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Non-perturbative production of fermionic DM from fast preheating

Based on 2209.02668 in collaboration with *Andrey Shkerin* and *Georgios Vacalis*

Introduction

- The nature of DM is one of the biggest questions in cosmology
- Wide range of possible *masses* and *interactions*
- DM has to interact *gravitationally* (rotation curves, galaxy clusters, lensing, CMB, large scale structure)
- What if there are no other interactions?
- Would it still be possible to produce the observed DM abundance?
- How would a tiny *inflaton coupling* change this picture?

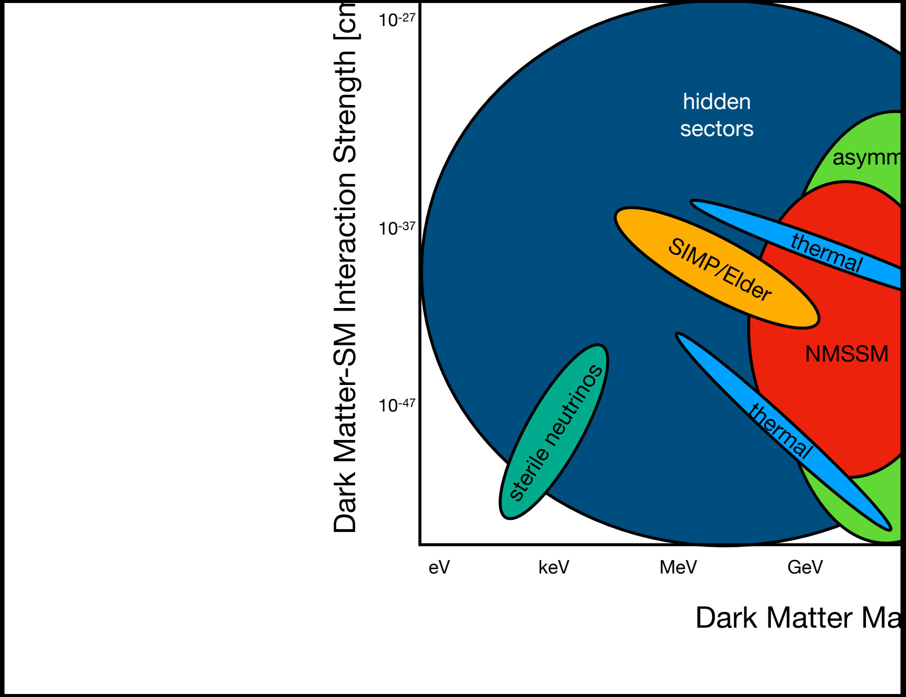


Figure from the Snowmass report [2209.07426]

Gravitational production of DM

- Gravitational mechanism especially important in the case of *non-perturbative* DM production
- Specifically, production is caused by the *varying geometry* of the expanding universe
- We consider the spatially-flat *FLRW* metric in conformal coordinates:

$$\mathrm{d}s^2 = a(\eta)^2(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2)$$

- Starting with the action:

$$S = \int \mathrm{d}^4x\sqrt{-g}\bar{\Psi}\left(i\gamma^\mu \mathcal{D}_\mu - M\right)\Psi$$

- We can substitute an *effective mass*:

$$M_{\text{eff}}(\eta) = M a(\eta)$$

- To obtain the equivalent theory in Minkowski space:

$$S = \int \mathrm{d}^4x\bar{\Psi}\left(i\gamma^\mu \mathcal{D}_\mu - M_{\text{eff}}\right)\Psi$$

