Primordial Black Holes as a dark matter candidate

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Motivation

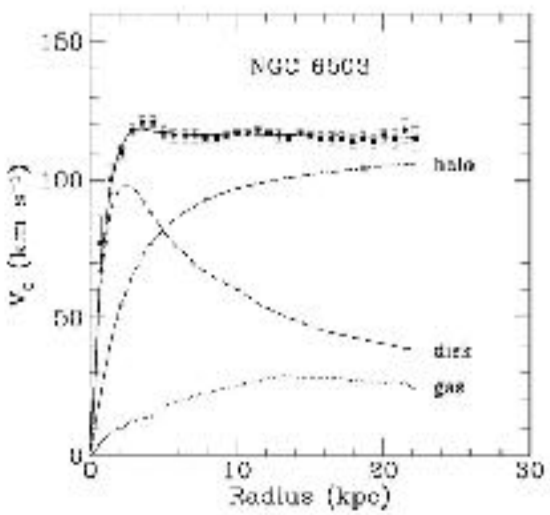
Formation

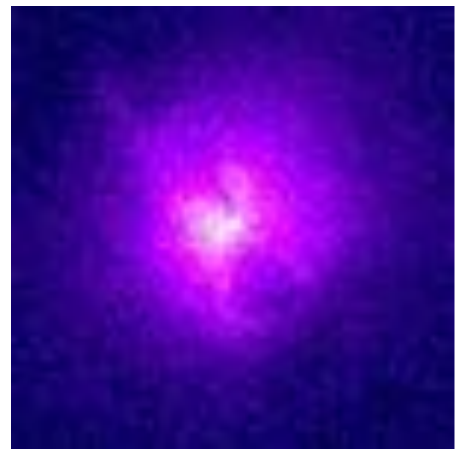
Constraints

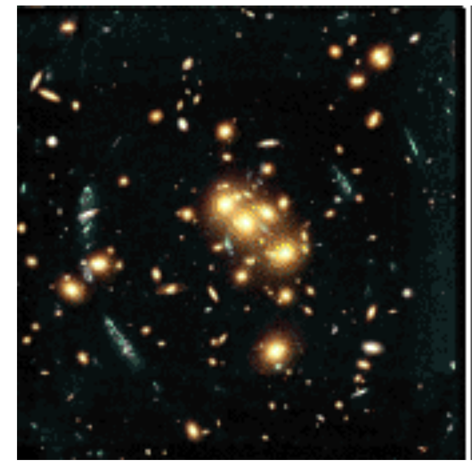
(If time permits: astrophysical uncertainties and extended mass functions)

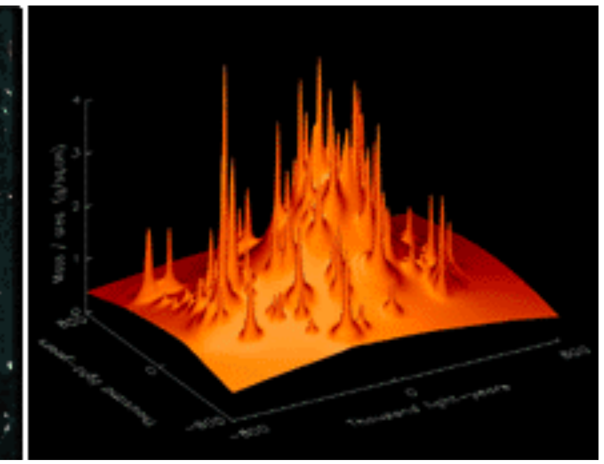
For further details on these topics (and also PBH binary mergers as source of GWs) see review by Sasaki, Suyama, Tanaka & Yokoyama arXiv:1801.05235.

Motivation





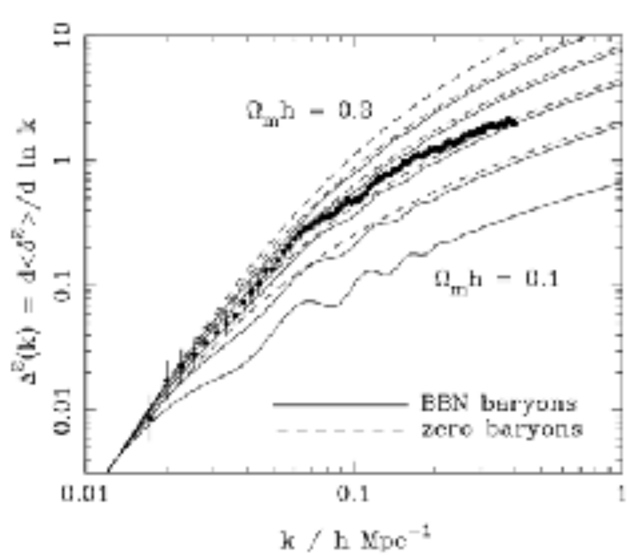


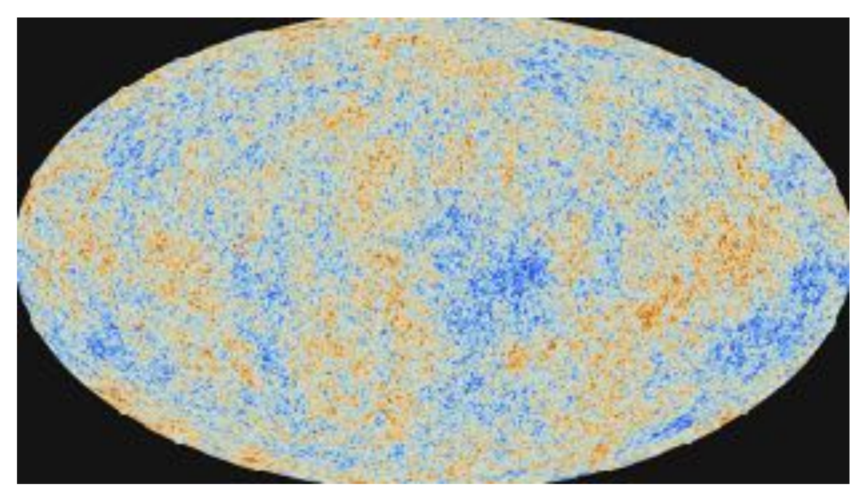


Lots of evidence for (non-baryonic cold) dark matter from diverse astronomical and cosmological observations

[galaxy rotation curves, galaxy clusters (galaxy velocities, X-ray gas, lensing), galaxy red-shift surveys, Cosmic Microwave Background]

assuming Newtonian gravity/GR is correct.



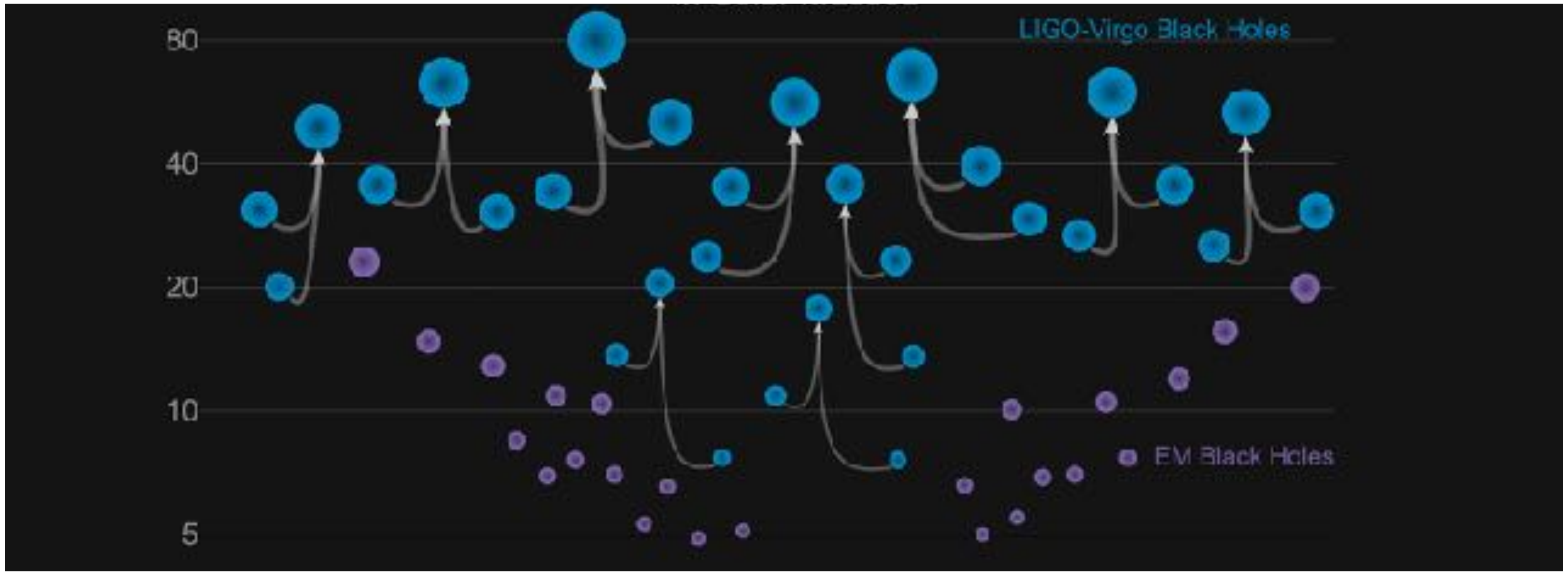


Primordial Black Holes (PBHs) form in the early Universe (before nucleosynthesis) and are therefore non-baryonic.

PBHs evaporate (Hawking radiation), lifetime longer than the age of the Universe for $M > 10^{15}$ g.

A DM candidate which (unlike WIMPs, axions, sterile neutrinos,…) isn't a new particle (however their formation does usually require Beyond the Standard Model physics, e.g. inflation).

LIGO has detected gravitational waves from mergers of $\sim 10 M_\odot$ BHs.



LIGO-Virgo, Elavsky

Could be formed by astrophysical processes, but such a large population possibly unexpected??

Could PBHs be the CDM?

And potentially also the source of the GW events?? Bird et al.; Sasaki et al.

Formation

Most 'popular' mechanism: during radiation domination an initially large (at horizon entry) density perturbation can collapse to form a PBH with mass of order the horizon mass. Zeldovich & Novikov; Hawking; Carr & Hawking

For gravity to overcome pressure forces resisting collapse, size of region at maximum expansion must be larger than Jean's length.

