Multi-phase Criticality of Dynamical Symmetry Breaking

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Dynamical Symmetry Breaking

- Scale-invariant models – a possible solution to the hierarchy problem, interesting for inflation, cosmological gravitational wave background
- Coleman-Weinberg mechanism: spontaneous symmetry breaking by quantum corrections

  Coleman & Weinberg, Phys. Rev. D7, 1888 (1973)
- Gildener-Weinberg approach to multi-field models: effective potential generated by quantum corrections in the flat direction

  Gildener & Weinberg, Phys. Rev. D13, 3333 (1976)

Dynamical Symmetry Breaking

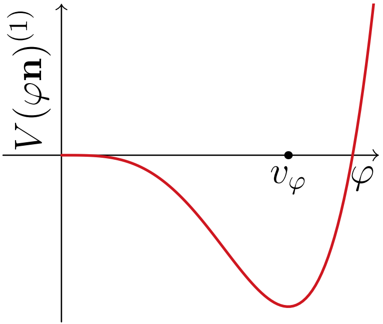
- Action classically scale invariant
  (zero mass and cubic terms)
  See e.g. Alexander-Nunneley & Pilaftsis,
  JHEP 09 (2010) 021 [arXiv:1006.5916]
- VEVs & masses generated
  via quantum corrections
- Does not work in the Standard Model
  due to a large top Yukawa coupling
- Needs new physics: e.g. a new scalar singlet

4 Dynamical Symmetry Breaking

- At one-loop level, the potential is

$$\mathrm{V}(\Phi) = \mathrm{V}^{(0)} + \mathrm{V}^{(1)}$$

- In a flat direction $\Phi = \varphi \mathrm{n}$, we have $\mathrm{V}(\Phi) = 0$
- $\mathrm{V}^{(1)}$ dominates in that direction



Our Work

- K.K., Marzola, Raidal, Strumia, *Light Higgs boson from multi-phase criticality in dynamical symmetry breaking*, Phys.Lett.B 816 (2021) 136241 [arXiv:2102.01084]
- K.K., Loos, Marzola, *Minima of classically scale-invariant potentials*, JHEP 06 (2021) 128 [arXiv:2011.12304]
- K.K., Kubarski, Marzola, *Geometry of Flat Directions in Scale-Invariant Potentials*, Phys.Rev.D 99 (2019) 11, 115034 [arXiv:1904.07867]

6 Field Content & Potential

- Vector of n real scalar fields

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_n \end{pmatrix}$$

- Tree-level potential $V^{(0)}$
- Fermions with Yukawa couplings to Φ
- Gauge bosons coupled to Φ via its covariant derivative

7 Biquadratic Potential

$$V^{(0)} = \sum_{i,j} \phi_i^2 \lambda_{ij} \phi_j^2 = \left(\Phi^{\circ 2}\right)^T \Lambda \Phi^{\circ 2}$$

- Hadamard product $(\mathbf{A} \circ \mathbf{B})_{ij} = A_{ij} B_{ij}$
- Hadamard power $(\mathbf{A}^{\circ n})_{ij} = A_{ij}^n$,
  e.g. $(\mathbf{\Phi}^{\circ 2})_i = (\mathbf{\Phi} \circ \mathbf{\Phi})_i = \phi_i^2$
- Potential symmetry is

$$\mathbb{Z}_2^n = (\mathbb{Z}_2)_1 \times \cdots \times (\mathbb{Z}_2)_n,$$

under which $\phi_i \to -\phi_i$

8 Hadamard Product Example

$$
\begin{aligned}
\left(\nabla_{\Phi} V^{(0)}\right)_k &= \frac{\partial}{\partial \phi_k} \sum_{i,j} \phi_i^2 \lambda_{ij} \phi_j^2 \\
&= 4\phi_k \sum_j \lambda_{kj} \phi_j^2
\end{aligned}
$$