Adding a fermion-inflaton coupling

- Besides the varying background, the fermion mass can also change due to the *coupling to the inflaton field*
- e.g. a Yukawa-like coupling:

$$\mathcal{L}_{int.} = c\,\Phi\bar{\Psi}\Psi$$

- Also leads to a *time-varying mass*

$$M(\eta) = M + c\,\Phi(\eta)$$

- The full effective mass is then:

$$M_{\text{eff}} = M(\eta)\,a(\eta)$$

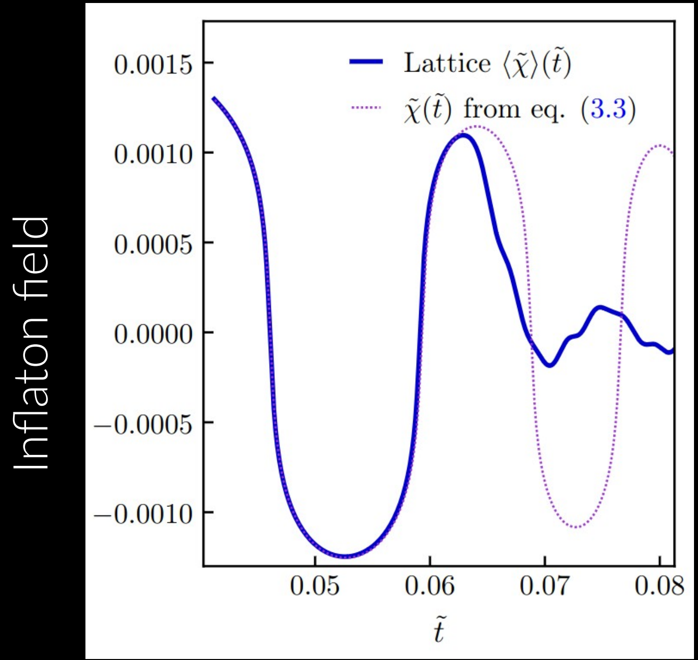
- Therefore, the two effects *cannot be disentangled*!

Instant preheating

- We consider the limiting case where preheating is *fast*, i.e. completes in *less than a Hubble time* $\tau H_{dS} \ll 1$

- For analytic estimates we further assume *“instant” preheating* [Felder/Kofman/Linde hep-ph/9812289] that takes less than one oscillation of the inflaton field

- Such properties are typical for various scenarios, e.g. in *Higgs inflation* [0710.3755, 0803.2664]



- Lattice simulation of preheating in Palatini-Higgs inflation, figure from [Dux/Florio/JK/Shkerin/Timiryasov 2203.13286]

The setup

- We assume a *rapid transition* between the inflationary stage and the radiation dominated stage

$$H_{dS} = a'_{dS}/a^2_{dS}\,,\; a_{dS} = \frac{1}{H_{dS}(\eta_R - \eta)}$$

$$H_{rd} = H_R/a^2_{rd}\,,\; a_{rd} = H_R(\eta_R + \eta)\,,$$

$$H_R = 1.66\sqrt{g_{*,eq}}T^2_{eq}a^2_{eq}/M_P$$

- The fermion *mass can vary* between the initial mass M_i in the inflationary epoch and radiation domination stage M_f

- The transition between the two stages can be *infinitely short*, or have a *finite duration* τ

How to compute the DM abundance?

- The solution of the resulting Dirac Equation in a co-moving volume can be decomposed as:

$$\Psi(\vec{x},\eta) = \frac{1}{\sqrt{V}} \sum_{\vec{k};s=\pm 1} \mathrm{e}^{i\vec{k}\cdot\vec{x}} \left(b_{\vec{k},s}\mathcal{U}_{k,s}(\eta) + d^{\dagger}_{-\vec{k},s}\mathcal{V}_{-k,s}(\eta) \right)$$

where $\mathcal{U}$, $\mathcal{V}$ are the normalized four-component spinors.

- These spinors can be further reduced to a pair of scalar mode functions:

$$\mathcal{U}_{k,s} = \left(i\gamma^0\partial_\eta - \vec{\gamma}\cdot\vec{k} + M_{\mathrm{eff}}(\eta) \right) f_k(\eta) u_s \qquad f_k'' + (k^2 + M_{\mathrm{eff}}^2 - iM_{\mathrm{eff}}')f_k = 0$$

$$\mathcal{V}_{-k,s} = \left(i\gamma^0\partial_\eta - \vec{\gamma}\cdot\vec{k} + M_{\mathrm{eff}}(\eta) \right) g_k(\eta) v_s \qquad g_k'' + (k^2 + M_{\mathrm{eff}}^2 + iM_{\mathrm{eff}}')g_k = 0$$

The in-states, the out-states...

