Model Building with Discrete Flavour Symmetries

based on collaborations with Stefan Antusch (Basel), Manfred Lindner (HD), Kher Sham Lim (HD), Michael A. Schmidt (Melbourne) and Martin Spinrath (SISSA/Karlsruhe)

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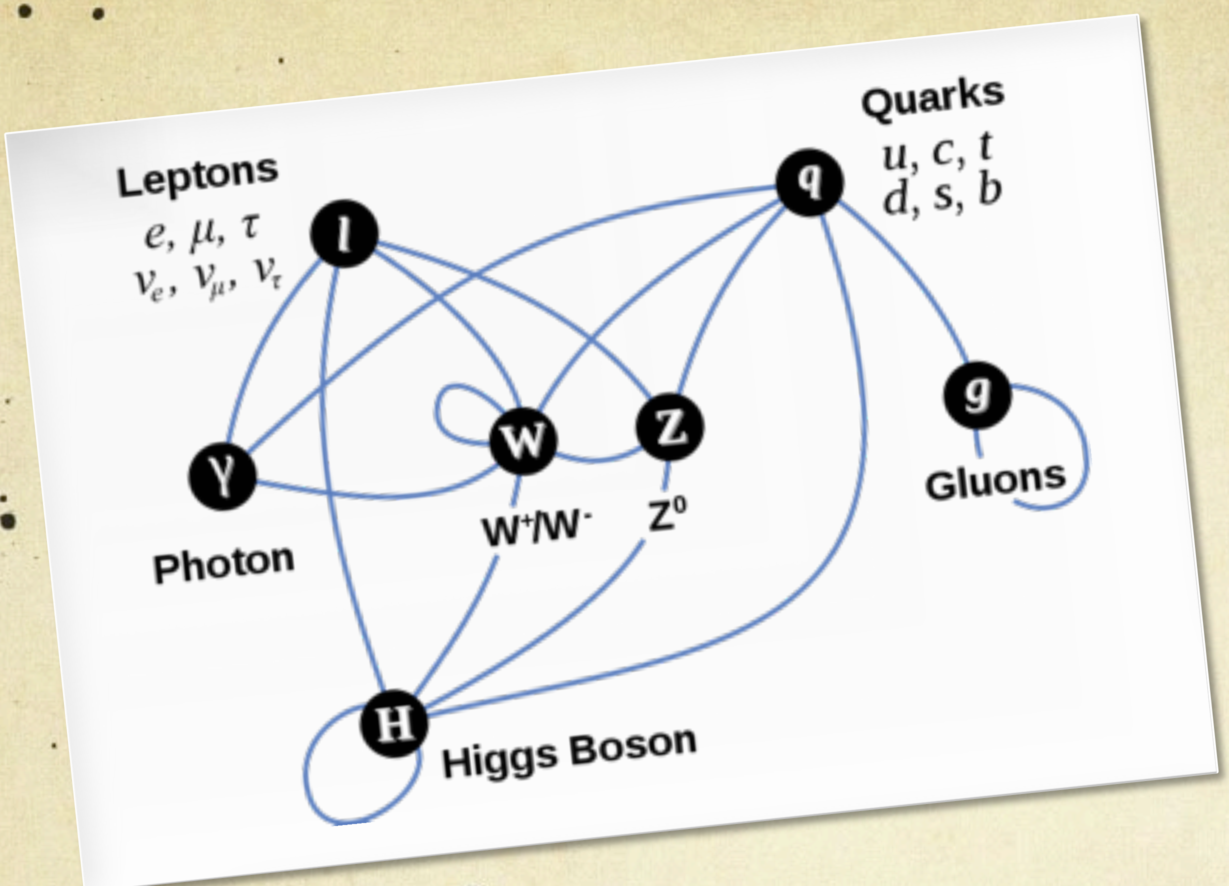
Outline

- Introduction
- Lepton Mixing from Discrete Groups
  - based on [MH, K.S. Lim, M. Lindner, PLB721(2013) 61-67; 1212.2411 and MH, K.S. Lim, PRD 88(2013) 033018; 1306.4356]
- A Model Building Challenge: Vacuum Alignment Problem
  - based on [MH, M.A. Schmidt *JHEP* **1201** (2012) 126;1111.1730 [hep-ph]] and [MH, M. Lindner, M.A. Schmidt PRD87 (2013)3, 033006; 1211.5143[hep-ph]]
- CP and Discrete Flavour Symmetries
  - based on [MH, M. Lindner and M.A. Schmidt JHEP 1304 (2013) 122, 1211.6953 and S. Antusch, MH, M.A. Schmidt, M. Spinrath, 1307.0710 (accepted by NPB)]
- Conclusions

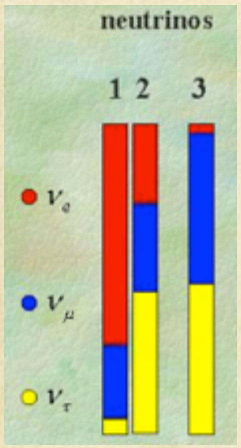


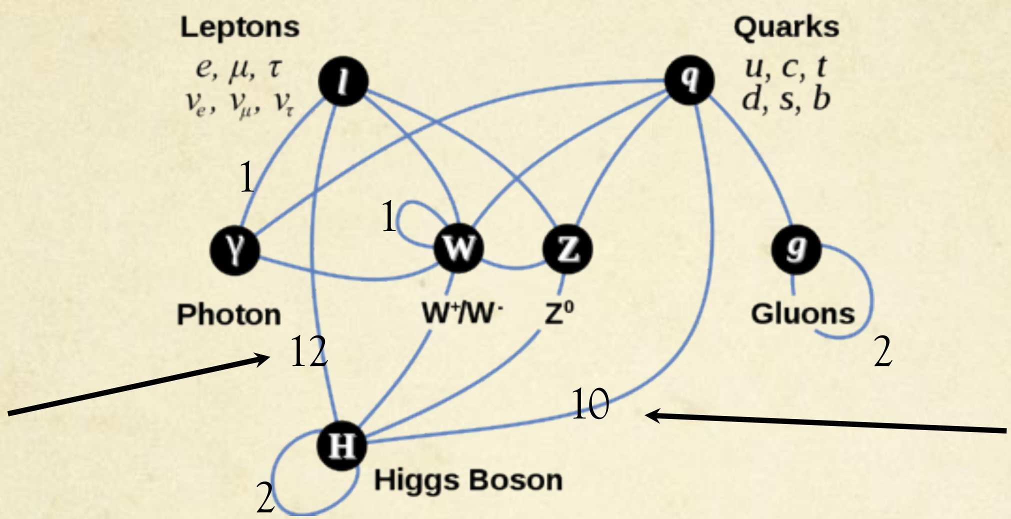
Introduction

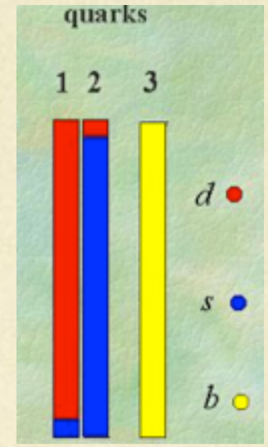
Motivation of Flavour Symmetries

Why Flavour Symmetry?

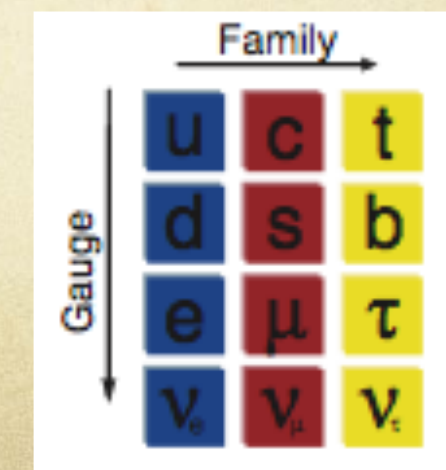
in SM (+Majorana neutrinos) there are a total of 28 parameters





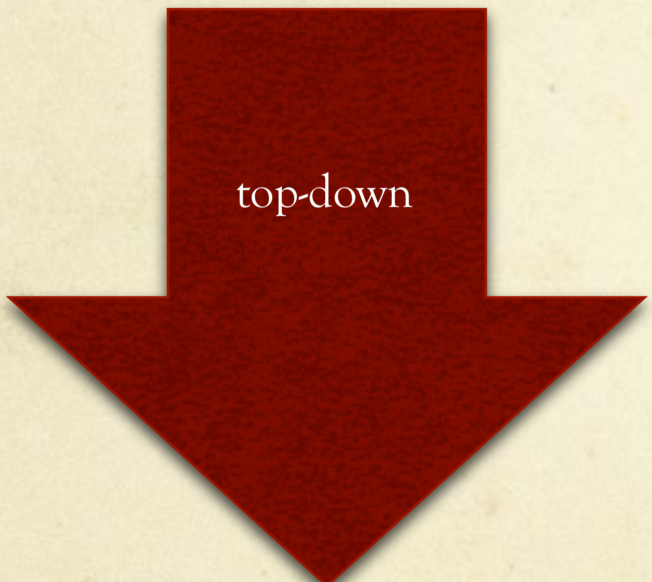


- most of the parameters stem from interactions with the Higgs field, i.e. flavour parameters
- other interactions tightly constrained by symmetry principles
- quarks small mixings; leptons large mixings
- this talk mostly leptons, for quarks many similar ideas are being pursued



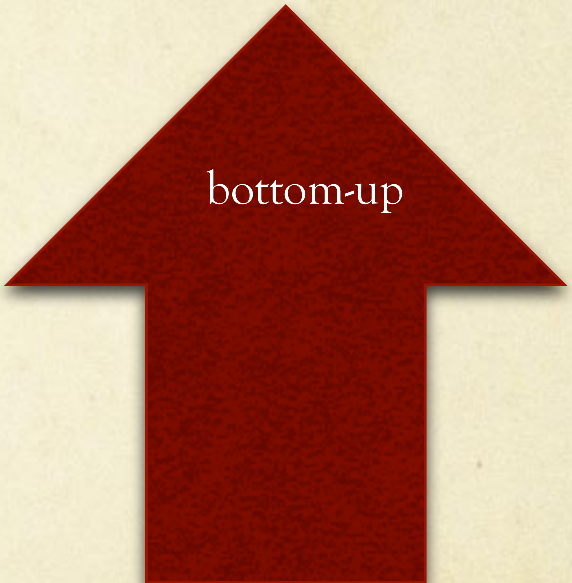
Two Theoretical Approaches to Flavour

String Theory, GUTs, etc.



SM

build models that reproduce the observed structures

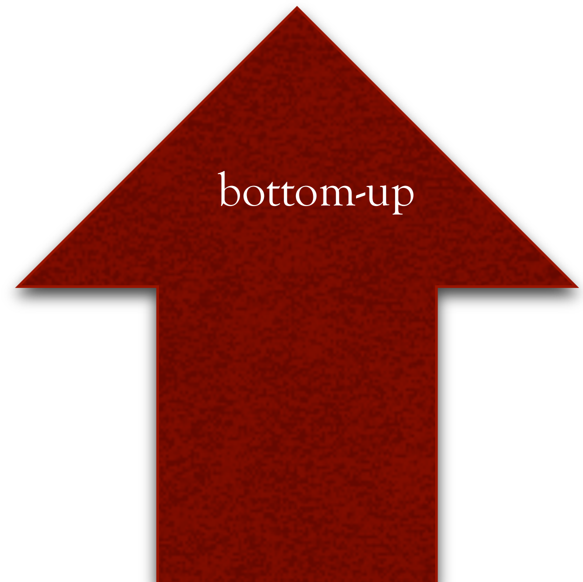


look at apparent regularities in masses and mixings etc.

Bottom Up Approach

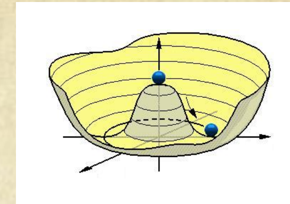
- there are many ways to go from low to high energies (effective field theory swampland)
- PLAN of Talk:
    - use remnant symmetries of mass matrices to guide model building
    - concrete model building challenge: vacuum alignment
    - applications to strong CP problem and CP phases

build models that reproduce the observed structures



look at apparent regularities in masses and mixings etc.

Leptonic Mixing



- in SM there are three generations of leptons, two mass matrices

$$\mathcal{L} \supset -L^T Y_e e^c \tilde{H} + \frac{(Y_\nu)_{ij}}{\Lambda}(L_i H)(L_j H) + \text{h.c.} \Rightarrow \mathcal{L} \supset e^T M_e e^c + \frac{1}{2}\nu^T M_\nu \nu + \text{h.c.}$$

- after diagonalization of two mass matrices

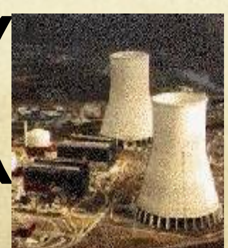
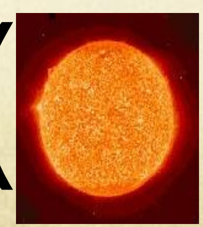
$$V_e^T M_e M_e^\dagger V_e^* = \text{diag}(m_e^2, m_\mu^2, m_\tau^2) \qquad \text{and} \qquad V_\nu^T M_\nu V_\nu = \text{diag}(m_1, m_2, m_3).$$

- flavour violation only in charged current interactions, analog of CKM

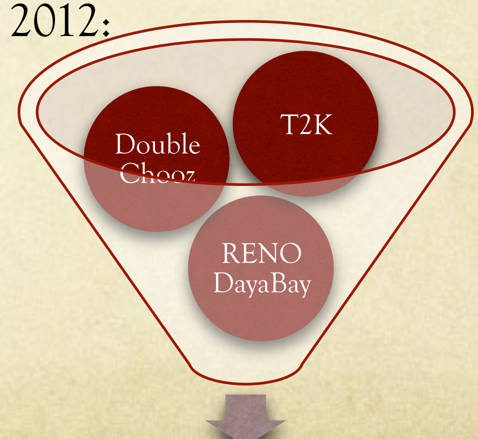
$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}}\left[e^\dagger \sigma^\mu U_{PMNS}\nu\right] W_\mu^+ + \text{h.c.}$$

$$\boxed{U_{PMNS} = V_e^\dagger V_\nu}$$

$$U_{\rm PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

= () x () x ()

2012:



$$\theta_{13} = \left(8.6^{+.44}_{-.46}\right)^\circ$$

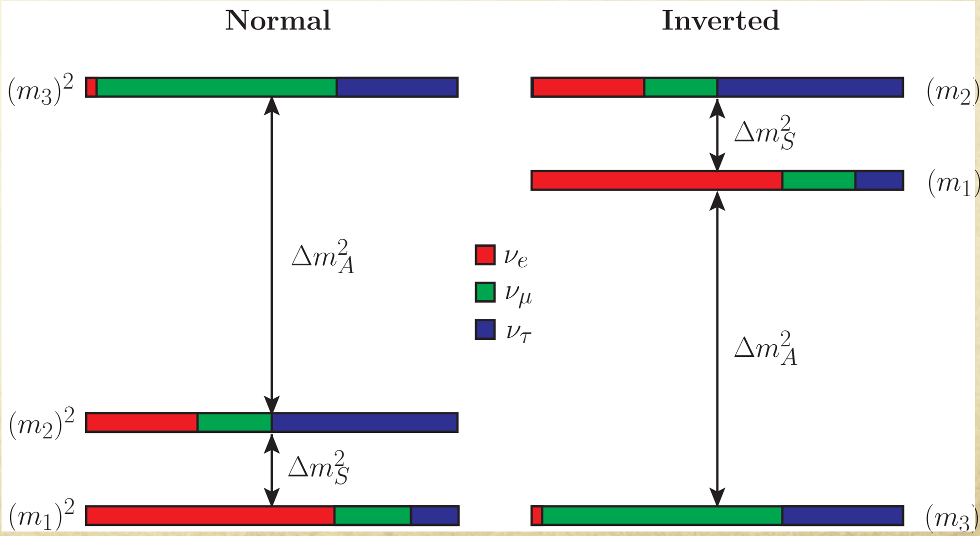
Standard 3-Flavour Picture

- Recent Progess in Determination of Leptonic Mixing Angles

global fit
[Forero, Tortola, Valle 2012]

	Δm^2_{21} $[10^{-5}\,\mathrm{eV}^2]$	$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\,\mathrm{eV}^2]$	$\sin^2\theta_{12}$ $[10^{-1}]$	$\sin^2\theta_{23}$ $[10^{-1}]$	$\sin^2\theta_{13}$ $[10^{-2}]$	δ $[\pi]$
best fit	$7.62^{+.19}_{-.19}$	$2.55^{+.06}_{-.09}$	$3.20^{+.16}_{-.17}$	$6.13^{+.22}_{-.40}$	$2.46^{+.29}_{-.28}$	$0.8^{+1.2}_{-.8}$
3σ range	$7.12 - 8.20$	$2.31 - 2.74$	$2.7 - 3.7$	$3.6 - 6.8$	$1.7 - 3.3$	$0 - 2$

- Largish Θ_{13} established
- Hint for non-maximal θ_{23}
- unknowns:
  - Normal or Inverted Hierarchy?
  - Majorana or Dirac (L-number conservation) ?
  - Is there CP violation in the lepton sector?
    - δ_{CP} & Majorana Phases
- are there sterile neutrinos that sizably mix with the active ones?



Lepton Mixing from Discrete Flavour Symmetries

based on [MH, K.S. Lim, M. Lindner 1212.2411(PLB)] and [MH, K.S. Lim, 1306.4356(PRD)]

Remnant Symmetries of Mass Matrices

the mass matrices are diagonalizable

$$V_e^T M_e M_e^\dagger V_e^* = \mathrm{diag}(m_e^2, m_\mu^2, m_\tau^2) \qquad \text{and} \qquad V_\nu^T M_\nu V_\nu = \mathrm{diag}(m_1, m_2, m_3).$$

which implies the following symmetries of the mass matrices

$$\boxed{\begin{aligned} U_e^T(\alpha) M_e M_e^\dagger U_e(\alpha)^* &= M_e M_e^\dagger \\ U_\nu^T(\epsilon) M_\nu U_\nu(\epsilon) &= M_\nu \end{aligned}}$$

with unitary matrices given by

$$\boxed{\begin{aligned} U_e(\alpha) &= V_e \mathrm{diag}(e^{\mathrm{i}\alpha_1}, e^{\mathrm{i}\alpha_2}, e^{\mathrm{i}\alpha_3}) V_e^\dagger \\ U_\nu(\epsilon) &= V_\nu \mathrm{diag}(\epsilon_1, \epsilon_2, \epsilon_3) V_\nu^\dagger \end{aligned}}$$

where α_i are real, implying $G_e = U(1)^3$, and $\varepsilon_i = \pm 1$, implying $G_\nu = Z_2^3$ (if neutrinos are Dirac $G_\nu = U(1)^3$)

- these symmetries are just reformulation of the fact that mass matrices are diagonalizable
  - for any mixing there are (different) symmetries

Given remnant symmetries, what can we learn about the mixing?

- symmetry generators commute with mass matrices, therefore can be diagonalized simultaneously
  - determine the mixing matrix up to permutations of rows and colums of the PMNS matrix
- remnant symmetries therefore encode information about the mixing angles, no information about mass ordering

Model → Mass Matrices → PMNS Matrix

- „remnant" symmetries can either be accidental or left-overs from the breakdown of a flavour symmetry G_f
  - since left-handed leptons are unified in SM, $L=(\nu, e)^T$, the flavour groups has to be broken to different subgroups G_e and G_ν

Lepton mixing from discrete groups

complete flavour group

$\mathrm{G_f}$

$e \to \rho(g_e)e$

$L \to \rho(g)L$

LH leptons 3-dim rep.

