

CP symmetry of order 4 and its phenomenological consequences

Igor Ivanov

CFTP, Instituto Superior Técnico, Universidade de Lisboa

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IPI, J. P. Silva, PRD 93, 095014 (2016)

A. Aranda, IPI, E. Jiménez, PRD 95, 055010 (2017)

P. Ferreira, IPI, E. Jiménez, R. Pasechnik, H. Serôdio, JHEP 1801 (2018) 065

IPI, JHEP 1802 (2018) 025

IPI, Laletin, JCAP 1905 (2019) 032

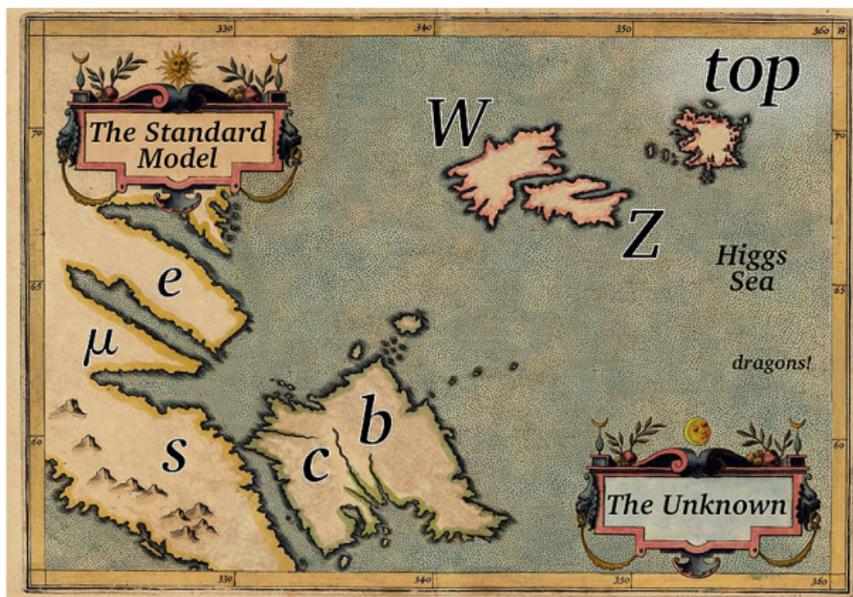


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- 1 Model building with exotic CP
- 2 DM from CP4
- 3 Flavored CP4 3HDM
- 4 Neutrino masses from CP4
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Standard Model



The Standard Model is just a **part of the real world!**

The Higgs sector of the SM

The minimal Higgs sector of the SM is **overstretched**: gives masses to W, Z , to down quarks, to up quarks, to leptons. As a result:

- does not explain the **hierarchical fermion masses** and the mixing patterns (flavour sector = the ugly part of the SM),
- does not explain tiny **neutrino masses**,
- “boring” flavor properties of the Higgs boson exchange: no FCNC, no LFV,
- **CP-violation** must be inserted by hand,
- unable to generate astroparticle and cosmological phenomena we observe: no **DM**, no sizable **baryon asymmetry**.

bSM model building

These features can be successfully reproduced in models with **extended scalar sectors**, see e.g. [LHCHXWG 1610.07922](#); [King, 1701.04413](#); [Ivanov, 1702.03776](#).

Many new fields \rightarrow many interaction terms \rightarrow lots of free parameters + often untractable analytically. **Problem!**

Imposing **extra global symmetries** is a popular way to proceed, e.g. [Ishimori et al, 1002.0211](#); [Altarelli, Feruglio, 1003.3552](#); [King, Luhn, 1301.1340](#).

Why imposing global symmetries?

- much fewer parameters, tractable analytically;
- **robust** way of achieving pheno features;
- **anticipation of unification** at high energy scales while staying conservative at low energies.

