The Higgs mass from a string-theoretic perspective

cf. 1204.2551 and 1304.2767 with **A. Knochel** and **T. Weigand**

Outline

- Intro I: Higgs mass and quartic coupling
- Intro II: Superstrings and their 10d effective theory
- The 'flux landscape' and its 4d effective theories
- High-scale SUSY and the Higgs mass
- 'Shift-symmetric' Higgs models and some potential phenomenological implications

Higgs mass and quartic coupling

$$V = -\mu^2|H|^2 + \lambda|H|^4$$

- Parameters: $(\mu, \lambda) \quad \longleftrightarrow \quad (v, m_h)$

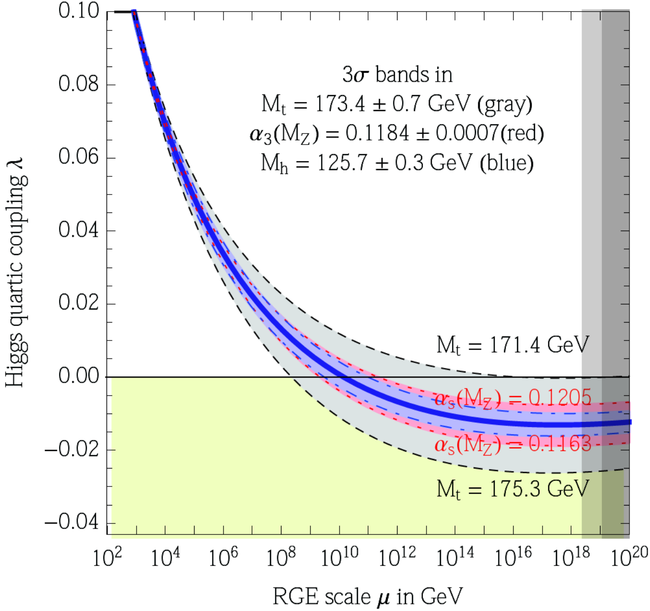
$$\boxed{\lambda = m_h^2/(2v^2) + \text{loops}}$$

- Well-known: for low m_h, λ runs to zero at some scale $< M_P$ (vacuum stability bound)

Lindner, Sher, Zaglauer '89
Froggatt, Nielsen '96
Gogoladze, Okada, Shafi '07
...
Shaposhnikov, Wetterich '09
Holthausen, Lim, Lindner '11
Redi, Strumia '12

- It has been attempted to turn this into an m_h prediction

Buttazzo et al. 1307.3536



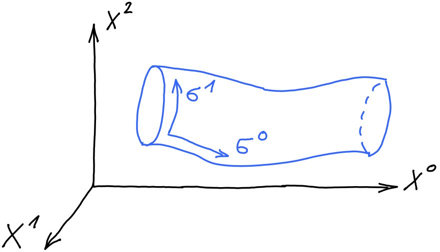
Jumping somewhat ahead:

Our proposal

- In spite of null-results from LHC, string-motivated high-scale SUSY remains theoretically well-motivated.
- In certain models, $\lambda = 0$ is a natural outcome of SUSY breaking.
- The 'vacuum stability scale' μ_λ is thus identified as the SUSY breaking scale.

String theory: ‘to know is to love’

- Let’s imagine particles were tiny closed string loops...

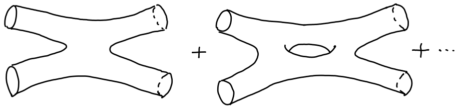


- The most natural action, the surface area of the ‘world sheet’, can be rewritten as

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \, (\partial_\alpha X^\mu) \, (\partial^\alpha X^\nu) \, \eta_{\mu\nu}$$

String theory: 'to know is to love'

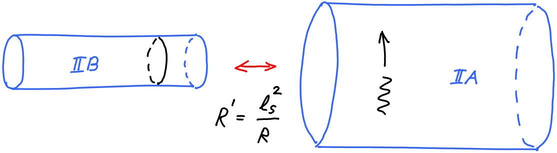
- Actually, only the supersymmetrised version of this 2d action leads to a stable target space vacuum.
- We now have a (perturbatively) finite theory of quantum gravity (in 10d target space)



- The (low-energy) effective field theory in 10d turns out to be supersymmetric.
- In fact, depending on fermionic boundary conditions we get four different 10d theories.