$\nu \to \rho(g_\nu)\nu$

residual symmetry of (Me Me⁺)

$\rho(g_e)^T M_e M_e^\dagger \rho(g_e)^* = M_e M_e^\dagger$

residual symmetry of M_ν

$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$

$\mathrm{G_e}$

- abelian (3 different masses)
- 3 different elements (to distinguish)
- smallest group Z_3

$$\Omega_e^\dagger \rho(g_e)\Omega_e = \rho(g_e)_{diag}$$

G_ν

- abelian (3 different masses)
- 3 different elements (to distinguish)
- if Majorana eigenvalues +/- 1
  - smallest group $Z_2 x Z_2$ that fixes all angles is also maximal

$$\Omega_\nu^\dagger \rho(g_\nu)\Omega_\nu = \rho(g_\nu)_{diag}$$

misaligned non-commuting symmetries lead to

$$U_{\mathrm{PMNS}} = \Omega_e^\dagger \Omega_\nu$$

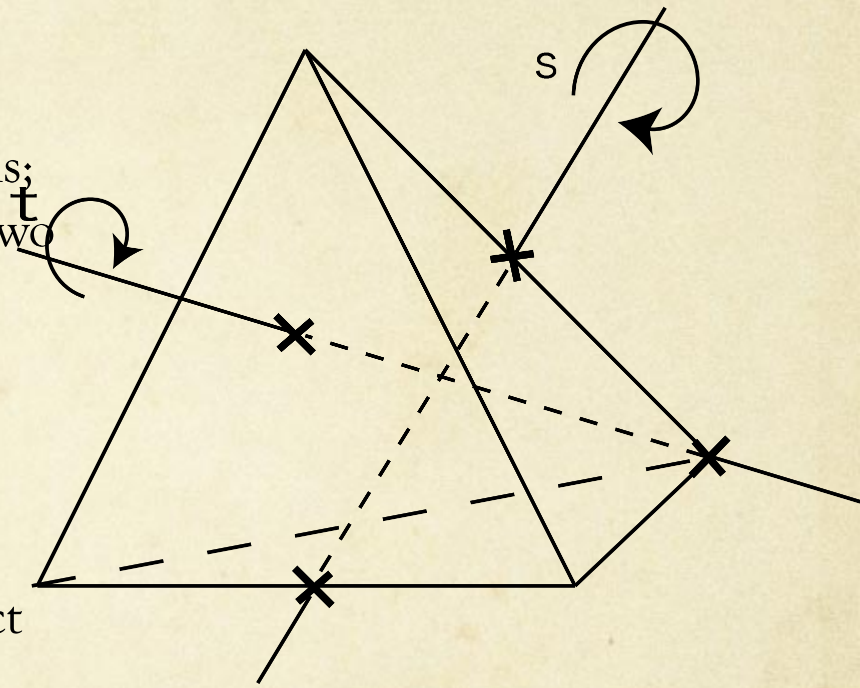
- mixing matrix determined from symmetry up to interchanging of rows/columns and diagonal phase matrix

- if $\mathrm{G_e}$ or G_ν do not have 3 different eigenvalues, the mixing matrix is not fully determined (free continuous parameter)

[He, Keum, Volkas '06; Lam' 07, '08; Altarelli,Feruglio' 05, Feruglio, Hagedorn,Toroop' 11]

What are discrete groups?

- consider the symmetry group of the regular tetrahedron A_4
- there are 12 symmetry transformations; they might be written as products of two generators S and T
- relations between generators (presentation) defines group
  - $S^2=T^3=(ST)^3=1$
- representations are maps from abstract group to matrices
  - A_4 has one three-dim. rep. **3** and three one-dim rep. $\mathbf{1}_1$, $\mathbf{1}_2$, $\mathbf{1}_3$
- groups with 3-dim. representations needed to account for 3 fermion generations



- origin of discrete flavour group
  - stringy [Nilles et al. 1204.2206]
  - breakdown of continuous SU(3) or SO(3) [Ovrut 1978,...]

Lepton mixing from discrete groups

famous example: S_4 gives TBM

complete flavour group

$$G_f = S_4$$

$$e \to \rho(g_e) e \qquad L \to \rho(g) L \qquad \nu \to \rho(g_\nu)\nu$$

LH leptons 3-dim rep.

residual symmetry of ($M_e M_e^+$)

$$\rho(g_e)^T M_e M_e^\dagger \rho(g_e)^* = M_e M_e^\dagger$$

residual symmetry of M_ν

$$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$$

$$G_e = \langle T \rangle = Z_3$$

$$\rho(T) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

(Z_3 smallest choice, but can also be continuous)

$$G_\nu = \langle S,U \rangle = Z_2 x Z_2$$

$$\rho(S) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \rho(U) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

($Z_2 x Z_2$ most general choice if mixing angles do not depend on masses & Majorana ν s)

misaligned non-commuting symmetries lead to

$$U_{\text{PMNS}} = \Omega_e^\dagger \Omega_\nu$$

$$\Omega_e^\dagger \rho(g_e) \Omega_e = \rho(g_e)_{diag}$$

$$\Omega_\nu^\dagger \rho(g_\nu) \Omega_\nu = \rho(g_\nu)_{diag}$$

mixing matrix determined from symmetry up to interchanging of rows/columns and diagonal phase matrix

[He, Keum, Volkas '06; Lam' 07, '08; Altarelli,Feruglio' 05, Feruglio, Hagedorn,Toroop' 11]

$G_\nu = \langle S,U \rangle = Z_2 x Z_2$

$$U_{\text{HPS}} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2\theta_{12} = \frac{1}{3}, \quad \sin^2\theta_{23} = \frac{1}{2}, \quad \sin^2\theta_{13} = 0$$

tri-bimaximal mixing (TBM)

$G_\nu = \langle S \rangle = Z_2$

$$U_{TMM} = U_{TBM} \begin{pmatrix} \cos\theta & 0 & e^{i\delta}\sin\theta \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta & 0 & \cos\theta \end{pmatrix}$$

tri-maximal mixing (TM2)

[Lin' 10, Shimizu et al. '11,Luhn,King' 11,...]

Lepton mixing from discrete groups

residual symmetry of

$\rho(g_e)^T M_e M_e^\dagger \rho(g$

$\mathrm{G_e} = \langle$

$\rho(T) =$

(Z3 smallest choice, b
continuous)

$\Omega_e^\dagger \rho(g_e)\Omega_e$

[He, Keum, Volkas '0
Lam' 07, '08;
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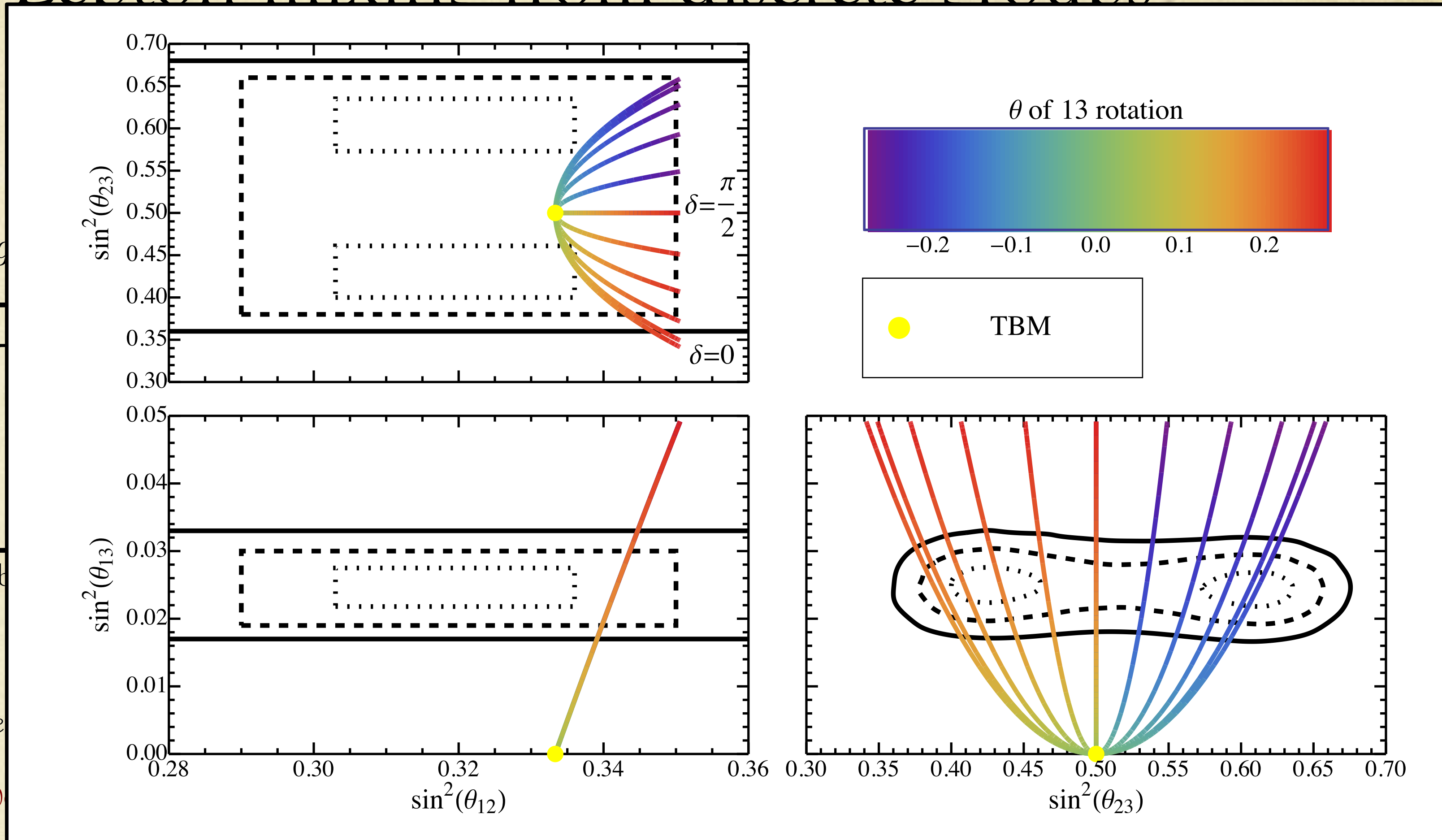

tri-bimaximal mixing (TBM)

$$U_{\rm HPS} \equiv \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\sin^2\theta_{12} = \frac{1}{3}, \quad \sin^2\theta_{23} = \frac{1}{2}, \quad \sin^2\theta_{13} = 0$$

$\mathrm{G}_\nu = \langle \mathrm{S} \rangle = \mathrm{Z}_2$

$$U_{TMM} = U_{TBM} \begin{pmatrix} \cos\theta & 0 & e^{i\delta}\sin\theta \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta & 0 & \cos\theta \end{pmatrix}$$

tri-maximal mixing (TM2)

[Lin' 10, Shimizu et al. '11,Luhn,King' 11,...]

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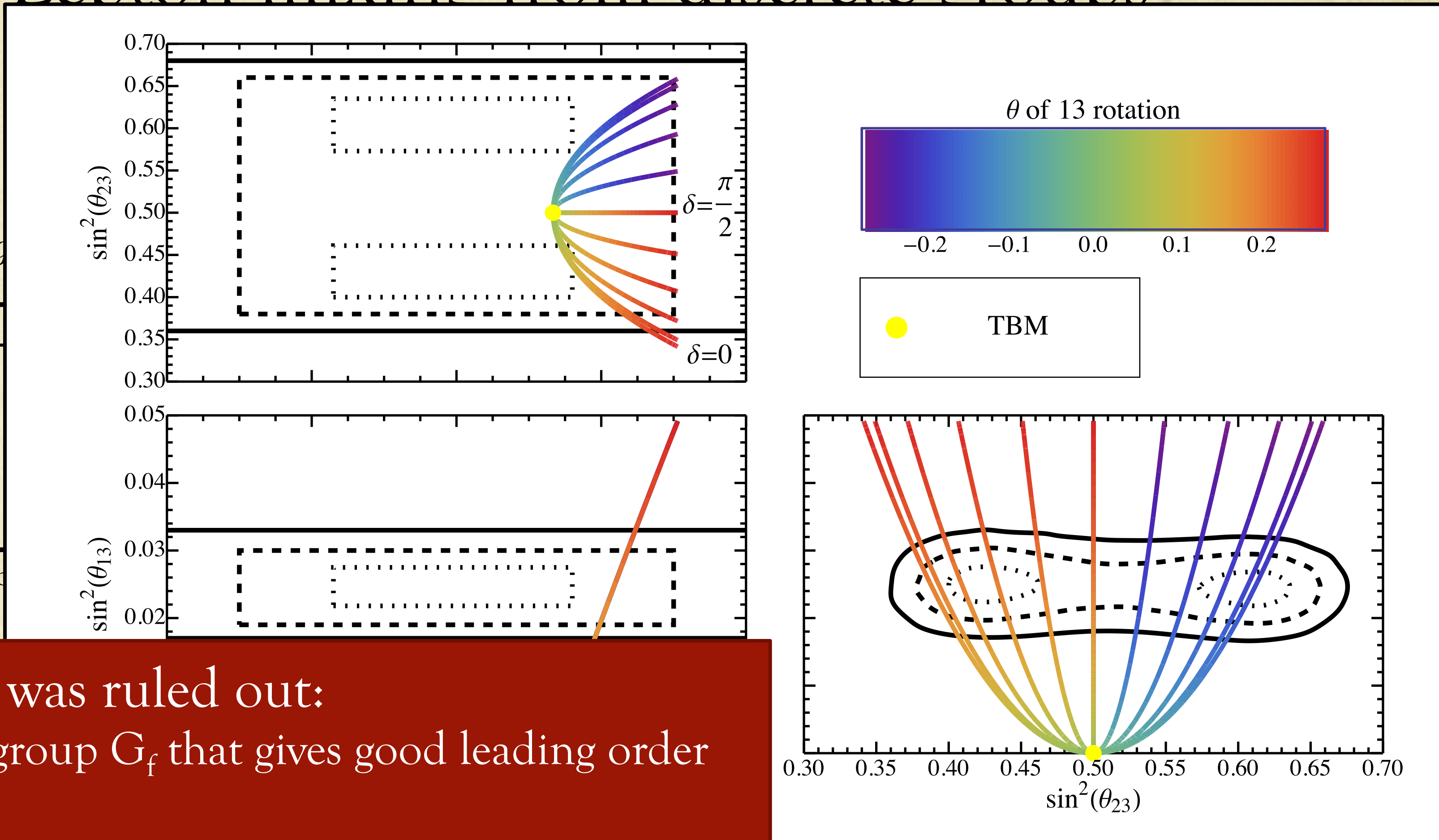


After TBM was ruled out:

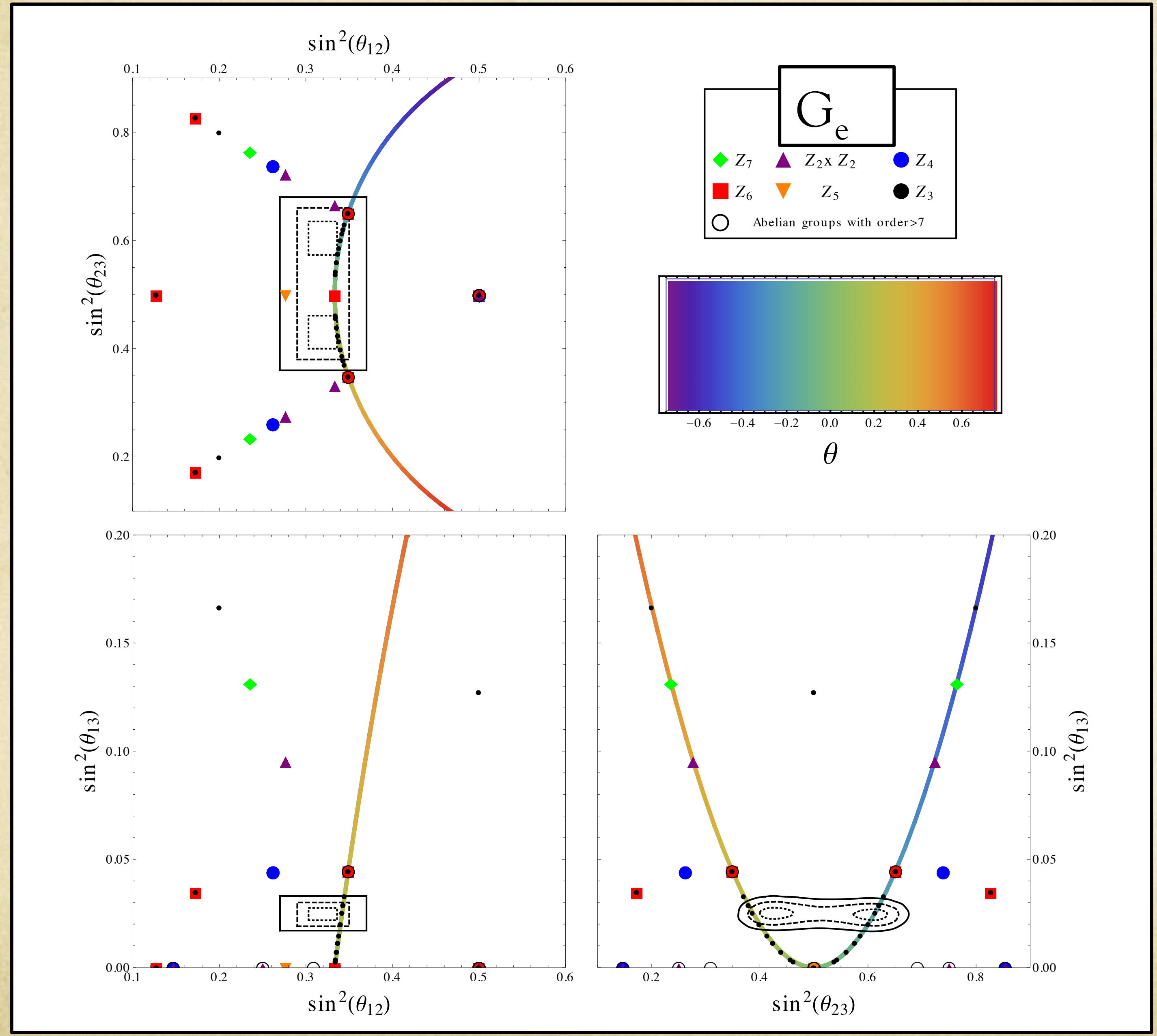
- is there a group G_f that gives good leading order mixing?
  - no predudices: take all groups up to certain order, all subgroups
  - implement on computer
    - GAP
  - scan for interesting groups

$G_\nu = \langle S \rangle = Z_2$

$$U_{TMM} = U_{TBM} \begin{pmatrix} \cos\theta & 0 & e^{i\delta}\sin\theta \\ 0 & 1 & 0 \\ -e^{i\delta}\sin\theta & 0 & \cos\theta \end{pmatrix}$$

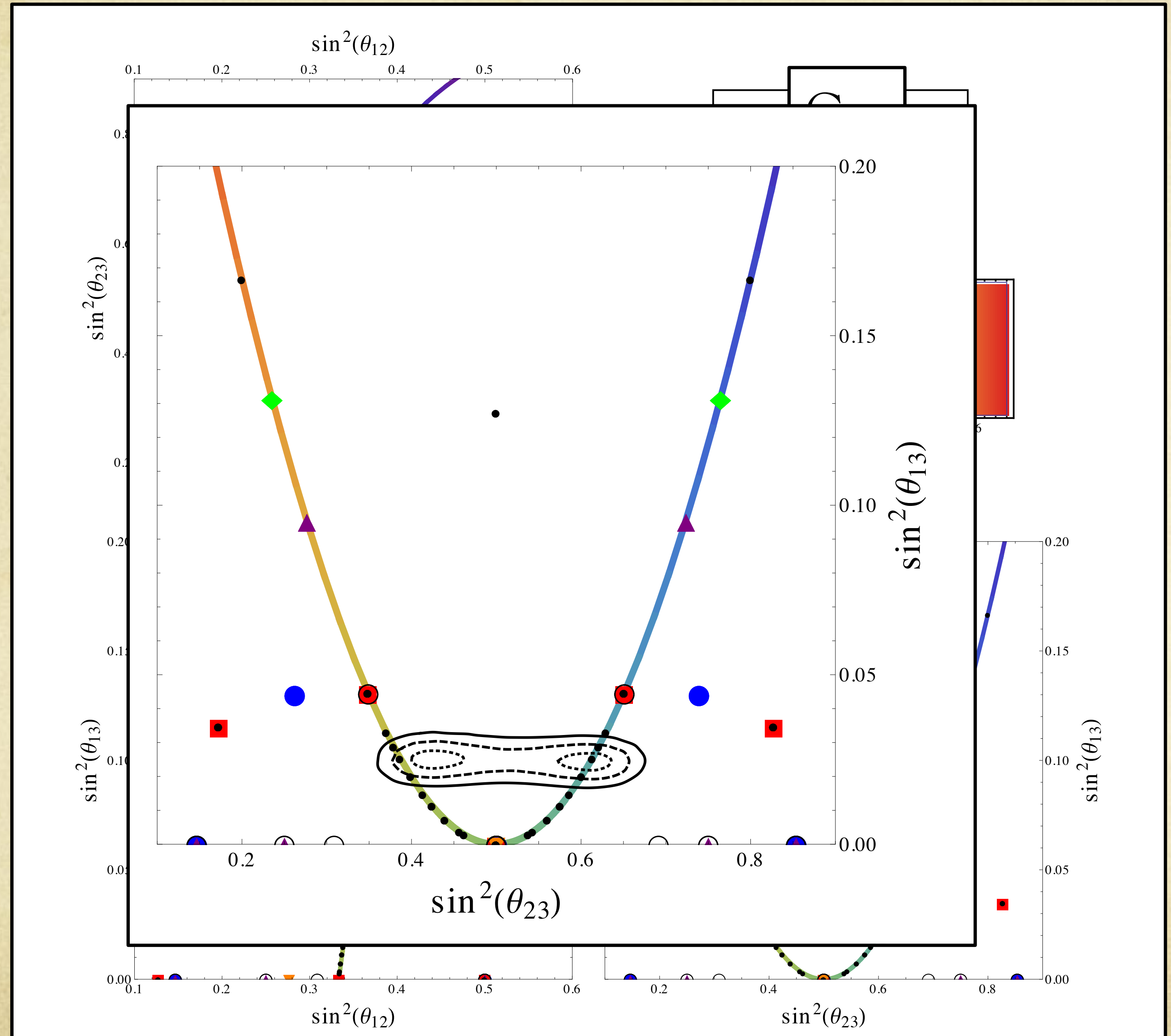
tri-maximal mixing (TM2)
[Lin' 10, Shimizu et al. '11,Luhn,King' 11,...]