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CP symmetry in the SM and beyond

- Within SM, the only (confirmed) source of *CP*-violation is the quark Yukawa sector \rightarrow CKM matrix.
- New opportunities for *CP* from extended Higgs sectors.
 - New Higgses may be **the source** of the complex CKM [T.D.Lee, 1973] and mediate additional *CP*-violation [Weinberg, 1976; Branco, 1979];
 - *CP* symmetry can be a member of a larger **flavour symmetry** group; see e.g. [King, 1701.04413];
 - **Exotic CP symmetry** with consequences for model-building.

Many bSM models with extended Higgs sectors are on the market, with *CP* playing various roles [Branco, Lavoura, Silva, 1999; Ivanov, 1702.03776].

CP4 3HDM

A dilemma in symmetry-based multi-Higgs model building:

- **Large symmetry groups** → very few free parameters, nicely calculable, very predictive, and unphysical.
- **Small symmetry groups** → many free parameters, compatible with experiment but not quite predictive.

I will show a peculiar model based on three Higgs doublets (**3HDM**) which

- **assumes very little**: the minimal model realizing a particular symmetry;
- this symmetry is unusual: **generalized CP-symmetry of order 4 (CP4)**;
- well **tractable analytically** and **quite predictive**.

In short, a good balance of minimality, predictiveness, and theoretical flair.

CP4 3HDM

Freedom of defining CP

In QFT, CP is not uniquely defined *a priori*.

- phase factors $\phi(\vec{r}, t) \xrightarrow{CP} e^{i\alpha} \phi^*(-\vec{r}, t)$ [Feinberg, Weinberg, 1959],
- with N scalar fields ϕ_i , the general CP transformation is

$$J: \quad \phi_i \xrightarrow{CP} X_{ij} \phi_j^*, \quad X \in U(N).$$

If \mathcal{L} is invariant under such J with whatever fancy X , it is explicitly CP-conserving [Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999].

- **NB:** The “standard” convention $\phi_i \xrightarrow{CP} \phi_i^*$ is basis-dependent!

Higher order CP

Squaring the CP transformation:

$$\phi_i \xrightarrow{CP} X_{ij} \phi_j^* \xrightarrow{CP} X_{ij} (X_{jk}^* \phi_k) = (XX^*)_{ik} \phi_k.$$

The family transformation XX^* does not have to be identity!

It may happen that $(CP)^k = \mathbb{I}$ for $k > 2$.

CP-symmetry can be of higher order!

The usual $CP = CP2$, the first non-trivial is $CP4$, then $CP8$, $CP16$, etc.

What is the **minimal multi-Higgs-doublet model** realizing $CP4$ without accidental symmetries?

CP4 3HDM

The answer was given in [Ivanov, Keus, Vdovin, 2012; Ivanov, Silva, 2016].

Consider 3HDM with the following potential $V = V_0 + V_1$ (notation: $i \equiv \phi_i$):

$$V_0 = -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2 \left[(2^\dagger 2)^2 + (3^\dagger 3)^2 \right] \\ + \lambda_3(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda_3'(2^\dagger 2)(3^\dagger 3) + \lambda_4 \left[(1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1) \right] + \lambda_4'(2^\dagger 3)(3^\dagger 2),$$

with all parameters real, and

$$V_1 = \lambda_5(3^\dagger 1)(2^\dagger 1) + \frac{\lambda_6}{2} \left[(2^\dagger 1)^2 - (3^\dagger 1)^2 \right] + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3) \left[(2^\dagger 2) - (3^\dagger 3) \right] + h.c.$$

with real $\lambda_{5,6}$ and complex $\lambda_{8,9}$. It is invariant under CP4 $J: \phi_i \xrightarrow{CP} X_{ij} \phi_j^*$ with

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix}, \quad J^2 = \text{diag}(1, -1, -1), \quad J^4 = \mathbb{I}.$$

Side remarks on CP4 and beyond

- A CP -conserving model based on CP4 can be distinguished from a CP -conserving model based on the usual CP [Haber, OGREID, OSLAND, REBELO, 1808.08629].
- An NHDM can have a **hidden** CP symmetry. Necessary and sufficient **basis-invariant conditions** detecting a symmetry are notoriously difficult to derive. In 2HDM, the problem was solved in [Davidson, Haber, 2005; Branco, Rebelo, Silva-Marcos, 2005; Gunion, Haber, 2005] and in [Ivanov, 2006; Nishi, 2006; Maniatis, von Manteuffel, Nachtmann, 2008].
- For CP4 3HDM, we solved this problem in [Ivanov, Nishi, Silva, Trautner, 1810.13396].
- Higher-order CPs are possible but require more Higgses; see examples of 5HDM with CP8 and CP16 in [Ivanov, Laletin, 1804.03083].