Simple analysis:

Carr; see Harada, Yoo & Kohri; Musco; Kalaja et al. … for refinements

density contrast: $$\delta \equiv \frac{\rho - \bar{\rho}}{\bar{\rho}}$$

threshold for PBH formation: $$\delta \geq \delta_{\mathrm{c}} \sim w = \frac{p}{\rho} = \frac{1}{3}$$

PBH mass: $$M \sim w^{3/2} M_{\mathrm{H}}$$

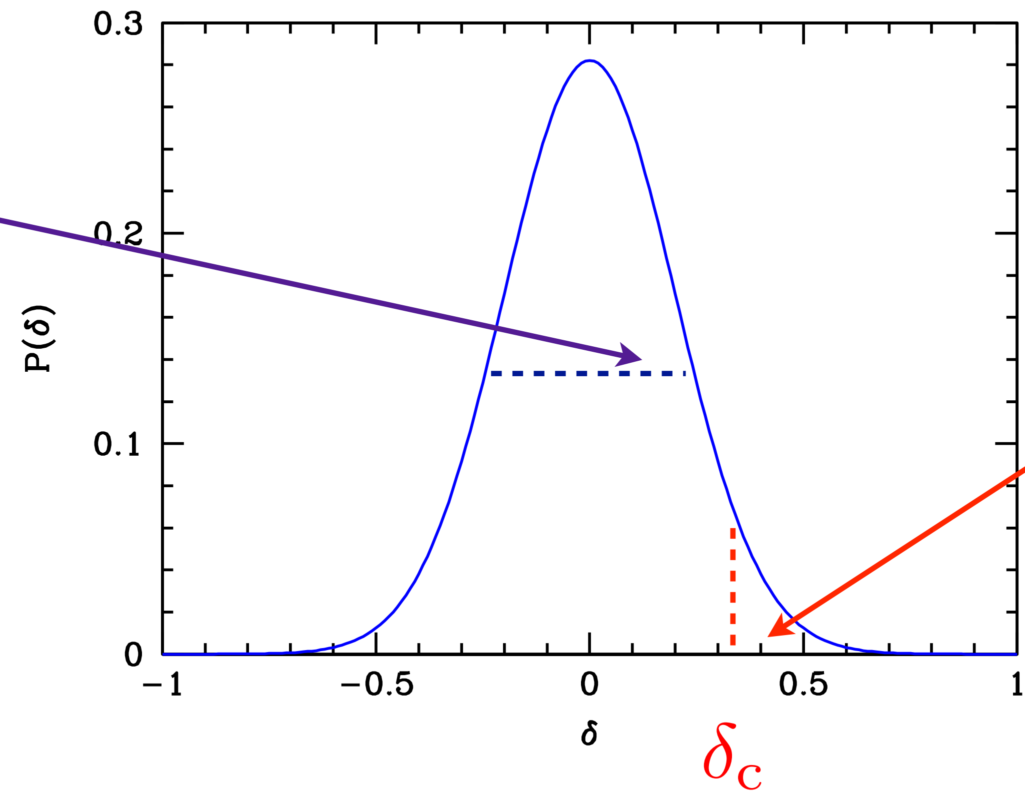
$$M_{\mathrm{H}} \sim 10^{15}\,\mathrm{g} \left(\frac{t}{10^{-23}\,\mathrm{s}} \right)$$

initial PBH mass fraction (fraction of universe in regions dense enough to form PBHs):

$$\beta(M) \sim \int_{\delta_{\mathrm{c}}}^{\infty} P(\delta(M_{\mathrm{H}}))\, \mathrm{d}\delta(M_{\mathrm{H}})$$

assuming a gaussian probability distribution: $\beta(M) = \mathrm{erfc}\left(\frac{\delta_{\mathrm{c}}}{\sqrt{2}\sigma(M_{\mathrm{H}})}\right)$

$\sigma(M_{\mathrm{H}})$ (mass variance)
typical size of fluctuations



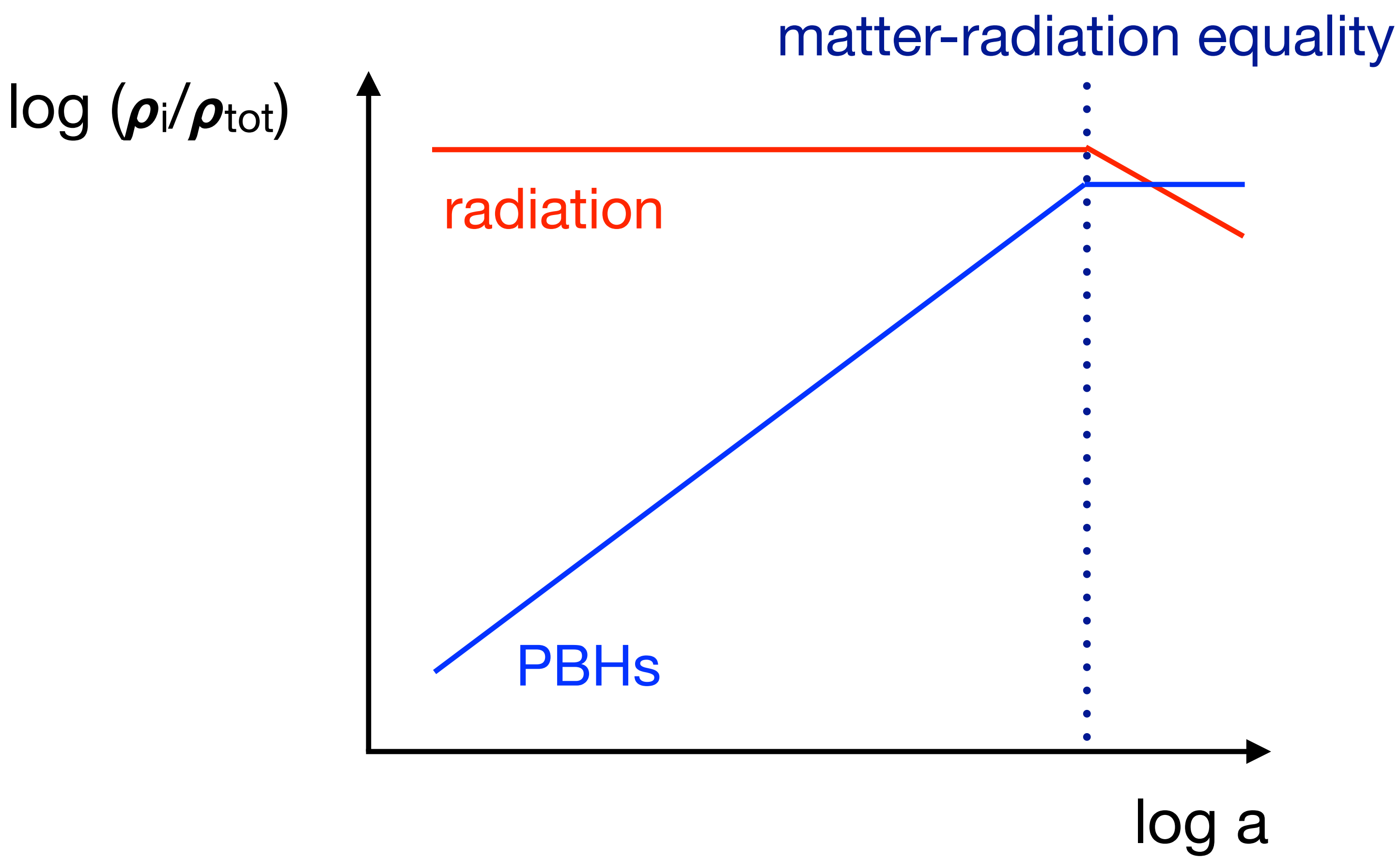
PBH forming fluctuations

but in fact β must be small, hence $\sigma \ll \delta_{\mathrm{c}}$

PBH abundance

Since PBHs are matter, during radiation domination the fraction of energy in PBHs grows with time:

$$\frac{\rho_{\rm PBH}}{\rho_{\rm rad}} \propto \frac{a^{-3}}{a^{-4}} \propto a$$



Relationship between **PBH initial mass fraction, β**, and **fraction of DM in form of PBHs, f**:

$$\beta(M) \sim 10^{-9} f \left(\frac{M}{M_\odot} \right)^{1/2}$$

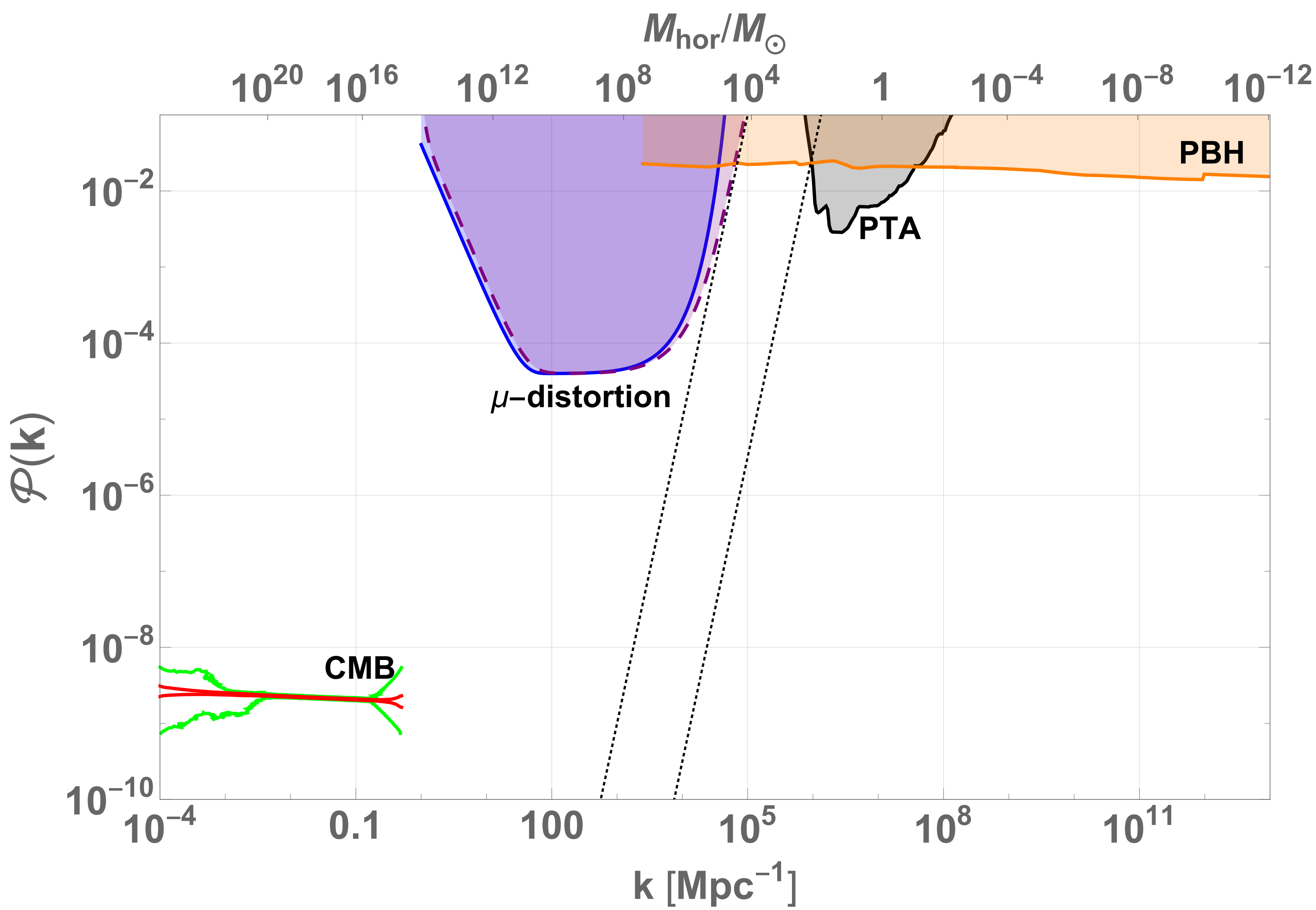
i.e. initial mass fraction must be small, but non-negligible.