No prejudice scan



- for general G_e the scan is performed up to order 511, for $G_e=Z_3$ up to 1536
- Majorana neutrinos $G_\nu=Z_2xZ_2$
- also for $G_e=Z_3$ not all groups lie on TM2 „parabola“
- known LO mixing pattern such as golden ratio etc. reproduced
- no exciting group for G_e>3

No prejudice scan



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TM2 "parabola"

complete flavour group

G_f

residual symmetry of ($M_e M_e^+$)

$G_e = \langle T \rangle = Z_3$

$$\rho(T) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$L \to \rho(g) L$

LH leptons 3-dim rep.

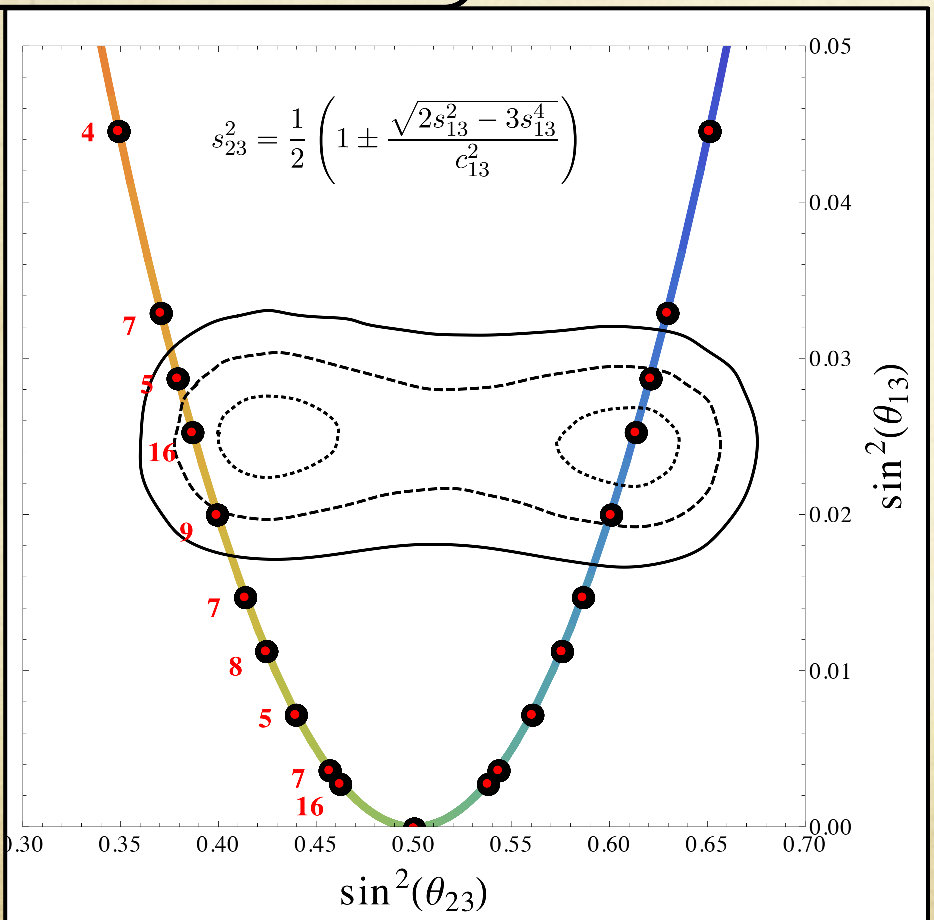
residual symmetry of M_ν

$$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$$

$G_\nu = \langle S, U_n \rangle = Z_2 x Z_2$

$$\rho(S) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \rho(U_n) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & z_n \\ 0 & z_n^* & 0 \end{pmatrix}$$

$$\langle z_n \rangle \cong Z_n$$



- Scan over all discrete groups of size smaller than 1556 with $G_e = Z_3$, $G_\nu = Z_2 x Z_2$
- all solutions which are close to exp. values have the TM2 form

$$U = U_{TBM} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

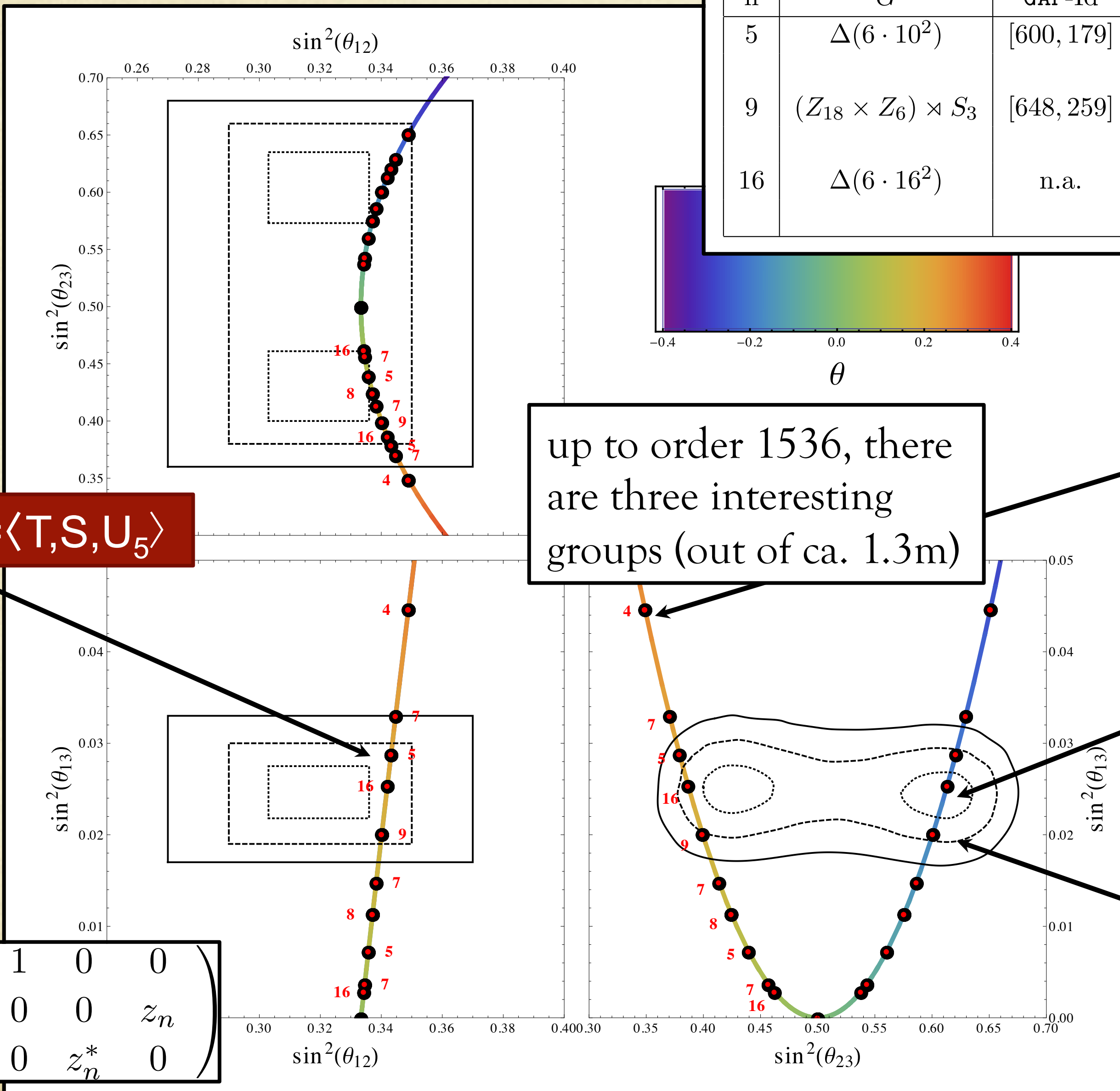
with $\theta = \frac{1}{2}\arg(z_n)$

- vanishing CP phase δ_{CP}

[MH, K.S. Lim, M. Lindner 1212.2411(PLB)]

Lepton mixing from discrete groups

n	G	GAP-Id	$\sin^2(\theta_{12})$	$\sin^2(\theta_{13})$	$\sin^2(\theta_{23})$
5	$\Delta(6\cdot 10^2)$	[600, 179]	0.3432	0.0288	0.3791
			0.3432	0.0288	0.6209
9	$(Z_{18}\times Z_6)\rtimes S_3$	[648, 259]	0.3402	0.0201	0.3992
			0.3402	0.0201	0.6008
16	$\Delta(6\cdot 16^2)$	n.a.	0.3420	0.0254	0.3867
			0.3420	0.0254	0.6134



up to order 1536, there are three interesting groups (out of ca. 1.3m)

[Lam 13]

$\Delta(6\cdot 10^2)=\langle T,S,U_5\rangle$

$\Delta(6\cdot 4^2)=\langle T,S,U_4\rangle$

[Torop, Hagedorn, Feruglio 11, King, Luhn, Steward 12, Ding 12]

$\Delta(6\cdot 16^2)=\langle T,S,U_{16}\rangle$

$(Z_{18}\times Z_6)\rtimes S_3 \cong \langle S_3, T_3, U_9\rangle$

$$\rho(U_n)=\begin{pmatrix}1&0&0\\0&0&z_n\\0&z_n^*&0\end{pmatrix}$$

$\langle z_n\rangle \cong Z_n$

[MH, K.S. Lim, M. Lindner 1212.2411(PLB)]

Lepton mixing from discrete groups

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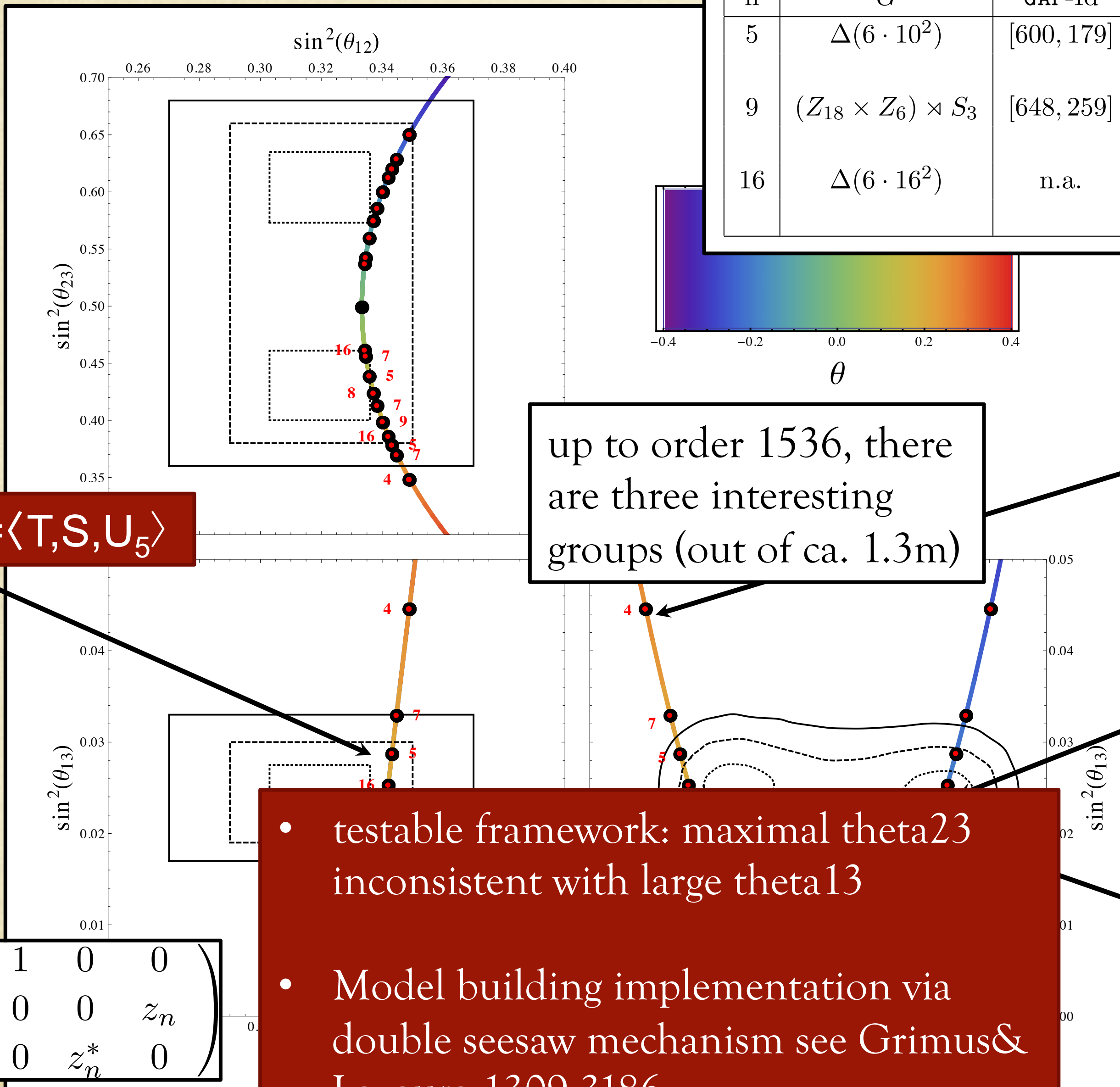

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$\Delta(6\cdot 4^2)=\langle T,S,U_4\rangle$

[Torop, Hagedorn, Feruglio 11, King, Luhn, Steward 12, Ding 12]

$\Delta(6\cdot 16^2)=\langle T,S,U_{16}\rangle$

$(Z_{18}\times Z_6)\rtimes S_3 \cong \langle S_3, T_3, U_9\rangle$

- testable framework: maximal theta23 inconsistent with large theta13
- Model building implementation via double seesaw mechanism see Grimus& Lavoura 1309.3186

$$\rho(U_n)=\begin{pmatrix}1&0&0\\0&0&z_n\\0&z_n^*&0\end{pmatrix}$$

$\langle z_n\rangle \cong Z_n$

[MH, K.S. Lim, M. Lindner 1212.2411(PLB)]

Quark Mixing from the same groups

$$
\begin{tikzcd}
G_e \arrow[rr, leftrightarrow, bend left, "\text{PMNS}"] & & G_\nu \\
& G_f \arrow[ul] \arrow[ur] \arrow[dl] \arrow[dr] & \\
G_u \arrow[rr, leftrightarrow, bend right, "\text{CKM}"'] & & G_d
\end{tikzcd}
$$

n	G	GAP-Id	$\sin^2(\theta_{12})$	$\sin^2(\theta_{13})$	$\sin^2(\theta_{23})$
5	$\Delta(6\cdot 10^2)$	[600, 179]	0.3432	0.0288	0.3791
			0.3432	0.0288	0.6209
9	$(Z_{18}\times Z_6)\rtimes S_3$	[648, 259]	0.3402	0.0201	0.3992
			0.3402	0.0201	0.6008
16	$\Delta(6\cdot 16^2)$	n.a.	0.3420	0.0254	0.3867
			0.3420	0.0254	0.6134

- breaking to different subgroups yields LO mixing pattern where only Cabibbo angle is produced

$$U_{\text{CKM}} = \begin{pmatrix} \cos\tilde\theta & \sin\tilde\theta & 0 \\ -\sin\tilde\theta & \cos\tilde\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

n	G_f	GAP-Id	$\sin\tilde\theta$	type
5	$\Delta(6\cdot 10^2)$	[600, 179]	0.156	A
			0.309	B
9	$(Z_{18}\times Z_6)\rtimes S_3$	[648, 259]	0.259	A
16	$\Delta(6\cdot 16^2)$	n.a.	0.195	A

- type A:

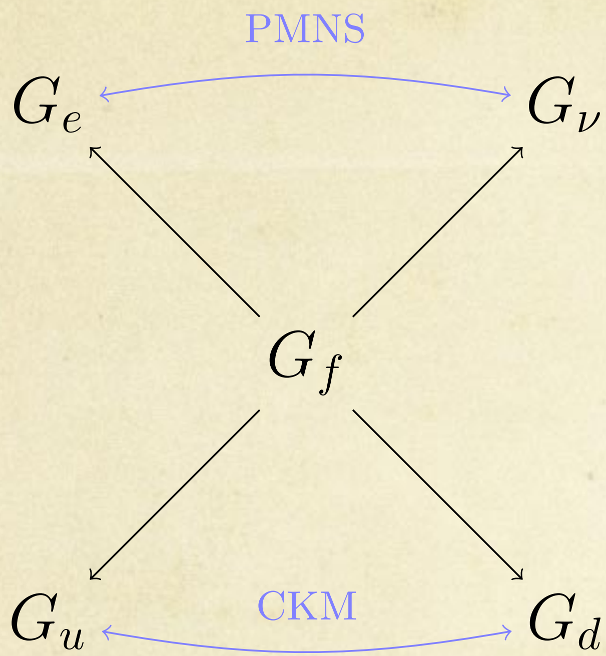
$$G_d = \langle S, U(n,p)\rangle \cong Z_2\times Z_2,$$
$$G_u = \langle (ST)^2 T U(n,m)\rangle \cong Z_4$$

- type B:

$$G_d = \langle S, U(n,p)\rangle \cong Z_2\times Z_2,$$
$$G_u = \langle S, (U(n,m)T^2)^2 (U(n,m)T)^2 U(n,m)\rangle$$
$$\cong Z_2\times Z_2$$

[MH, K.S. Lim, 1305.4356(PRD)]

Quark Mixing from the same groups



n	G	GAP-Id	$\sin^2(\theta_{12})$	$\sin^2(\theta_{13})$	$\sin^2(\theta_{23})$
5	$\Delta(6\cdot 10^2)$	[600, 179]	0.3432	0.0288	0.3791
			0.3432	0.0288	0.6209
9	$(Z_{18}\times Z_6)\rtimes S_3$	[648, 259]	0.3402	0.0201	0.3992
			0.3402	0.0201	0.6008
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16	$\Delta(6\cdot 16^2)$	n.a.	0.195	A

- all other quark angles are NLO effects
- smaller NLO corrections possible

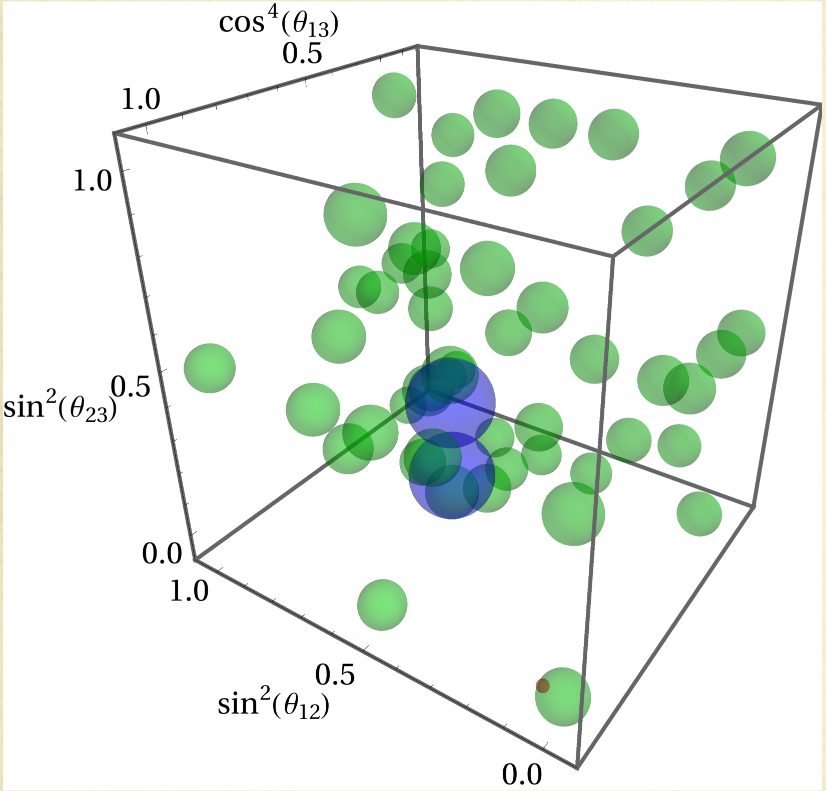
G_d

$$G_u = \langle S, (U(n,m)T^2)^2 (U(n,m)T)^2 U(n,m)\rangle \cong Z_2\times Z_2$$

[MH, K.S. Lim, 1305.4356(PRD)]

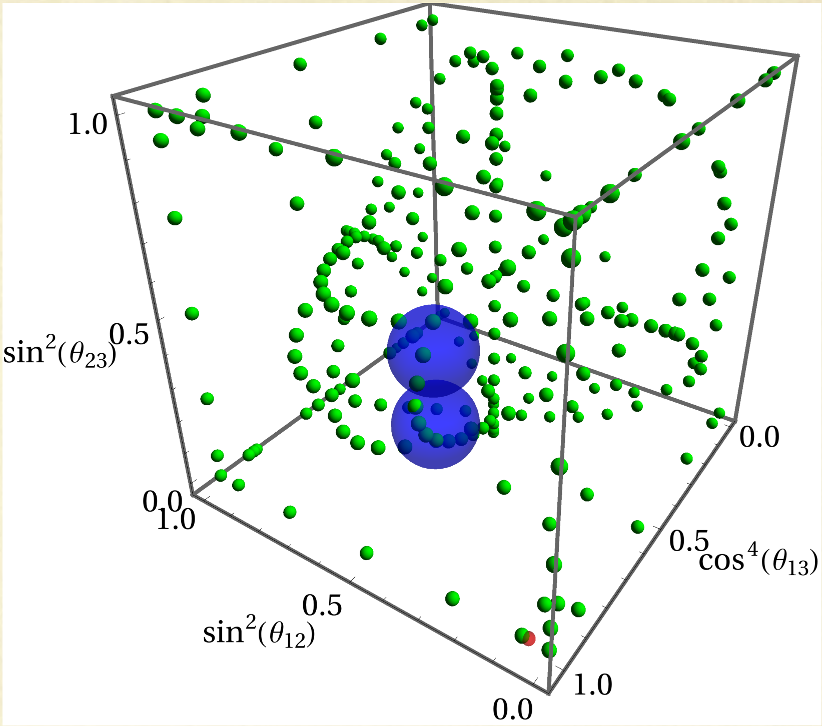
Quo vadis?