- We require the in-states to be in the *Bunch-Davies vacuum*

$$f_{\mathrm{in},k} \to \mathrm{e}^{-ik\eta}\,, \quad g_{\mathrm{in},k} \to f^*_{\mathrm{in},k}\,, \quad \eta \to -\infty$$

- With an analytic solution in terms of the Hankel function:

$$f_{\mathrm{in},k} = \sqrt{\frac{\pi k(\eta_R - \eta)}{2}}\,\mathrm{e}^{\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)} H^{(1)}_{\nu}(k(\eta_R - \eta))\,,$$

$$\nu = 1/2 + iM_i/H_{dS}$$

- Whereas the out-states satisfy the boundary condition

$$f_{\mathrm{out},k} \to \mathrm{e}^{-i\int^{\eta} \mathrm{d}\eta' \omega_{\eta',k}}\,, \quad g_{\mathrm{out},k} \to f^*_{\mathrm{out},k}\,,$$

$$\eta \gtrsim \eta_*\,, \quad \omega^2_{\eta,k} = k^2 + M_f^2 a_{rd}^2$$

- With the solution in terms of the parabolic cylinder function:

$$f_{\mathrm{out},k}(\eta) = D_{\alpha}(\sqrt{2}\mathrm{e}^{\frac{i\pi}{4}}\sqrt{M_f H_R}(\eta_R + \eta))\,,$$

$$\alpha = -1 - ik^2/(2M_f H_R)$$

… and the Bogoliubov coefficients

- The two sets of solutions need to be *matched* at the time of transition
- The coefficients connecting the two sets of solution are known as the *Bogoliubov coefficients*
- The resulting coefficient is computed by evaluating:

$$|\beta_k|^2 = \frac{1}{2}|\bar{f}_k|^2 \frac{\left|\bar{f}'_k/\bar{f}_k - f'_{\mathrm{out},k}/f_{\mathrm{out},k} - i(M_{\mathrm{in}} - M_{\mathrm{out}})\right|^2}{k^2 + \left|f'_{\mathrm{out},k}/f_{\mathrm{out},k} - iM_{\mathrm{out}}\right|^2}$$

- Which can be used to determine the *particle number* and *energy densities*:

$$n = \frac{2}{\pi^2 a^3}\int_0^\infty dk\; k^2|\beta_k|^2 \qquad \rho = \frac{2}{\pi^2 a^4}\int_0^\infty dk\; k^2\omega_{\eta,k}|\beta_k|^2$$

The limit of infinitely short preheating

- We compute the $|\beta_k|^2$ coefficients in the limit of infinitely short preheating
- *Discontinuities* in the derivatives of $M_{\text{eff}}(\eta)$ lead to *power-like high-momentum asymptotics*

$$|\beta_k|^2 \propto k^{-N}$$

- To extract the asymptotics we consider two relevant momentum scales:

$$k_1 = \sqrt{MH_R}$$

separates *relativistic and non-relativistic* modes at $H = M$

$$k_2 = \sqrt{H_{dS}H_R}$$

corresponds to the *size of the horizon* at the end of inflation

- These momentum scales determine the final shape of the DM spectrum

Constant mass: Asymptotic shape of the DM spectrum

- Qualitatively we find three regimes

$$|\beta_k|^2_{k \lesssim k_1} = \frac{1}{2}$$

$$|\beta_k|^2_{k_1 \lesssim k \lesssim k_2} = \frac{k_1^4}{16k^4}$$

$$|\beta_k|^2_{k_2 \lesssim k} = \frac{k_1^4 k_2^2}{16k^6}$$

for $k_1 \ll k_2$

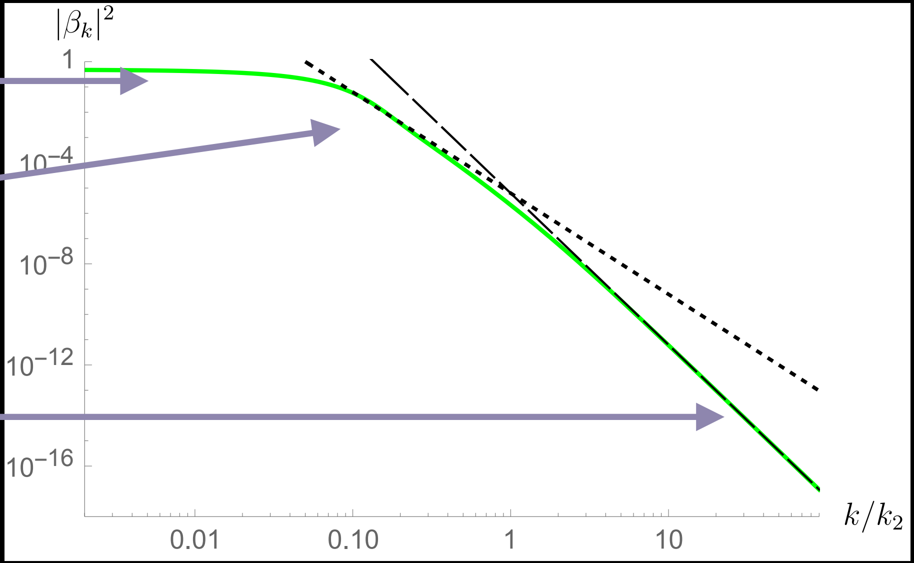


Figure from [JK/Shkerin/Vacalis 2203.13286]

Constant mass: DM abundance

- Based on these asymptotics we can estimate the final abundance of DM in the case when the mass is constant

$$n = \frac{0.2 M^{3/2} H_R^{3/2}}{\pi^2 a^3}\ , \quad \rho = M n$$

- This gives the DM abundance *consistent with previous studies* of gravitational production:

$$\frac{\Omega_\Psi}{\Omega_{DM}} = \left(\frac{M}{1.9 \cdot 10^8\ \text{GeV}}\right)^{5/2}$$

See e.g. [1109.2524, 2005.00391]

Varying mass due to inflaton coupling

- In the more general case we have $M_i \neq M_f$
- The fermion mass during inflation is induced by the inflaton background, so we generically expect that $M_i \gg M_f$
- The final mass can therefore largely be neglected in the computations, and only enters when computing the final non-relativistic energy density
- To account for the varying mass

$$k_1 = \sqrt{M_f H_R}$$

- We also notice the appearance of another scale

$$k_3 = (M_i - M_f)\sqrt{\frac{H_R}{H_{dS}}}$$

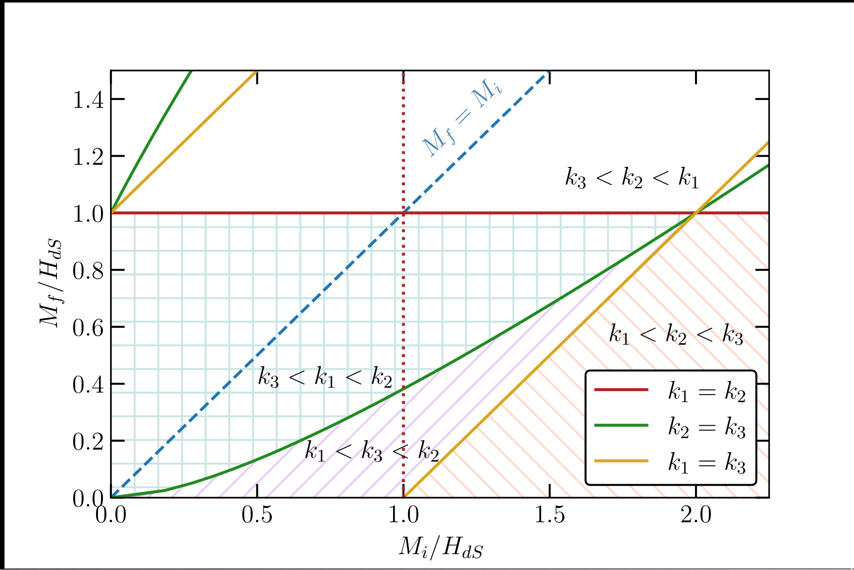


Figure from [JK/Shkerin/Vacalis 2203.13286]

