Electroweak Baryogenesis and Dark Matter from a Complex Singlet

Da Huang

IFT, University of Warsaw

@ Max-Planck-Institut fur Kernphysik, Heidelberg

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In collaboration with Bohdan Grzadkowski

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Motivation

- In spite of the great success of the Standard Model (SM) of particle physics, there are still many puzzles needing to be explained. Among others, two important questions are

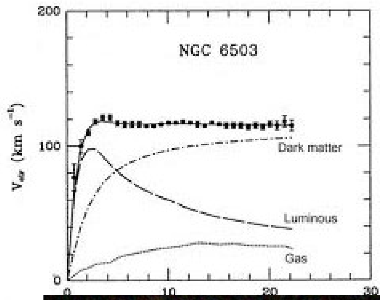
  - Dark Matter : In the SM, there is no DM candidate.
  - Matter-Antimatter Asymmetry in our Universe

- Both problems require the physics beyond the SM.

Motivation

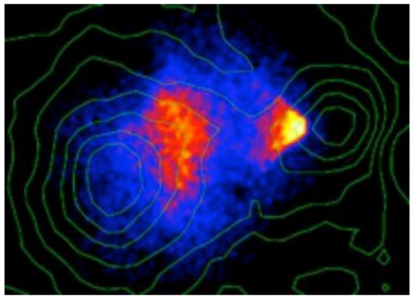
➢ There are already many established evidences for the existence of dark matter

- Rotation Curves of Spiral Galaxies
  Babcock, 1939, Bosma, 1978; Rubin & Ford, 1980

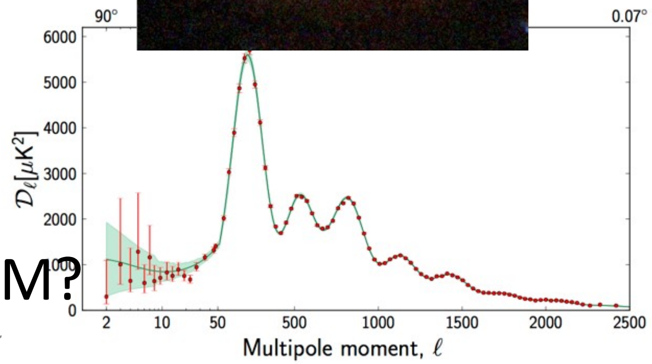


- Gravitational Lensing

- CMB



- Bullet Clusters



But , what is the particle nature of DM?

Motivation

- Observed Baryon Asymmetry: Planck Collaboration, arXiv: 1502.01589

$$\eta_B \equiv \frac{n_B}{s} = (8.61 \pm 0.09) \times 10^{-11}$$

- Three Sakharov criteria for baryogenesis:

A. D. Sakharov, 1967

✓ B violation
✓ C and CP violation
✓ Thermal non-equilibrium

- Situation in the SM:

F. R. Klinkhamer & N.S. Manton 1984

✓ B violation: weak sphaleron process

M. E. Shaposhnikov, 1987

✓ The CP violation due to CKM phase is inadequate
✓ EW phase transition is actually a cross-over, rather than being of strongly first-order.

K. Kajantie et al, hep-ph/9605288

Motivation

- EW Baryogenesis:

V. A. Kuzmin, V.A. Rubakov, M.E. Shaposhnikov, 1985;
A. G. Cohen, D. B. Kaplan and A. E. Nelson, 1990

✓ new CPV sources

✓ adding new particles with masses of EW scale in order to make the EWPT of strongly first-order, which provides the necessary deviation from an equilibrium.

- Problem: The new CPV source required by the baryogenesis is strongly constrained by the EDMs of electrons and neutrons.

ACME Collaoration, 1310.7534; PDG 2016;

- Possible solution: If the CP is spontaneously broken at high temperatures before the EWPT while restored afterward, then the CPV constraints can be evaded!

J. McDonald, 1994; W. Chao, 1706.01041

The Model

- Extend the SM by an EW singlet complex scalar

$$S = (s+ia)/\sqrt{2}$$

with a Z_2 symmetry: $S \leftrightarrow -S$ and CP symmetry related to S

J. McDonald, 1994, 1995; G.C. Branco et al, 9805302; S. Profumo et al, 0705.2425; …

- The scalar potential at zero temperature:

$$H = (0, h/\sqrt{2})^T$$

REAL couplings

$$\begin{aligned}
V_0(H,S) &= \lambda_H \left(|H|^2 - \frac{v_0^2}{2}\right)^2 - \mu_1^2 (S^*S)^2 - \frac{\mu_2^2}{2}(S^2 + S^{*2}) \\
&+ \lambda_1 (S^*S)^2 + \frac{\lambda_2}{4}(S^2 + S^{*2})^2 + \frac{\lambda_3}{2}|S|^2(S^2 + S^{*2}) \\
&+ |H|^2 \left[\kappa_1 (S^*S) + \frac{\kappa_2}{2}(S^2 + S^{*2})\right] \\
&= -\frac{1}{2}\lambda_H v_0^2 h^2 + \frac{1}{4}\lambda_H h^4 - \frac{1}{2}(\mu_1^2 + \mu_2^2)s^2 - \frac{1}{2}(\mu_1^2 - \mu_2^2)a^2 \\
&+ \frac{1}{4}(\lambda_1 + \lambda_2 + \lambda_3)\, s^4 + \frac{1}{4}(\lambda_1 + \lambda_2 - \lambda_3)\, a^4 \\
&+ \frac{1}{4}(\kappa_1 + \kappa_2) h^2 s^2 + \frac{1}{4}(\kappa_1 - \kappa_2) h^2 a^2 + \frac{1}{2}(\lambda_1 - \lambda_2) s^2 a^2 + \text{const.}\,.
\end{aligned}$$