3 sigma values of exp. measured lepton/ quark angles
possible values from G_f



(a) $\mathcal{S}_4$

small groups, large NLO corrections



(b) $(Z_{18} \times Z_6) \rtimes \mathcal{S}_3$

large groups, small NLO corrections

for a possible measure comparing the ‚relative instrinsic precitvitiy' of flavour groups, see [MH, K.S. Lim, 1305.4356(PRD)]

A Model Building Challenge: Vacuum Alignment Problem

based on
[MH, M.A. Schmidt *JHEP* **1201** (2012) 126, 1111.1730 [hep-ph]]
and
[MH, M. Lindner, M.A. Schmidt 1211.5143 (PRD)]

A_4 Model Building Boot Camp

- predictions for mixing angles are a result of remnant symmetries
- in a concrete model one therefore needs a peculiar breaking pattern

- (A_4,Z_4) charge assignments: L~ (3,i), e^c~ (1_1,-i), μ^c~ (1_2,-i), τ^c~ (1_3,-i) ,χ ~(3,1), Φ ~(3,-1), ξ ~(1,-1)
- auxiliary Z_4 separates neutral and charged lepton sectors at LO

$$y_e(L\chi)_{\underline{1}_1}e^c + y_\mu(L\chi)_{\underline{1}_3}\mu^c + y_\tau(L\chi)_{\underline{1}_2}\tau^c$$

$$A_4 = \langle S,T|S^2 = T^3 = (ST)^3 = E\rangle$$

$$(LL)_{\underline{3}}\phi + (LL)_{\underline{1}_1}\xi$$

$\langle\chi\rangle \sim (1,1,1)$

$\langle \Phi \rangle$ ~(1,0,0)

$$\rho(T) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Z_3=⟨T⟩

Z_2=⟨S⟩

$$\rho(S) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$M_E \sim \begin{pmatrix} y_e & y_\mu & y_\tau \\ y_e & \omega y_\mu & \omega^2 y_\tau \\ y_e & \omega^2 y_\mu & \omega y_\tau \end{pmatrix}$$

TBM

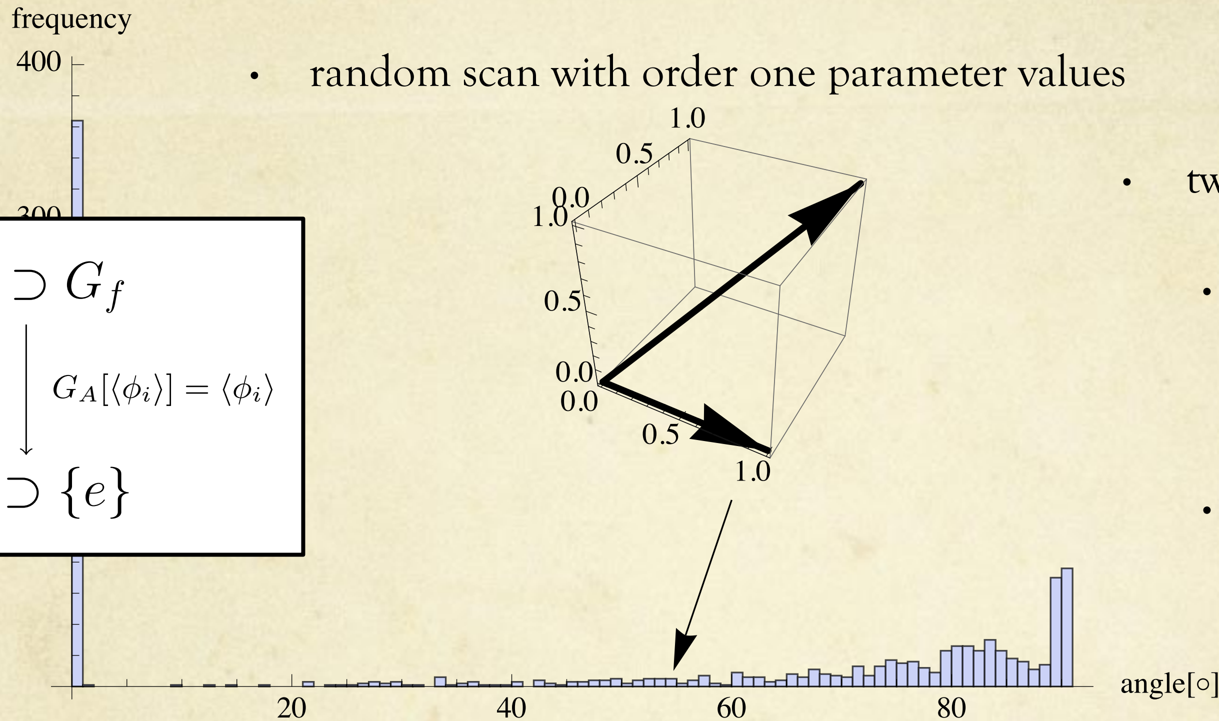
$$M_\nu \sim \begin{pmatrix} \tilde{a} & 0 & 0 \\ 0 & \tilde{a} & \tilde{d} \\ 0 & \tilde{d} & \tilde{a} \end{pmatrix}$$

(symmetry U accidental)

Vacuum alignment crucial!

[e.g. Ma,Rajasekaran' 01, Babu, Ma, Valle '03, Altarelli,Feruglio, '05,' 06]

Can Vacuum Alignment be realized?



$$\begin{array}{ccc} & G_V \supset G_f & \\ \langle\phi_i\rangle \neq 0 & \Big\downarrow & G_A[\langle\phi_i\rangle] = \langle\phi_i\rangle \\ & G_A \supset \{e\} & \end{array}$$

- random scan with order one parameter values

- two solutions:
  - both flavons preserved same subgroup, angle zero
  - no conserved subgroup, TBM vaccum has to be fine-tuned

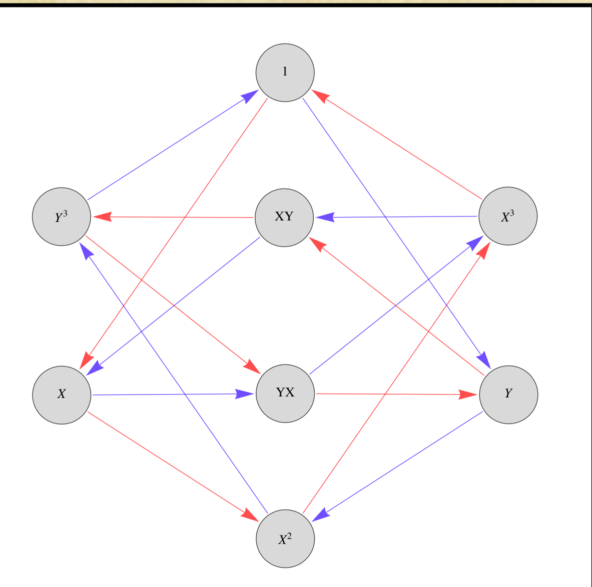
[for a general discussion of breakdown to discrete subgroups. see Michel 1980]

$$\boxed{\kappa_{\underline{\mathbf{3}}_\mathbf{1}}(\phi\phi)_{\underline{\mathbf{3}}_\mathbf{1}}(\chi\chi)_{\underline{\mathbf{3}}_\mathbf{1}} + \left(\kappa_{\underline{\mathbf{1}}_\mathbf{2}}(\phi\phi)_{\underline{\mathbf{1}}_\mathbf{2}}(\chi\chi)_{\underline{\mathbf{1}}_\mathbf{3}} + \text{h.c.}\right)}$$

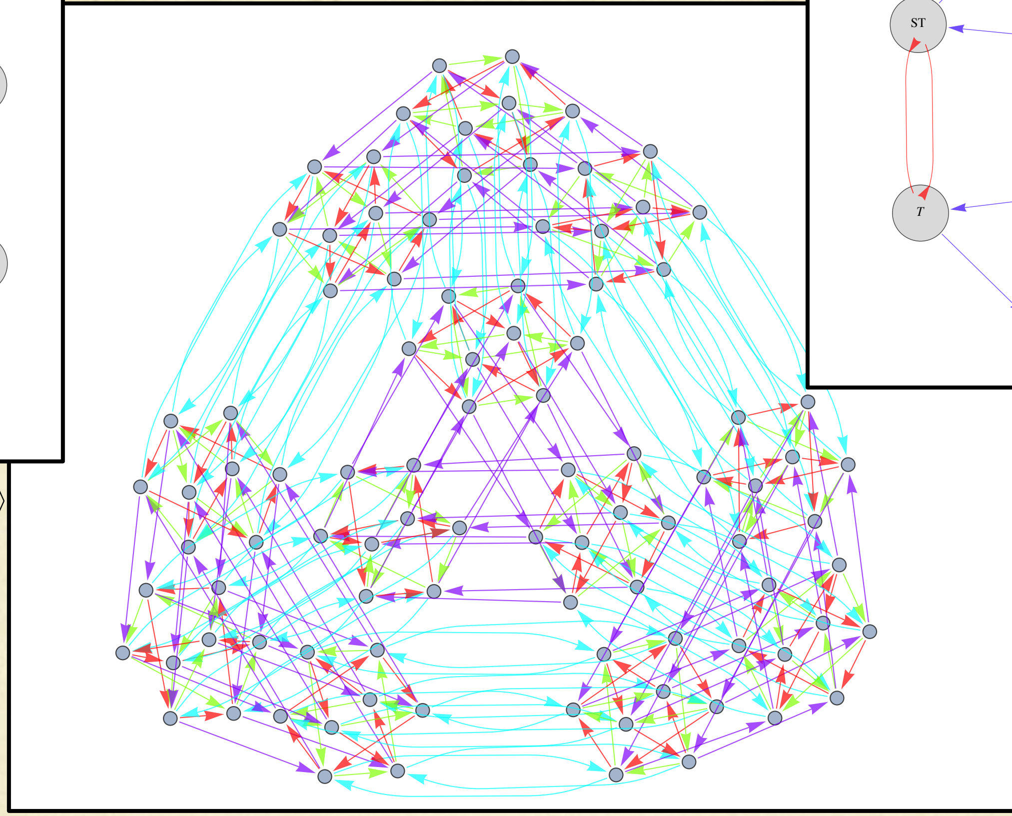
need a natural explanation for absence of dangerous cross-coupling terms

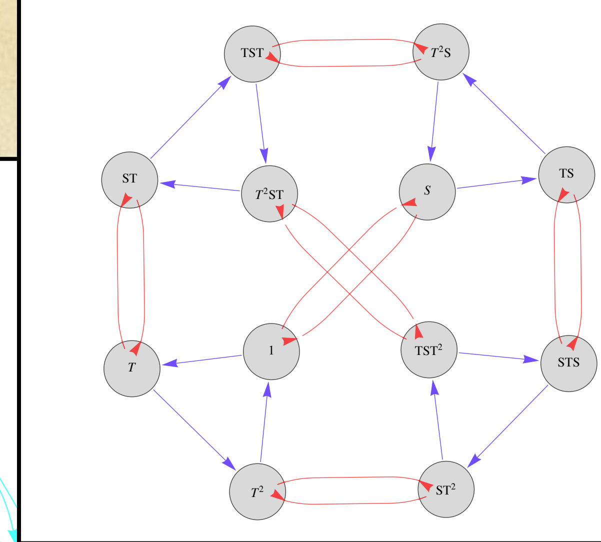
- usually quite complicated costructions based on extra dimensions or continuous R symmetries in SUSY
- ALWAYS: engineered accidental $A_4 x A_4$ symmetry ONLY of potential Φ ~(3,1) & χ ~(1,3)
- What is the smallest solution within 4D non-SUSY QFT?

Answer: $Q_8 \rtimes A_4$



$\langle X, Y | X^4 = 1,\ X^2 = Y^2,\ Y^{-1}XY = X^{-1} \rangle$





$\langle S, T | S^2 = T^3 = (ST)^3 = 1 \rangle$

$$SXS^{-1} = X, \qquad SYS^{-1} = Y^{-1}, \qquad TXT^{-1} = YX, \qquad TYT^{-1} = X$$

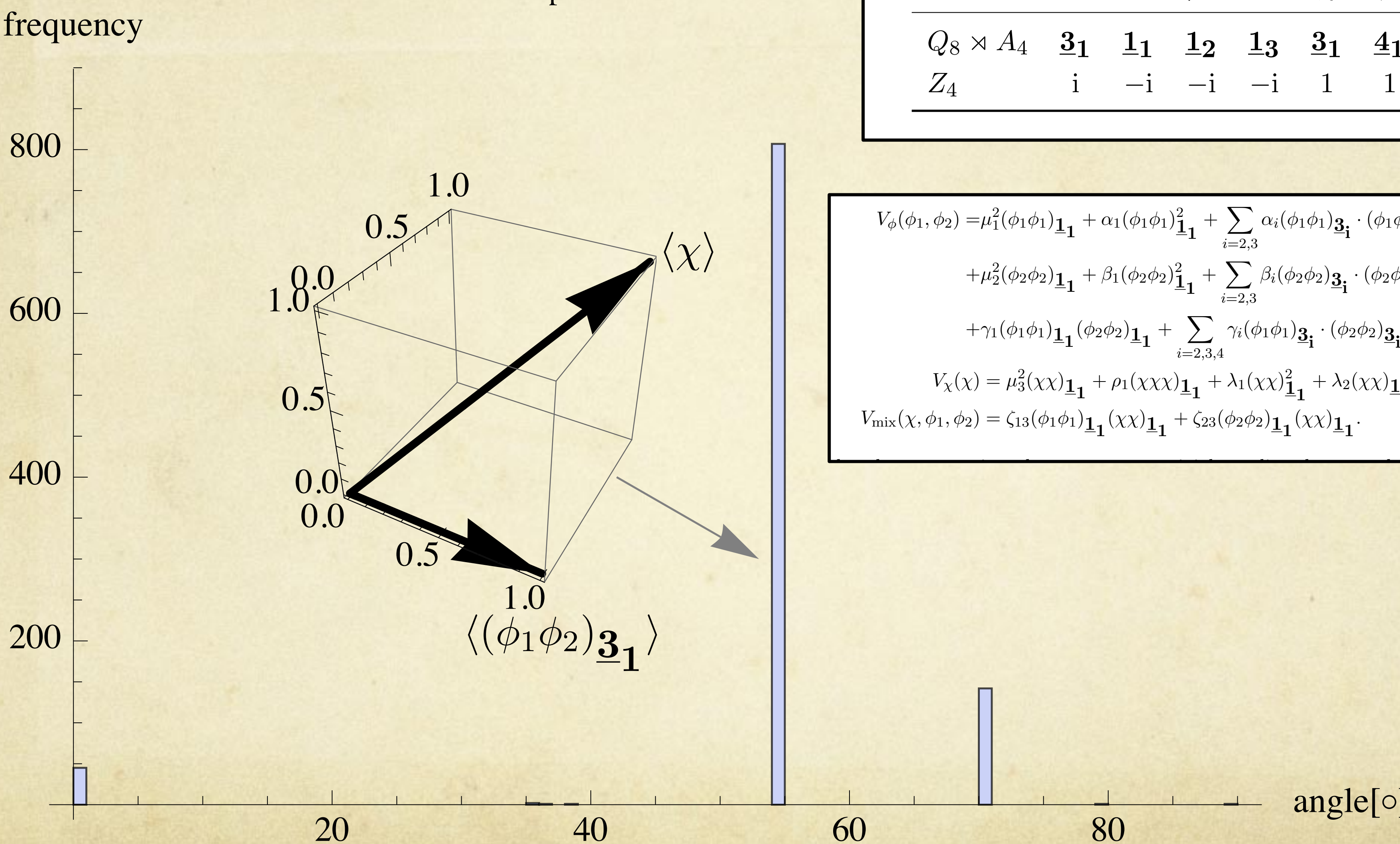
- there is one representation which does not contain an A_4 representation in is symmetric product

$$\underline{\mathbf{4}}_{\mathbf{1}} \times \underline{\mathbf{4}}_{\mathbf{1}} = \underline{\mathbf{1}}_{\mathbf{1}S} + \underline{\mathbf{3}}_{\mathbf{1}A} + \underline{\mathbf{3}}_{\mathbf{2}S} + \underline{\mathbf{3}}_{\mathbf{3}S} + \underline{\mathbf{3}}_{\mathbf{4}S} + \underline{\mathbf{3}}_{\mathbf{5}A}$$

- therefore the coupling between a scalar transforming as 3_1 and 4_1 is non-renormalizable, which leads to an accidental symmetry $V=V_\Phi(\Phi)+V_\chi(\chi)+(\Phi\,\Phi)_1(\chi\chi)_1$