Different versions of CP4 3HDM

- **DM CP4 3HDM**: unbroken CP4 with scalar DM candidates, similar to the inert doublet model in 2HDM. We assume that ϕ_2, ϕ_3 don't get vevs \rightarrow **scalar DM candidates** with peculiar properties [Ivanov, Silva, 2016; Ivanov, Laletin, 2018].

Scalar DM **stabilized by a CP-symmetry!**

- **flavored CP4 3HDM**: CP4 is extended to the Yukawa sector and must be spontaneously broken \rightarrow patterns in the flavor sector [Ferreira, Ivanov, Jimenez, Pasechnik, Serodio, 2017]

DM from CP4

DM CP4 3HDM

CP4-conserving minimum: $v_i = (v, 0, 0)$. Expand the doublets as

$$\phi_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h_{SM} + iG^0) \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}(H + ia) \end{pmatrix}, \quad \phi_3 = \begin{pmatrix} H_3^+ \\ \frac{1}{\sqrt{2}}(h + iA) \end{pmatrix}.$$

In the basis with $\lambda_5 = 0$, these fields are mass eigenstates:

$$m_{H^+}^2 = -m_{22}^2 + \frac{v^2}{2}\lambda_3, \quad M^2, m^2 = -m_{22}^2 + \frac{v^2}{2}(\lambda_3 + \lambda_4 \pm \lambda_6).$$

where $M \equiv m_{H,A}$ and $m \equiv m_{h,a}$.

If $\lambda_6 > 0$ and $\lambda_6 > \lambda_4$, then h and a are the **DM candidates**.

DM from CP4 3HDM

These real neutrals are not CP-eigenstates:

$$H \xrightarrow{CP} A, \quad A \xrightarrow{CP} -H, \quad h \xrightarrow{CP} -a, \quad a \xrightarrow{CP} h.$$

They can be combined into **neutral complex CP-eigenstate fields**

$$\Phi = \frac{1}{\sqrt{2}}(H - iA), \quad \varphi = \frac{1}{\sqrt{2}}(h + ia), \quad \Phi \xrightarrow{CP} i\Phi, \quad \varphi \xrightarrow{CP} i\varphi.$$

Conserved quantum number: **not CP-parity** but **CP-charge q** defined mod 4.

Absence of conjugation comes from the enhanced freedom of basis change transformations $U(N) \rightarrow O(2N)$ for mass-degenerate zero-charge fields [Aranda, Ivanov, Jimenez, 2017].

Notice that $\varphi^*|0\rangle$ is **not** the antiparticle of $\varphi|0\rangle$ but is a **different** one-particle state with the same mass \rightarrow an example of spectrum doubling beyond Kramers degeneracy mentioned e.g. in [Weinberg, vol. 1, app. 2C].

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CP4 3HDM vs Inert doublet model (IDM)

The spectrum reproduces the one of IDM [Despande, Ma, 1978; Ma, 2006; etc.] if we define $(\lambda_5)_{IDM} = (\lambda_6)_{CP4}$.

This correspondence can be extended to the entire lagrangian:

CP4 3HDM = duplicated IDM + inert self-interaction

Symmetric initial conditions $n(\varphi) = n(\varphi^*)$: CP4 3HDM \simeq IDM \times 2 [Köpke, 2018].

Asymmetric DM regime

Suppose at $T \sim m$, a non-thermal process generates $n(\varphi) \neq n(\varphi^*) \rightarrow$ **asymmetric DM**. Will this asymmetry survive?