The Model

- Leading-order finite-temperature corrections in the high-T expansion

$$V_T = \frac{1}{2}c_h T^2 h^2 + \frac{1}{2}c_s T^2 s^2 + \frac{1}{2}c_a T^2 a^2$$

where

$$c_h = \frac{3g^2}{16} + \frac{g'^2}{16} + \frac{y_t^2}{4} + \frac{\lambda_H}{2} + \frac{\kappa_1}{12}$$

$$c_s = \frac{1}{6}(2\lambda_1 + \kappa_1 + \kappa_2) + \frac{\lambda_3}{4},$$

$$c_a = \frac{1}{6}(2\lambda_1 + \kappa_1 - \kappa_2) - \frac{\lambda_3}{4}.$$

- Total Potential:

$$V_{\rm tot} = V_0 + V_T.$$

EW Phase Transition

- Rewrite the total scalar potential $\langle S \rangle = w_c e^{i\alpha}/\sqrt{2}$

$$V_{\rm tot} = \frac{\lambda_{hs}}{4}\left(h^2 - v_c^2 + \frac{v_c^2 s^2}{w_c^2 \cos^2\alpha}\right)^2 + \frac{\lambda_{ha}}{4}\left(h^2 - v_c^2 + \frac{v_c^2 a^2}{w_c^2 \sin^2\alpha}\right)^2$$
$$+\frac{\lambda_{sa}}{4}\left(s^2 \sin^2\alpha - a^2\cos^2\alpha\right)^2 + \frac{\kappa_{hs}}{4}h^2 s^2 + \frac{\kappa_{ha}}{4}h^2 a^2$$
$$+\frac{1}{2}(T^2 - T_c^2)[c_h h^2 + c_s s^2 + c_a a^2]$$

- Two vacua: (h, s, a) = (v_c, 0, 0) and (0, w_c cosα, w_c sinα)

- Critical Temperature:

$$T_c^2 = \lambda_H(v_0^2 - v_c^2)/c_h$$

EW Phase Transition

- Further Consistency Constraints:

✓ Strongly First-Order EWPT:

$$v_c/T_c > 1$$

G. D. Moore, hep-ph/9805264

✓ Potential Stability: assume positive couplings

✓ Correct EWPT direction from $(0, w_c \cos\alpha, w_c \sin\alpha)$ to $(v_c, 0, 0)$

$$c_h v_c^2 > c_s w_c^2 \cos^2\alpha + c_a w_c^2 \sin^2\alpha$$

✓ Z_2 symmetry: $\alpha \in (-\pi/2, \pi/2)$

✓ Perturbativity: $|\lambda_{1,2,3}, \kappa_{1,2}| \leqslant 5$ M. Nebot et al, 0711.0483

Dark Matter Physics

- Depending on the mass ordering, either s or a can be DM candidate X

- The DM pheno. only depends on Higgs portal coupling

$$\boxed{\lambda_{hX} h^2 X^2/4}$$

J. M. Cline & K. Kainulainen, 1210.4196

with

$$\lambda_{hX} = \begin{cases} \kappa_{hs} + \dfrac{2\lambda_{hs} v_c^2}{w_c^2 \cos^2\alpha}, & X = s \\ \kappa_{ha} + \dfrac{2\lambda_{ha} v_c^2}{w_c^2 \sin^2\alpha}, & X = a \end{cases}$$

- The DM relic density is obtained by the freeze-out mechanism, and is calculated with MicrOMEGAs code.
- In order to consider the case with subdominant DM, we define the DM fraction: $f_X = \dfrac{\Omega_X h^2}{\Omega_{\rm DM,obs} h^2}$ with $\Omega_{\rm DM,obs} h^2 = 0.1186$

Dark Matter Physics

- DM Constraints:

  - ✓ DM direct detection: XENON1T
  - ✓ DM Indirect detection: Fermi-LAT, Planck, and AMS-02
  - ✓ SM Higgs Invisible Decay: $\mathrm{Br}(h \to XX) \leqslant 0.24$ PDG 2016
  - ✓ Monojet searches: CMS

High-T CP Violation

- S can acquire a complex VEV before EWPT

$$\langle S \rangle = w_c e^{i\alpha}/\sqrt{2}$$

- With the following dim-6 operator

$$\mathcal{O}_6 = \frac{S^2}{\Lambda^2}\bar{Q}_{3L}\tilde{H}t_R + \text{H.c.}$$

J. R. Espinosa et al., 1110.2876;
J.M. Cline & K. Kainulainen, 1210.4196;
V. Vaskonen, 1611.02073

the CP symmetry is spontaneously broken, which is manifested by the induced complex-valued top quark Yukawa coupling

$$\frac{w_c^2 e^{i2\alpha}}{2\Lambda^2}\bar{Q}_{3L}\tilde{H}t_R + \text{H.c.}$$

- Together with top Yukawa, we have a complex top-quark mass

EW Baryogenesis

- For a first-order EWPT, the PT proceeds via the bubble nucleation.

- Near the bubble wall, the top yields a spatially varying complex mass

$$m_t(z) = \frac{y_t}{\sqrt{2}} h(z) \left(1 + \frac{S(z)^2}{y_t \Lambda^2}\right) \equiv |m_t(z)| e^{i\theta(z)}$$

M. Joyce, et al., hep-ph/9410282; J.M. Cline et al., hep-ph/9708393, hep-ph/0006119

- This top mass would generate CPV force that acts on tops and anti-tops differently when they pass through the wall.

$$F_z = -\frac{(m^2)'}{2E_0} \pm s\frac{(m^2\theta')'}{2E_0 E_{0z}} \mp s\frac{\theta' m^2 (m^2)'}{4E_0^3 E_{0z}}$$

L. Fromme & S.J. Huber, hep-ph/0604159

which is the source of CPV for the EW baryogenesis.

EW Baryogenesis

- Approximate solution of bubble wall profile:

$$S(z) \equiv \frac{w_c e^{i\alpha}}{2\sqrt{2}}[1 + \tanh(z/L_w)] ,$$
$$h(z) \equiv \frac{v_c}{2}[1 - \tanh(z/L_w)] ,$$

J. R. Espinosa, et al, arXiv: 1110.2876

where L_w is the bubble wall width given by

$$L_w = \frac{v_c^2 + w_c^2}{6V_\times}$$

Here $V_\times$ is the energy density at the top of the barrier, which can be determined with the scalar potential.

EW Baryogenesis

- The CP asymmetry created around the bubble wall would transport to the EW symmetric phase deeply, where it biases the EW sphaleron process to generate baryon asymmetry.