On CMB scales the primordial perturbations have amplitude $\sigma(M_{\rm H}) \sim 10^{-5}$

If the primordial perturbations are very close to scale-invariant the number of PBHs formed will be completely negligible: $\beta(M) \sim {\rm erfc}(10^5) \sim \exp\left(-10^{10}\right)$

To form an interesting number of PBHs the primordial perturbations must be significantly larger ($\sigma^2(M_{\rm H})\sim0.01$) on small scales than on cosmological scales.

$\sim \sigma^2$



Byrnes, Cole & Patil

deviations from simple scenario:

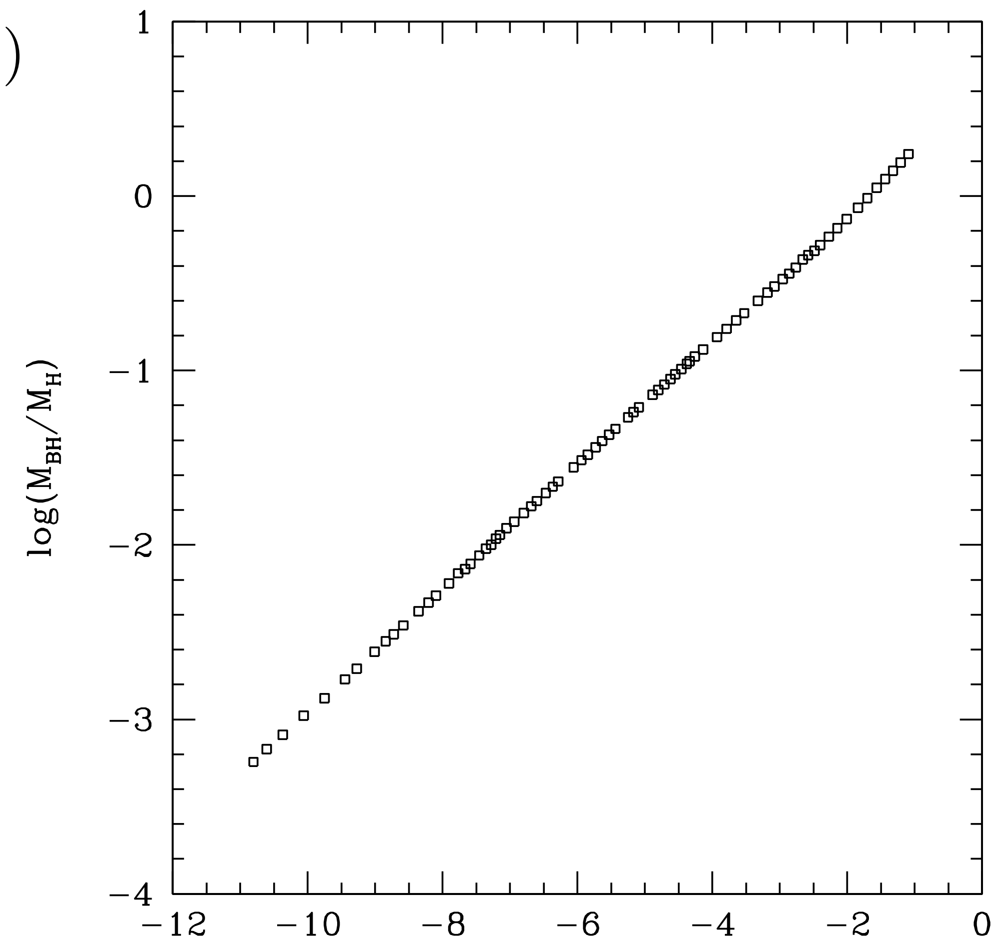
i) critical collapse

Choptuik; Evans & Coleman; Niemeyer & Jedamzik

BH mass depends on size of fluctuation it forms from:

$$M = k M_{\mathrm{H}} (\delta - \delta_{\mathrm{c}})^{\gamma}$$

$\log_{10}(M_{\mathrm{BH}}/M_{\mathrm{H}})$



log(δ-δc)

Musco, Miller & Polnarev

using numerical simulations (with appropriate initial conditions) find k=4.02, γ=0.357, δ_c = 0.45

Get PBHs with range of masses produced even if they all form at the same time i.e. we don't expect the PBH MF to be a delta-function

ii) non-gaussianity

Since PBHs are formed from rare large density fluctuations, changes in the shape of the tail of the probability distribution (i.e. non-gaussianity) can significantly affect the PBH abundance. Bullock & Primack; Ivanov;… Byrnes, Copeland & Green;…

Relationship between density perturbations and curvature perturbations is non-linear, so even if curvature perturbations are gaussian (large) density perturbations won't be. Kawasaki & Nakatsuka; De Luca et al.; Young, Musco & Byrnes

iii) clustering

Is potentially important (affects merger rate and also many of the abundance constraints), but hard to calculate. Is currently a somewhat open issue.

PBHs don't form in clusters Ali-Haïmoud (previous work Chisholm extrapolated an expression for the normalized correlation beyond its range of validity).

Clustering depends on shape of power spectrum, small if PBHs form from spike in power spectrum. Desjacques & Riotto; Ballesteros, Serpico & Taoso; …

Inflation: a crash course

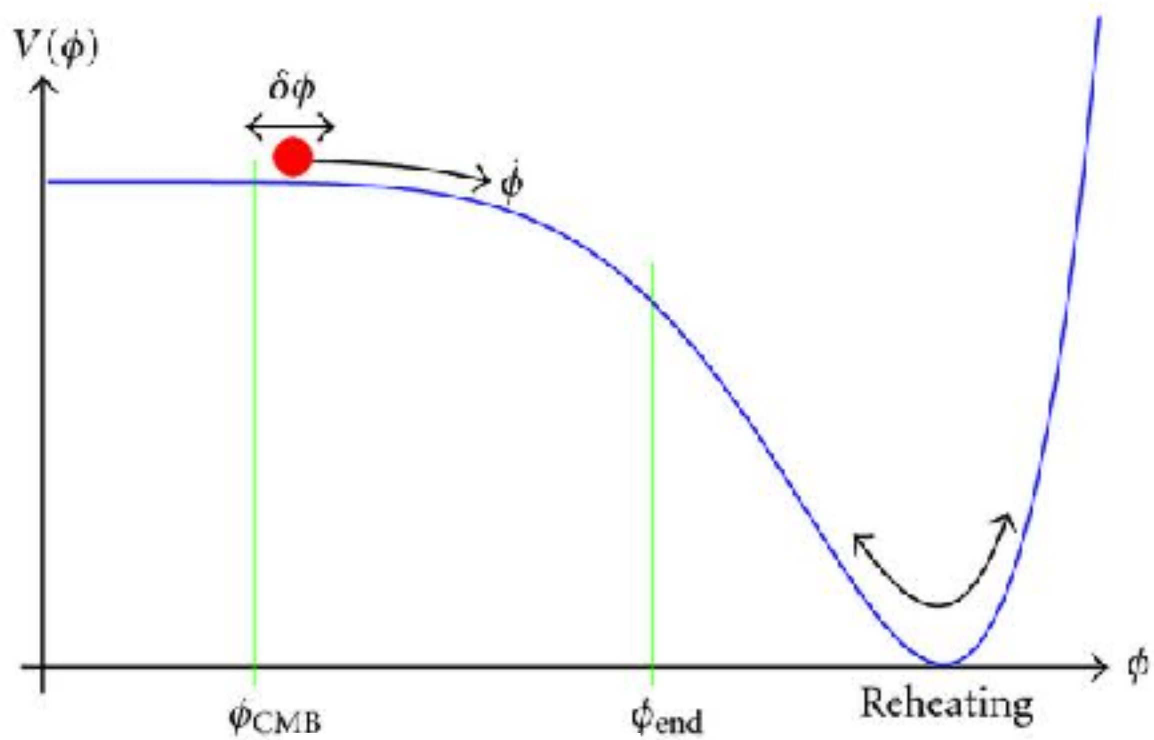
A postulated period of accelerated expansion in the early Universe, proposed to solve various problems with the Big Bang (flatness, horizon & monopole).

Driven by a ‘slowly rolling’ scalar field.

Quantum fluctuations in scalar field generate density perturbations.

Scale dependence of primordial perturbations depends on shape of potential:

Yadav & Wandelt



in slow-roll approx

$$\sigma^2(M_{\rm H}) \propto \frac{V^3}{(V')^2}$$

Scales probed by:

Large scale structure & the CMB

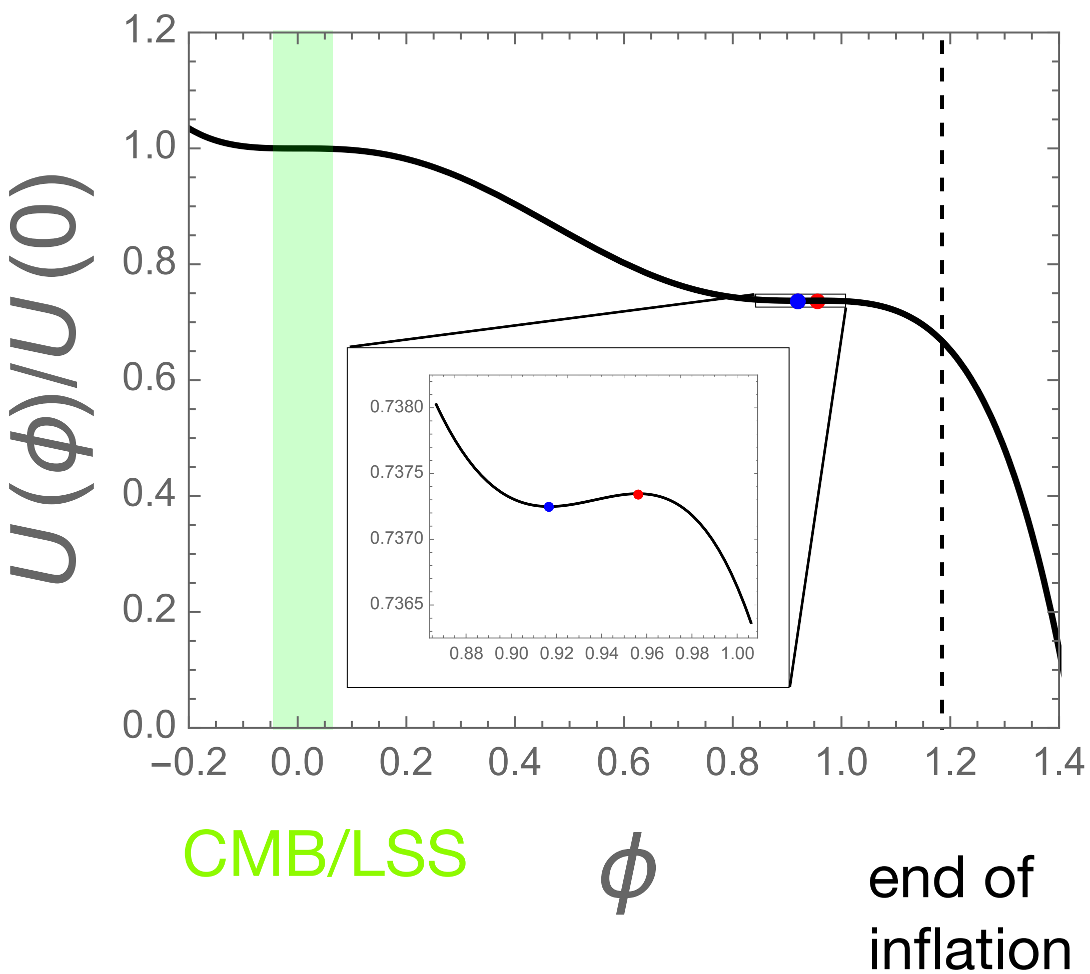
Primordial Black Holes

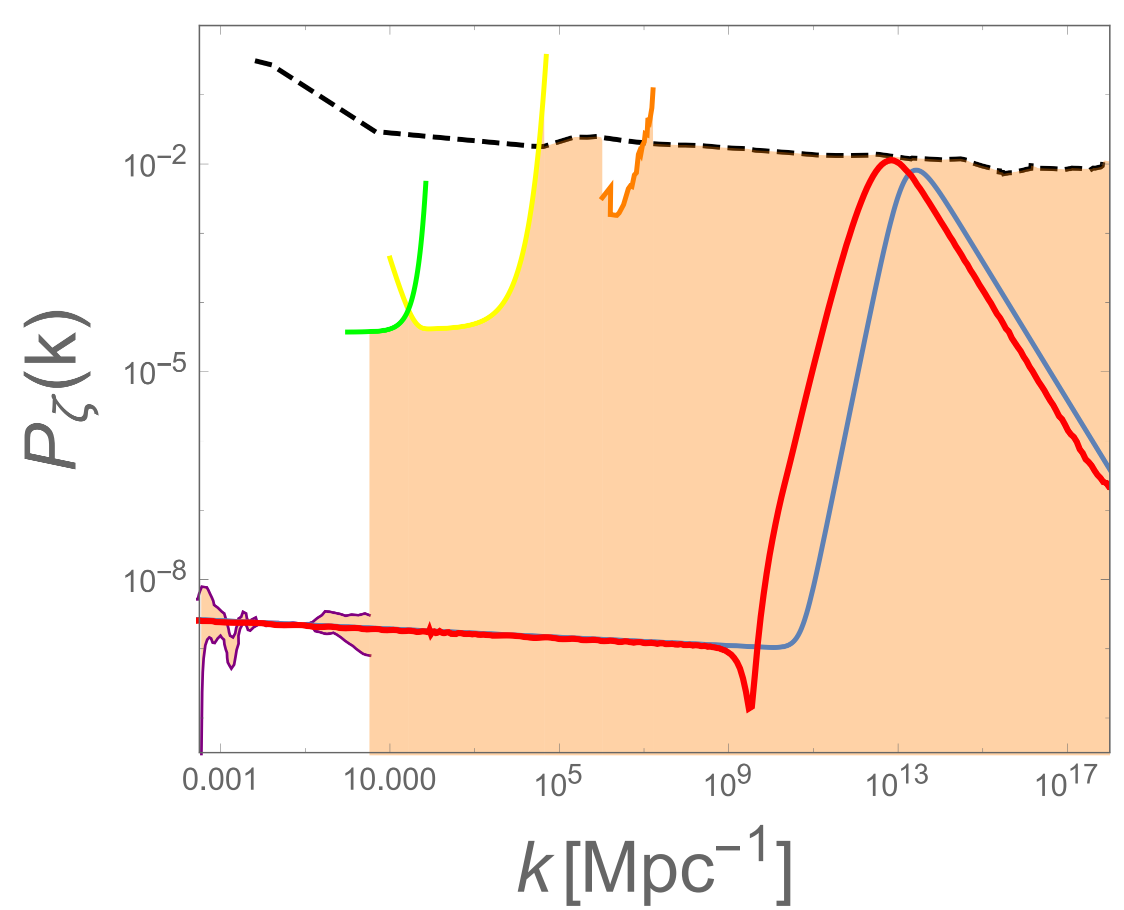
(some) Inflation models with (potentially) large perturbations on small scales

a) over-shoot a local minimum

Ballesteros & Taoso; Herzberg & Yamada

Potential fine-tuned so that field goes past local min, but with reduced speed



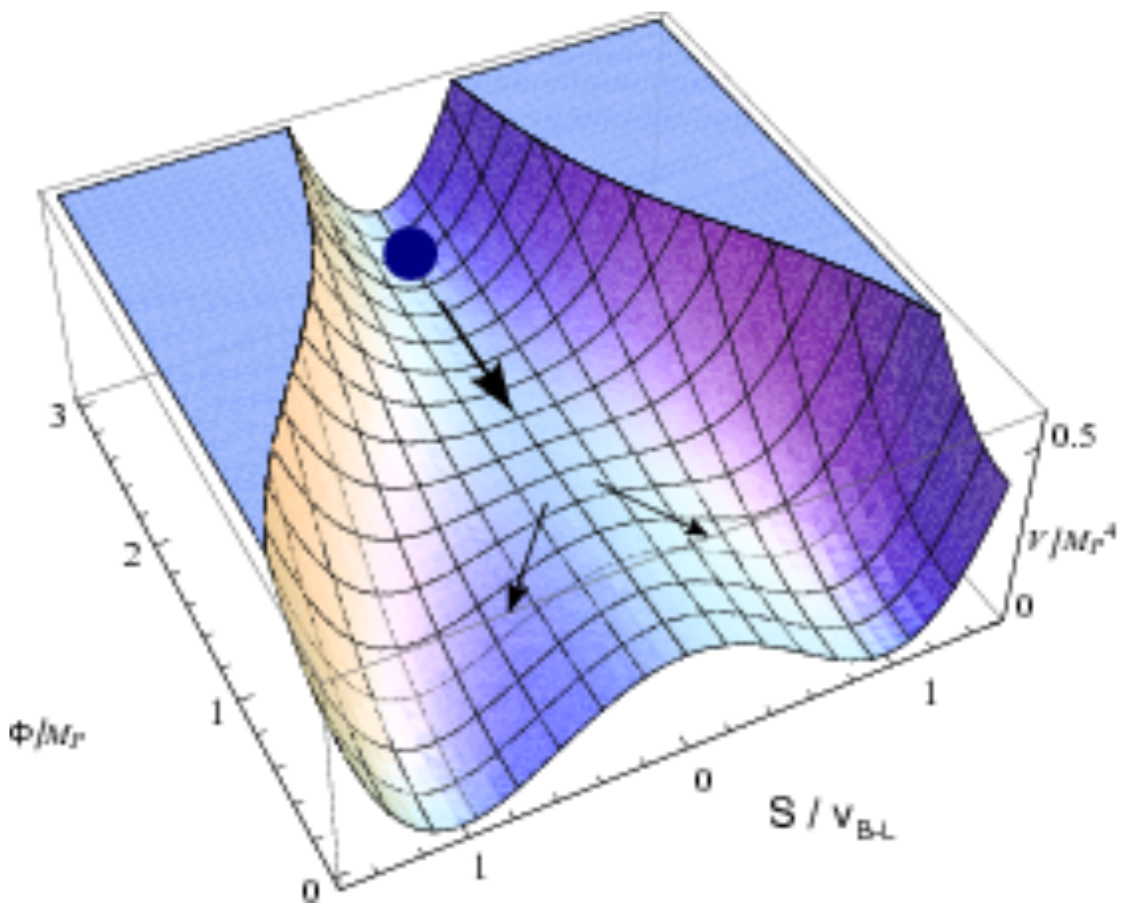


b) multi-field models

hybrid inflation with a mild waterfall transition

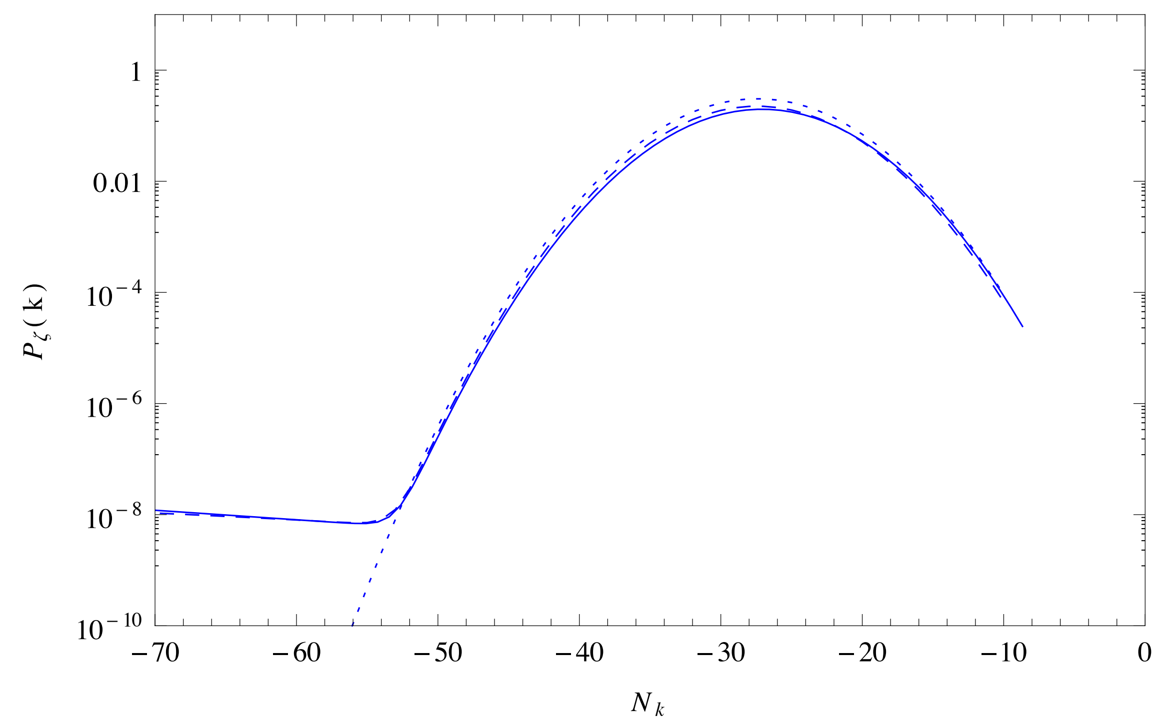
Garcia-Bellido, Linde & Wands

potential



Buchmuller

primordial power spectrum



Clesse & Garcia-Bellido

c) various others

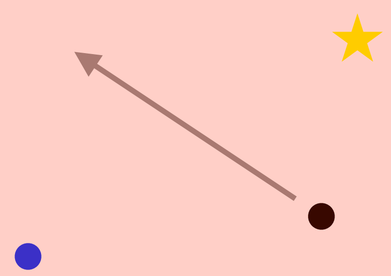
running mass, double inflation, axion-like curvaton, …

Constraints

[Initially all constraints assume a delta-function PBH mass function.]