- Annihilation: $\varphi\varphi^* \rightarrow \text{SM}$ but $\varphi\varphi \not\rightarrow \text{SM}$ due to the conserved CP4.
- But unlike in typical asymmetric DM models, $n(\varphi) - n(\varphi^*)$ is not fixed due to the **regeneration process** $\varphi\varphi \leftrightarrow \varphi^*\varphi^*$ driven by

$$\frac{\lambda_{\text{conv}}}{4!} [\varphi^4 + (\varphi^*)^4].$$

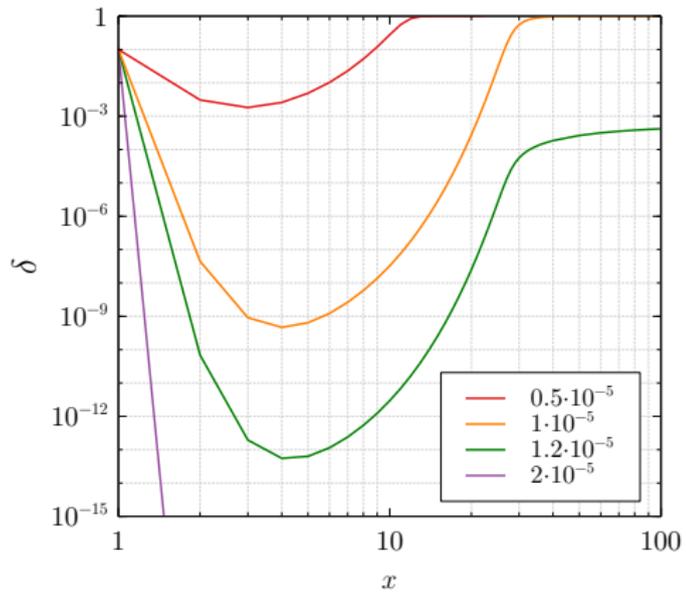
- Thermal evolution of the relic density and DM asymmetry comes from the non-trivial competition of these two processes [[Ivanov, Laletin, 1812.05525](#)].

Asymmetric DM regime

The competition between the annihilation $\varphi\varphi^* \rightarrow \text{SM}$ and conversion $\varphi\varphi \leftrightarrow \varphi^*\varphi^*$ affects the thermal evolution of the asymmetry:

$$\delta = \frac{n_\varphi - n_{\varphi^*}}{n_\varphi + n_{\varphi^*}}.$$

If evolution starts at $x = 1$, the boundary is at $\lambda_{\text{conv}} \sim 10^{-5}$.



Flavored CP4 3HDM

CP4-symmetric quark sector

Extending CP4 to the Yukawa sector: $\psi_i \rightarrow Y_{ij} \psi_j^{CP}$, where $\psi^{CP} = \gamma^0 C \bar{\psi}^T$.

$$-\mathcal{L}_Y = \bar{q}_L \Gamma_a d_R \phi_a + \bar{q}_L \Delta_a u_R \tilde{\phi}_a + h.c.$$

is invariant under CP4 with known X_{ab} if

$$(Y^L)^\dagger \Gamma_a Y^d X_{ab} = \Gamma_b^*, \quad (Y^L)^\dagger \Delta_a Y^u X_{ab}^* = \Delta_b^*.$$

Matrix Y can be always brought to

$$Y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} \\ 0 & e^{-i\alpha} & 0 \end{pmatrix},$$

with $\alpha_L, \alpha_{dR}, \alpha_{uR}$ being free parameters.

CP4-symmetric quark sector

In [Ferreira, Ivanov, Jimenez, Pasechnik, Serodio, 2017], we solved these equations = found Yukawa matrices Γ 's and Δ 's and mixing matrices Y^L , Y^d , Y^u , which satisfy all these conditions and do not lead to immediate problems with masses and mixing.

Consider only down-sector first. We found only four possibilities: A , B_1 , B_2 , B_3 .