- The transportation of the CP asymmetry is described by the transport equations of chemical potentials and velocity perturbations of t_L, t_R, b_L and SM Higgs. L. Fromme & S.J. Huber, hep-ph/0604159

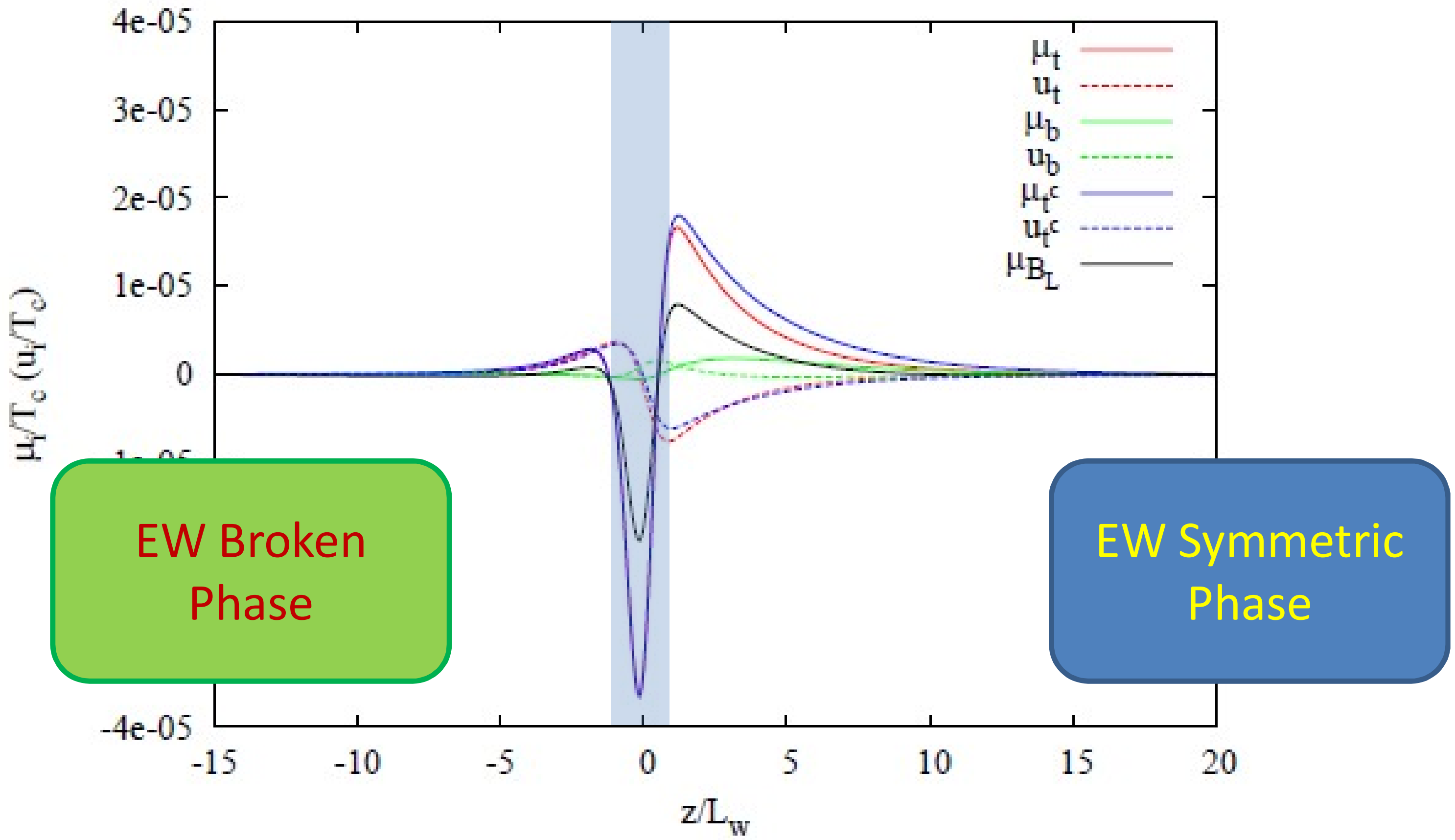
- The final baryon asymmetry is predicted to be

$$\eta_B = \frac{n_B}{s} = \frac{405\Gamma_{\rm sph}}{4\pi^2 v_w g_* T}\int_0^\infty dz \mu_{B_L}(z) e^{-45\Gamma_{\rm sph}|z|/(4v_w)}$$

where $\mu_{B_L} = \frac{1}{2}(1+4K_{1,t_L})\mu_{t_L} + \frac{1}{2}(1+4K_{1,b_L})\mu_{b_L} + 2K_{1,t_R}\mu_{t_R}$, $v_w \approx 0.1$is the bubble wall velocity in the plasma, and $\Gamma_{\rm sph} \simeq 10^{-6}T$ is the sphaleron rate in the symmetric phase. J.M. Cline et al., hep-ph/0006119

EW Baryogenesis

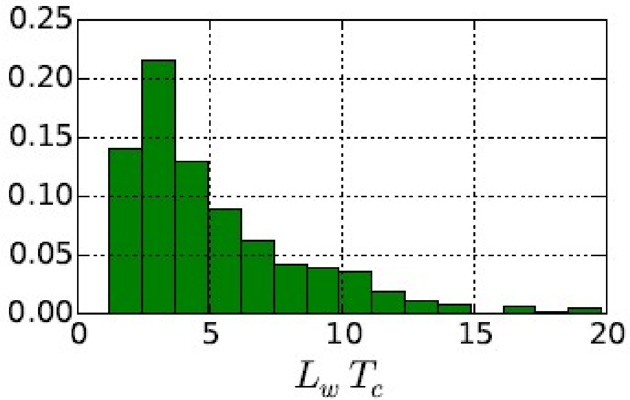
- Solution to the Transport Equations



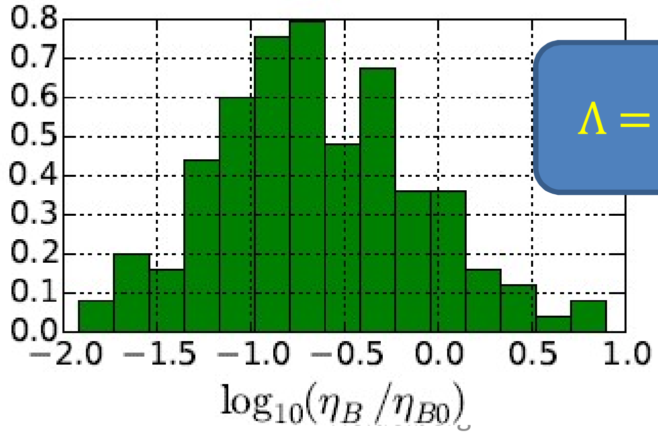
EW Baryogenesis

➢ Additional Constraints:

✓ Validity of semiclassical framework ➡ $L_w T_c \geqslant 3$



✓ Positive baryon asymmetry ➡ CPV phase α < 0

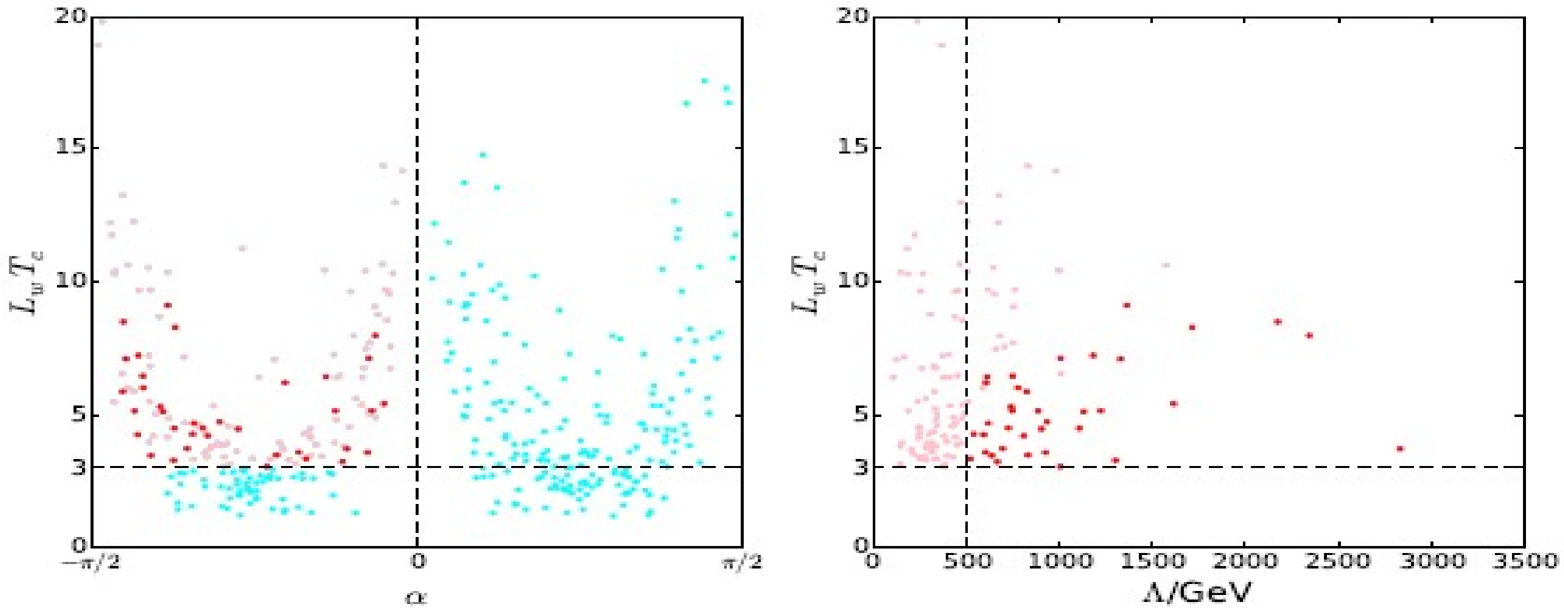


$\Lambda = 1$ TeV

EW Baryogenesis

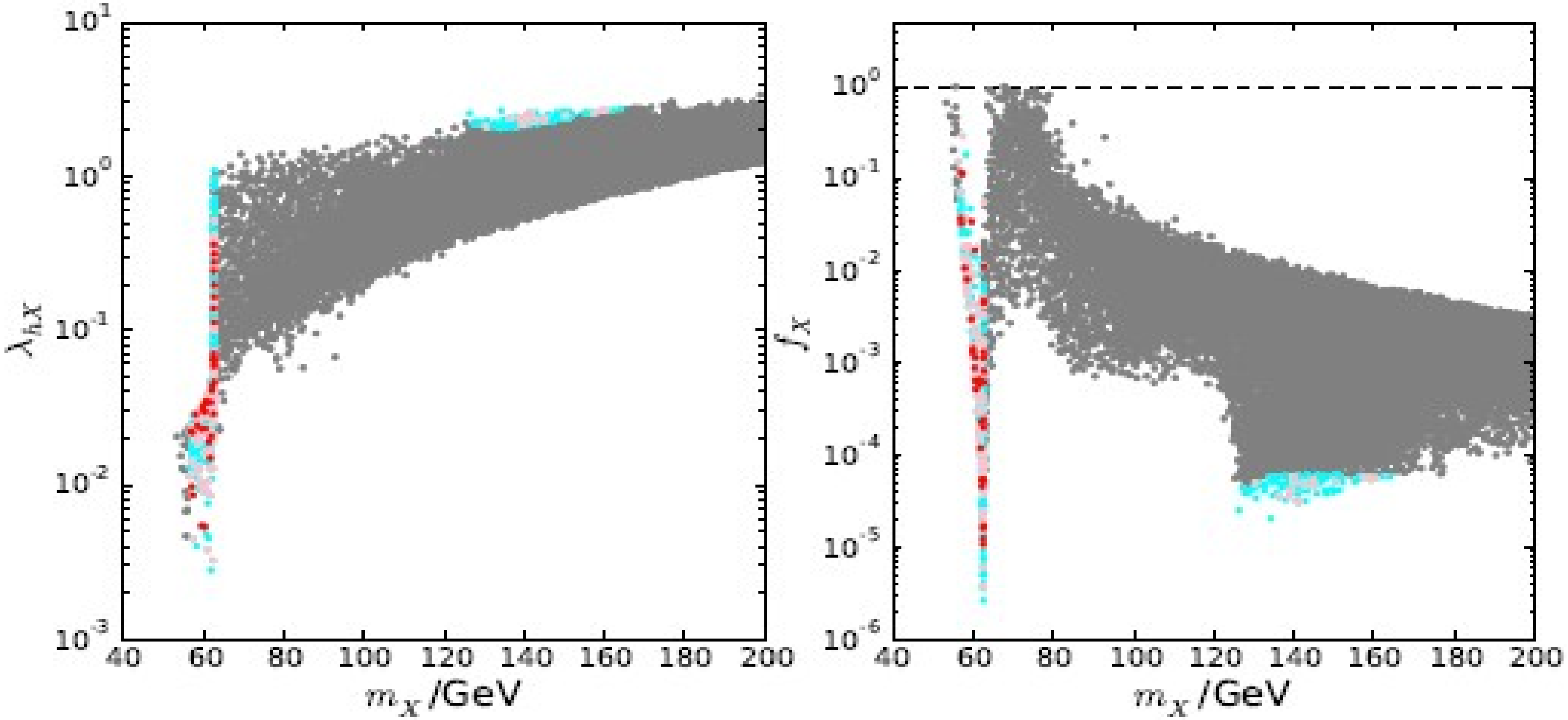
- ➢ We can also represent our results in terms of Λ by fixing $\eta_B = \eta_B^{obs}$, which enables us to place some other constraints.