String theory: 'to know is to love'

- These turn out to be all possible 10d supergravity theories.
- They are related by dualities and are thus part of a single, more general theory.



- We focus on one of the four options: Type IIB.

The 10d type IIB lagrangian

$$\mathcal{L} \sim \frac{1}{l_s^8}\left[e^{-2\varphi}\left(\mathcal{R} + 4(\partial\varphi)^2 - \frac{1}{2}H_3^2\right) - \frac{1}{2}\left(F_1^2 + F_3^2 + F_5^2\right)\right] + \cdots$$

where, for example:

$$F_3 = dC_2\,, \quad \text{i.e.} \quad (F_3)_{\mu\nu\rho} \sim \partial_{[\mu} C_{\nu\rho]}$$

and analogously for the other 'form fields'.

- This theory has non-perturbative 'solitons' (D-branes), on which (e.g. 8-dimensional) gauge theories are localized.

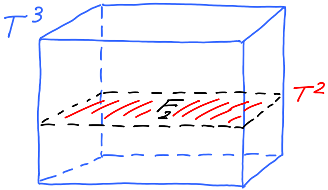
Towards the real world...

> ...our mistake is not that we take our theories too seriously, but that we do not take them seriously enough.
>
> S. Weinberg

- Thus, we have to compactify 6 of the 10 dimensions.
- To solve the Einstein equations, our compact space should have $\mathcal{R}_{\mu\nu} = 0$.
- Such 6d spaces are called Calabi-Yaus, of which $\sim 10^4$ are known.
- The resulting 4d theories are supersymmetric and have many 'moduli'.
- These moduli are scalars with exactly vanishing potential (they parametrise size and shape of the compact space).

Fluxes

- However, we are missing one important ingredient: Fluxes.
- Fluxes are background VEVs of the higher-form field-strenghts e.g. F_3, H_3.
- To understand this better, take T^3 as our compact space. Let's also assume that our toy model has an F_2-field:



- Here we sketched a T^2-submanifold (2-cycle) of our our T^3.
- We know (from Dirac) that $\int_{T^2} F_2$ is quantized.

The 'Landscape'

- Now, Calabi-Yaus have hundreds of 3-cycles, allowing us to choose very many 'flux-quanta' for F_3 and H_3.
  (This is a geometric choice, just like choosing one or the other of the 10^4 Calabi-Yaus).

- We now have $\sim 10^{500}$ 4d theories, with moduli stabilized and (sometimes) with supersymmetry broken in a controlled way.

- These are all different solutions of the single 10d quantum gravity theory we started from.

- They are best described in terms of
  (spontaneously broken) 4d $\mathcal{N} = 1$ supergravity.

4d Supergravity

- (The simplest) supergravity models are defined by a (real) Kähler potential
$$K(\varphi^i, \overline{\varphi}^{\bar{\imath}})$$
and a (holomorphic) superpotential
$$W(\varphi^i).$$

- The 4d lagrangian reads
$$\mathcal{L} = \frac{1}{2}\mathcal{R} + K_{i\bar{\jmath}}(\partial\varphi^i)(\partial\varphi^{\bar{\jmath}}) - V(\varphi^i, \overline{\varphi}^{\bar{\imath}}) + \cdots$$
with
$$V = e^K \left[K^{i\bar{\jmath}} D_i W \overline{D_j W} - 3|W|^2\right] \qquad \text{and} \qquad D_i \equiv \partial_i + K_i \, .$$

- The crucial point is that K and W are calculable in terms of Calabi-Yau geometry and flux choice.

- Simplest example:

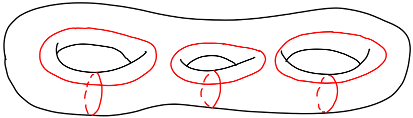
$$K = -3\ln(T + \overline{T}) + \cdots \qquad \text{with} \qquad T = R^4 + i\int C_4$$

where R is a typical radius of the Calabi-Yau.