Varying mass I: Asymptotic shape of the DM spectrum

- The IR asymptotics match the result without mass variation
- Irrespective of the ordering of the scales, we find that the UV asymptotics is the same:

$$|\beta_k|^2 = \frac{1}{2}|\bar{f}_k|^2 \frac{\left|\cancel{\bar{f}'_k/\bar{f}_k - f'_{\text{out},k}/f_{\text{out},k}} - i(M_{\text{in}} - M_{\text{out}})\right|^2}{k^2 + \left|\cancel{f'_{\text{out},k}/f_{\text{out},k}} - iM_{\text{out}}\right|^2}$$

$$|\beta_k|^2_{k \gtrsim \max(k_2, k_3)} = \frac{k_3^2}{4k^2}$$

$$n = \frac{2}{\pi^2 a^3} \int_0^\infty dk\, k^2 |\beta_k|^2$$

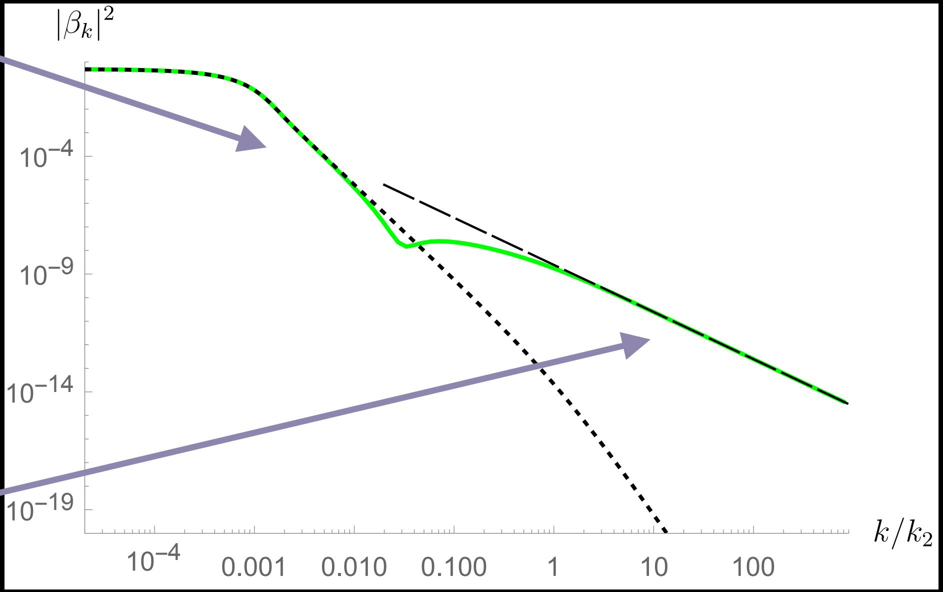


Figure from [JK/Shkerin/Vacalis 2203.13286]

Crucially, the order of the UV singularity changes – the DM abundance diverges!