- case A : $\alpha_L = 0$, $\alpha_{dR} = 0 \rightarrow \Gamma_1 \simeq$ is an arbitrary real matrix, $\Gamma_{2,3} = 0$.
- case B_1 : $\alpha_L = \pi/2$, $\alpha_{dR} = 0$.
- case B_2 : $\alpha_L = 0$, $\alpha_{dR} = \pi/2$.
- case B_3 : $\alpha_L = \pi/2$, $\alpha_{dR} = \pi/2$.

CP4-symmetric quark sector

case B_1

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{31} & g_{31}^* & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} -g_{22}^* & -g_{21}^* & -g_{23}^* \\ g_{12}^* & g_{11}^* & g_{13}^* \\ 0 & 0 & 0 \end{pmatrix}.$$

case B_2

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{13}^* \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} g_{22}^* & -g_{21}^* & 0 \\ g_{12}^* & -g_{11}^* & 0 \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}.$$

case B_3

$$\Gamma_1 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ -g_{12}^* & g_{11}^* & 0 \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{23} \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & -g_{23}^* \\ 0 & 0 & g_{13}^* \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}.$$

CP4-symmetric quark sector

When combining up and down quarks, need to match α_L : **8 combinations**.

$$(A^{\text{down}}, A^{\text{up}}), \quad (A^{\text{down}}, B_2^{\text{up}}), \quad (B_2^{\text{down}}, A^{\text{up}}), \quad (B_2^{\text{down}}, B_2^{\text{up}}),$$

$$(B_1^{\text{down}}, B_1^{\text{up}}), \quad (B_1^{\text{down}}, B_3^{\text{up}}), \quad (B_3^{\text{down}}, B_1^{\text{up}}), \quad (B_3^{\text{down}}, B_3^{\text{up}}).$$

- case (A, A) implies real CKM, with CPV arising only in the scalar sector.
- cases B_1, B_2, B_3 : quark mass matrices

$$M_d = \frac{1}{\sqrt{2}} \sum \Gamma_a v_a, \quad M_u = \frac{1}{\sqrt{2}} \sum \Delta_a v_a^*.$$

All vevs v_1, v_2, v_3 must be nonzero to avoid mass-degenerate quarks.

- No built-in suppression of FCNC! Avoiding FCNC from h_{125} via scalar alignment condition: $m_{11}^2 = m_{22}^2$.

Numerical scan

Numerical scan procedure

Procedure:

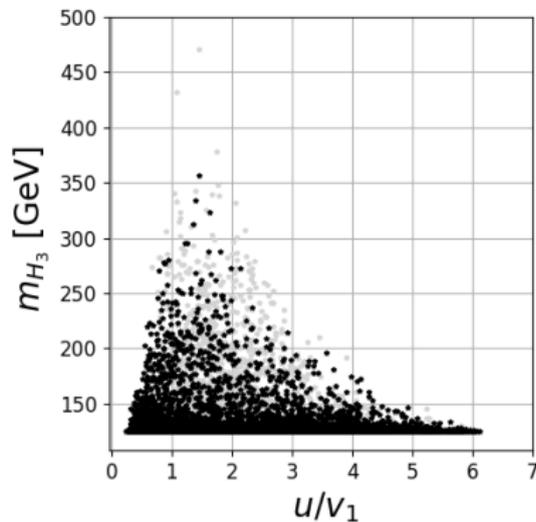
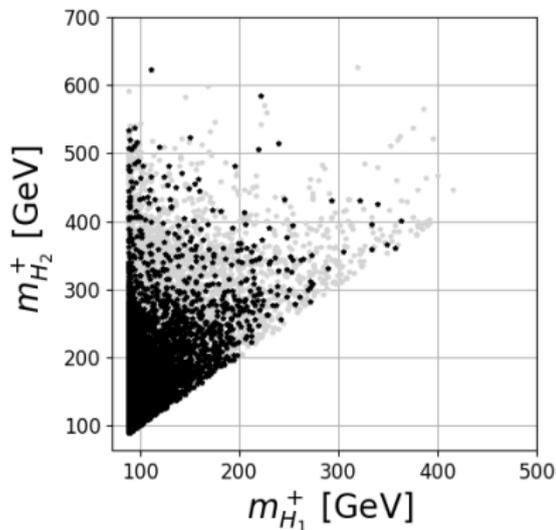
1 Scalar sector scan:

- stick to the scalar alignment, take h_{125} to be the lightest scalar, vary 9 free parameters: v_3/v_2 , u/v_1 , and 7 λ 's;
- simplified checks of boundedness from below and perturbativity (all $|\lambda| < 5$; the exact conditions exist [Bento, Haber, Romao, Silva, 2017]);
- check that S , T , U parameters are within 3σ of expt.