- More generally:

$$K = -2\ln\left[\kappa_{ijk}(t+\overline{t})^i(t+\overline{t})^j(t+\overline{t})^k\right] + \cdots \quad \text{where} \quad t^i = R_i^2 + \cdots$$

and R_i are radii of a certain 2-cycle-basis (while κ_{ijk} are the intersection numbers of the dual 4-cycle basis).



- Very symbolically:

$$W \sim N_i z^i + \cdots$$

where N_i are the 'flux numbers' and z^i are further moduli, this time parametrizing the sizes of 3-cycles of the Calabi-Yau.

- As a side remark:

After minimization, the vacuum value $W_0 \equiv \langle W \rangle$ can take $\sim 10^{500}$ values roughly within the unit circle.

This enters directly into the vacuum energy and is one of best-studied instances of the possible fine-tuning in the landscape.

Denef/Douglas '04

High scale SUSY and the Higgs mass

- In this setting, the gravitino mass ($\equiv$ SUSY breaking scale) is

$$m_{3/2} = e^{K/2}|W_0|$$

  which can be anywhere between string and TeV scale.

- Matter fields are usually localized on D-branes wrapping some of the CY cycles.

- Their Kähler potential, while very important for applications (e.g. SUSY breaking masses) is usually hard to get.

- Let's start by recalling that (canonical Kähler potential):

$$K \supset Q\overline{Q} \qquad \Rightarrow \qquad \mathcal{L} \supset (\partial_\mu Q)(\partial^\mu \overline{Q})$$

- String-derived Kähler potentials are usually more complex, e.g.

$$K = -3\ln(T + \overline{T} + Q\overline{Q} + U\overline{U}) - \ln(S + \overline{S} + H_u\overline{H}_u) + \cdots$$

  together with

$$W = W_0 + y\,QH_uU$$

- Part of our specific work was identifying cases where the Higgs Kähler potential is

$$K \supset -\ln[S + \overline{S} + |H_u + \overline{H}_d|^2]$$

  where S can be viewed as a constant.

- The crucial point here is the shift symmetry

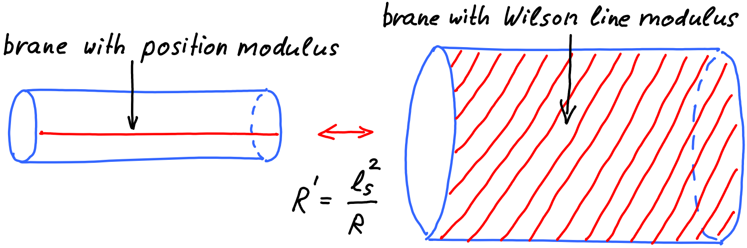
$$H_u \rightarrow H_u + c \qquad \text{and} \qquad H_d \rightarrow H_d - \overline{c}$$

Origin of shift symmetry

- Let's start with a D6 brane stack with SU(6) gauge group.
- This corresponds to a 7d gauge theory,
  living on a 3d submanifold of our Calabi-Yau.
- It can be broken to a GUT as SU(6)$\to$SU(5)$\times$U(1).
- The adjoint decomposes as $\mathbf{35} \to \mathbf{24} + \mathbf{5} + \overline{\mathbf{5}} + \mathbf{1}$.
- The $\mathbf{5} + \overline{\mathbf{5}}$ contain the $\mathbf{2} + \overline{\mathbf{2}}$ MSSM Higgs doublets.
- Secretly, these Higgses are Wilson lines on our 3d submanifold.
- Their shift symmetry is just the $A_{5,6} \to A_{5,6} + c$ gauge trf.
- This enforces the special form of above Kähler potential.

- By T-duality ($\equiv$ mirror symmetry) this structure is transported to the type IIB setup.
  (This is where moduli stabilization is better understood.)

- The Higgses now correspond to transverse brane motion, but they still have the shift symmetry.