✓ Reliable use of O_6 ⟹ $\Lambda > 500$ GeV and $w_c^2/\Lambda^2 < 0.5$



Scanning Results

➢ Implications of EWBG on the DM properties

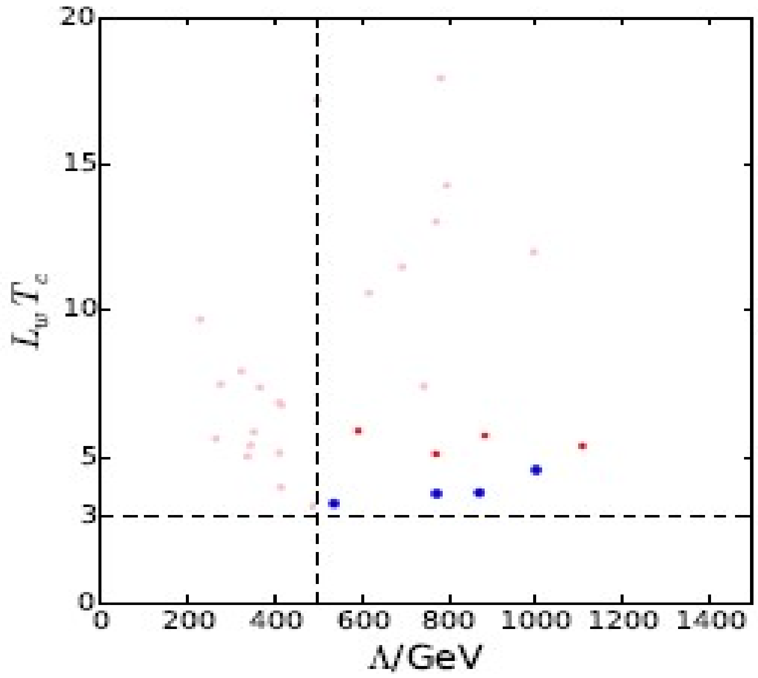


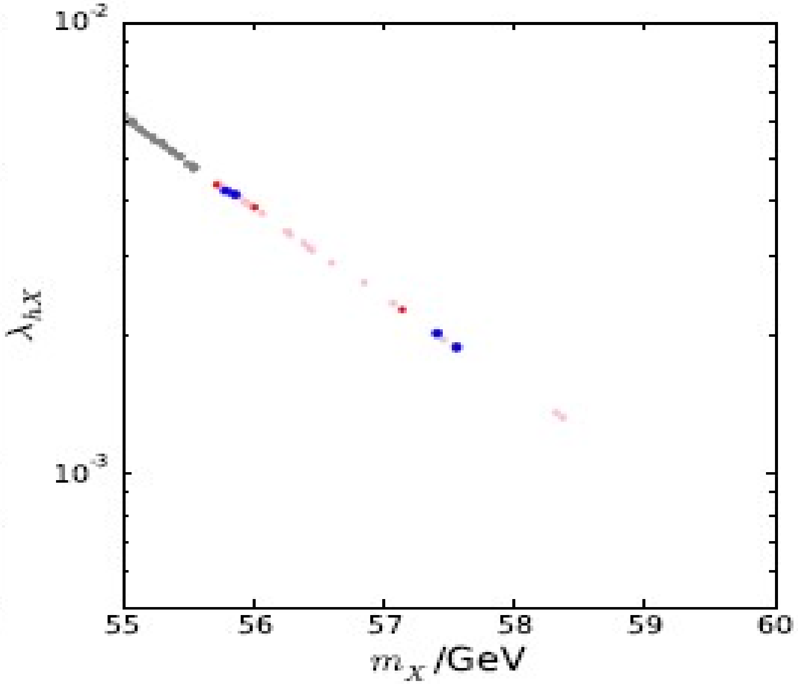
➢ Only SM Higgs resonance region can generate the enough cosmological baryon asymmetry without violating any bounds.

Models with Correct DM Density

➢ Question: Can this simple model explain the DM relic density and baryon asymmetry simultaneously?

➢ Zoom-in Scan near SM Higgs Resonance





Red: $w_c^2/\Lambda^2 < 0.5$

Blue: $w_c^2/\Lambda^2 < 0.2$

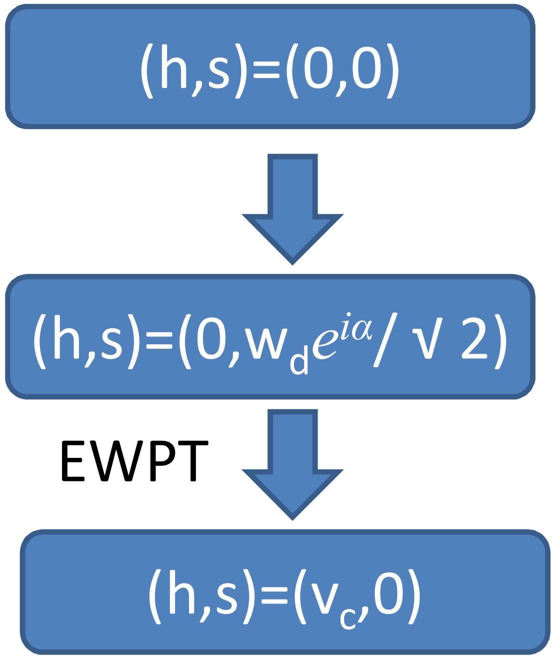
Problems with Exact CP Symmetry

- Previously, we assumed that at the time before the EWPT, the Universe is filled with a single vacuum with (h,S) = (0, $w_c e^{i\alpha}/\sqrt{2}$).