2 Yukawa sector scan

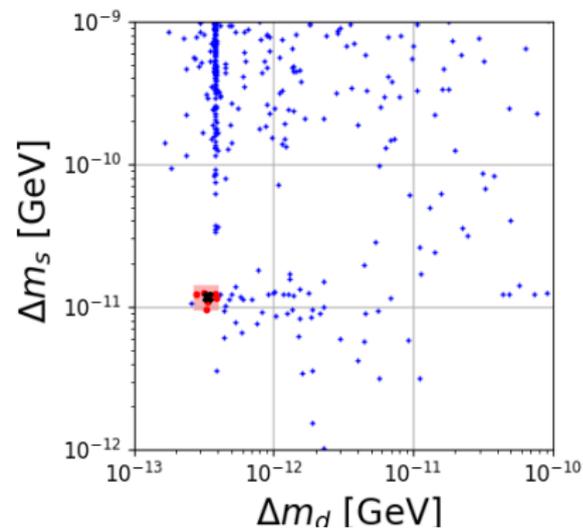
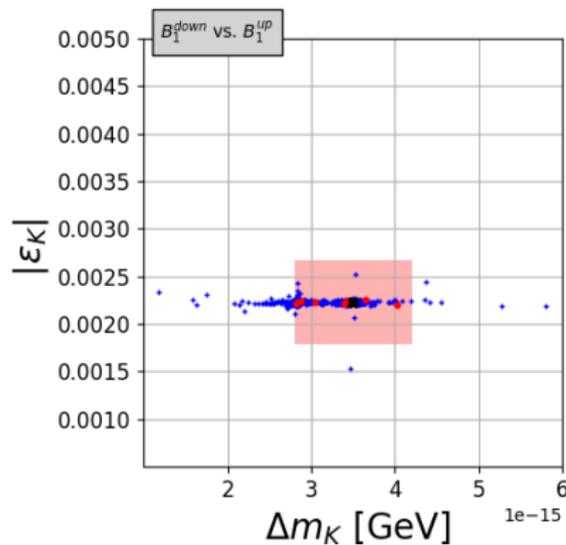
- fit all quark masses, mixing, and CPV phase (easy);
- add K and B oscillation parameters $|\epsilon_K|$, Δm_K , Δm_{B_d} , Δm_{B_s} via expressions from [Buras et al, 2013] (tree-level contributions from neutral Higgses only).