Scalar Potential & Vacuum Alignment

- random scan over order one parameter values



	L	e^c	μ^c	τ^c	χ	ϕ_1	ϕ_2
$Q_8 \rtimes A_4$	$\underline{\mathbf{3}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{1}}$	$\underline{\mathbf{1}}_{\mathbf{2}}$	$\underline{\mathbf{1}}_{\mathbf{3}}$	$\underline{\mathbf{3}}_{\mathbf{1}}$	$\underline{\mathbf{4}}_{\mathbf{1}}$	$\underline{\mathbf{4}}_{\mathbf{1}}$
Z_4	i	−i	−i	−i	1	1	−1

$$\begin{aligned}
V_\phi(\phi_1,\phi_2) =& \mu_1^2(\phi_1\phi_1)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \alpha_1(\phi_1\phi_1)^2_{\underline{\mathbf{1}}_{\mathbf{1}}} + \sum_{i=2,3}\alpha_i(\phi_1\phi_1)_{\underline{\mathbf{3}}_{\mathbf{i}}}\cdot(\phi_1\phi_1)_{\underline{\mathbf{3}}_{\mathbf{i}}} \\
&+\mu_2^2(\phi_2\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \beta_1(\phi_2\phi_2)^2_{\underline{\mathbf{1}}_{\mathbf{1}}} + \sum_{i=2,3}\beta_i(\phi_2\phi_2)_{\underline{\mathbf{3}}_{\mathbf{i}}}\cdot(\phi_2\phi_2)_{\underline{\mathbf{3}}_{\mathbf{i}}} \\
&+\gamma_1(\phi_1\phi_1)_{\underline{\mathbf{1}}_{\mathbf{1}}}(\phi_2\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \sum_{i=2,3,4}\gamma_i(\phi_1\phi_1)_{\underline{\mathbf{3}}_{\mathbf{i}}}\cdot(\phi_2\phi_2)_{\underline{\mathbf{3}}_{\mathbf{i}}}, \\
V_\chi(\chi) =& \mu_3^2(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \rho_1(\chi\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \lambda_1(\chi\chi)^2_{\underline{\mathbf{1}}_{\mathbf{1}}} + \lambda_2(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}}(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{3}}}, \\
V_{\text{mix}}(\chi,\phi_1,\phi_2) =& \zeta_{13}(\phi_1\phi_1)_{\underline{\mathbf{1}}_{\mathbf{1}}}(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}} + \zeta_{23}(\phi_2\phi_2)_{\underline{\mathbf{1}}_{\mathbf{1}}}(\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{1}}}.
\end{aligned}$$

- no fine tuning needed to obtain TBM vacuum
- check out Mathematica package Discrete [http://discrete.hepforge.org/]

Accidental symmetries as the origin of vacuum alignment mechanisms

- usual flavour symmetry setup: a flavour group G_f is broken to different subgroups by different VEVs; in total the group is completely broken
- in general, this VEV state is unprotected by any symmetry and one therefore has to tune parameters to get VEVs
  - kills predictivity of models
- theories that solve the vacuum alignment problem realize the VEV as symmetric solution of an accidental symmetry of the potential
  - VEV config symm. solution under unbroken accidental symmetry

$$
\begin{tikzcd}
 & A_4 \arrow[dl, bend right, "{\langle\phi_1\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}}"'] \arrow[d, "{\langle\phi_2\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}}"'] \arrow[dr, bend left, "{\langle\phi_3\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}}"] & \\
\langle S\rangle & \langle T^2ST\rangle & \langle TST^2\rangle
\end{tikzcd}
$$

group completely broken

$$
\begin{tikzcd}
\text{(sym. of pot.) } G_V \supset G_f \text{ (flavour sym. gr.)} \arrow[d, "{\langle\phi_i\rangle \neq 0}"', "{G_A[\langle\phi_i\rangle] = 0}"] \\
\text{(acc. sym.) } G_A \supset \{e\}
\end{tikzcd}
$$

CP and Discrete Flavour Symmetries

based on

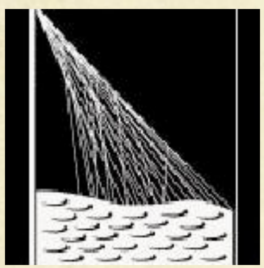
[MH, M. Lindner and M.A. Schmidt 1211.6953 (*JHEP*)]

and

[S. Antusch, MH, M.A. Schmidt, M. Spinrath, 1307.0710]

θ_{13} and Leptonic CP Violation

$$U_{\mathrm{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

= () x () x ()

$$\begin{aligned} J_{CP} &= \mathrm{Im}\left[U_{PMNS,11}U^*_{PMNS,13}U^*_{PMNS,31}U_{PMNS,33}\right] \\ &= \frac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta \end{aligned}$$

- Largish Θ_{13} means CP violation can be observed in oscillations in not-so-distant future

Predicting the CP phase

- natural next step: try to predict the CP phase using CP and discrete flavour symmetries (G. Ross: last chance)
- however, there was some confusion about how to implement CP in theories with discrete flavour symmetries

$$A_4 = \langle S, T | S^2 = T^3 = (ST)^3 = E \rangle$$

$$\underline{\mathbf{3}} : S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} , T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\underline{\mathbf{1}}_3 : S = 1 \, , T = \omega^2;$$

Example: consider the group A_4 with a triplet scalar $\chi = (\chi_1, \chi_2, \chi_3)^T \sim \underline{\mathbf{3}}$ and and a non-trivial singlet ξ transforming as $\mathbf{1}_3$

Under the CP transformation $\chi \to \chi^* \quad \xi \to \xi^*$

$$\omega = e^{\mathrm{i}\frac{2\pi}{3}}$$

the A4 invariant

$$I = \xi \left(\chi_1 \chi_1 + \omega^2 \chi_2 \chi_2 + \omega \chi_3 \chi_3 \right) \sim \underline{\mathbf{1}}_{\mathbf{1}}$$

is mapped to sth. not invariant:

$$CP[I] = \xi^* \left(\chi_1^* \chi_1^* + \omega^2 \chi_2^* \chi_2^* + \omega \chi_3^* \chi_3^* \right) \sim \underline{\mathbf{1}}_{\mathbf{2}}$$

some confusion in literature

- CP extends the group A4 and forbids this invariant??
- Is is possible to impose CP without enlarging the group?

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$$= 1 \, , T = \omega^2 ;$$

$$\omega = e^{\mathrm{i}\frac{2\pi}{3}}$$

Group Theoretical Origin of CP Violation

Mu-Chun Chen[1,*] and K.T. Mahanthappa[2,†]

[1]*Department of Physics & Astronomy, University of California, Irvine, CA 92697-4575, USA*

[2]*Department of Physics, University of Colorado at Boulder, Boulder, CO 80309-0390, USA*

Abstract

We propose the complex group theoretical Clebsch-Gordon coefficients as a novel origin of CP violation. This is manifest in our model based on SU(5) combined with the T' group as the family symmetry. The complex CG coefficients in T' lead to explicit CP violation which is thus geometrical in origin. The predicted CP violation measures in the quark sector are consistent with the current experimental data. The corrections due to leptonic Dirac CP violating phase gives the experimental best fit value for the solar mixing angle, and we also gets the right amount of the baryonic asymmetry.

2

some confusion in literature

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PHYSICAL REVIEW D **86,** 113003 (2012)

A supersymmetric $SU(5) \times T'$ unified model of flavor with large θ_{13}

Aurora Meroni,[1,*] S. T. Petcov,[1,2,†] and Martin Spinrath[1,‡]

[1]*SISSA/ISAS and INFN, Via Bonomea 265, I-34136 Trieste, Italy*

[2]*Kavli IPMU, University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa 277-8583, Japan*

(Received 12 September 2012; published 3 December 2012)

We present a SUSY $SU(5) \times T'$ unified flavor model with type I seesaw mechanism of neutrino mass generation, which predicts the reactor neutrino angle to be $\theta_{13} \approx 0.14$ close to the recent results from the Daya Bay and RENO experiments. The model predicts also values of the solar and atmospheric neutrino mixing angles, which are compatible with the existing data. The T' breaking leads to tribimaximal mixing in the neutrino sector, which is perturbed by sizeable corrections from the charged lepton sector. The model exhibits geometrical *CP* violation, where all complex phases have their origin from the complex Clebsch-Gordan coefficients of T'. The values of the Dirac and Majorana *CP* violating phases are predicted. For the Dirac phase in the standard parametrization of the neutrino mixing matrix we get a value close to 90°: $\delta \cong \pi/2 - 0.45\theta^c \cong 84.3°$, θ^c being the Cabibbo angle. The neutrino mass spectrum can be with normal ordering (2 cases) or inverted ordering. In each case the values of the three light neutrino masses are predicted with relatively small uncertainties, which allows one to get also unambiguous predictions for the neutrinoless double beta decay effective Majorana mass.

DOI: 10.1103/PhysRevD.86.113003 PACS numbers: 14.60.Pq, 12.10.Dm, 12.15.Ff, 12.60.Jv

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Important Question for Model Building:

How can CP be defined consistently in a theory with a discrete flavour symmetry?

How to define CP consistently

- A generalized CP acts upon the vector of fields

[Bernabeu, Branco, Gronau 86]

$$\boxed{CP : \phi(t, \vec{x}) \to U \phi^*(t, -\vec{x})}$$

where U is unitary, to leave the kinetic term invariant.

How to define CP consistently

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for gauge groups this has been investigated by [Grimus, Rebelo 95]

CONSISTENCY CONDITION:

- If G is the complete symmetry group, CP has to close in G:

$$\phi \xrightarrow{CP} U\phi^*$$

How to define CP consistently

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\phi \arrow[ur, bend left, "CP"] \arrow[dr, bend right, "g'"'] & & & U\rho(g)^*\phi^* \arrow[dl, bend left, "CP^{-1}"] \\
& \rho(g')\phi = & U\rho(g)^*U^{-1}\phi &
\end{tikzcd}$$

$$\boxed{U\rho(g)^*U^{-1} \in \mathrm{Im}\rho}$$

CP and the automorphism group

- The consistency condition $U\rho(g)^{*}U^{-1} \in \mathrm{Im}\rho$ defines an automorphism

$$\begin{tikzcd}
 & \rho(g)^{*} \arrow[r] & U\rho(g)^{*}U^{-1} = \rho(g') \arrow[dr, bend left, "\rho^{-1}"] & \\
g \in G \arrow[ur, bend left, "\rho"] \arrow[rrr, "u : G \to G"] & & & u(g) = g' \in G
\end{tikzcd}$$

$$\boxed{U\rho(g)^{*}U^{-1} = \rho(u(g))}$$

- the matrcies {U} furnish a representation of the automorphism group

$$\begin{aligned}
\rho\left((a \circ b)(g)\right) = \rho(a(b(g))) &= U(a)\rho(b(g))^{*}U(a)^{-1} \\
&= U(a)W\rho(b(g))W^{-1}U(a)^{-1} \\
&= U(a)WU(b)\rho(g)^{*}U(b)^{-1}W^{-1}U^{-1}(a)
\end{aligned}$$

$$\boxed{U(a \circ b) = U(a)WU(b)}$$

remember $\rho(g)^{*} = W\rho(g)W^{-1}$

neutral: $U(id) = W$

inverse: $U(u^{-1}) = WU^{-1}(u)W^{-1}$

CP and the automorphism group

Inverse Direction: : Each automorphism u of G may be represented by such a matrix U.

$$U\rho(g)^*U^{-1} = \rho(u(g))$$

Proof:

- Construct group extended by automorphism u (u^n=id)

$$G' = G \rtimes_\theta Z_n \qquad \theta : \{0, ..., n-1\} \to Aut(G) \qquad \theta(1) = u$$

$$(g_1, z_1) \star (g_2, z_2) = (g_1 \theta_{z_1}(g_2), z_1 + z_2)$$

- u acts as conjugation within this group

$$(E, 1) \star (g, 0) \star (E, 1)^{-1} = (u(g), 0)$$

- Consider representation $\rho' : G' \to U(M)$ induced via $\rho'(g, 0) = \rho(g)$

- automorphism u is represented by matrix

$$U(u) = \rho'((E, 1))W$$

where $\rho(g)^* = W\rho(g)W^{-1}$

$$\begin{aligned}
\rho(u(g)) &= \rho'(u(g), 0) \\
&= \rho'((E, 1) \star (g, 0) \star (E, 1)^{-1}) \\
&= \rho'((E, 1))\rho'((g, 0))\rho'((E, 1))^{-1} \\
&= \rho'((E, 1))W\rho(g)^*W^{-1}\rho'((E, 1))^{-1}
\end{aligned}$$

An application: ‚Calculable Phases‘

- in general one expects two different kinds of vacua of a CP conserving potential
  - either VEV is real, conserves CP and phase does not depend on potential parameters
  - or VEV is complex, breaks CP and phase depends on potential parameters

An application: ‚Calculable Phases‘

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  - or VEV is complex, breaks CP and phase depends on potential parameters

Example:

all parameters real

$$\begin{aligned} V &= m_1^2 \varphi^* \varphi + m_2^2 (\varphi^2 + {\varphi^*}^2) + \lambda_1 (\varphi^* \varphi)^2 + \lambda_2 (\varphi^4 + {\varphi^*}^4) \\ &= m_1^2 A^2 + m_2^2 A^2 \cos 2\alpha + \lambda_1 A^4 + \lambda_2 A^4 \cos 4\alpha \end{aligned}$$

invariant under $\varphi \to \varphi^*$

$\varphi = A e^{\mathrm{i}\alpha}$

$$\alpha = 0$$

$$A = -\frac{\sqrt{-\mathrm{m1}^2 - 2\mathrm{m2}^2}}{\sqrt{2}\sqrt{\lambda 1 + 2\lambda 2}}$$

$$\cos^2 \alpha = \frac{2\lambda_2 m_1^2 + \lambda_1 m_2^2 - 2\lambda_2 m_2^2}{4\lambda_2 m_1^2}$$

$$A = \frac{m_1}{\sqrt{2}\sqrt{2\lambda_2 - \lambda 1}}$$

An application: ‚Calculable Phases‘

- in general one expects two different kinds of vacua of a CP

nd

Exampl

arameters real

V

$\varphi^{*4})$

invar

Highly desireable for model building:

- non-trivial phase that does not depend on potential parameters
  - same reason as VEV alignment, makes predictive model
  - called ‚calculable phases‘ or ‚geometric CP violation‘
- calculable phases are a result of an accidental CP symmetry of potential
  - explicit example $\Delta(27)$ [Branco, Gerard, Grimus 1984]

$$A = -\frac{\sqrt{-\mathrm{m1}^2 - 2\mathrm{m2}^2}}{\sqrt{2}\sqrt{\lambda 1 + 2\lambda 2}}$$

$2\lambda_2 m_2^2$

$$A = \frac{m_1}{\sqrt{2}\sqrt{2\lambda_2 - \lambda 1}}$$

Possible CP transformations in $\Delta\,(27)$

$$\Delta(27) = \langle A, B | A^3 = B^3 = (AB)^3 = E \rangle$$

automorphism group generated by

$u_2 : (A, B) \to (ABAB, B^2)$ red

$u_1 : (A, B) \to (ABA^2, B^2AB)$ blue

	E	$BABA$	ABA	A	BAB	AB	A^2	B^2	B	BA^2BAB	AB^2ABA
$\underline{\mathbf{1}}_1$	1	1	1	1	1	1	1	1	1	1	1
$\underline{\mathbf{1}}_2$	1	ω	ω^2	1	ω	ω^2	1	ω	ω^2	1	1
$\underline{\mathbf{1}}_3$	1	ω^2	ω	1	ω^2	ω	1	ω^2	ω	1	1
$\underline{\mathbf{1}}_4$	1	ω	ω	ω^2	ω^2	ω^2	ω	1	1	1	1
$\underline{\mathbf{1}}_5$	1	ω^2	1	ω^2	1	ω	ω	ω	ω^2	1	1
$\underline{\mathbf{1}}_6$	1	1	ω^2	ω^2	ω	1	ω	ω^2	ω	1	1
$\underline{\mathbf{1}}_7$	1	ω^2	ω^2	ω	ω	ω	ω^2	1	1	1	1
$\underline{\mathbf{1}}_8$	1	1	ω	ω	ω^2	1	ω^2	ω	ω^2	1	1
$\underline{\mathbf{1}}_9$	1	ω	1	ω	1	ω^2	ω^2	ω^2	ω	1	1
$\underline{\mathbf{3}}$	3	.	.	.	.	.	.	.	.	3ω	$3\omega^2$
$\underline{\mathbf{3}}^*$	3	.	.	.	.	.	.	.	.	$3\omega^2$	3ω

Calculable Phases in Δ (27)

- consider again a triplet of Higgs doublets $H = (H_1, H_2, H_3) \sim \underline{\mathbf{3}}$ which transforms as

$$\rho(A) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \qquad \rho(B) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

- the potential only contains one phase dependent term

$$I \equiv (H_1^\dagger H_2)(H_1^\dagger H_3) + (H_2^\dagger H_3)(H_2^\dagger H_1) + (H_3^\dagger H_1)(H_3^\dagger H_2)$$

  - if coupling λ_4 multiplying I is positive, the global minimum is at $\langle H \rangle = \frac{v}{\sqrt{3}}(\omega^2, 1, 1)$
  (or a configuration that can be obtained by acting on this vacuum with a group element)
  - if coupling λ_4 is negative, the global minimum is at $\langle H \rangle = \frac{v}{\sqrt{3}}(1, \omega, \omega^2)$
  (or a configuration that can be obtained by acting on this vacuum with a group element)
- These phases do not depend on potential parameters!
  - can this be used to predict (leptonic) CP phases?
  - can they be understood in terms of generalized CP?

Calculable Phases in Δ (27)

- The vacuum of the form $\langle H \rangle = \frac{v}{\sqrt{3}}(1, \omega, \omega^2)$ leaves invariant the CP transformation

$$H \to \rho(B^2) H^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix} H^*$$

- which is a symmetry of I+I*
  - no surprise there, CP symmetric potential has CP symmetric ground state
- for the other solution $\langle H \rangle = \frac{v}{\sqrt{3}}(\omega^2, 1, 1)$ there is no group element that leaves H invariant $\langle H \rangle = \rho(g) \langle H \rangle^*$
  - this was called geometrical CP violation

GEOMETRICAL *T*-VIOLATION

G.C. BRANCO
Instituto Nacional de Investigação Científica, Av. do Prof. Gama Pinto 2, Lisbon, Portugal

and

J.-M. GERARD [1] and W. GRIMUS
CERN, Theory Division, Geneva, Switzerland

[Branco, Gerard and Grimus 1984; de Medeiros Varzielas, Emmanuel-Costa 2011; Battacharyya, de Medeiros Varzielas, Leser 2012, Ivanov,Lavoura 2013]

Calculable Phases as a Result of an accidental generalized CP transformation

- every automorphism corresponds to a generalized CP transformation
  - automorphism group of $\Delta(27)$ is of order 432, generated by $u_1 : (A,B) \to (ABA^2, B^2AB)$ $\quad u_2 : (A,B) \to (ABAB, B^2)$
- this allows one to search for CP transformation that leaves $\langle H \rangle = \frac{v}{\sqrt{3}}(\omega^2, 1, 1)$ invariant and gives a real λ_4
- indeed there is such a CP transformation:

corresponds to outer automorphism

$$H \to \tilde{U} H$$

$$\tilde{U} = \begin{pmatrix} 0 & 0 & \omega^2 \\ 0 & 1 & 0 \\ \omega & 0 & 0 \end{pmatrix}$$

$$\Rightarrow$$

$$CP_u[\langle H \rangle] = \langle H \rangle$$

$$CP_u[I] = I$$

CP character of trafo apparent when you look at 1-dim reps.

$$u : (A,B) \to (AB^2AB, AB^2A^2)$$

$$\underline{\mathbf{1}}_{\mathbf{2}} \leftrightarrow \underline{\mathbf{1}}_{\mathbf{3}}, \qquad \underline{\mathbf{1}}_{\mathbf{5}} \leftrightarrow \underline{\mathbf{1}}_{\mathbf{9}}, \qquad \underline{\mathbf{1}}_{\mathbf{6}} \leftrightarrow \underline{\mathbf{1}}_{\mathbf{8}},$$

Calculable Phases as a Result of an accidental generalized CP transformation

- it seems that geometric CP violation can always be explained as the result of an accidental generalized CP symmetry of the potential
- a symmetric potential can have a symmetric ground state
  - phases are dictated by accidental CP symmetry
  - explains the independence from potential parameters
- this setup is interesting for phenomenlogy:
  - if accidental symmetry only of potential, not of Yukawas, it can be used to predict phases etc.
- need groups with large outer automorphism group
  - note that the automorphism group of shaping symmetries(to be discussed shortly) is huge

$$|\mathrm{Out}\, Z_4^4| = 1321205760$$

The strong CP problem & the right unitarity triangle

- QCD could violate CP via

$$\mathcal{L} \supset \frac{\bar{\theta}}{32\pi^2}\tilde{G}_{\mu\nu}G^{\mu\nu}$$

- there are 2 contributions

$$\bar{\theta} = \theta + \arg\det(M_u M_d)$$

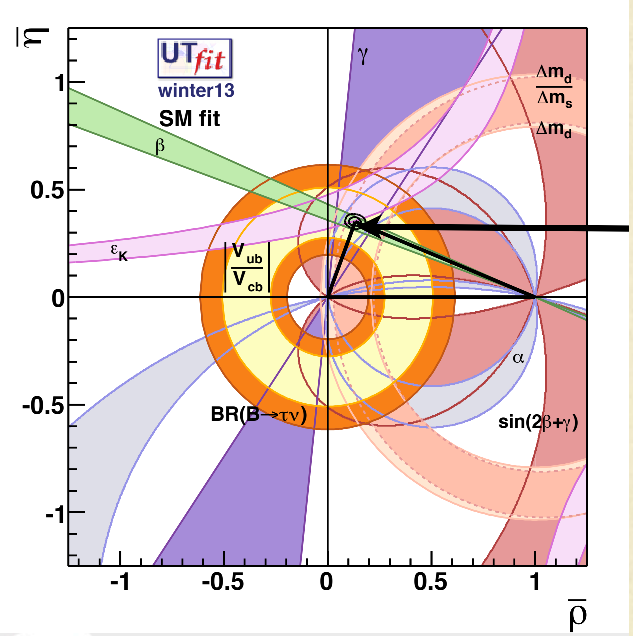
- bound from experiment

$$\bar{\theta} \lesssim 10^{-11}$$

- Why is there near-perfect cancellation?

use discrete flavour symmetries and spontaneous CP violation in model to explain both

- CP is violated by quark masses



α = (88.7 ± 3.1)° [UTFit Winter 13]

- right unitarity triangle
  - accident or sign of sponteneous CP violation?