- However, in the present model, the transition has two steps.

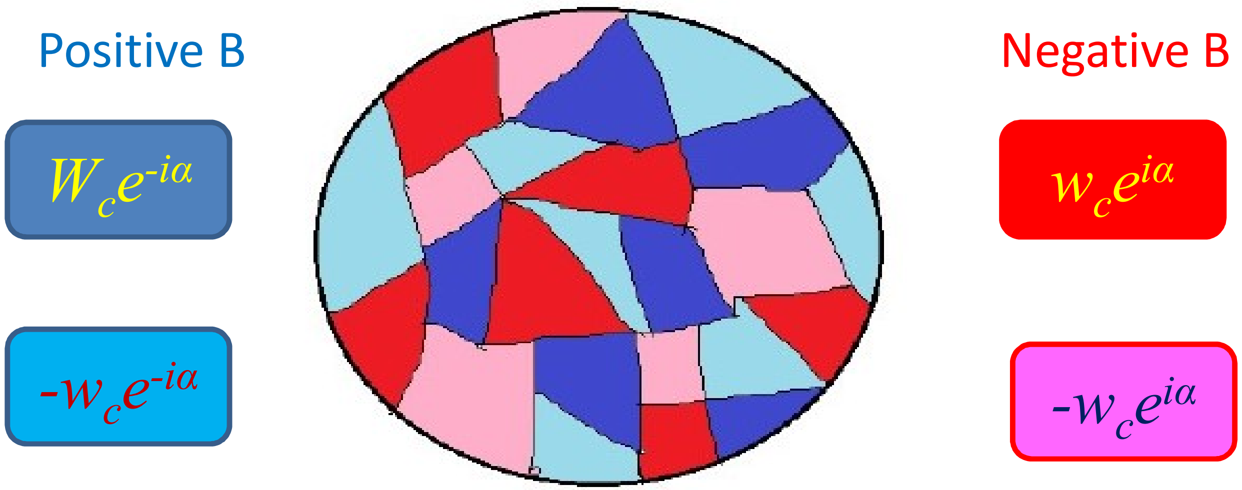
- In the first step of phase transition, Z_2 and CP symmetries are broken, while the EW gauge symmetries are kept.

- The second step is the usual EWPT, in which baryon asymmetry can be produced via bubble nucleation.



Problems with Exact CP Symmetry

- If Z_2 and CP symmetries are exact, it is expected that there are 4 vacua with $\langle S \rangle = \pm w_d e^{\pm i\alpha}$ in the Universe, each with the same volume.



- The vacua with positive CPV phase would produce negative baryon asymmetry during EWPT, which would totally cancel the positive baryon numbers created in vacua of negative phase.

D. Comelli, et al., arXiv: 9304267; J.McDonald, PLB **323**, 339 (1994); PLB **357**, 19 (1995);

Possible Solution with Explicit CPV

- One possible solution is to introduce a small explicit CPV phase in the scalar potential, which uplifts the vacuum degeneracy so that the ones with negative phase are favored.

- Example: Explicit CPV in quartic term S^4

$$V_4 = \frac{\lambda_2 e^{i\delta}}{4} S^4 + \frac{\lambda_2 e^{-i\delta}}{4} S^{*4} + \frac{\lambda_2}{2} |S|^4 ,$$

So that the vacua $(0, \pm w_d e^{i\alpha}/\sqrt{2})$ have the potential density

$$V_T^+ = \frac{1}{8}\lambda_2 w_d^4 \cos(\delta + 4\alpha) + V_T^{CP} ,$$

while the potential for vacua $(0, \pm w_d e^{-i\alpha}/\sqrt{2})$ is

$$V_T^- = \frac{1}{8}\lambda_2 w_d^4 \cos(\delta - 4\alpha) + V_T^{CP} ,$$

D. Comelli, et al., arXiv: hep-ph/9304267; J.McDonald, PLB **323**, 339 (1994); PLB **357**, 19 (1995);

- Potential difference: $\Delta V_T = -\frac{1}{4}\lambda_2 w_d^4 \sin(4\alpha) \sin\delta$

Possible Solution with Explicit CPV

➢ It is well known that the wrong-sign vacua can be eliminated via the movement of the domain walls interpolating between the wrong- and right-sign vacua.

H. Lew and A. Riotto, arXiv: hep-ph/9304203; J.McDonald, PLB **357**, 19 (1995);

➢ The domain walls begin moving when the energy scale of the potential difference approaches that of domain wall's surface energy $\eta_{\mathrm{DM}} \sim w_d^3$. Thus, the time for domain wall movement is

$$t_{\mathrm{DW}} \approx \frac{\eta_{\mathrm{DW}}}{|\Delta V_T|} \sim \frac{1}{|\lambda_2 \sin(4\alpha) \sin\delta| w_d} .$$

Possible Solution with Explicit CPV

- Our picture of EWBG requires to eliminate the wrong-sign domains at least before the EWPT with the time $t_{\rm EW} \sim M_{\rm Pl}/T_c^2$

$$|\sin\delta| > \frac{T_c^2}{|\lambda_2 \sin(4\alpha)| w_d M_{\rm Pl}} \sim \frac{T_c^2}{|\lambda_2 \sin(4\alpha)| w_c M_{\rm Pl}},$$

- Typical EWPT parameters:

$$T_c \sim 100 \text{ GeV},\ w_c \sim 100 \text{ GeV},\ |\sin(4\alpha)| \sim 0.1,\ |\lambda_2| \sim \mathcal{O}(0.1)$$

the explicit CPV phase can be as small as $O(10^{-15})$, which agrees with the results given in J.McDonald, PLB **357**, 19 (1995).