$$
\nabla_{\Phi} V^{(0)} = 4\Phi \circ \Lambda \Phi^{\circ 2}
$$

9 Biquadratic Potential

$$
\begin{aligned}
V &= \lambda_H |H|^4 + \lambda_{HS} |H|^2 \frac{s^2}{2} + \lambda_S \frac{s^4}{4} \\
&= \frac{1}{4}\lambda_H h^4 + \frac{1}{4}\lambda_{HS} h^2 s^2 + \frac{1}{4}\lambda_S s^4 \\
&= \left(\Phi^{\circ 2}\right)^T \Lambda \Phi^{\circ 2}
\end{aligned}
$$

where

$$
\Phi = \begin{pmatrix} h \\ s \end{pmatrix}, \quad \Lambda = \frac{1}{4} \begin{pmatrix} \lambda_H & \frac{1}{2}\lambda_{HS} \\ \frac{1}{2}\lambda_{HS} & \lambda_S \end{pmatrix}
$$

10 Biquadratic Potential

$$
\begin{aligned}
V &\approx \frac{1}{4}\lambda_H(t)h^4 + \frac{1}{4}\lambda_{HS}(t)h^2s^2 + \frac{1}{4}\lambda_S(t)s^4 \\
&= \left(\Phi^{\circ 2}\right)^T \Lambda(t)\Phi^{\circ 2}
\end{aligned}
$$

where

$$
\Phi = \begin{pmatrix} h \\ s \end{pmatrix}, \quad \Lambda(t) = \frac{1}{4}\begin{pmatrix} \lambda_H(t) & \frac{1}{2}\lambda_{HS}(t) \\ \frac{1}{2}\lambda_{HS}(t) & \lambda_S(t) \end{pmatrix}
$$

- Couplings run as

$$
\frac{d\lambda_i}{dt} = \beta_{\lambda_i}
$$

11 Vacuum Stability

- In order to have a finite minimum,
  the potential must be bounded from below
- $V > 0$ if Λ is copositive, i.e. $(\Phi^{\circ 2})^T \Lambda \Phi^{\circ 2}$ for $\Phi^{\circ 2} \geqslant 0$

  K. Kannike, Eur. Phys. J. C72, 2093 (2012) [arXiv:1205.3781]
- Potential bounded from below at large scales
  is a necessary condition for a finite minimum to exist
- In a finite range, the running $\Lambda(t)$ must *violate*
  the conditions to have a radiative minimum

12 Flat Direction(s)

- In the minimum $V < 0$
- For large fields $V > 0$
- Somewhere in between, $V = 0$
- *Each* minimum has a related flat direction

13 Vacuum Stability Conditions

Cottle-Habetler-Lemke theorem:
Suppose that the order $n-1$ principal submatrices of a real symmetric matrix $\mathbf{A}$ of order n are copositive.
Then $\mathbf{A}$ is copositive if and only if

$$\det(\mathbf{A}) \geqslant 0 \quad \vee \quad \text{some element(s) of } \operatorname{adj}(\mathbf{A}) < 0.$$

- The adjugate $\operatorname{adj}(\mathbf{A})$ defined by $\mathbf{A}\operatorname{adj}(\mathbf{A}) = \det(\mathbf{A})\,\mathbf{I}$
- $\mathbf{A}^{-1} = \dfrac{\operatorname{adj}(\mathbf{A})}{\det(\mathbf{A})}$ if $\det(\mathbf{A}) \neq 0$

14 Vacuum Stability

$$\Lambda = \begin{pmatrix} \lambda_H & \frac{1}{2}\lambda_{HS} \\ \frac{1}{2}\lambda_{HS} & \lambda_S \end{pmatrix}$$

- Self-couplings $\lambda_H \geqslant 0$ and $\lambda_S \geqslant 0$
- We have

$$\det(\Lambda) = \lambda_H \lambda_S - \frac{1}{4}\lambda_{HS}^2, \qquad \operatorname{adj}(\Lambda) = \begin{pmatrix} \lambda_H & -\frac{1}{2}\lambda_{HS} \\ -\frac{1}{2}\lambda_{HS} & \lambda_S \end{pmatrix}$$