- Very schematically, we now have

$$K = -3\ln[T+\overline{T}] - \ln[S_0+\overline{S}_0+|H_u+\overline{H}_d|^2]+\cdots$$

$$W = W_0 + y\,QH_uU$$

- A straightforward supergravity analysis gives (at tree-level):

$$\boxed{m_1^2 = m_2^2 = m_3^3 = 2m_{3/2}^2}$$

where m_i^2 are the entries of the Higgs mass matrix.

Phenomenological details

- Of course, high-scale SUSY has been considered before

  Giudice, Romanino '04
  Arkani-Hamed, Dimopoulos, Arvatinaki, Kaplan,.. '04..'12
  Hall, Nomura '09

- Quartic coupling λ at SUSY-breaking scale m_s:

$$\lambda(m_s) = \frac{g^2(m_s) + g'^2(m_s)}{8} \cos^2(2\beta)$$

- β is the rotation angle needed to diagonalize the mass matrix

$$M_H^2 = \begin{pmatrix} |\mu|^2 + m_{H_d}^2 & B\mu \\ B\mu & |\mu|^2 + m_{H_u}^2 \end{pmatrix} = \begin{pmatrix} m_1^2 & m_3^2 \\ m_3^2 & m_2^2 \end{pmatrix}$$

- We have thus provided a symmetry reason for

$$M_H^2 \sim \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

- In other words, we have a symmetry reason for

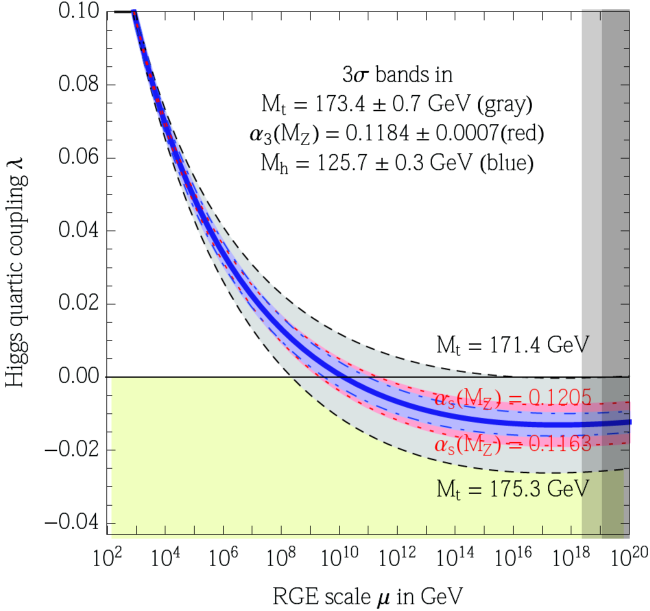
$$\tan(\beta) = 1 \qquad \text{and therefore} \qquad \lambda(m_s) = 0$$

- This can be interpreted in two ways:

  1) We have a setting predicting the SUSY-breaking scale based on the observables m_h, m_t and α_s (cf. plot).

  2) We have a setting realising the highest SUSY breaking scale that's allowed in a high-scale MSSM.

Buttazzo et al. 1307.3536



Corrections? Precision?

- The structure

$$M_H^2 \sim \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

predicts $\lambda(m_s) = 0$ and one exactly massless Higgs doublet (since $\det(M_H) = 0$).

- However, loops correct both λ and the mass matrix:

$$M_H^2 \sim \begin{pmatrix} 1+\beta & 1+\delta \\ 1+\delta & 1+\gamma \end{pmatrix}$$

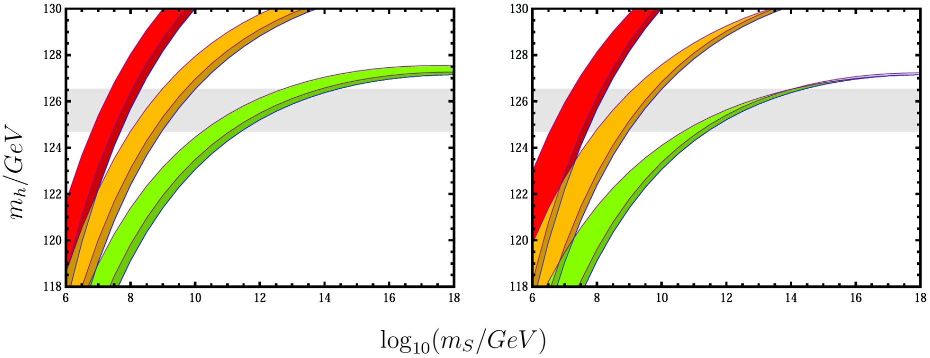
- The loop corrections have to be (and can be!) tuned to insure $\det(M_H) \sim m_h \ll m_s$.
- This affects λ only very mildly (see plot).