[S. Antusch, MH, M.A. Schmidt, M. Spinrath, 1307.0710(NPB)]

Strategy to Suppress the Strong CP Phase

- Make CP fundamental: Θ=0
- Break CP spontaneously while maintaining arg det($M_u M_d$)=0

$$M_d = \begin{pmatrix} 0 & * & 0 \\ * & \mathrm{i}\, * & * \\ 0 & 0 & * \end{pmatrix}, \quad M_u \text{ real}$$

arg det($M_u M_d$)=0

α = 90°

$$\boxed{\delta\bar{\theta} \approx \arg\det(\delta M\, M^{-1})}$$

to realize this in a model we need

- special VEV alignment
- high control over NLO corrections, have to avoid
  - higher dim. operators, which would spoil structure of mass matrices
  - SUSY breaking terms

[S. Antusch, MH, M.A. Schmidt, M. Spinrath 1307.0710]

Model Overview

- symmetry of the model

$$\underbrace{SU(3)_C \times SU(2)_L \times U(1)_Y}_{\text{gauge sym.}} \times \underbrace{A_4 \times Z_2 \times Z_4^5}_{\text{discrete family/shaping sym.}} \times \underbrace{U(1)_R}_{R\text{ sym.}}$$

- only consider the quark sector (d_R is A_4 triplet)
- 5 singlet flavons with real vevs, 4 triplets:

$$\langle\phi_1\rangle \sim \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \langle\phi_2\rangle \sim \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \langle\phi_3\rangle \sim \begin{pmatrix}0\\0\\1\end{pmatrix}, \quad \langle\tilde{\phi}_2\rangle \sim \mathrm{i}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

[S. Antusch, MH, M.A. Schmidt, M. Spinrath, 1307.0710(NPB)]

Coupling to Matter

- the effective superpotential reads

$$\mathcal{W}_d = Q_1 \bar{d} H_d \frac{\phi_2 \xi_d}{\Lambda^2} + Q_2 \bar{d} H_d \frac{\phi_1 \xi_d + \tilde{\phi}_2 \xi_s + \phi_3 \xi_t}{\Lambda^2} + Q_3 \bar{d} H_d \frac{\phi_3}{\Lambda}$$

$$\mathcal{W}_u = Q_1 \bar{u}_1 H_u \frac{\xi_u^2}{\Lambda^2} + Q_1 \bar{u}_2 H_u \frac{\xi_u \xi_c}{\Lambda^2} + Q_2 \bar{u}_2 H_u \left(\frac{\xi_c}{\Lambda} + \frac{\xi_t^2}{\Lambda^2} \right) + (Q_2 \bar{u}_3 + Q_3 \bar{u}_2) H_u \frac{\xi_t}{\Lambda} + Q_3 \bar{u}_3 H_u$$

- giving the desired mass matrix structure

$$M_d = \begin{pmatrix} 0 & b_d & 0 \\ b'_d & \mathrm{i}c_d & d_d \\ 0 & 0 & e_d \end{pmatrix} \text{ and } M_u = \begin{pmatrix} a_u & b_u & 0 \\ 0 & c_u & d_u \\ 0 & d'_u & e_u \end{pmatrix}$$

[S. Antusch, MH, M.A. Schmidt, M. Spinrath, 1307.0710(NPB)]

Flavon Alignment

- the flavon superpotential is given by

$$\mathcal{W} = A_i \cdot (\phi_i \star \phi_i) + O_{i;j}(\phi_i \cdot \phi_j) + \frac{P}{\Lambda^2}\left(\phi_i^4 \pm M_F^4\right)$$

- it has the accidental symmetries

	ϕ_1	ϕ_2	ϕ_3	$\tilde{\phi}_2$	A_i	$O_{1;3}$	$O_{2;3}$	$\tilde{O}_{1;2}$	$\tilde{O}_{2;3}$
Z_2	S	$-S$	$-S$	$-S$	S	-	+	-	+
Z_2	$-T^2ST$	T^2ST	$-T^2ST$	T^2ST	T^2ST	+	-	-	-
Z_2	$-TST^2$	$-TST^2$	TST^2	$-TST^2$	TST^2	-	-	+	-

which are conserved by the VEV configuration

$$\langle\phi_1\rangle \sim \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \langle\phi_2\rangle \sim \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \langle\phi_3\rangle \sim \begin{pmatrix}0\\0\\1\end{pmatrix}, \quad \langle\tilde{\phi}_2\rangle \sim \mathrm{i}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$\begin{array}{c} G_V \supset G_f \\ \langle\phi_i\rangle \neq 0 \;\Big\downarrow\; G_A[\langle\phi_i\rangle] = \langle\phi_i\rangle \\ G_A \supset \{e\} \end{array}$$

- the phases are are a result of the accidental CP transformation $\tilde{\phi}_2 \to -\tilde{\phi}_2^*\,,\ \tilde{O}_{i;j} \to -\tilde{O}_{i;j}^*\,,\ \varphi \to \varphi^*$

Flavon Alignment

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- it has the accidental symmetries

	ϕ_1	ϕ_2	ϕ_3	$\tilde{\phi}_2$	A_i	$O_{1;3}$	$O_{2;3}$	$\tilde{O}_{1;2}$	$\tilde{O}_{2;3}$
Z_2	S	$-S$	$-S$	$-S$	S	-	$+$	-	$+$
Z_2	$-T^2ST$	T^2ST	$-T^2ST$	T^2ST	T^2ST	$+$	-	-	-
Z_2	$-TST^2$	$-TST^2$	TST^2	$-TST^2$	TST^2	-	-	$+$	-

which are conserved by the VEV configuration

$$\langle\phi_1\rangle \sim \begin{pmatrix}1\\0\\0\end{pmatrix}, \quad \langle\phi_2\rangle \sim \begin{pmatrix}0\\1\\0\end{pmatrix}, \quad \langle\phi_3\rangle \sim \begin{pmatrix}0\\0\\1\end{pmatrix}, \quad \langle\tilde{\phi}_2\rangle \sim \mathrm{i}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

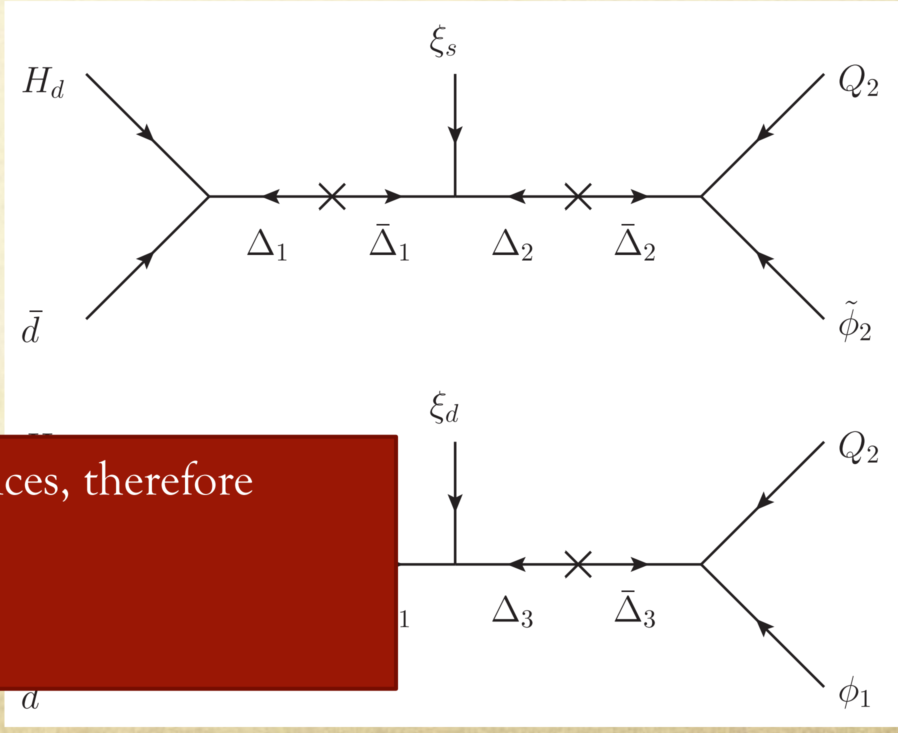
$$\begin{tikzcd} G_V \supset G_f \arrow[d, "\langle\phi_i\rangle \neq 0"', "{G_A[\langle\phi_i\rangle] = \langle\phi_i\rangle}"] \\ G_A \supset \{e\} \end{tikzcd}$$

- the phases are are a result of the accidental CP transformation $\tilde{\phi}_2 \to -\tilde{\phi}_2^*\,, \quad \tilde{O}_{i;j} \to -\tilde{O}_{i;j}^*\,, \quad \varphi \to \varphi^*$

Higher Dimensional Operators

- flavon sector corrections
  - at least dim. seven, directions and phases unchanged
- up sector corrections
  - suppressed (real) corrections to 1-1, 1-2, 2-2 elements of M_u
- down sector corrections

- we give a "UV completion" of the model giving full control over the effective operators!



no corrections to structure of mass matrices, therefore

- arg det$(M_u M_d)=0$
- $\alpha = 90°$

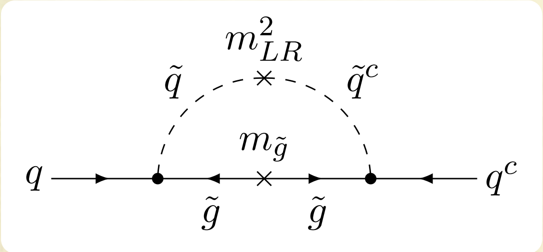
survives

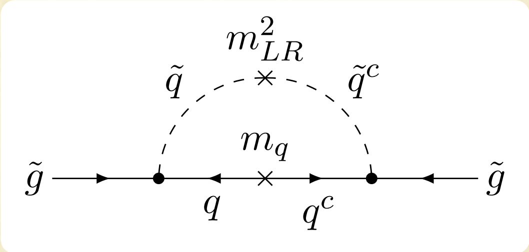
Possible Corrections from SUSY Breaking

- as long as SUSY is unbroken, non-renormalization theorems guarantee theta=0
- SUSY breaking might give corrections

$$\delta\bar{\theta} = 3\arg(m_{\tilde{g}})$$

- also LR sfermion mixing gives corrections





- if SUSY breaking conserves CP and is (nearly) minimal flavour violating, corrections are potentially small enough
  - note that SUSY breaking has to be flavour (and CP) non-generic, if SUSY@TeV

Summary

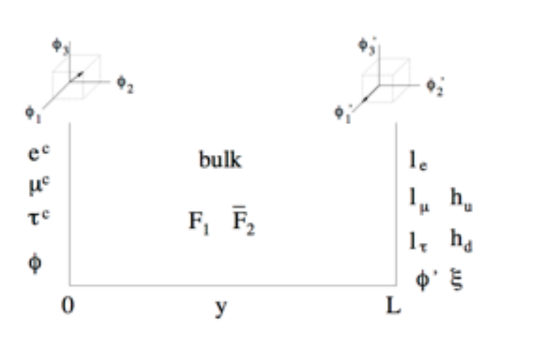
- Discrete Flavour Groups may still be a viable approach to the SM flavour puzzle
  - either NLO corrections or large symmetry groups needed to account for large value of θ_{13}
  - 3 candiate groups found in scan over 1.3m groups, testable predictions
- the vacuum alignment problem of such flavour models can be solved by a non-trivial extension of the flavour group
- Consistency Conditions should be kept in mind when constructing models that contain CP and Flavour Symmetries
  - generalized CP transformations may be viewed as furnishing a representation of the automorphism group
  - geometrical CP violation may be interpreted as a consequence of (accidental) generalized CP symmetries of the potential
- possible solution of strong CP problem with discrete flavour symmetries and CP

Backup Slides

Solutions in the Literature

In models with extra dimensions(ED), it is possible to locate the various fields at different locations in the ED, thereby forbidding the cross-couplings.

Altarelli, Feruglio 2005



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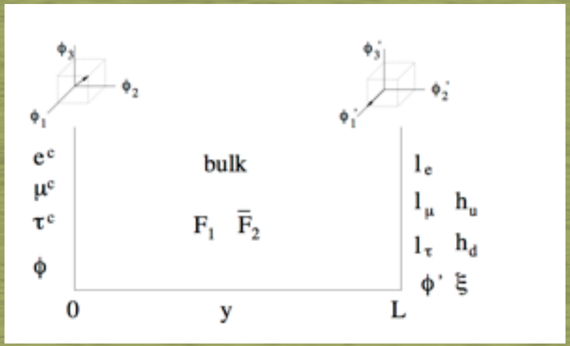

Altarelli, Feruglio 2005

In SUSY, one has to introduce a continuous R-symmetry and additional fields with R-charge 2(driving fields). These fields enter the superpotential only linearly and allow the vacuum alignment.

Field	φ_T	φ_S	ξ	$\tilde{\xi}$	φ_0^T	φ_0^S	ξ_0
A_4	3	3	1	1	3	3	1
Z_3	1	ω	ω	ω	1	ω	ω
$U(1)_R$	0	0	0	0	2	2	2

Altarelli, Feruglio 2006

Solutions in the Literature

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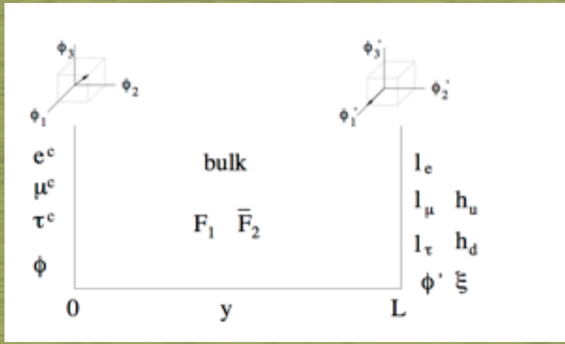


ALWAYS:

engineered accidental symmetry of potential

$A_4 \times A_4$: $\Phi \sim (3,1)$, $\chi \sim (1,3)$

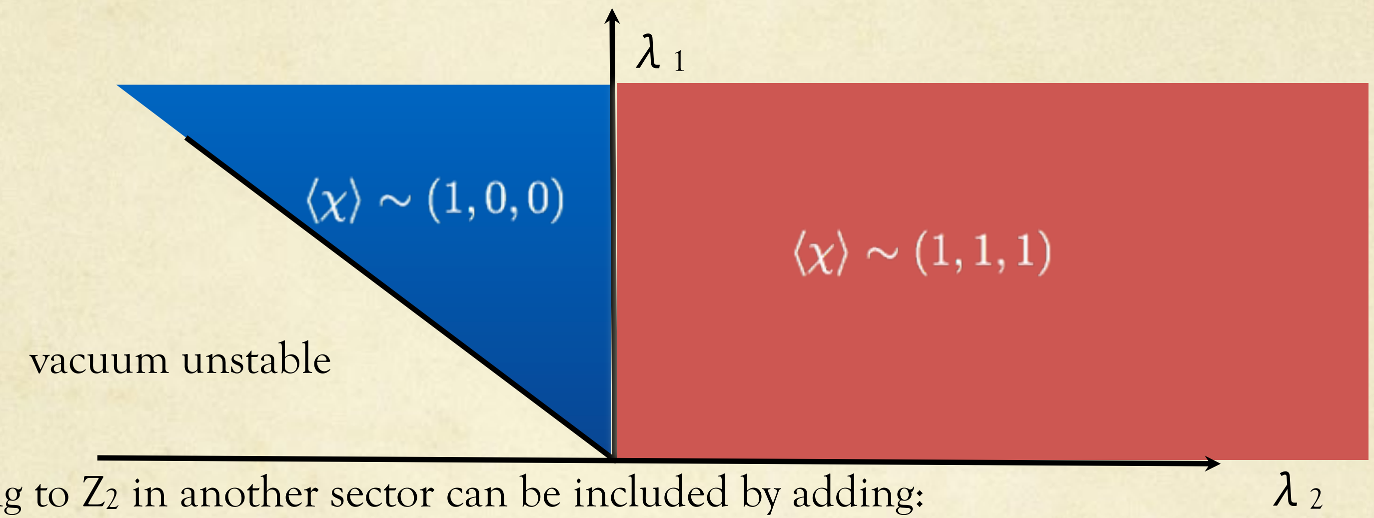
What is the minimal amount of engineering possible?

[MH, M.A. Schmidt *JHEP* **1201** (2012) 126, 1111.1730 [hep-ph]]

- if one takes $\Phi \sim 16$, vacuum alignment possible as $V=V(\Phi)+V(\chi)+(\Phi\ \Phi)_1(\chi\chi)_1$
- neutrino masses then generated by coupling to $\langle \Phi^4 \rangle \sim (1,0,0)$

Can Vacuum Alignment be realized?

$$V_\chi = m_0^2\,(\chi\chi)_{\underline{\mathbf{1}}_1} + \lambda_1\,(\chi\chi)_{\underline{\mathbf{1}}_1}(\chi\chi)_{\underline{\mathbf{1}}_1} + \lambda_2\,(\chi\chi)_{\underline{\mathbf{1}}_2}(\chi\chi)_{\underline{\mathbf{1}}_3}$$



Effect of breaking to Z_2 in another sector can be included by adding:

$$V_{soft,Z_2} = m_A^2\chi_1^2 + m_B^2\chi_2^2 + m_C^2\chi_2\chi_3$$

Minimization conditions then give:

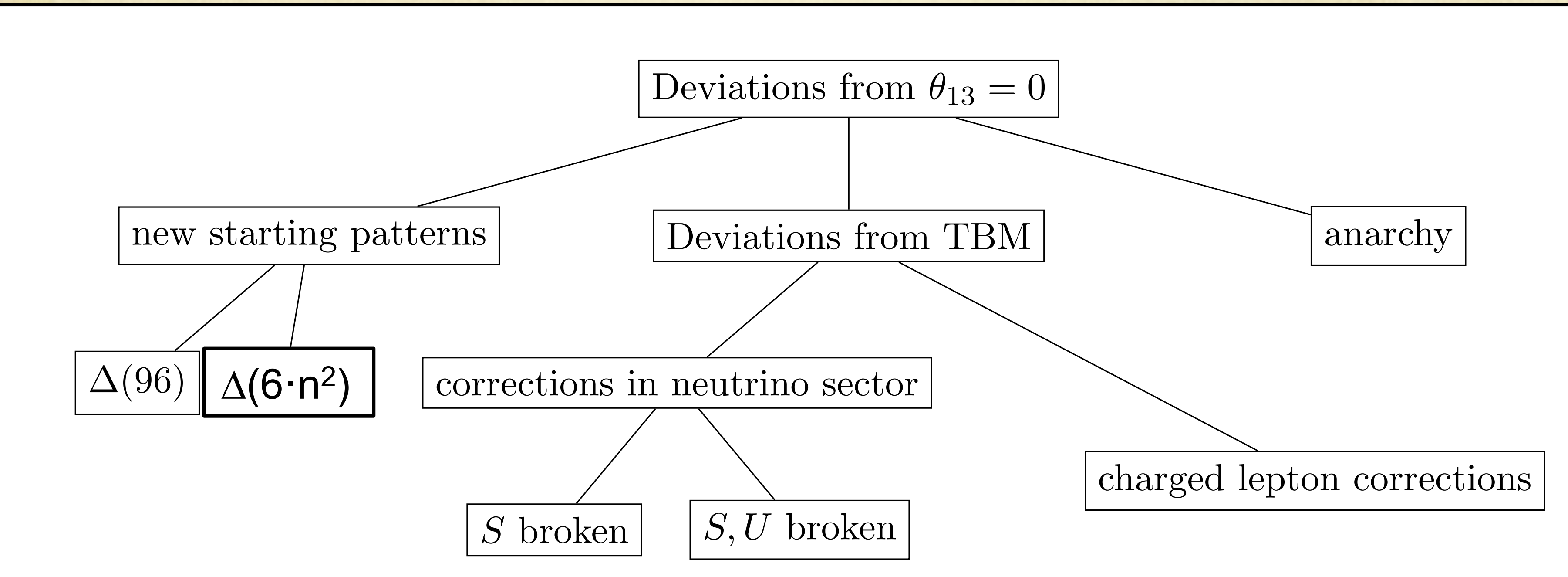
$$0 = \left[\frac{\partial V}{\partial\chi_1}\right]_{\chi_i=v'} = \frac{2}{\sqrt{3}}\left(m_0^2 + \sqrt{3}m_A^2\right)\,v' + 4\lambda_1 v'^3$$

$$0 = \left[\frac{\partial}{\partial\chi_2}V - \frac{\partial}{\partial\chi_3}V\right]_{\chi_i=v'} = 2\,m_B^2\,v'$$

$$0 = \left[\frac{\partial}{\partial\chi_1}V - \frac{\partial}{\partial\chi_3}V\right]_{\chi_i=v'} = \left(2\,m_A^2 - m_C^2\right)\,v'$$

- This thus requires $m_A = m_B = m_C = 0$, i.e. all non-trivial contractions between Φ and χ have to vanish in the potential.
- potential should be of the form $V = V_\Phi(\Phi) + V_\chi(\chi) + (\Phi\Phi)_1(\chi\chi)_1$.