Microlensing

stars, supernovae, quasars



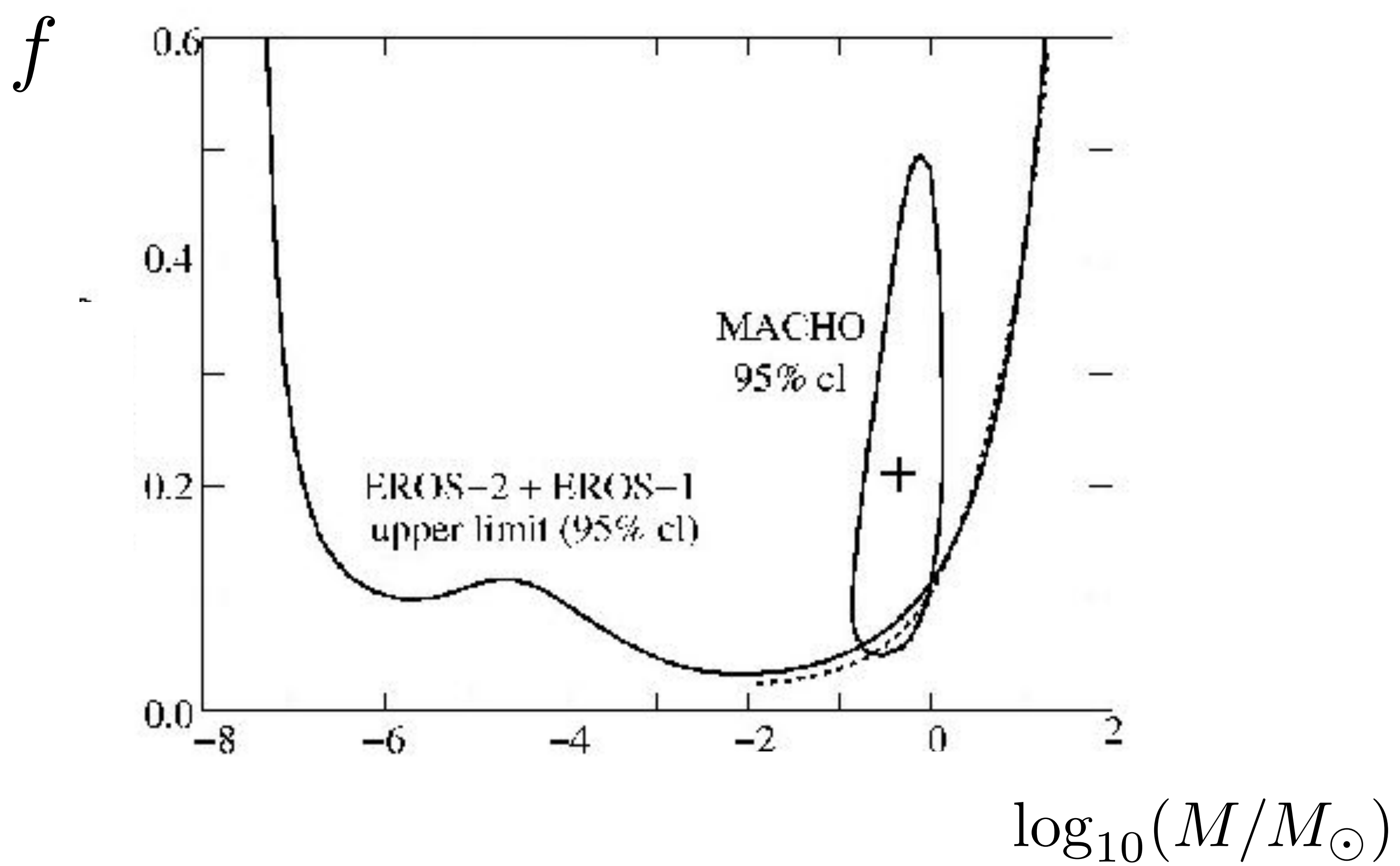
Constraints

[Initially all constraints assume a delta-function PBH mass function.]

Microlensing of stars in Magellanic Clouds

Stellar microlensing: temporary (achromatic) brightening of background star when compact object passes close to the line of sight.

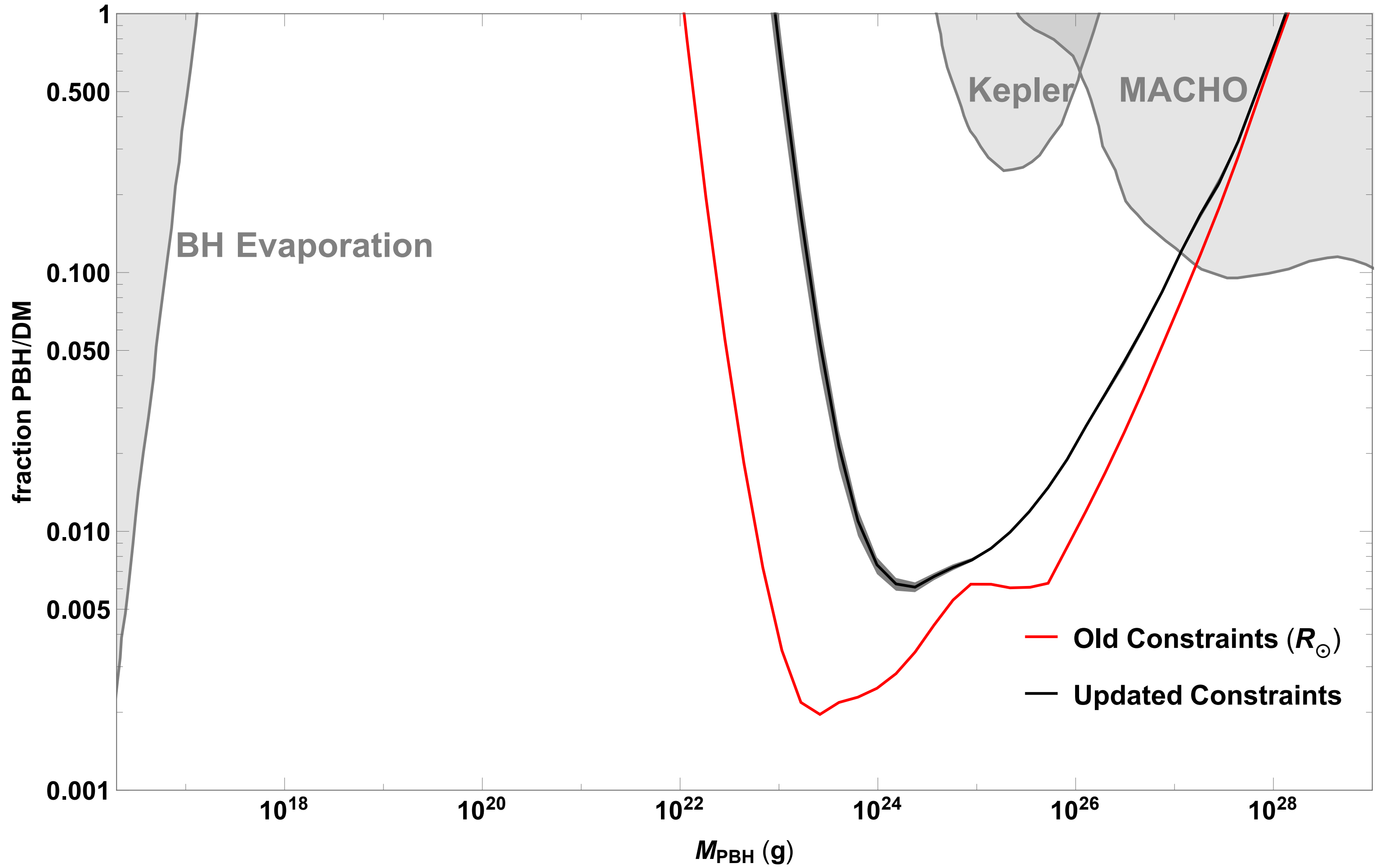
EROS constraints on fraction of DM in compact objects:



MACHO has very similar limits for $M > 3M_\odot$.

Microlensing of stars in M31

Subaru HSC observations have higher cadence than EROS/MACHO, so sensitive to shorter duration events and hence lighter compact objects. Niikura et al.



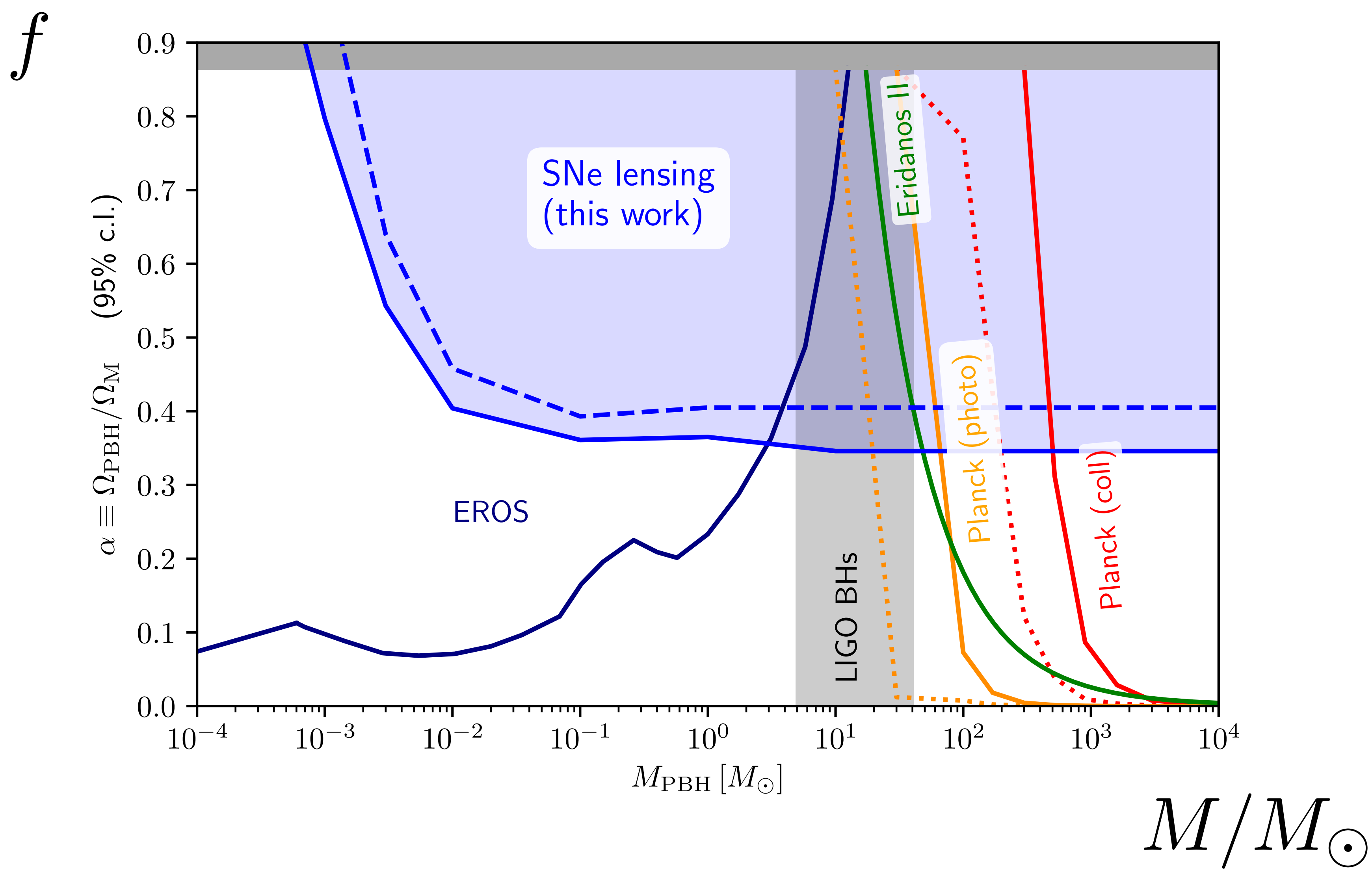
Smyth et al.