Fast preheating

- The apparent divergence of the dark matter abundance is regulated by the finite *duration of preheating* τ
- We model the preheating period by interpolating between the initial and final masses:

$$M = \frac{M_i + M_f}{2} + \frac{M_f - M_i}{2} \,\mathrm{th}\, \frac{\eta}{\tau_\eta}$$

- With the conformal preheating time:

$$\tau_\eta = \tau\sqrt{H_{dS}/H_R}$$

- The analytic solutions are no longer valid
- We solve the *mode equation numerically*
- If we further assume that preheating is fast:

$$\tau_\eta \ll k_2^{-1} \rightarrow \tau H_{dS} \ll 1$$

- We can also analytically solve the *intermediate preheating period*
- Effectively, we patch three solutions:
  - Inflation in dS, before the start of preheating
  - Preheating effectively in Minkowski
  - Radiation domination, after preheating

Varying mass II: Asymptotic shape of the DM spectrum

- For $\tau_\eta^{-1} \gg k_3 \to \tau^{-1} \gg M_i$
- IR modes still have the solution corresponding to $\tau \to 0$
- UV modes are described by the Minkowski solution:

$$|\beta_k|^2_{k \gtrsim \max(k_2, k_3)} = \frac{\sinh^2\left(\frac{\pi \tau_\eta k_3}{2}\right)}{\sinh^2(\pi \tau_\eta k)}$$

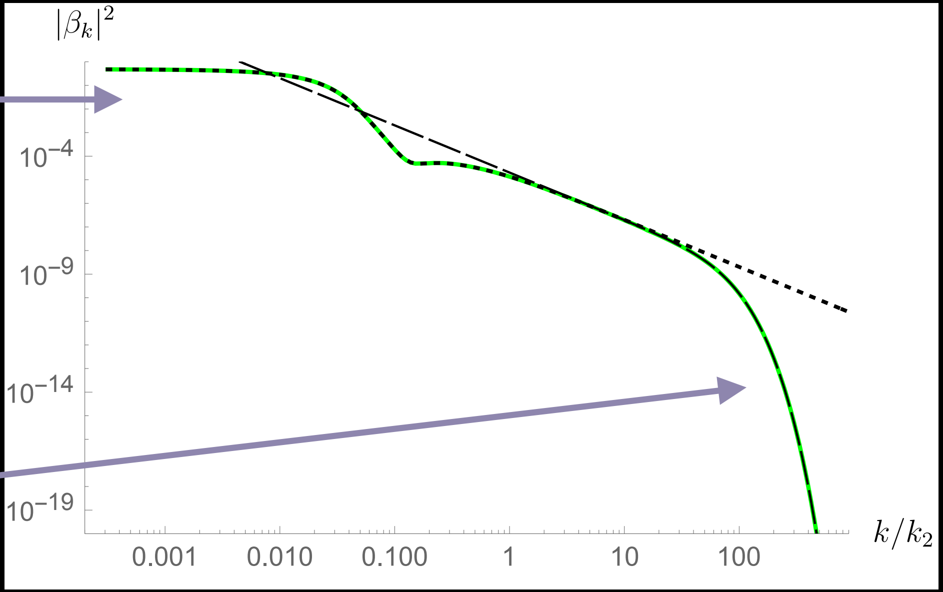


Figure from [JK/Shkerin/Vacalis 2203.13286]

Most importantly, the produced DM abundance is finite!

Varying mass II: DM abundance

- Which leads to the DM number and energy density inversely proportional to the duration of preheating:

$$n = \frac{k_3^2}{12\pi\tau_\eta a^3}\ , \quad \rho = M_f n$$

- With the DM abundance given by:

$$\frac{\Omega_\Psi}{\Omega_{DM}} = \frac{0.3}{\tau H_{dS}}\left(\frac{H_{dS}}{10^{10}\ \mathrm{GeV}}\right)^{-1/2}\left(\frac{M_i}{10^{10}\ \mathrm{GeV}}\right)^2\frac{M_f}{10^5\ \mathrm{GeV}}$$