- Therefore

$$\det(\Lambda) \geqslant 0 \quad \vee \quad -\lambda_{HS} < 0,$$

equivalent to

$$\lambda_{HS} + 2\sqrt{\lambda_H \lambda_S} \geqslant 0$$

Phases

s) $s \neq 0$ and $h = 0$ arises when the critical boundary

$$\lambda_S = 0, \quad \beta_{\lambda_S} > 0$$

is crossed ($\lambda_{HS} > 0$ to give $m_h^2 > 0$)

h) $h \neq 0$ and $s = 0$ arises when

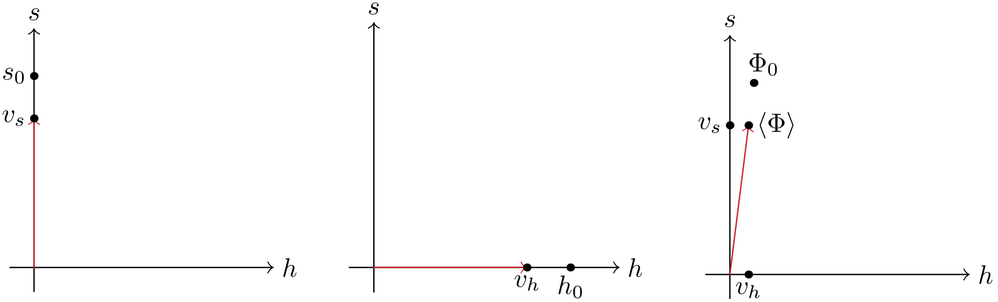
$$\lambda_H = 0, \quad \beta_{\lambda_H} > 0$$

sh) $s, h \neq 0$ arises when

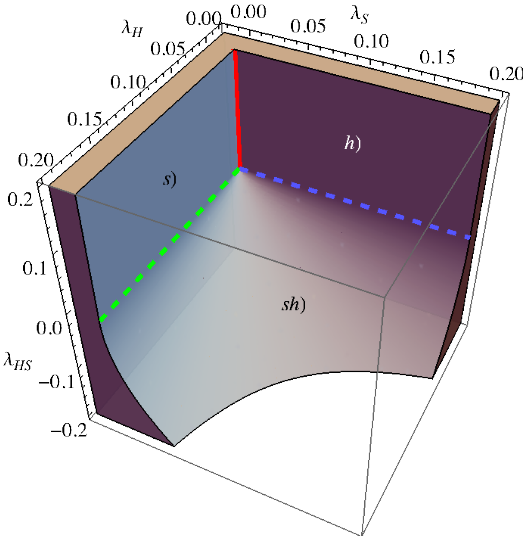
$$2\sqrt{\lambda_H \lambda_S} + \lambda_{HS} = 0, \quad \lambda_S \beta_{\lambda_H} + \lambda_H \beta_{\lambda_S} - \frac{1}{2}\lambda_{HS}\beta_{\lambda_{HS}} > 0$$

is crossed; flat direction given by $s/h = (\lambda_H/\lambda_S)^{1/4}$

16 Phases



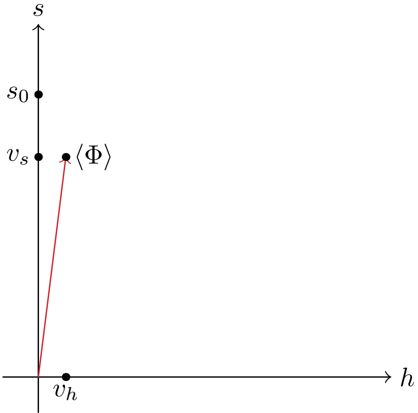
17 Phases



λ_H
0.00
0.05
0.10
0.15
0.20
0.00
0.05
λ_S
0.10
0.15
0.20
0.2
0.1
0.0
λ_{HS}
−0.1
−0.2
$s)$
$h)$
$sh)$

