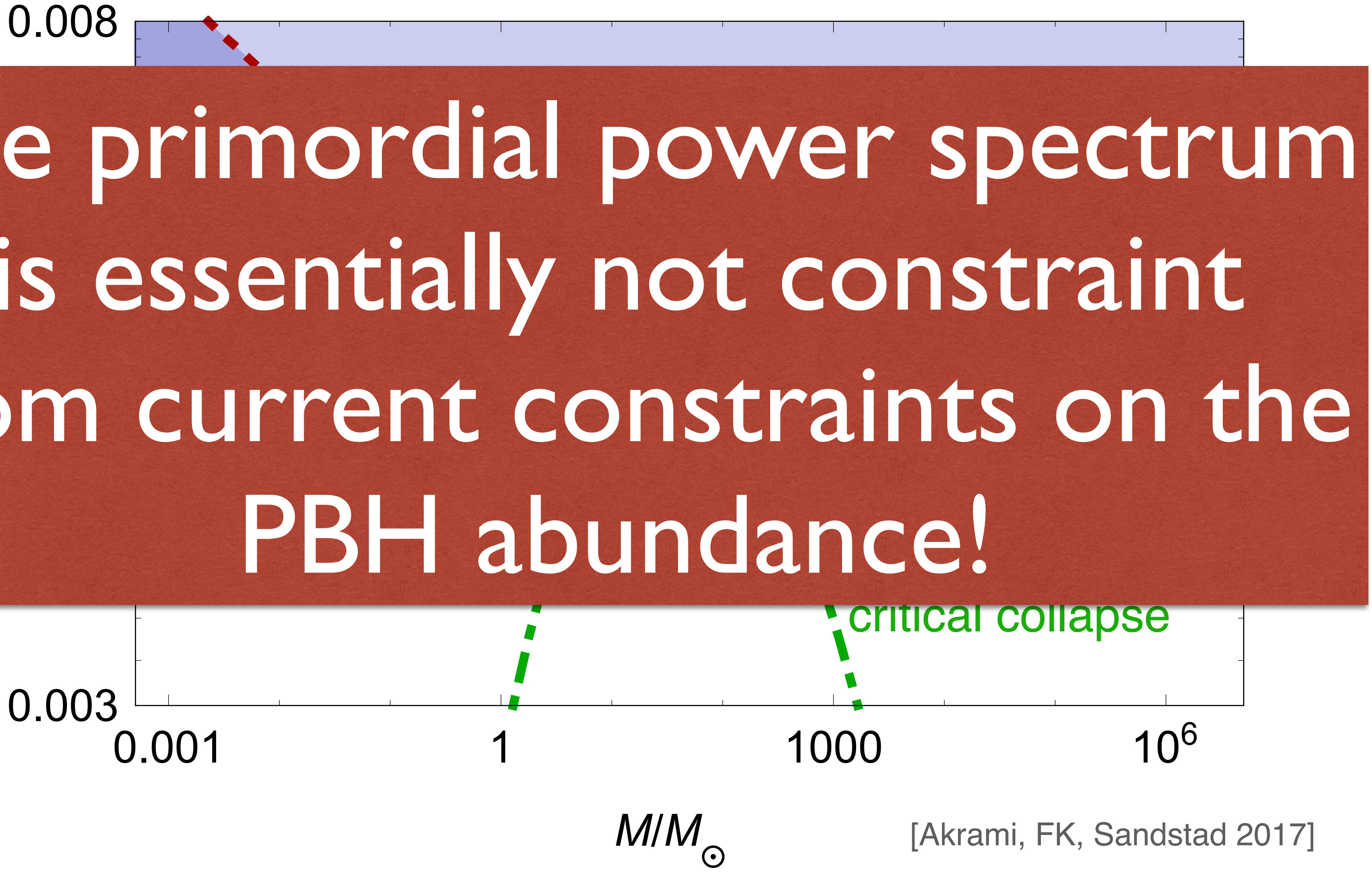

The primordial power spectrum is essentially not constraint from current constraints on the PBH abundance!

[Akrami, FK, Sandstad 2017]

Conclusion

- Primordial black holes are very interesting!
  - They are unique probes of their formation scenarios.
  - They could provide the entire dark matter.
  - A detailed understanding their formation is crucial.
  - Extended mass spectra require special care when comparing to constraints.
  - Most these constraints rely on rather unconfirmed assumptions.
  - LISA might detect PBHs!
  - Also, combined dark-matter scenarios (PBHs + WIMPs or sterile neutrinos) might be well constraint in the near future.