Corrections? Precision?

- The two main theoretical errors come from

  (I) non-SUSY-loops at m_S correcting λ (left)

  (II) SUSY-loops above m_S correcting M_H (right).



Predictivity/Applications

- Clearly, we eventually need more phenomenological implications of 'stringy high-scale SUSY'

- Among others, axion(s), cosmological moduli, gauge unification and proton decay can be potentially related to the high SUSY-breaking scale

  Chatzistavrakidis, Erfani, Nilles, Zavala '12
  Ibanez, Marchesano, Regalado, Valenzuela '12

- Particularly interesting point: The term $H_u H_d \subset K$, which is potentially controlled by the shift symmetry, is crucial for reheating and and hence dark radiaton abundance

  Higaki, Kamada, Takahashi '12
  Cicoli, Conlon, Quevedo,... Angus,... '12...'13

An interesting footnote...

- Amusingly, SUSY can be broken even far above the scale where $\lambda = 0$
- One first needs to enforce $\lambda = 0$ 'from the Kähler potential'
- This leading-order result can then be corrected 'via the superpotential', using an NMSSM-like scalar:

$$W = \kappa S H_u H_d + \frac{M^2}{2} S^2 + \cdots$$

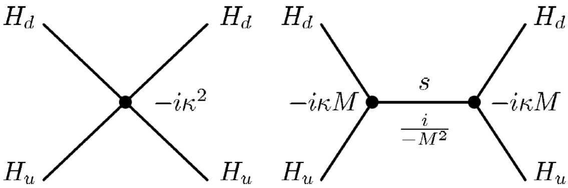
- If S also has a SUSY-breaking mass-quared $m_s^2 < 0$, after integrating out S one has:

$$V_F = \frac{\kappa^2 m_s^2}{M^2 + m_s^2} |H_0|^4 < 0$$

Giudice, Strumia '11

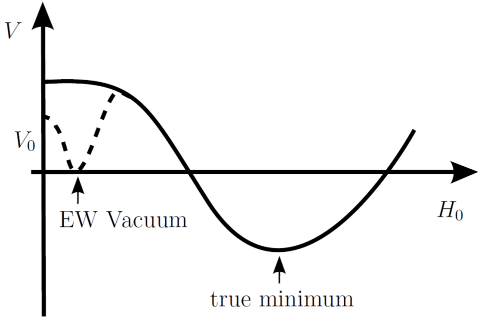
- This gives $\lambda < 0$ below the SUSY-breaking scale.

Diagrammatic view of the generation of the quartic term



- In unbroken SUSY, the cancellation is perfect.
- In broken SUSY, $M^2 \to (M^2 + m_s^2)$ upsets the cancellation.

- Directly below the SUSY breaking scale, the point $H_0 = 0$ is (quartically) unstable.



- 'Our' minimum is generated only radiatively (since λ runs to positive values)
- The SU(2)-breaking minimum is a tiny extra effect
- This can be viewed as a microscopic realization of the metastability scenario

Conclusions / Summary

- In the absence of new electroweak physics at a TeV, the 'vacuum stability scale' μ_λ may be a hint at new physics
- Well-motivated guess: SUSY broken with $\tan\beta = 1$ at μ_λ
- Possible reason: Shift symmetry in Higgs sector
- Specific settings include the bulk-type Higgs in type IIB/F-theory GUTs

  Weigand, Palti, Mayrhofer,...
- But: SUSY breaking above μ_λ with $\lambda < 0$ is also possible; cosmological challenges need further study

  Abel/Chu/Jaeckel/Khoze '06
  Lebedev/Westphal '12