18 Multi-phase criticality

- $\lambda_S(t_0) = \lambda_{HS}(t_0) = 0$
- The s and sh flat directions coincide on the s-axis: a 'double' flat direction
- Minimum is *off* axis



19 Effective Potential

At one-loop level,

$$V = V^{(0)} + V^{(1)},$$

where

$$V^{(1)} = \frac{1}{64\pi^2}\left\{\mathrm{tr}\left[\mathbf{m}_S^4\left(\ln\frac{\mathbf{m}_S^2}{\mu^2} - \frac{3}{2}\right)\right] + 3\,\mathrm{tr}\left[\mathbf{m}_V^4\left(\ln\frac{\mathbf{m}_V^2}{\mu^2} - \frac{5}{6}\right)\right] - 4\,\mathrm{tr}\left[\mathbf{m}_F^4\left(\ln\frac{\mathbf{m}_F^2}{\mu^2} - \frac{3}{2}\right)\right]\right\}$$

with dimensional regularisation in the $\overline{\mathrm{MS}}$ scheme

20 Effective Potential

The one-loop potential can be written as

$$V^{(1)} = \frac{1}{64\pi^2} \operatorname{Str} m^4 \left(\ln \frac{m^2}{\mu^2} - c \right) = \mathbb{A} + \mathbb{B} \ln \frac{\mathcal{M}^2}{\mu^2},$$

where

$$\mathbb{A} = \frac{1}{64\pi^2} \operatorname{Str} M^4 \left(\ln \frac{M^2}{\mathcal{M}^2} - c \right),$$

$$\mathbb{B} = \frac{1}{64\pi^2} \operatorname{Str} M^4$$

One-Loop Potential

- $\mathcal{M}$ is a 'pivot scale' with the dimension of mass
  Chataigner, Prokopec, Schmidt & Świeżewska,
  JHEP 08 (2018) 083 [arXiv:1805.09292]
- We choose $\mathcal{M}^2 = \mathrm{e}_{\mathcal{M}}^{\mathrm{T}} \Phi^{\circ 2}$,
  where $\mathrm{e}_{\mathcal{M}}$ is a constant vector
- Radial coordinate $\varphi^2 \equiv \Phi^{\mathrm{T}} \Phi = \mathrm{e}^{\mathrm{T}} \Phi^{\circ 2}$,
  where vector $\mathrm{e} = \begin{pmatrix} 1 & \dots & 1 \end{pmatrix}^{\mathrm{T}}$

Running

Callan-Szymanzik equation tells that

$$\frac{\mathrm{d}V^{(0)}}{\mathrm{d}t} = (4\pi)^2\mathbb{B} = (\Phi^{\circ 2})^{\mathrm{T}}\beta\Phi^{\circ 2} - \Phi^{\mathrm{T}}\gamma\nabla_{\Phi}V^{(0)}$$

where

$$\beta = \frac{\mathrm{d}\Lambda}{\mathrm{d}t}, \qquad \mathrm{t} = \frac{\ln(\mu^2/\mu_0^2)}{(4\pi)^2}$$

23 Running

- Set $\mu = \mathcal{M}$, so that $\mathcal{M}_0 = \mu_0$
- The effective potential becomes

$$V(t) = V^{(0)}(t) + \mathbb{A}$$

- The running parameter is now also $\mathcal{M}$-dependent:

$$t = \frac{1}{(4\pi)^2} \ln \frac{\mathcal{M}^2}{\mathcal{M}_0^2}$$

24 Minimisation Equation

The stationary point equation is

$$0 = \nabla_\Phi V = 4\Phi \circ \Lambda(t)\Phi^{\circ 2} + \nabla_\Phi \mathbb{A} + \frac{dV}{dt}\nabla_\Phi t$$