Finite size of source stars and effects of wave optics (Schwarzschild radius of BH comparable to wavelength of light) leads to reduction in maximum magnification for $M \lesssim 10^{-7} M_\odot$ and $M \lesssim 10^{-11} M_\odot$ respectively. Witt & Mao; Gould; Nakamura; Sugiyama, Kurita & Takada

And only large stars are bright enough for microlensing to be observed. Montero-Camacho et al.; Smyth et al.

supernova microlensing

Lensing magnification distribution of type 1a SNe affected (most lines of sight are demagnified relative to mean, plus long-tail of high magnifications): Zumalacarregui & Seljak

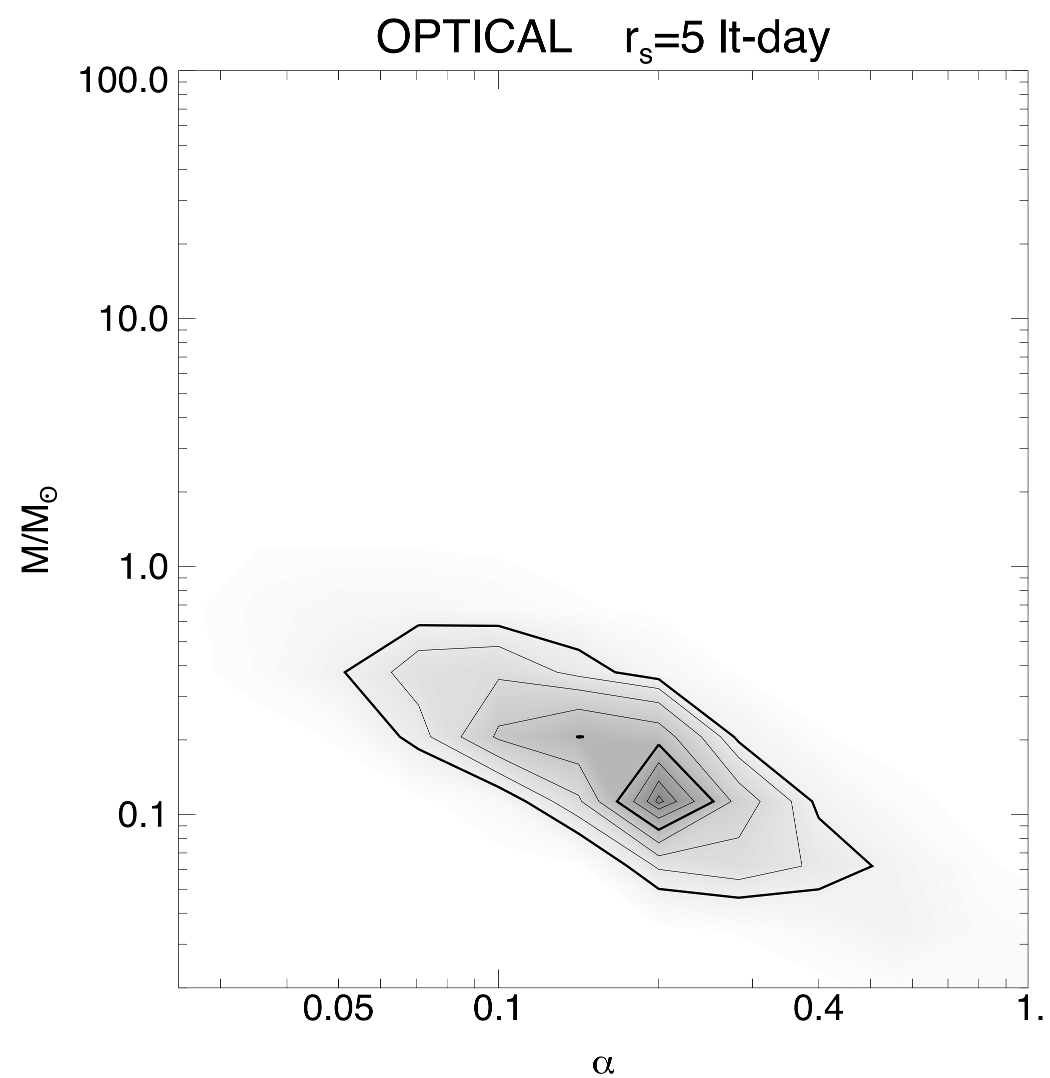


Garcia-Bellido, Clesse & Fleury. argue priors on cosmological parameters are overly restrictive and physical size of supernovae have been underestimated.

quasar microlensing

Quasar microlensing by compact objects in lens galaxy leads to variation in brightness of images in multiply lensed quasars. Chang & Refusal

α= 0.2 ± 0.05 of the mass is in compact objects with $0.05\,M_\odot < M < 0.45\,M_\odot$, consistent with abundance of stars. Mediavilla et al. However no constraint on f (fraction of mass in dark compact objects) published.

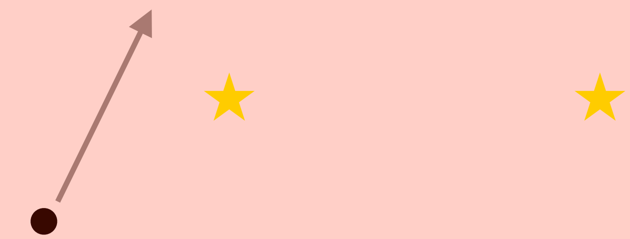


Microlensing

stars, supernovae, quasars

Dynamical effects

dwarf galaxies, wide binary stars



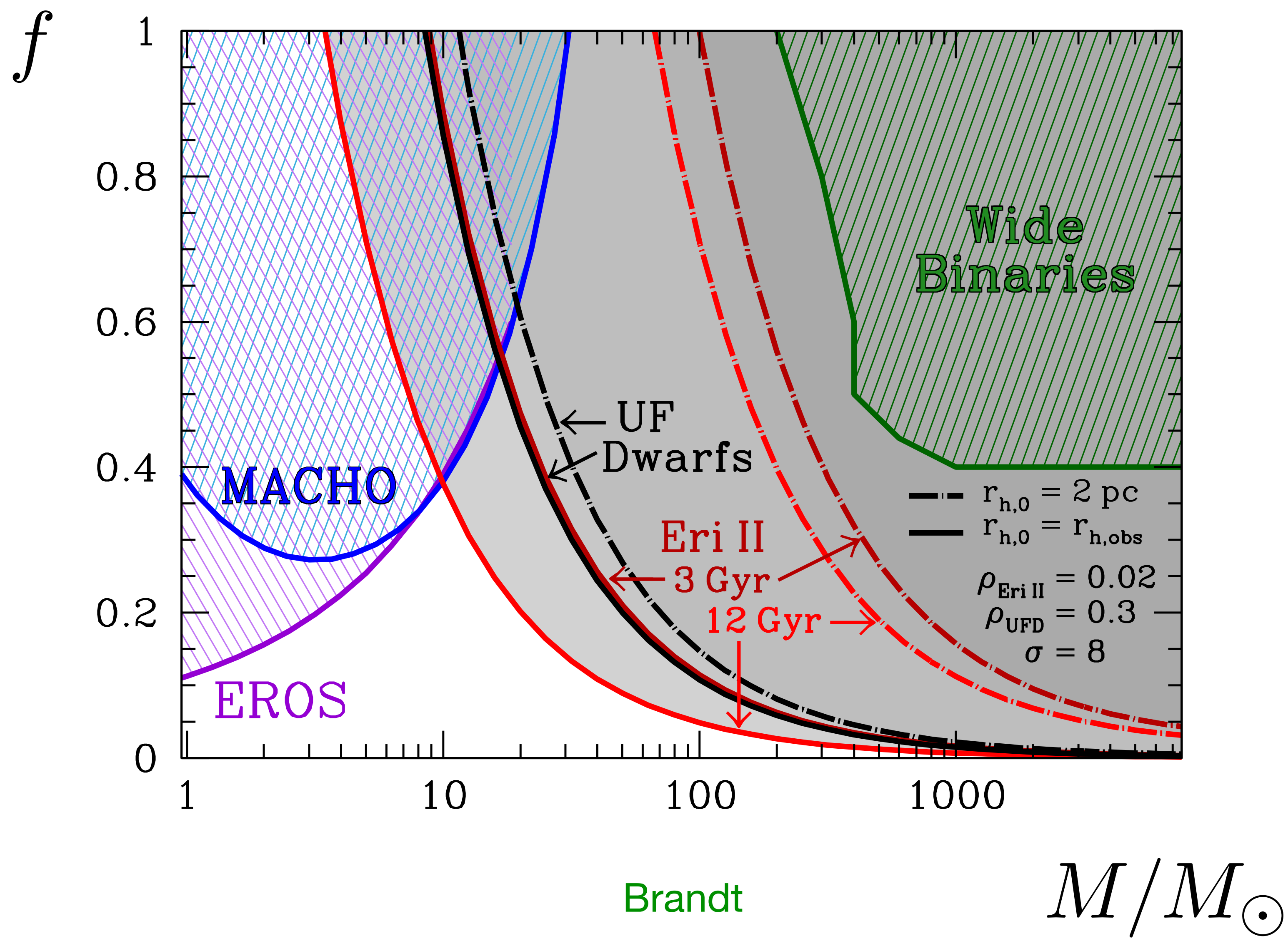
Constraints

[Initially all constraints assume a delta-function PBH mass function.]

Dwarf heating

multi-Solar mass PBHs dynamically heat stars and increase size of stellar component

Brandt; Koushiappas & Loeb; Zhu et al.; Stegmann et al.