Discrete groups with non-vanishing θ_{13}



- new starting patterns: large groups
  - maybe ‚indirect' origin [e.g. King, Luhn 09]
- if one starts from TBM, large NLO corrections are needed
  - charged lepton corrections & GUT relations [e.g. Antusch,King 04, ...]
  - TM1, TM2,.... [for a review e.g. King, Luhn 13]
- anarchy [Hall, Murayama, Weiner 1999]
  - possible, works better for neutrinos than for quarks

CP and discrete flavour symmetries

- consistency conditions for CP transformations
- CP transformations = representations of automorphisms
- application to A_4 model building

CP vs. A_4

- the ‚CP transformation‘ that is trivial with regard to A_4 runs into trouble if one considers a non-trivial singlet $\xi \sim \underline{\mathbf{1}}_{\mathbf{3}}$ in addition to the triplet $\chi \sim \underline{\mathbf{3}}$
- if one would use $\chi \to \chi^*$ and $\xi \to \xi^*$ one finds that the invariant is mapped to sth. non-invariant

$$\underline{\mathbf{1}}_{\mathbf{1}} \sim (\chi\chi)_{\underline{\mathbf{1}}_{\mathbf{2}}}\,\xi \to (\chi^*\chi^*)_{\underline{\mathbf{1}}_{\mathbf{2}}}\,\xi^* \sim \underline{\mathbf{1}}_{\mathbf{2}}$$

with $(\phi\phi)_{\underline{\mathbf{1}}_{\mathbf{2}}} = \frac{1}{\sqrt{3}}\left(\phi_1\phi_1 + \omega^2\phi_2\phi_2 + \omega\phi_3\phi_3\right)$

- this can be readily understood if one looks at how this ‚CP transformation‘ $\phi \to U\phi^*$ acts upon $\phi = (\xi, \xi^*, \chi)^T$
  - naive CP corresponds to $U=1_5$
  - A_4 does not close under this CP:

$$U\rho(T)^*U^{-1} = \rho(T)^* \notin \rho(G)$$

$$\rho(T) = \mathrm{diag}(\omega, \omega^2, T_3)$$
$$\rho(S) = \mathrm{diag}(1, 1, S_3)$$

- the real flavour group is larger, this has to be considered when constructing Lagrangian

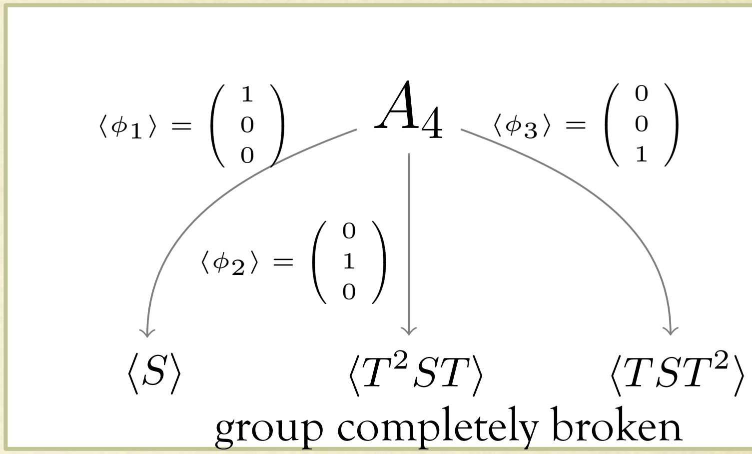
often overlooked in literature [Toorop et. al. 2011, Ferreira, Lavoura 2011,....]

Alignments, calculable phases and all that

- highly symmetric VEV configurations are the result of accidental symmetry transformations of scalar potential in flavour space
- calculable phases (a.k.a geometrical CP violation) are a result of accidental CP symmetries of the potential

Accidental symmetries as the origin of vacuum alignment mechanisms

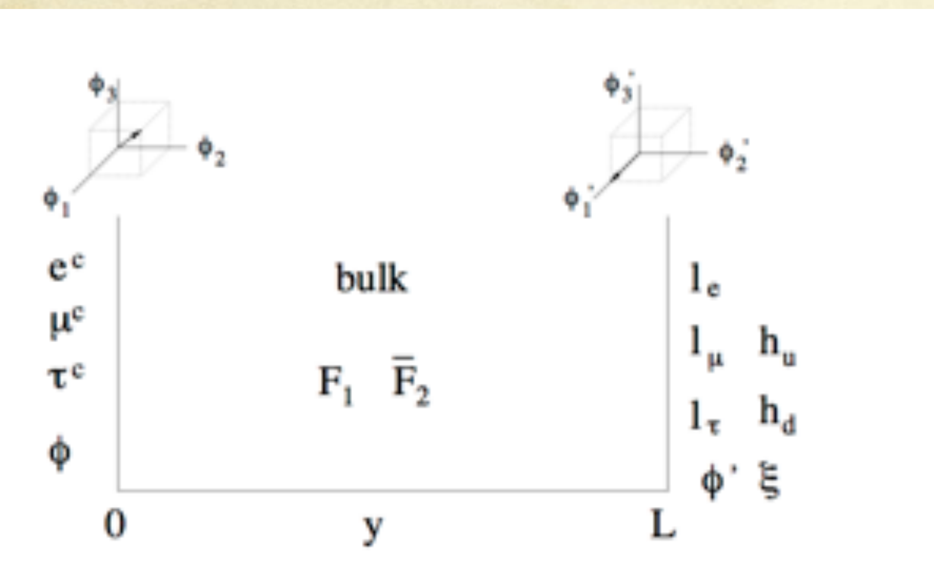
- usual flavour symmetry setup: a flavour group Gf is broken to different subgroups by different VEVs; in total the group is completely broken
- in general, this VEV state is unprotected by any symmetry and one therefore has to tune parameters to get VEVs
  - kills predictivity of models



Accidental symmetries of popular VEV alignment models

- Extra dimensions: $G_F = A_4$ but $G_V = A_4 x A_4$

$$\begin{array}{c}
G_V \supset G_f \\
\langle\phi_i\rangle \neq 0 \;\Big\downarrow\; G_A[\langle\phi_i\rangle] = 0 \\
G_A \supset \{e\}
\end{array}$$



Accidental symmetries of popular VEV alignment models

- Extra dimensions: $G_F = A_4$ but $G_V = A_4 x A_4$
- AF-type driving fields: $G_F = A_4$ but $G_V = A_4 x A_4$

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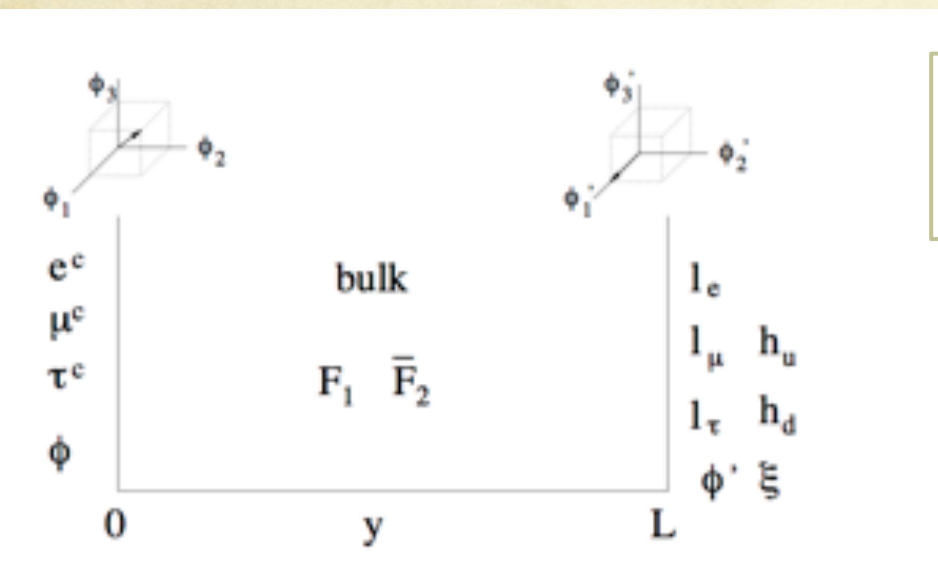
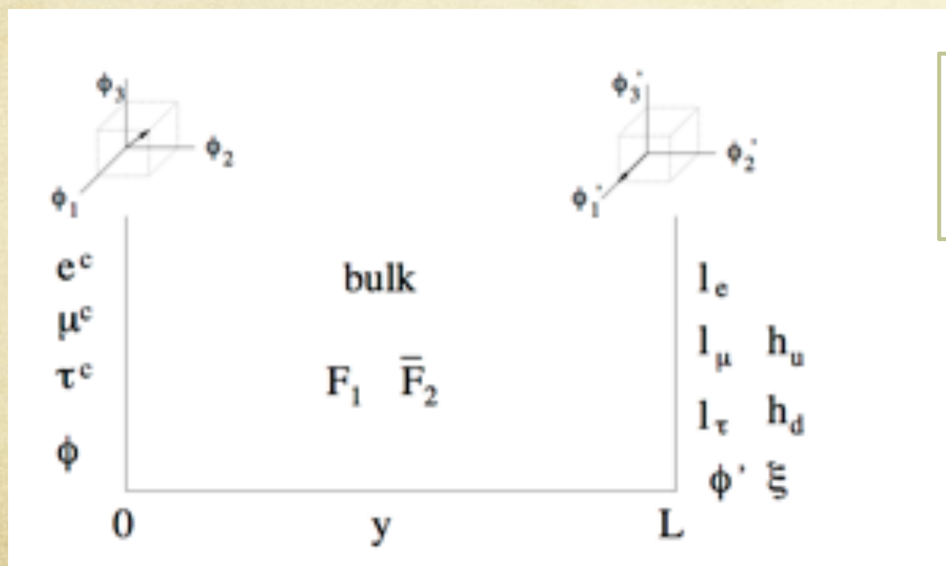


$$\begin{aligned} w_d &= M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) \\ &+ g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0(\varphi_S \varphi_S) \end{aligned}$$

Accidental symmetries of popular VEV alignment models

- Extra dimensions: $G_F=A_4$ but $G_V=A_4 \times A_4$
- AF-type driving fields: $G_F=A_4$ but $G_V=A_4 \times A_4$
- minimal realization discussed in [MH, M.A. Schmidt *JHEP* **1201** (2012)], $G_F=Q_8 \rtimes A_4$ but $G_V=(Q_8 \rtimes A_4) \times A_4$

$$
\begin{tikzcd}
G_V \supset G_f \arrow[d, "\langle\phi_i\rangle \neq 0"', "G_A[\langle\phi_i\rangle] = \langle\phi_i\rangle"] \\
G_A \supset \{e\}
\end{tikzcd}
$$

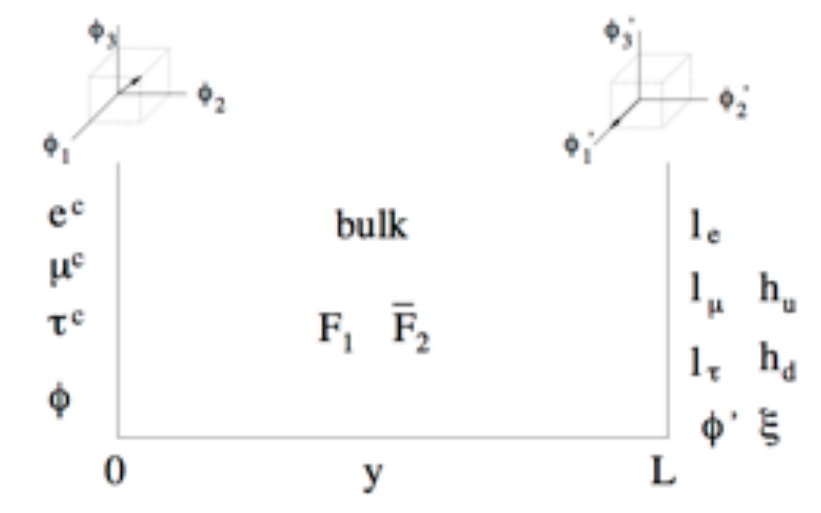


$$
\begin{aligned}
w_d &= M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) \\
&+ g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0 (\varphi_S \varphi_S)
\end{aligned}
$$

Accidental symmetries of popular VEV alignment models

- Extra dimensions: $G_F = A_4$ but $G_V = A_4 x A_4$
- AF-type driving fields: $G_F = A_4$ but $G_V = A_4 x A_4$
- minimal realization discussed in [MH, M.A. Schmidt *JHEP* **1201** (2012)], $G_F = Q_8 \rtimes A_4$ but $G_V = (Q_8 \rtimes A_4) x A_4$
- model of last talk: $G_A = Z_2 x Z_2 x Z_2$

$$
\begin{array}{ccc}
& G_V \supset G_f & \\
\langle \phi_i \rangle \neq 0 & \downarrow & G_A[\langle \phi_i \rangle] = 0 \\
& G_A \supset \{e\} &
\end{array}
$$



$$
\begin{aligned}
w_d &= M(\varphi_0^T \varphi_T) + g(\varphi_0^T \varphi_T \varphi_T) \\
&+ g_1(\varphi_0^S \varphi_S \varphi_S) + g_2 \tilde{\xi}(\varphi_0^S \varphi_S) + g_3 \xi_0 (\varphi_S \varphi_S)
\end{aligned}
$$

$$
\mathcal{W} = A_i \cdot (\phi_i \star \phi_i) + O_{i;j}(\phi_i \cdot \phi_j) + \frac{P}{\Lambda^2}\left(\phi_i^4 \pm M_F^4\right)
$$

	ϕ_1	ϕ_2	ϕ_3	$\tilde{\phi}_2$	A_i	$O_{1;3}$	$O_{2;3}$	$\tilde{O}_{1;2}$	$\tilde{O}_{2;}$
Z_2	S	$-S$	$-S$	$-S$	S	-	+	-	+
Z_2	$-T^2ST$	T^2ST	$-T^2ST$	T^2ST	T^2ST	+	-	-	-
Z_2	$-TST^2$	$-TST^2$	TST^2	$-TST^2$	TST^2	-	-	+	-

CP vs. A_4

	E	T	T^2	S
$\underline{\mathbf{1}}_{\mathbf{1}}$	1	1	1	1
$\underline{\mathbf{1}}_{\mathbf{2}}$	1	ω	ω^2	1
$\underline{\mathbf{1}}_{\mathbf{3}}$	1	ω^2	ω	1
$\underline{\mathbf{3}}$	3	0	0	-1

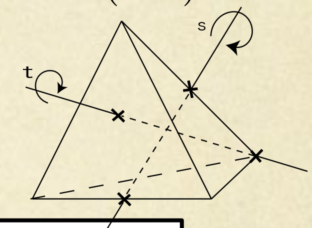
outer automorphism group Z_2, there is one outer automorphism:

$$u:(S,T)\to(S,T^2).$$

on 3-dim representation

$$\rho_{\underline{\mathbf{3}}_{\mathbf{1}}}(S)=S_3\equiv\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-1\end{pmatrix}\qquad \rho_{\underline{\mathbf{3}}_{\mathbf{1}}}(T)=T_3\equiv\begin{pmatrix}0&1&0\\0&0&1\\1&0&0\end{pmatrix}$$

$$A_4=\langle S,T|S^2=T^3=(ST)^3=E\rangle$$



as long as the theory only contains 3-dimensional representations, there are 2 possibilities for $U\rho(g)^*U^{-1}\in\mathrm{Im}\rho$

$$U=U_3\equiv\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

$$U_3\rho(T)^*U_3^{-1}=\rho(T^2)$$

$$u:(S,T)\to(S,T^2).$$

$$\underline{\mathbf{3}}\to U_3\underline{\mathbf{3}}^*$$

outer automorphism

$$U=\mathbb{1}_3$$

$$\rho(T)^*=\rho(T)$$

trivial map

$$\underline{\mathbf{3}}\to\underline{\mathbf{3}}^*$$

inner automorphism

CP vs. A_4

if one does not want to extend the group one therefore has the options

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & U_3 \end{pmatrix} .$$

$$u : (S,T) \to (S,T^2).$$

$$\chi \to U_3 \chi^*$$

$$\xi \to \xi^*$$

$$U = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \mathbb{1}_3 \end{pmatrix} .$$

trivial map

$$\chi \to \chi^*$$

$$\xi \to \xi$$

to fulfil the consistency condition

$$U \rho(g)^* U^{-1} = \rho(u(g))$$

Note that complex VEVs of the type (1,z,z*) conserve this CP

$$U_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

CP vs. A$_4$ - Application

- consider a triplet of Higgs doublets $\chi = (\chi_1, \chi_2, \chi_3)^T \sim \underline{\mathbf{3}}$
- there is one phase-dependent term in the potential

$$\lambda_5\ (\chi^\dagger \chi)_{\underline{\mathbf{3}}_{\mathbf{1}}} \left(\chi^\dagger \chi\right)_{\underline{\mathbf{3}}_{\mathbf{1}}} + \text{h.c.} = \lambda_5 \left[\left(\chi_1^\dagger \chi_2\right)^2 + \left(\chi_2^\dagger \chi_3\right)^2 + \left(\chi_3^\dagger \chi_1\right)^2\right] + \text{h.c.}$$

- the CP trafo $\chi \to \chi^*$ would restrict the phase to be zero
- even for non-vanishing phase, the VEV configuration $\langle \chi \rangle = V(1,1,1)$, $V \in \mathbb{R}$ can be obtained. [Toorop et. al. 2011]
- Spontaneous CP restoration??
- This can be understood if one considers the CP transformation $\chi \to U_3 \chi^*$
    - this is a symmetry of the potential for any phase of λ_5
    - also the VEVs preserve the CP transformation
    - therefore this CP is conserved in this case
- accidental CP transformations seem to be origin of ‚calculable phases‘

$$U_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Outer automorphism group

- if U is solution of $U\rho(g)^* U^{-1} = \rho(u(g))$ then so is $\rho(g')U$
  - corresponds to performing a CP transformation followed by a group transformation described by ρ (g)
  - The group transformation corresponds to an inner homomorphism, which does not pose any new restrictions
- therefore interesting generalized CP transformations correspond to

$$\mathrm{Out}(G) \equiv \mathrm{Aut}(G)/\mathrm{Inn}(G)$$

where $\mathrm{Inn}(G) = \{u \in \mathrm{Aut}(G) | u(g) = AgA^{-1} \text{for some } A \in G\}$

- aside: continuous groups

  $\mathrm{Out}(G) = E, Z_2$ except for $\mathrm{Out}(\mathrm{SO}(8)) = S_3$

- outer automorphism groups of small groups can be more involved:

$$\mathrm{Out}(\Delta(27)) \cong \mathrm{GL}(2,3)$$

geometrical CP violation in T‘?