We have $\nabla_\Phi \mathcal{M}^2 = \nabla_\Phi e_\mathcal{M}^T \Phi^{\circ 2} = 2e_\mathcal{M} \circ \Phi$ and consequently

$$\nabla_\Phi t = \frac{2}{(4\pi)^2}\frac{1}{\mathcal{M}^2} e_\mathcal{M} \circ \Phi$$

25 Radial Minimisation Equation

Project along the field vector:

$$
\begin{aligned}
0 &= \Phi^{T} \nabla_{\Phi} V \\
&= 4V^{(0)}(t) + 4\mathbb{A} + 2\mathbb{B} \\
&= 4V + 2\mathbb{B},
\end{aligned}
$$

where we used $\Phi^{T} \nabla_{\Phi} \mathbb{A} = 4\mathbb{A}$, which holds because $\mathbb{A}$ is a homogenous function of order four.

26 Radial Minimisation Equation

- Let $V(t_0) = 0$ at a scale t_0
- $\mathbb{B} > 0$ so the potential is bounded from below at higher scales
- Expanding to linear order in t,

$$0 \approx 4\left(V(t_0) + \frac{dV^{(0)}}{dt}t\right) + 2\mathbb{B}$$

  at the minimum
- Using $\frac{dV^{(0)}}{dt} = (4\pi)^2\mathbb{B}$, we recover the GW relation $\mathcal{M}^2 = e^{-1/2}\mathcal{M}_0^2$, so $t = t_0 - 1/[2(4\pi)^2]$

27 Minimisation Equation

$$0 = \Phi \circ \left(4\Lambda(t)\Phi^{\circ 2} + 2\mathbb{B}\frac{e_{\mathcal{M}}}{\mathcal{M}^2} \right),$$

$$\Lambda(t)\Phi^{\circ 2} = \frac{V}{\mathcal{M}^2} e_{\mathcal{M}},$$

solved by

$$\Phi^{\circ 2} = \frac{1}{\det(\Lambda(t))} \frac{V}{\mathcal{M}^2} \operatorname{adj}(\Lambda(t)) e_{\mathcal{M}}.$$

Approximating V by $V^{(0)}$ this becomes

$$\Phi^{\circ 2} = \frac{\mathcal{M}^2}{e_{\mathcal{M}}^T \operatorname{adj}(\Lambda(t)) e_{\mathcal{M}}} \operatorname{adj}(\Lambda(t)) e_{\mathcal{M}}.$$

Mass Matrix

The mass matrix around the minimum is then given by

$$\begin{aligned} m_S^2 &= \nabla_\Phi \nabla_\Phi^T V \\ &= M_S^2 + (4\pi)^2 \nabla_\Phi \mathbb{B} \nabla_\Phi^T t + (4\pi)^2 \nabla_\Phi t \nabla_\Phi^T \mathbb{B} \\ &+ (4\pi)^2 \mathbb{B} \nabla_\Phi \nabla_\Phi^T t + \nabla_\Phi \nabla_\Phi^T \mathbb{A}, \end{aligned}$$

where M_S^2 is the tree-level scalar mass matrix

- The term proportional to $\mathbb{B}$ is canceled by a similar term resulting from $\nabla_\Phi \nabla_\Phi^T \mathbb{A}$

Calculation of the Potential Minimum

- Choose $\mathcal{M} = s$ along the flat direction

$$e_{\mathcal{M}} = e_s = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Then $t = \ln(s^2/s_0^2)/(4\pi)^2$
- Tree-level flat direction $V = 0$ at s_0
- Minimum given by the usual Gildener-Weinberg relation

$$s^2 = e^{-\frac{1}{2}} s_0^2$$

30 Calculation of the Potential Minimum

- Expand $\mathbb{A}$ in h^2/s^2, giving $\Lambda(t) \to \Lambda(t) + \Delta\Lambda$
- Define $t_S = t(s = s_S)$ and $t_{HS} = t(s = s_{HS})$, the expansion can absorbed in the field-dependent couplings, yielding

$$\lambda_S(t) = \beta_S(t - t_S),$$
$$\lambda_{HS}(t) = \beta_{HS}(t - t_{HS}),$$