- It is obvious that such a small explicit CPV phase cannot have any visible effects under the current experimental status.

Problem with Z_2 Domain Walls

- For the domain walls separating the two right-sign vacua $(0, \pm w_d e^{-i\alpha}/\sqrt{2})$ whose stability is guaranteed by the broken Z_2 symmetry, one would worry that they might dominate the energy density and change the evolution of the Universe.

- However, these domain walls would decay immediately after the Z_2 symmetry is restored at the EWPT with $T_c \sim$ 100 GeV, which is well before they dominate the energy at $T \sim 10^{-7}$ GeV.

J. R. Espinosa, et al. arXiv: 1110.2876;
J. M. Cline and K. Kainulainen, arXiv: 1210.4196

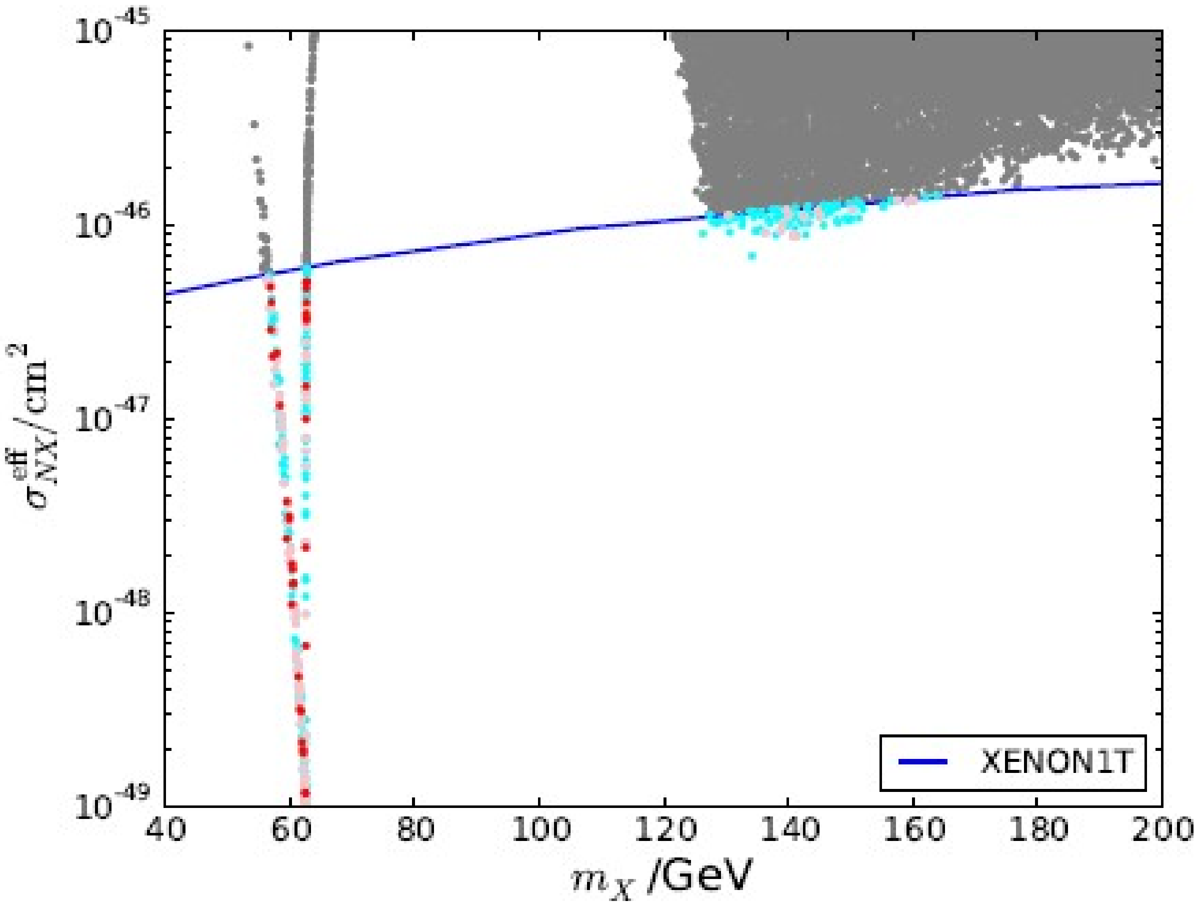
Summary

- We have explored a new connection between DM and EWBG in a simple complex singlet extension of the SM.
- The model is appealing in that the large CPV necessary for the EWBG is only spontaneously generated at temperatures higher than the EWPT, while the CP symmetry is restored at present time, so that the low-energy electron and neutron EDM constraints can be evaded.
- We show that the model can generate the DM relic density and baryon asymmetry with the DM mass near the SM Higgs resonance.
- The explicit CPV necessary to eliminate the wrong-sign domains can be so small that it has no observable effects.

Thanks for your attention!