- if we consider just one doublet $\psi \sim \underline{\mathbf{2}}_{\mathbf{2}}$ there is only one phase dependent term in the potential

$$\lambda \frac{\tilde{\omega}^2}{\sqrt{3}} \left(\psi_1(\psi_1^3 - (2-2\mathrm{i})\psi_2^3)\right) + \mathrm{h.c.}$$

- one gets solutions of the type $\langle\psi\rangle = (Ve^{\mathrm{i}\alpha}, 0)^T$ which conserve T

$\lambda < 0$

$\langle\psi\rangle = \{1, \mathrm{i}, -1, -\mathrm{i}\}(e^{\mathrm{i}\pi 11/24}, 0)^T$

$\psi \to \{1, -1, 1, -1\} CP[\psi]$

$\lambda > 0$

$\langle\psi\rangle = \{1, \mathrm{i}, -1, -\mathrm{i}\}(e^{\mathrm{i}\pi 5/24}, 0)^T$

$\psi \to -\mathrm{i}\{1, -1, 1, -1\} CP[\psi]$

- again the phases are a result of a generalized CP symmetry of the potential
- similar discussion holds for potentials of the type $P\left(\frac{\xi^n}{\Lambda^{n-2}} \mp M^2\right)$

TM2 “parabola”

complete flavour group

G_f

residual symmetry of (Me Me⁺)

$$G_e = \langle T \rangle = Z_3$$

$$\rho(T) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

$L \to \rho(g) L$

LH leptons 3-dim rep.

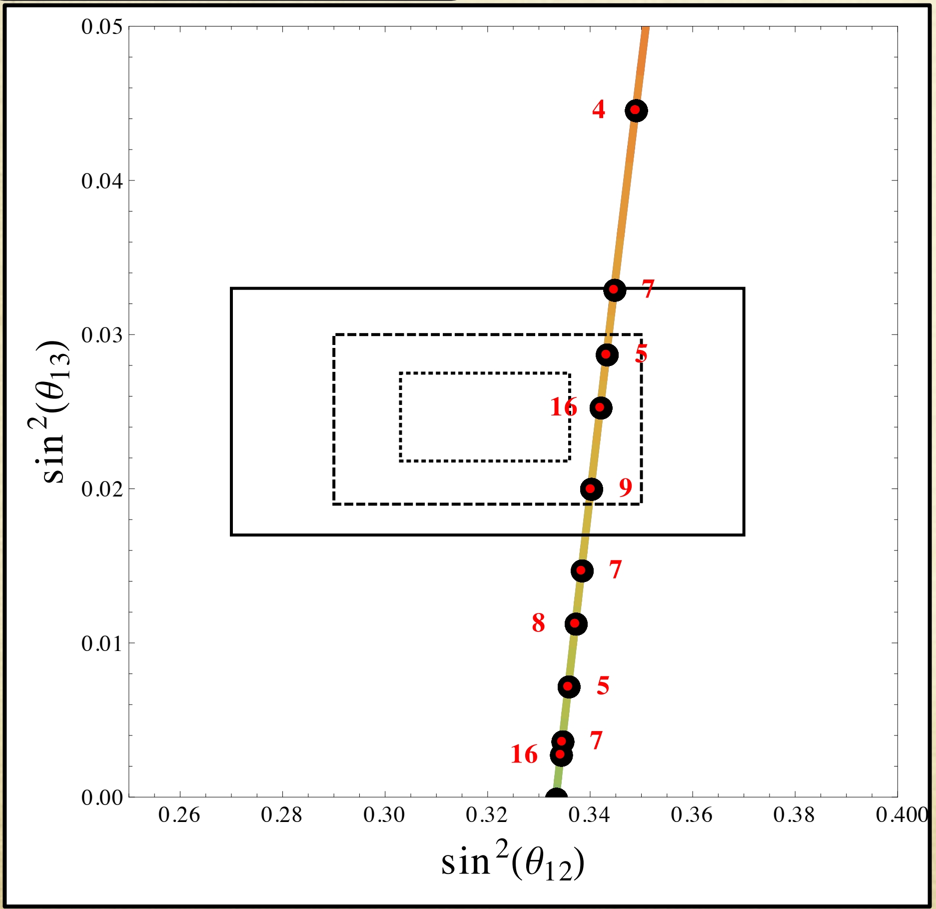
residual symmetry of M_ν

$$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$$

$$G_\nu = \langle S, U_n \rangle = Z_2 \mathrm{x} Z_2$$

$$\rho(S) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \rho(U_n) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & z_n \\ 0 & z_n^* & 0 \end{pmatrix}$$

$$\langle z_n \rangle \cong Z_n$$



- Scan over all discrete groups of size smaller than 1556 with $G_e = Z_3$, $G_\nu = Z_2 \mathrm{x} Z_2$
- all solutions which are close to exp. values have the TM2 form

$$U = U_{TBM} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}$$

with $\theta = \frac{1}{2} \arg(z_n)$

- vanishing CP phase δ_{CP}

[MH, K.S. Lim, M. Lindner 1212.2411(PLB)]

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$$\rho(g_\nu)^T M_\nu \rho(g_\nu) = M_\nu$$

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$$\rho(S) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad \rho(U_n) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & z_n \\ 0 & z_n^* & 0 \end{pmatrix}$$

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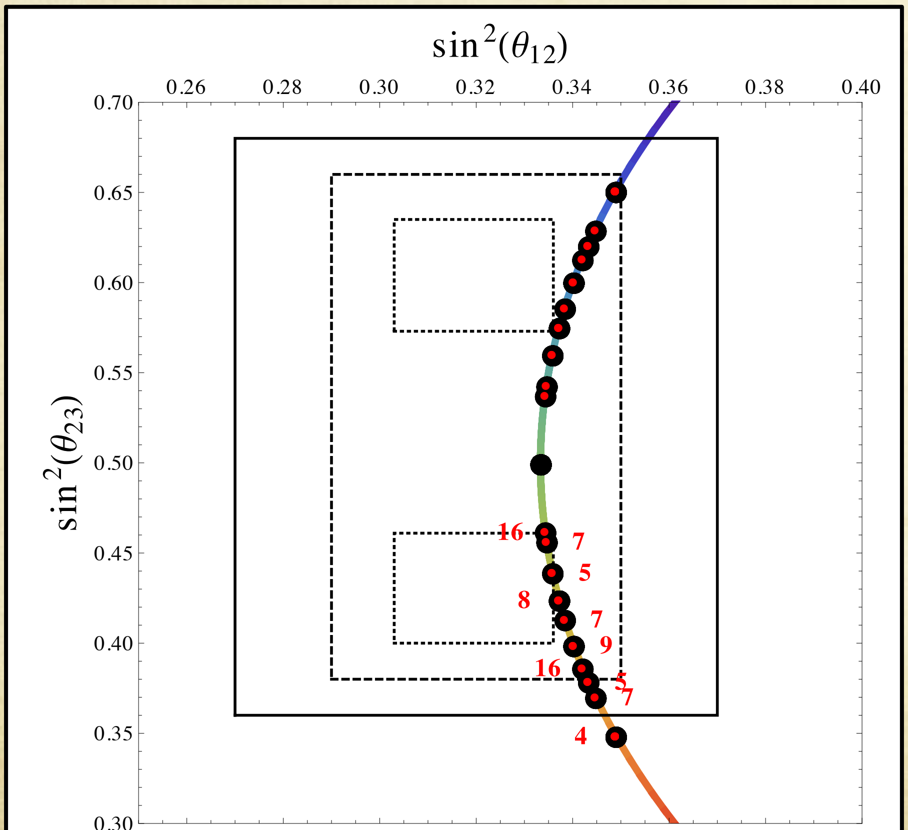

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[MH, K.S. Lim, M. Lindner 1212.2411(PLB)]

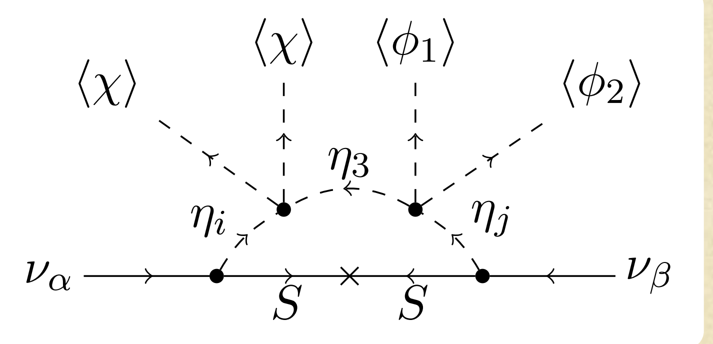
Flavour Breaking at the Electroweak Scale

Input

- VEV alignment mechanism based on group theory allows for low scale flavour breaking
- implement model at EW scale
  - make χ an EW doublet
  - add messenger fields to make it renormalizable
- do not add any new symmetry

fermion	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
S	1	0	$\underline{\mathbf{3}}_{\mathbf{2}}$	-1

scalars	$SU(2)_L$	$U(1)_Y$	$Q_8 \rtimes A_4$	Z_4
η_1	2	1/2	$\underline{\mathbf{3}}_{\mathbf{5}}$	i
η_2	2	1/2	$\underline{\mathbf{3}}_{\mathbf{4}}$	i
η_3	2	1/2	$\underline{\mathbf{3}}_{\mathbf{5}}$	$-$i



Output

- neutrino masses are generated at one-loop level, therefore small

$$m_\nu \sim \frac{1}{16\pi^2} h^2 \left(\frac{\delta M_\eta^2}{M_\eta^2}\right)^2 \frac{M_\eta^2}{M_S} \qquad \text{for} \qquad h \sim \frac{\delta M_\eta^2}{M_\eta^2} \sim 10^{-2}, M_\eta \sim 100\,\text{GeV}\, M_S \sim 100\,\text{GeV}$$

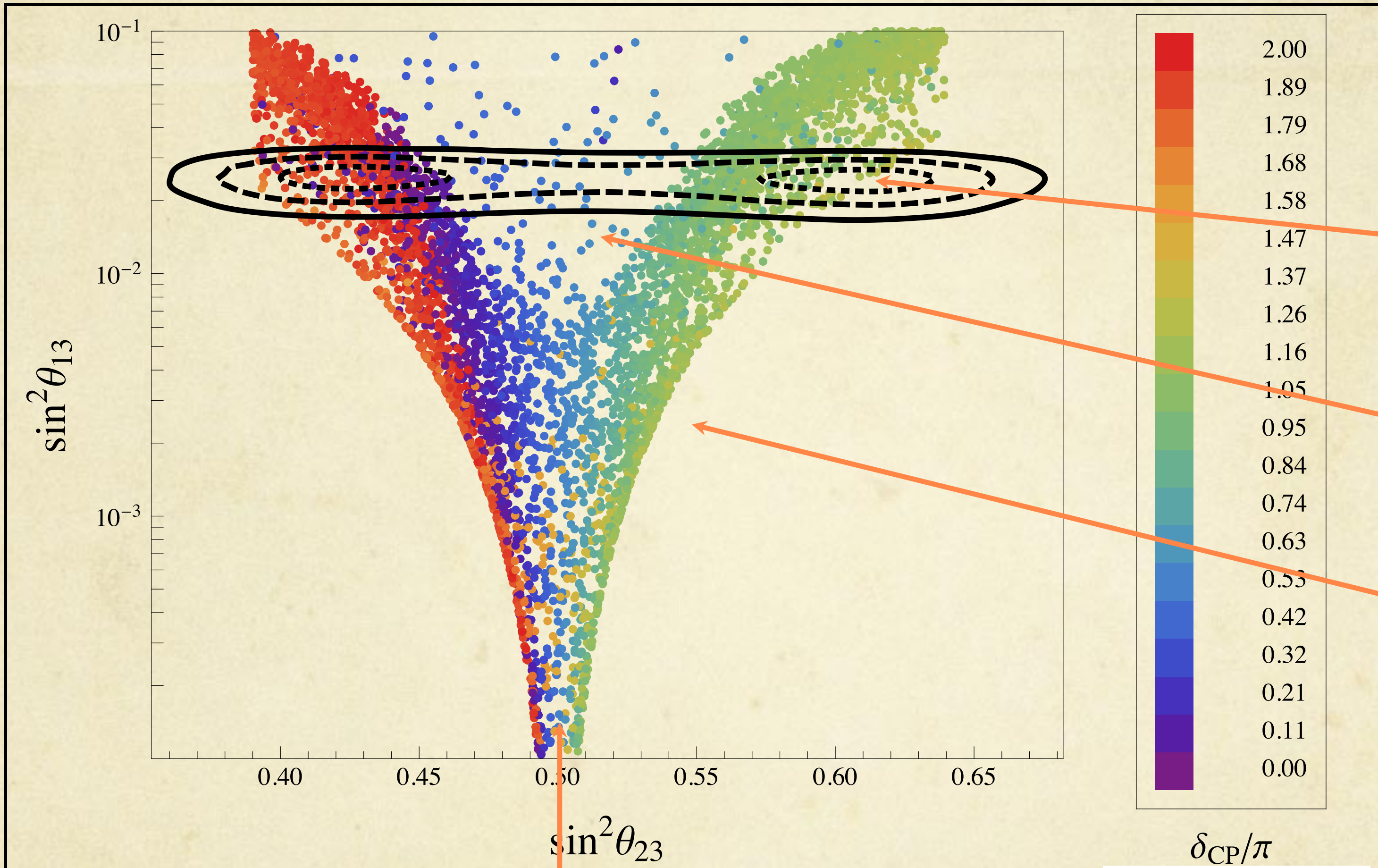
- at LO, 4 real parameters & one phase in neutrino mass matrix

$$M_\nu = \begin{pmatrix} \hat{a} & \hat{e}\, e^{i\alpha_\lambda} & \hat{e}\, e^{i\alpha_\lambda} \\ . & \hat{a} + \hat{b}\, e^{i\alpha_\lambda} & \hat{d} + \hat{e}\, e^{i\alpha_\lambda} \\ . & . & \hat{a} \end{pmatrix}$$

- DM candidate from flavour symmetry, protection from flavour violation

[MH, M. Lindner, M.A. Schmidt, 1211.5143 (PRD)]

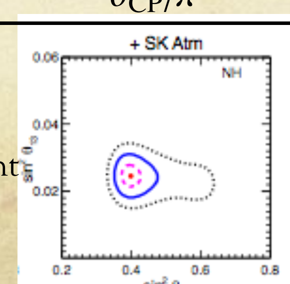
Flavour Breaking at the EW Scale-Mixing

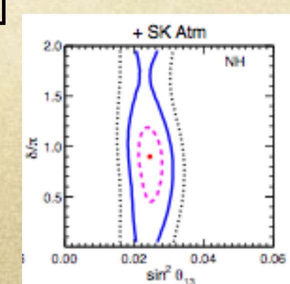


- $\sin^2\theta_{13}\approx.03$ needs e/a≈.1
- $\sin^2\theta_{12}$ too small
- $\sin^2\theta_{12}$ too large

- TBM for b=e=0

Fogli et al. see a hint for $\sin^2\theta_{23}\approx.4$ & delta$_{CP}\approx$pi

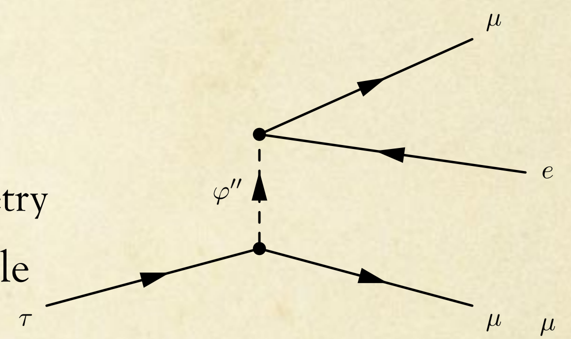




Flavour Breaking at the EW Scale-LFV&Higgs

- in the charged lepton sector, the VEV (1,1,1) leaves the Z_3 subgroup generated by T invariant
    - go to charged lepton basis where T is diagonal $\quad (L_e, L_\mu, L_\tau) \sim (1, \omega^2, \omega)$
- only H gets a VEV and plays the role of the SM Higgs $\quad (H, \varphi', \varphi'') \sim (1, \omega^2, \omega)$

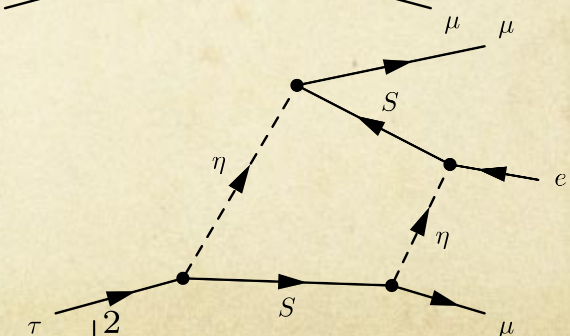
$$-\mathcal{L}_e = \tilde{H}\left(y_e L_e e^c + y_\mu L_\mu \mu^c + y_\tau L_\tau \tau^c\right) + \tilde{\varphi}'\left(y_e L_\mu e^c + y_\mu L_\tau \mu^c + y_\tau L_e \tau^c\right) + \tilde{\varphi}''\left(y_e L_\tau e^c + y_\mu L_e \mu^c + y_\tau L_\mu \tau^c\right) + \text{h.c.}$$



- other doublets have flavour off-diagonal couplings
    - this is usually extremely dangerous, saved by flavour symmetry
- this generates LFV 4 fermion operators (with the selection rule $\Delta L_e \Delta L_\mu \Delta L_\tau = \pm 2$)
- the most constraining process is

exp. bound

$$\mathrm{Br}(\tau^- \to \mu^- \mu^- e^+) \sim 2.3 \cdot 10^{-8} \left(\frac{21\,\mathrm{GeV}}{M_{\varphi''}}\right)^4$$



- suppressed by small Yukawa couplings, for the process mediated by eta we get

$$\mathrm{Br}(\tau^- \to \mu^- \mu^- e^+) \sim 2.3 \cdot 10^{-8} \cdot \left|\left(\frac{140\,\mathrm{GeV}}{M_\eta}\right)^2 (h_1^4 + h_2^4 - h_1^2 h_2^2)\right|^2$$

[Ma,Rajasekaran 2001, Toorop, Bazzocchi, Merlo, Paris,2010]

How to define CP consistently

- Consider the vector made up out of all real(R), pseudo-real (P) and complex (C) representations of a given model

$$\phi = \begin{pmatrix} \varphi_R, & \varphi_P, & \varphi_P^*, & \varphi_C, & \varphi_C^* \end{pmatrix}^T$$

- under the group G it transforms as $\phi \xrightarrow{G} \rho(g)\phi, \qquad g \in G.$
- the (reducible) representation $\rho : G \to U(N)$ is assumed to be faithful and complex
  - if not faithful then real symmetry group of theory is $G/\ker\rho$
  - ρ is homomorphism: ρ (a*b)= ρ (a) ρ (b)
- definition implies the existence of matrix W

$\phi^* = W\phi$ or

$\rho(g)^* = W\rho(g)W^{-1}$

$$P : \varphi(t, \vec{x}) \to \varphi(t, -\vec{x})$$

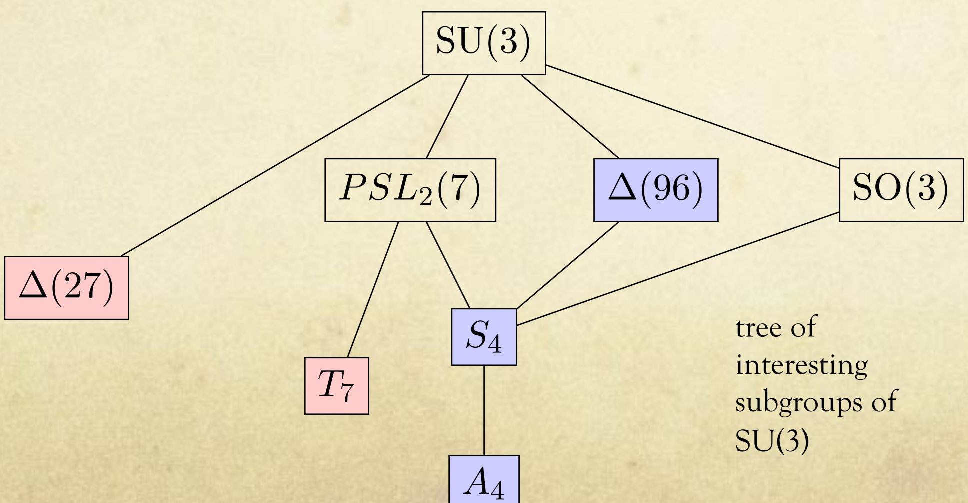
$$C : \varphi(t, \vec{x}) \to \varphi^*(t, \vec{x})$$

$$CP : \varphi(t, \vec{x}) \to \varphi^*(t, -\vec{x})$$

- here only Lorentz-scalars, generalization straightforward

A bottom up approach

- start with a smallish flavour group
- no SUSY at LHC, try non-superymmetric
  - new solution to VEV alignment problem needed
- high scale models are hard to test
  - to make it testable, try to break symmetry at the electroweak scale or TeV scale



tree of interesting subgroups of SU(3)