- Flat direction given by

$$\begin{aligned} 0 &= \det(\Lambda(t_0)) \\ &= \beta_{HS}^2 t_0^2 - (4\lambda_H\beta_S + 2\beta_{HS}^2 t_{HS})t_0 + 4\lambda_H\beta_S t_S \end{aligned}$$

31 Calculation of the Potential Minimum

- Ignore terms proportional to β_{HS}^2 (irrelevant unless $|t_{HS} - t_S|$ is large)
- We get $\lambda_S(t_0) \approx 0$ and $s_0 \approx s_S$
- Minimum direction

$$\Phi^{\circ 2} = e^{-\frac{1}{2}s_0^2} \frac{\mathrm{adj}(\Lambda(t))e_s}{e_s^T \mathrm{adj}(\Lambda(t))e_s},$$

where

$$\mathrm{adj}(\Lambda)(t) = \begin{pmatrix} \lambda_H(t) & -\frac{1}{2}\lambda_{HS}(t) \\ -\frac{1}{2}\lambda_{HS}(t) & \lambda_S(t) \end{pmatrix}$$

Calculation of the Potential Minimum

- We have that $t = t_0 - (1/2)/(4\pi)^2 \approx t_S - (1/2)/(4\pi)^2$ at the minimum
- Define $\ln R \approx (4\pi)^2(t - t_{HS})$ or $R = e^{-1/2}\frac{s_S^2}{s_{HS}^2}$
- Expressing t_{HS} in terms of $\ln R$, the potential minimum is at

$$\Phi^{\circ 2} \approx e^{-\frac{1}{2}} s_S^2 \begin{pmatrix} -\beta_{\lambda_{HS}} \ln R/2\lambda_H(4\pi)^2 \\ 1 \end{pmatrix}$$

Calculation of the Potential Minimum

- Mixing angle

$$\theta \approx \frac{(m_S^2)_{hs}}{m_S^2 - m_h^2} \approx \frac{\beta_{HS}(1 + \ln R)}{2\beta_S + \beta_{HS} \ln R} \times \frac{h}{s}$$

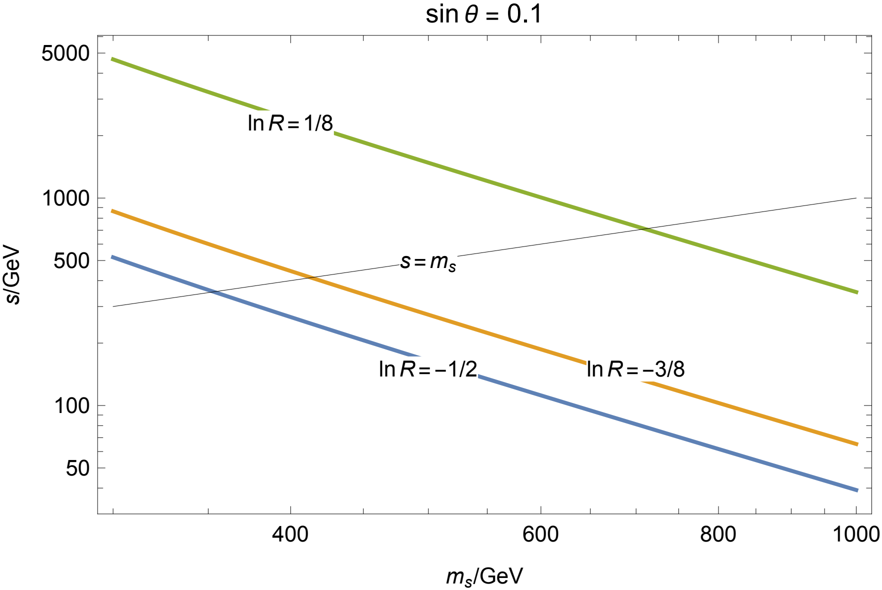
- Higgs mass (since $\nabla_\Phi t \propto e_s$)

$$m_h^2 = 3\lambda_H(t)h^2 + \frac{1}{2}\lambda_{HS}(t)s^2$$
$$\approx \frac{-s^2 \beta_{\lambda_{HS}} \ln R}{(4\pi)^2} = 2\lambda_H h^2$$