Varying mass III: The superheavy case

- In the limit:

$$k_2 \ll \tau_\eta^{-1} \lesssim k_3 \to \tau^{-1} \ll M_i$$

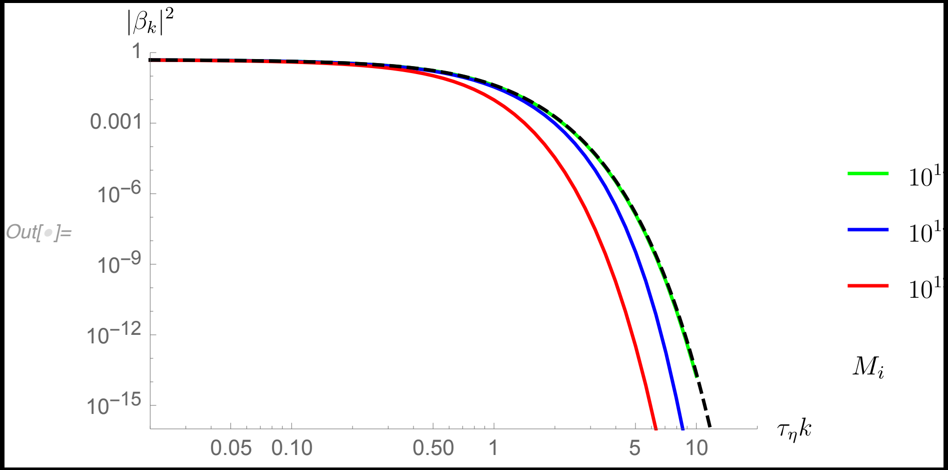
- The dependence on the initial mass drops out and we find a Fermi-Dirac like spectrum:

$$|\beta_k|^2_{k_2 \ll k \ll k_3} = \frac{1}{1 + e^{\pi \tau_\eta k}}$$

- And an effective temperature:

$$T_{eff} = \frac{1}{\pi \tau}$$

Figure from [JK/Shkerin/Vacalis 2203.13286]

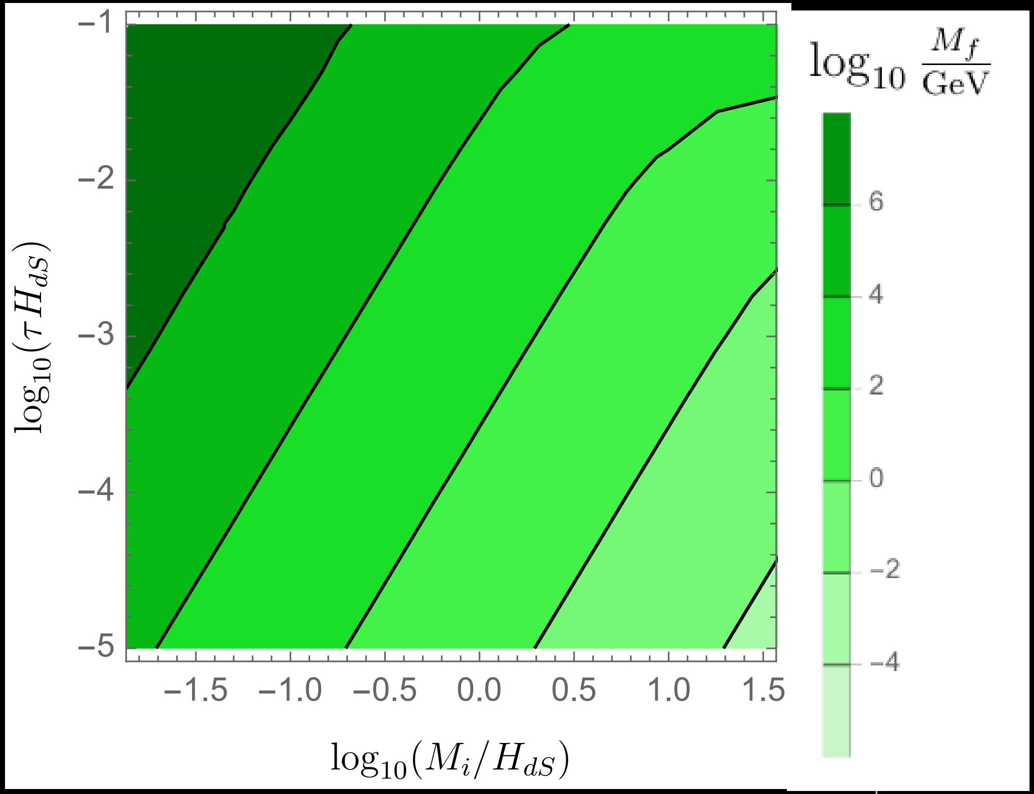


With a DM abundance:

$$\frac{\Omega_\Psi}{\Omega_{DM}} = \frac{0.1}{(\tau H_{dS})^3} \left(\frac{H_{dS}}{10^{10}\ \mathrm{GeV}} \right)^{3/2} \frac{M_f}{10^5\ \mathrm{GeV}}$$

Varying mass: putting everything together

- The production mechanism can give a wide range of possible DM masses
- For fixed preheating parameters, the DM mass is completely fixed



Application to Palatini-Higgs inflation

- The action for the fermion field is given by:

$$S = \int \mathrm{d}^4x \sqrt{-g} \left\{ -\frac{1}{2}(M_P^2 + \xi\varphi^2) g^{\mu\nu} R_{\mu\nu}(\Gamma, \partial\Gamma) + \frac{1}{2} g^{\mu\nu} \partial_\mu\varphi \partial_\nu\varphi - \frac{\lambda\varphi^4}{4} \right.$$
$$\left. + i\bar{\Psi}_R \gamma^\mu \mathcal{D}_\mu \Psi_R - \frac{M_R}{2} \bar{\Psi}_R^C \Psi_R - \frac{c_5}{2\Lambda} \varphi^2 \bar{\Psi}_R^C \Psi_R + \text{h.c.} \right\}$$

- The production from the Yukawa term is too small for couplings compatible with observational bounds
- We include a dimension-5 operator instead

- The a cutoff is chosen to match the one of the gravitational sector:

$$\Lambda(\varphi) = \sqrt{M_P^2 + \xi\varphi^2}$$

- For a fixed value of the Higgs field at the end of inflation: $\varphi_e \approx M_P$

with: $\xi = 10^7$, $\lambda = 10^{-3}$

- The initial and final masses can be read off:

$$M_i = \frac{c_5 M_P}{\xi} \, , \quad M_f = M_R$$

DM abundance in Palatini-Higgs inflation

- The duration of preheating can be taken from the lattice computation:

$$\tau = \frac{2\tilde{t}\xi}{\sqrt{\lambda}M_P}\,, \quad \text{with} \quad \tilde{t} = 0.14$$

[Dux/Florio/JK/Shkerin/Timiryasov 2203.13286]

- Which leads to the DM abundance:

$$\frac{\Omega_\Psi}{\Omega_{DM}} \sim 3 \cdot 10^9 \frac{c_5^2}{\lambda^{1/4}\xi^{3/2}} \frac{M_R}{1\ \text{GeV}}$$

$$\sim \left(\frac{c_5}{10^{-1}}\right)^2 \frac{M_R}{100\ \text{GeV}}$$

- This is of the ***same order of magnitude*** as the abundance obtained through ***thermal production***:

$$\frac{\Omega_\Psi}{\Omega_{DM}} = \left(\frac{c_5}{10^{-1}}\right)^2 \frac{M_R}{176\ \text{GeV}}$$

From [Bezrukov/Gorbunov/Shaposhnikov 0812.3622]

Conclusions

- Non-perturbative processes offer an exciting way to produce DM
- In the absence of additional coupling we confirm that the fermions are produced abundantly and can account for *all observed DM for M=10^8 GeV*
- In the case where a *fermion-inflaton* coupling is included, the effective mass varies by many orders of magnitude, *greatly enhancing the production* mechanism
- We find that the produced DM *abundance depends on the duration of preheating*
- For *superheavy* initial *masses* we find a *thermal spectrum* with a temperature determined by the duration of preheating
- We illustrated these results on the specific example of *sterile neutrino* DM in the framework of *Palatini Higgs inflation*, and find that this non-perturbative mechanism is *same order of magnitude as the thermal production*

Thank you!