Scan: extra Higgses

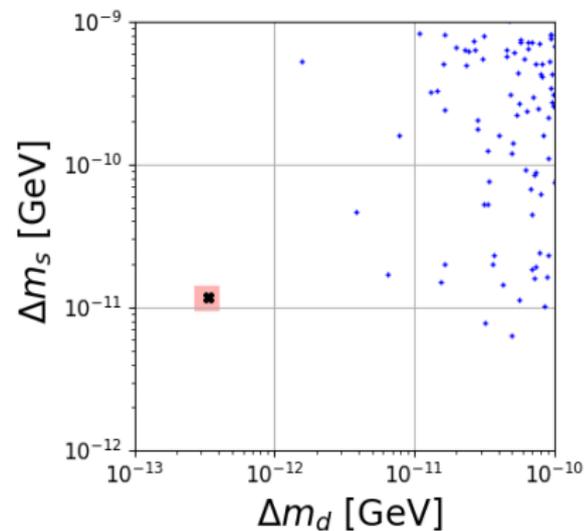
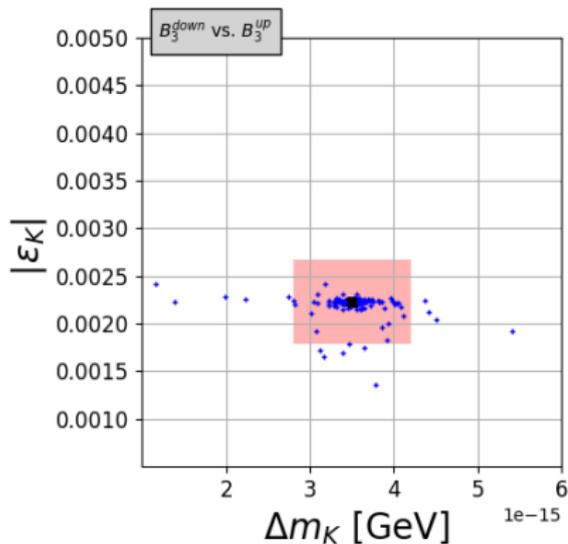


Scan: flavor observables

case (B_1^{down} , B_1^{up})



Scan: flavor observables

case (B_3^{down} , B_3^{up})

Overall results

- Four cases produced good points:

$$(A, B_2), \quad (B_2, B_2), \quad (B_1, B_1), \quad (B_1, B_3).$$

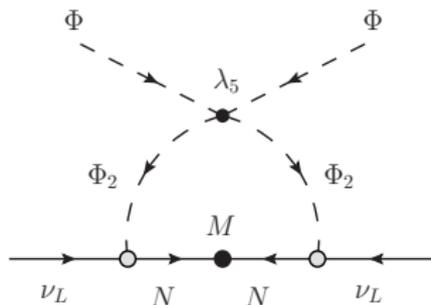
- Typical points have **light Higgses** (< 150 GeV); a few points have moderately heavy Higgses.
- Higgs spectrum in a benchmark point for (B_1, B_3) :

(ν_1, ν_2, ν_3) [GeV]:	(142.8,	66.1,	74.6)	
neutral Higgses [GeV]:	220.4,	304.4,	318.9,	352.2
charged Higgses [GeV]:	209.3,	242.1.		

Neutrino masses from CP4

Scotogenic model

In 2006, Ma proposed [scotogenic model](#) for Majorana neutrino masses: radiative origin of \mathcal{M}_ν within IDM generated by inert neutral scalars.



It uses Φ_2 and three RH neutrinos N_j , all of them being \mathbb{Z}_2 -odd:

$$-\mathcal{L}_{\text{lept.}} = \Gamma_{\alpha\beta} \bar{L}_\alpha \Phi_1 \ell_{R\beta} + Y_{\alpha k} \bar{L}_\alpha \tilde{\Phi}_2 N_k + \frac{1}{2} M_{ij} \bar{N}_i^c N_j + h.c.,$$

and a special interaction term in the Higgs potential:

$$V = \dots + \frac{\lambda_5}{2} \left[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right].$$

Scotogenic model

The resulting light neutrino mass matrix is

$$(\mathcal{M}_\nu)_{\alpha\beta} = \frac{1}{2} \sum_k Y_{\alpha k} Y_{\beta k} \cdot J(m, M; M_k),$$

where $m \equiv m_H$, $M \equiv m_A \neq m$, and

$$J(m, M; M_k) = \frac{M_k}{16\pi^2} \left(\frac{m^2}{M_k^2 - m^2} \log \frac{M_k^2}{m^2} - \frac{M^2}{M_k^2 - M^2} \log \frac{M_k^2}{M^2} \right).$$

Since $m^2 - M^2 = \lambda_5 v^2$ and can be small, we get extra suppression w.r.t. the usual seesaw.