Brandt

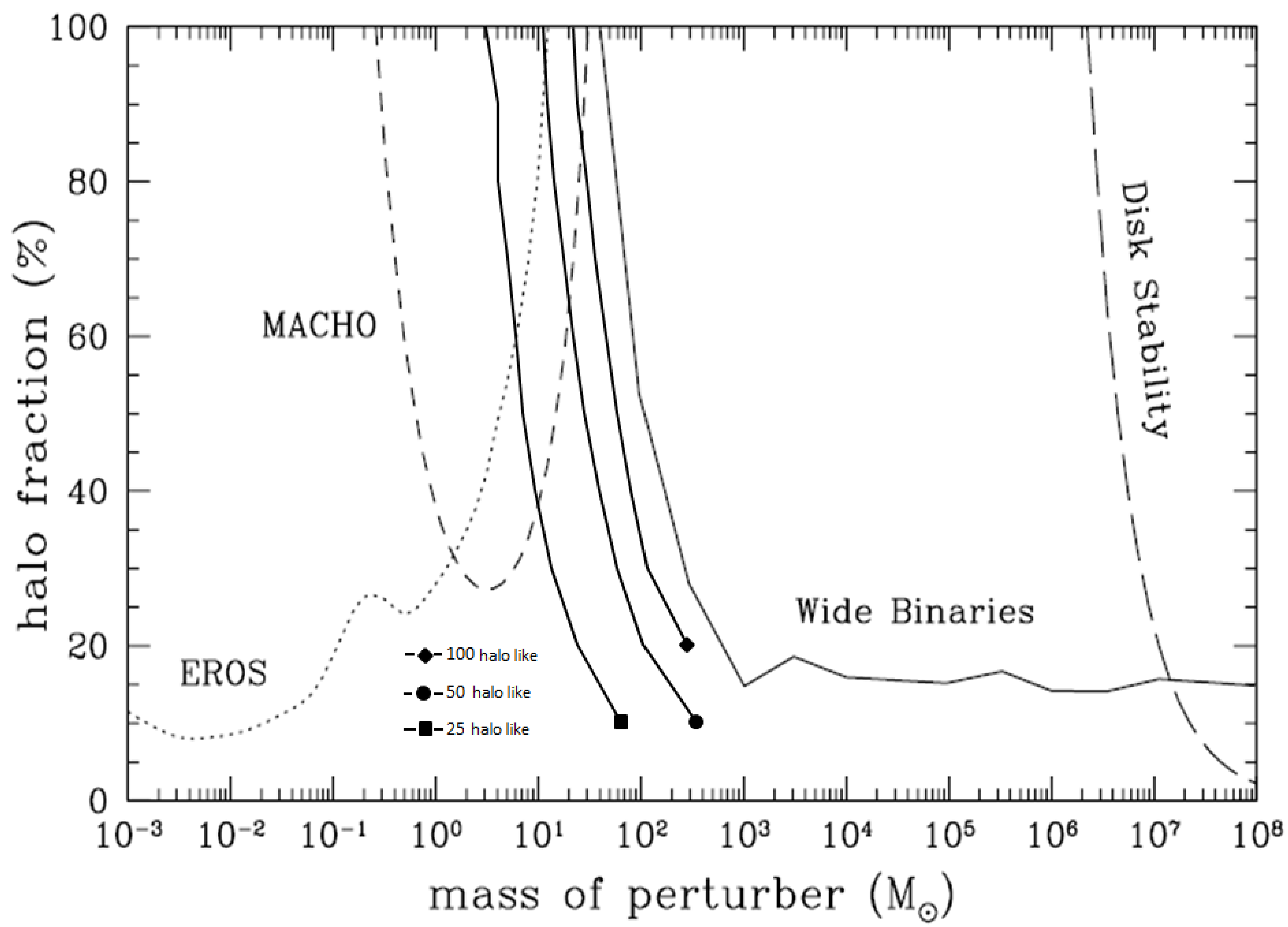
For any PBH mass in the range (1-100) $M_{\odot}$ there is an ultra-faint dwarf galaxy that disfavours f=1. Stegmann et al.

Wide binary disruption

Chaname & Gould; Yoo, Chaname & Gould; Quinn et al.; Monroy-Rodriguez & Allen

Massive compact objects perturb wide binaries.

$100f$



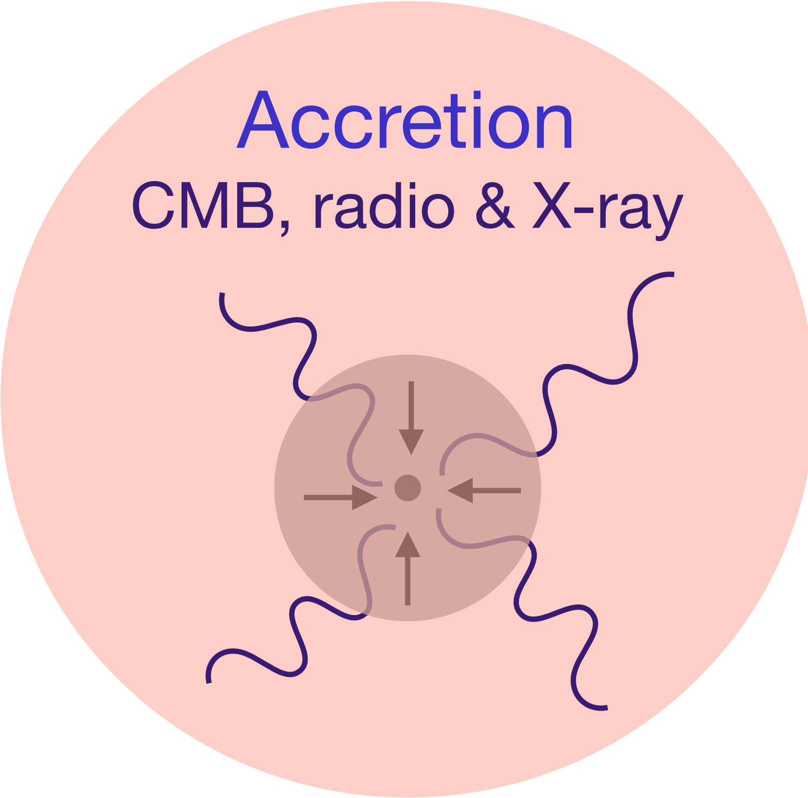
$M/M_\odot$

Monroy-Rodriguez & Allen

Microlensing
stars, supernovae, quasars

Dynamical effects
dwarf galaxies, wide binary stars

Constraints



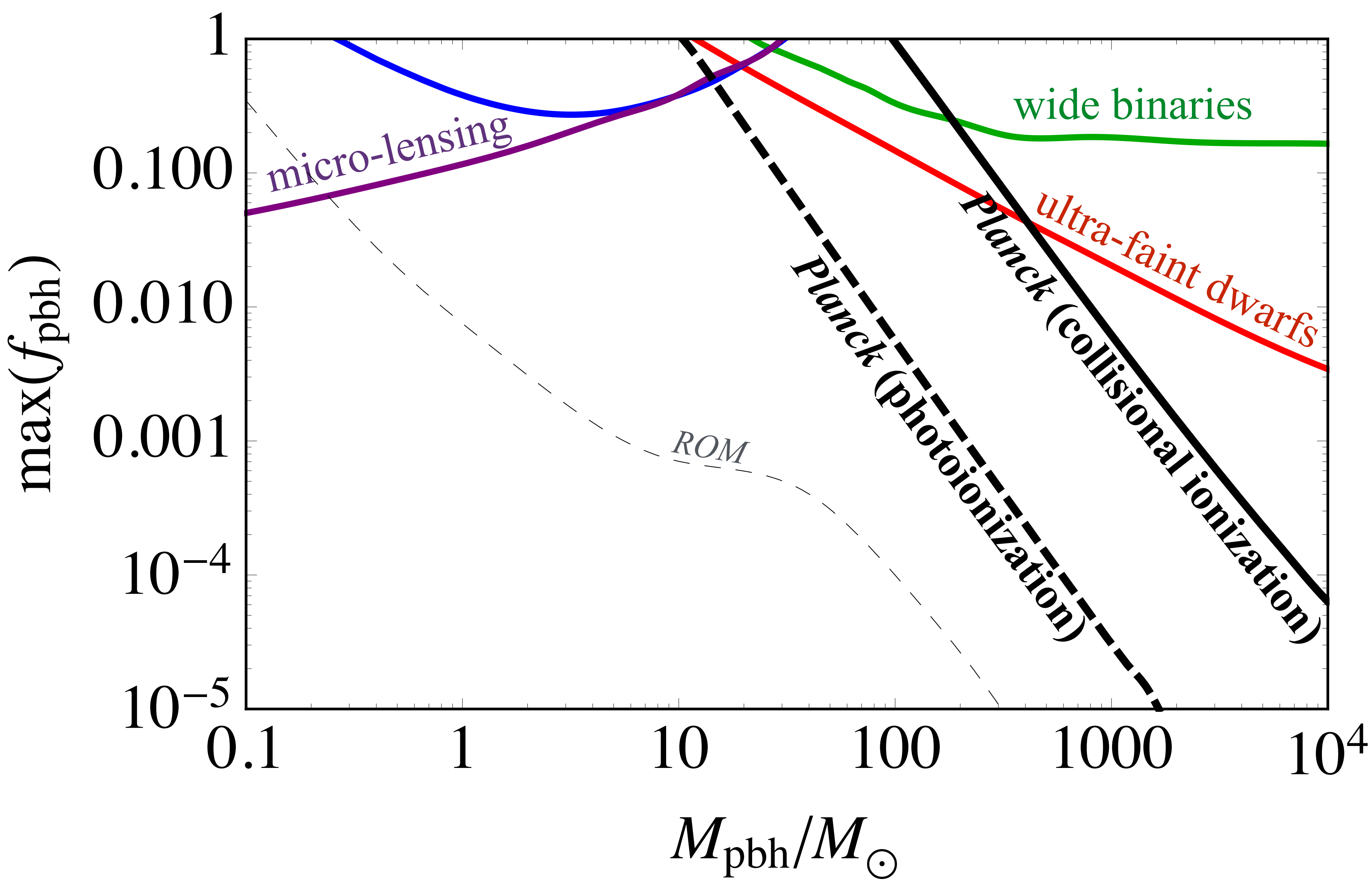
[Initially all constraints assume a delta-function PBH mass function.]

Cosmic Microwave Background distortions

Ricotti et al; Ali-Haïmoud & Kamionkowski; Horowitz; Blum, Aloni & Flauger

Accretion onto PBH leads to emission of X-rays which can distort the spectrum (FIRAS) and anisotropies (WMAP/Planck) in the CMB.

Significant uncertainties in constraint due to modelling of complex astrophysical processes.