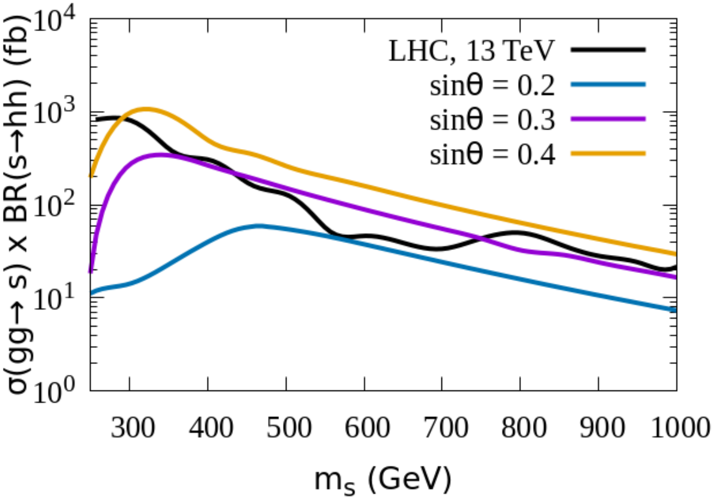
- Since $\Phi^T(M_S^2 + \nabla_\Phi \nabla_\Phi^T \mathbb{A})\Phi = 12V$, the scalon mass is

$$m_s^2 \approx \frac{1}{s^2}\Phi^T m^2 \Phi \approx \frac{\beta_{\lambda_S}}{(4\pi)^2} 2s^2$$

Numerical Results



Numerical Results



Huitu, K.K., Koivunen, Marzola, Mondal, Raidal, to appear

Conclusions

- Matrix formalism
  with Hadamard product:
  compact ‘prepackaged’ expressions
- Multi-phase criticality:
  loop-suppressed mass both
  for scalon and the Higgs boson

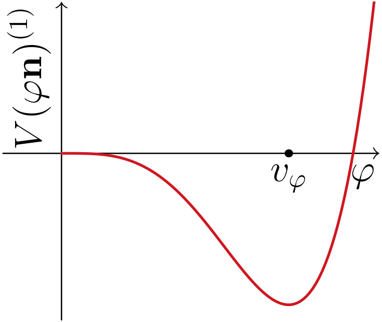
Lowest Order Solution

- Approximating
  $$\Lambda \approx \Lambda_{\min} + \beta \ln \frac{\varphi^2}{v_\varphi^2},$$ we have
  $$V(\varphi) = \mathbb{B}(n) v_\varphi^4 \left(\ln \frac{\varphi^2}{v_\varphi^2} - \frac{1}{2} \right)$$

  Sher, Phys. Rept. 179 (1989) 273

- The field φ along the flat direction obtains a mass $m_\varphi^2 = 8\mathbb{B}(n) v_\varphi^2$
  Gildener & Weinberg,

  Phys. Rev. D13, 3333 (1976)



Anomalous Dimensions

- Fields in must be scaled with anomalous dimensions as $\Phi(t) = \exp(\Gamma(t))\Phi(t_0)$, where $\Gamma(t) = -\int_{t_0}^{t} \gamma(s)ds$
- Denote $\Phi \equiv \Phi(t_0)$ and take the anomalous dimensions into account by scaling $\Lambda \to \exp(2\Gamma(t))\Lambda\exp(2\Gamma(t))$ and $\beta \to \exp(2\Gamma(t))\beta\exp(2\Gamma(t))$