Scanning Results

➤ Constraining power of DM direct searches



Transport Equations for EWBG

- Transport equations: 2[nd] -order perturbation: tL, tR, bL, h

$$
\begin{aligned}
3v_w K_{1,t}\mu'_{t,2} + 3v_w K_{2,t}(m_t^2)'\mu_{t,2} + 3u'_{t,2}& \\
-3\Gamma_y(\mu_{t,2}+\mu_{t^c,2}+\mu_{h,2}) - 6\Gamma_m(\mu_{t,2}+\mu_{t^c,2}) - 3\Gamma_W(\mu_{t,2}-\mu_{b,2})& \\
-3\Gamma_{ss}[(1+9K_{1,t})\mu_{t,2}+(1+9K_{1,b})\mu_{b,2}+(1-9K_{1,t})\mu_{t^c,2}] &= 3K_{7,t}\theta'_t m_t^2 \mu'_{t,1} \\
3v_w K_{1,b}\mu'_{b,2} + 3u'_{b,2}& \\
-3\Gamma_y(\mu_{b,2}+\mu_{t^c,2}+\mu_{h,2}) - 3\Gamma_W(\mu_{b,2}-\mu_{t,2})& \\
-3\Gamma_{ss}[(1+9K_{1,t})\mu_{t,2}+(1+9K_{1,b})\mu_{b,2}+(1-9K_{1,t})\mu_{t^c,2}] &= 0 \\
3v_w K_{1,t}\mu'_{t^c,2} + 3v_w K_{2,t}(m_t^2)'\mu_{t^c,2} + 3u'_{t^c,2}& \\
-3\Gamma_y(\mu_{t,2}+\mu_{b,2}+2\mu_{t^c,2}+2\mu_{h,2}) - 6\Gamma_m(\mu_{t,2}+\mu_{t^c,2})& \\
-3\Gamma_{ss}[(1+9K_{1,t})\mu_{t,2}+(1+9K_{1,b})\mu_{b,2}+(1-9K_{1,t})\mu_{t^c,2}] &= 3K_{7,t}\theta'_t m_t^2 \mu'_{t^c,1} \\
2v_w K_{1,h}\mu'_{h,2} + 2u'_{h,2}& \\
-3\Gamma_y(\mu_{t,2}+\mu_{b,2}+2\mu_{t^c,2}+2\mu_{h,2}) - 2\Gamma_h\mu_{h,2} &= 0 \qquad (4.3)
\end{aligned}
$$

$$
\begin{aligned}
-3K_{4,t}\mu'_{t,2} + 3v_w\tilde{K}_{5,t}u'_{t,2} + 3v_w\tilde{K}_{6,t}(m_t^2)'u_{t,2} + 3\Gamma_t^{\rm tot}u_{t,2} &= \\
= -3v_w K_{8,t}(m_t^2\theta'_t)' + 3v_w K_{9,t}\theta'_t m_t^2(m_t^2)' - 3\tilde{K}_{10,t}m_t^2\theta'_t u'_{1,t}& \\
-3K_{4,b}\mu'_{b,2} + 3v_w\tilde{K}_{5,b}u'_{b,2} + 3\Gamma_b^{\rm tot}u_{b,2} &= 0 \\
-3K_{4,t}\mu'_{t^c,2} + 3v_w\tilde{K}_{5,t}u'_{t^c,2} + 3v_w\tilde{K}_{6,t}(m_t^2)'u_{t^c,2} + 3\Gamma_t^{\rm tot}u_{t^c,2} &= \\
= -3v_w K_{8,t}(m_t^2\theta'_t)' + 3v_w K_{9,t}\theta'_t m_t^2(m_t^2)' - 3\tilde{K}_{10,t}m_t^2\theta'_t u'_{1,t^c}& \\
-2K_{4,h}\mu'_{h,2} + 2v_w\tilde{K}_{5,h}u'_{h,2} + 2\Gamma_h^{\rm tot}u_{h,2} &= 0 \qquad (4.4)
\end{aligned}
$$

Determination of $V_\times$

$$V_\times = \frac{N_\times}{D_\times}\,,$$

$$
\begin{aligned}
N_\times =\ & v_c^4 w_c^2 \Big(\kappa_{ha} + \kappa_{hs} + (\kappa_{hs} - \kappa_{ha})\cos(2\alpha)\Big)^2 \Big(128\lambda_{hs}\lambda_{ha} v_c^4 + 3(\lambda_{hs} + \lambda_{ha})\lambda_{sa} w_c^4 \\
& + (\lambda_{hs} + \lambda_{ha})\lambda_{sa} w_c^4 \big(\cos(8\alpha) - 4\cos(4\alpha)\big)\Big)\,, \\
D_\times =\ & 4096\lambda_{hs}\lambda_{ha}(\kappa_{hs} + \kappa_{ha}) v_c^6 + 768(\kappa_{hs}^2\lambda_{ha} + \kappa_{ha}^2\lambda_{hs}) v_c^4 w_c^2 \\
& + 96(\kappa_{hs} + \kappa_{ha})(\lambda_{ha} + \lambda_{hs})\lambda_{sa} v_c^2 w_c^4 + 2\lambda_{sa}(7\kappa_{ha}^2 + 10\kappa_{ha}\kappa_{hs} + 7\kappa_{hs}^2) w_c^6 \\
& - 8\cos(2\alpha)\Big(512(\kappa_{ha} - \kappa_{hs})\lambda_{hs}\lambda_{ha} v_c^6 + 128(\kappa_{ha}^2\lambda_{hs} - \kappa_{hs}^2\lambda_{ha}) v_c^4 w_c^2 \\
& + 4(\kappa_{ha} - \kappa_{hs})(\lambda_{ha} + \lambda_{hs})\lambda_{sa} v_c^2 w_c^4 + (\kappa_{ha}^2 - \kappa_{hs}^2)\lambda_{sa} w_c^6\Big) \\
& - w_c^2 \cos(4\alpha)\Big(-256(\kappa_{hs}^2\lambda_{ha} + \kappa_{ha}^2\lambda_{hs}) v_c^4 + 128(\kappa_{ha} + \kappa_{hs})(\lambda_{ha} + \lambda_{hs})\lambda_{sa} v_c^2 w_c^2 \\
& + (17\kappa_{ha}^2 + 30\kappa_{ha}\kappa_{hs} + 17\kappa_{hs}^2)\lambda_{sa} w_c^4\Big) \\
& + \lambda_{sa} w_c^4 \Big(12\cos(6\alpha)(\kappa_{ha} - \kappa_{hs})\big(4(\lambda_{ha} + \lambda_{hs}) v_c^2 + (\kappa_{hs} + \kappa_{ha}) w_c^2\big) \\
& + \cos(8\alpha)\big(32(\kappa_{ha} + \kappa_{hs})(\lambda_{ha} + \lambda_{hs}) v_c^2 + 2(\kappa_{ha}^2 + 6\kappa_{ha}\kappa_{hs} + \kappa_{hs}^2) w_c^2\big) \\
& + 8\cos(10\alpha)(\kappa_{ha} - \kappa_{hs})\big(4(\lambda_{ha} + \lambda_{hs}) v_c^2 + (\kappa_{ha} + \kappa_{hs}) w_c^2\big) \\
& - 2\cos(12\alpha)(\kappa_{ha} - \kappa_{hs})^2 w_c^2\Big)\,. \qquad (26)
\end{aligned}
$$