Scotogenic model with CP4

Repeating the same with **unbroken** CP4 [Ivanov, 1712.02101]:

$$-\mathcal{L}_{\text{lept.}} = \Gamma_{\alpha\beta}^{(a)} \overline{L}_\alpha \Phi_a \ell_{R\beta} + Y_{\alpha k}^{(a)} \overline{L}_\alpha \tilde{\Phi}_a N_k + \frac{1}{2} M_{ij} \overline{N}_i^c N_j + h.c.$$

with $M = \text{diag}(M_0, M_0, M'_0)$ and

$$Y^{(1)} = \begin{pmatrix} 0 & 0 & y_{13} \\ 0 & 0 & y_{23} \\ 0 & 0 & y_{33} \end{pmatrix}, \quad Y^{(2)} = \begin{pmatrix} y_{11} & y_{12} & 0 \\ y_{21} & y_{22} & 0 \\ y_{31} & y_{32} & 0 \end{pmatrix}, \quad Y^{(3)} = \begin{pmatrix} -iy_{12}^* & iy_{11}^* & 0 \\ -iy_{22}^* & iy_{21}^* & 0 \\ -iy_{32}^* & iy_{31}^* & 0 \end{pmatrix}.$$

Dirac term $m_D = Y^{(1)} \nu / \sqrt{2}$ of rank 1 leading to

$$\mathcal{M}_{\alpha\beta}^{\text{seesaw}} = -m_D M^{-1} m_D^T = -\frac{v^2}{2M'_0} y_{3\alpha} y_{3\beta}.$$

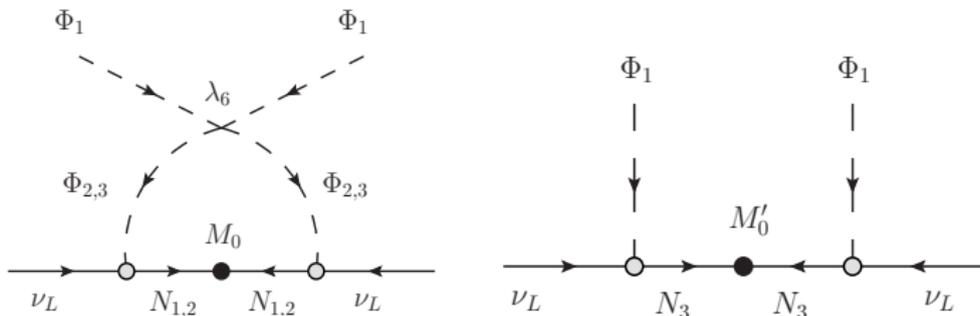
Scotogenic contribution

$$\mathcal{M}_{\alpha\beta}^s = 2\text{Re} \left[\sum_k Y_{\alpha k}^{(2)} Y_{\beta k}^{(2)} \right] \cdot J(m, M; M_0)$$

of **rank 3** despite involving just two RH neutrinos.

Scotogenic model with CP4

Final result: $\mathcal{M}_\nu = \mathcal{M}^S + \mathcal{M}^{\text{seesaw}}$, a hybrid seesaw-scotogenic mechanism.



It naturally predicts two mass scales with mild hierarchy:

$$m_1 \sim m_2 \sim \frac{\lambda_6}{32\pi^2} \frac{v^2}{M_0} [Y^{(2)}]^2 \log\left(\frac{M_0^2}{m^2}\right), \quad m_3 \sim \frac{v^2}{M'_0} [Y^{(1)}]^2.$$

The idea that NLO corrections can modify the LO seesaw result is not new (see e.g. [Grimus, Neufeld, 2000; Hehn, Ibarra, 2013; Wegman, 2017]); here it comes just from the requirement of exotic CP.

Conclusions: what's done

- CP4 3HDM is the minimal model implementing higher-order CP without accidental symmetries.
- Unbroken CP4 leads to scalar DM with novel DM features, especially in the asymmetric DM regime, and can naturally produce neutrino masses with moderate hierarchy.
- Spontaneously broken CP4 can be extended to the Yukawa sector → very characteristic flavor sector.
- (Rich) pheno awaits further exploration!

Framework for conservative multi-Higgs model building

- based on a single symmetry assumption,
- quite predictive with rich phenomenology,
- tractable analytically.