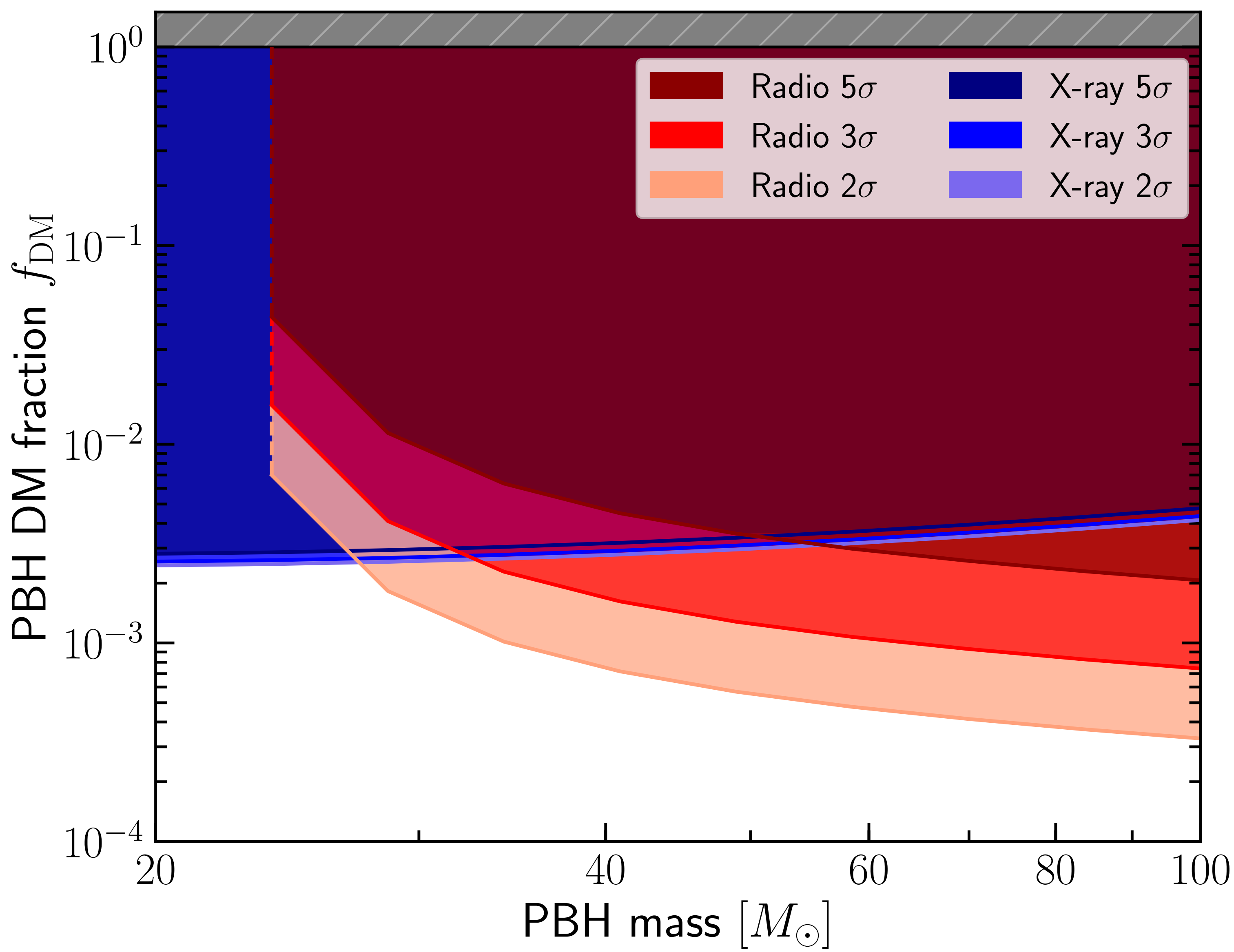


Ali-Haïmoud & Kamionkowski

X-ray and radio emission

Gaggero et al; Inoue & Kusenko; Manshanden et al.

Accretion onto PBH leads to X-rays and radio emission.



Manshanden et al.

Microlensing
stars, supernovae, quasars

Dynamical effects
dwarf galaxies, wide binary stars

Accretion
CMB, radio & X-ray

Constraints

Mergers
gravitational waves

[Initially all constraints assume a delta-function PBH mass function.]

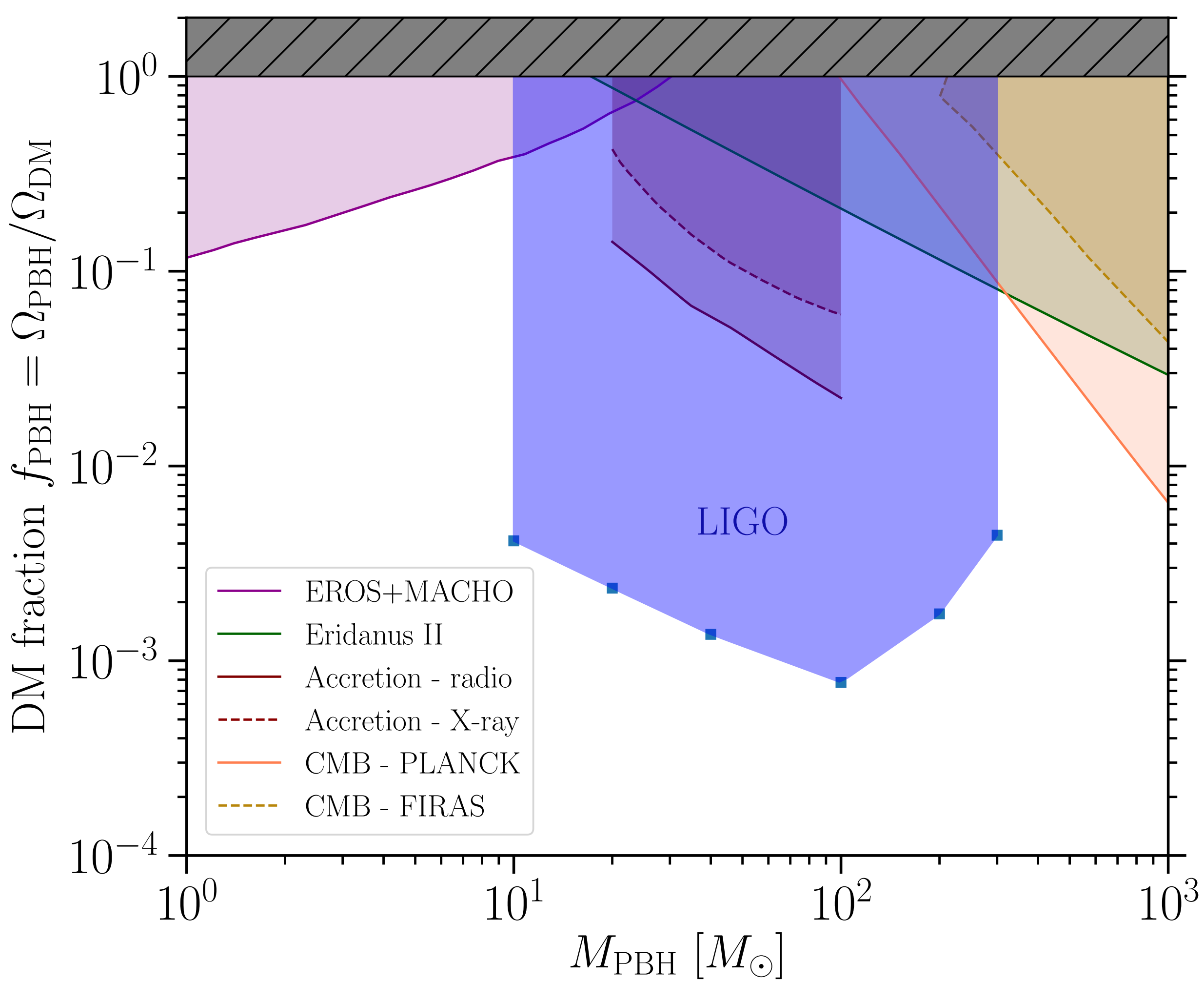
gravitational waves

PBH binaries can form:

i) in the early Universe (from chance proximity), Nakamura, Sasaki, Tanaka & Thorne

ii) via gravitational capture in present-day halos. Bird et al.

If PBH binaries formed in the early Universe survive to the present day then their mergers are dominant, and orders of magnitude larger than the merger rate measured by LIGO. Nakamura et al.; Ali-Haïmoud, Kovetz & Kamionkowski



Kavanagh, Gaggero & Bertone, taking into account PBH's dark 'dresses'

Microlensing
stars, supernovae, quasars

Dynamical effects
dwarf galaxies, wide binary stars

Accretion
CMB, radio & X-ray

Constraints

Effects on stars
white dwarfs, neutron stars

Mergers
gravitational waves

[Initially all constraints assume a delta-function PBH mass function.]

PBH interactions with stars

Stars can capture asteroid mass PBHs through dynamical friction, accretion onto PBH can then destroy the star. Capela, Pshirkov & Tinyakov; Pani & Loeb; **Montero-Camacho et al.**

Transit of asteroid mass PBH through white dwarf heats it, due to dynamical friction, causing it to explode. Graham, Rajendran & Varela;

Montero-Camacho et al. No current current constraints, but potential future constraints from

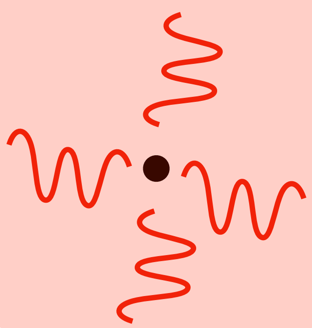
i) survival of neutron stars in globular clusters **if** they're shown to have formed in DM halos (need high DM density, low velocity-dispersion environment)

ii) signatures of star being destroyed

Microlensing
stars, supernovae, quasars

Dynamical effects
dwarf galaxies, wide binary stars

Evaporation
gamma-rays



Constraints

Accretion
CMB, radio & X-ray

Effects on stars
white dwarf, neutron stars

Mergers
gravitational waves

[Initially all constraints assume a delta-function PBH mass function.]

extragalactic gamma-rays background

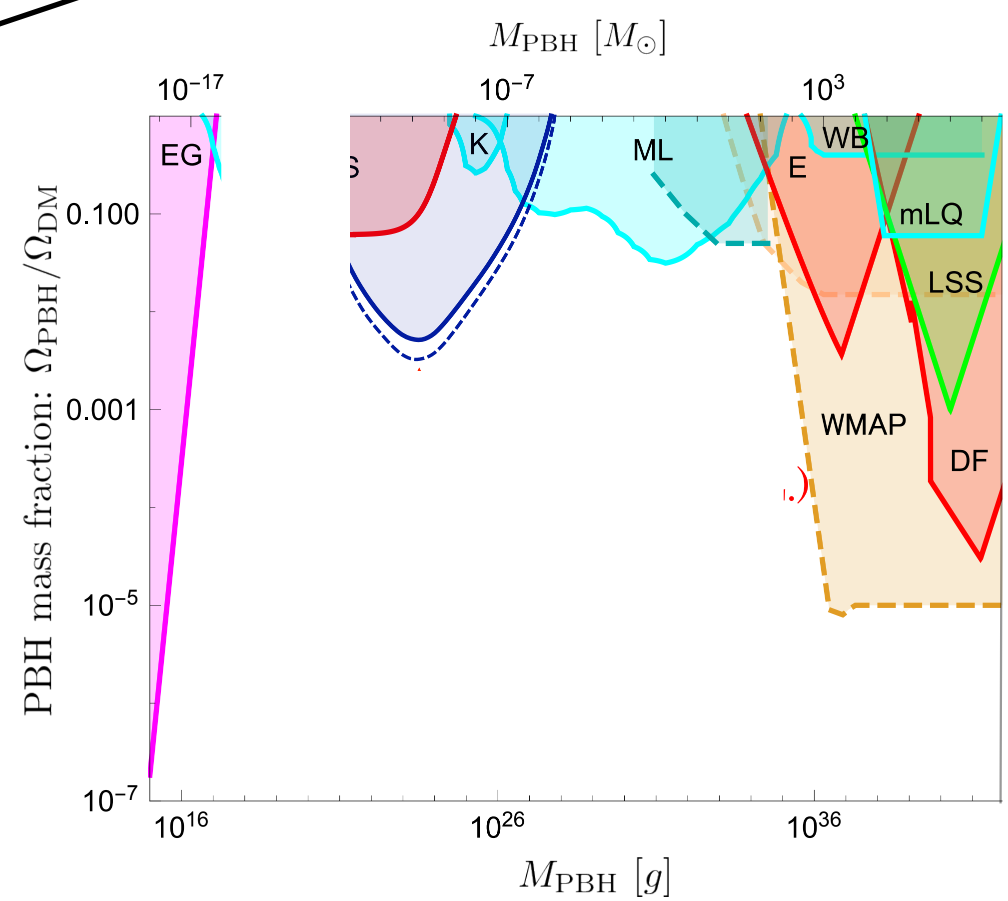
Page & Hawking; … ; Carr, Kohri, Sendouda & Yokoyama

Gamma-rays produced by evaporation can not exceed intensity of gamma-ray background measured by EGRET/Fermi.

$$f(M) \lesssim 2 \times 10^{-8} \left(\frac{M}{5 \times 10^{14}\,\mathrm{g}} \right)^{3.1-3.4}$$

uncertainty in energy dep. of measured intensity

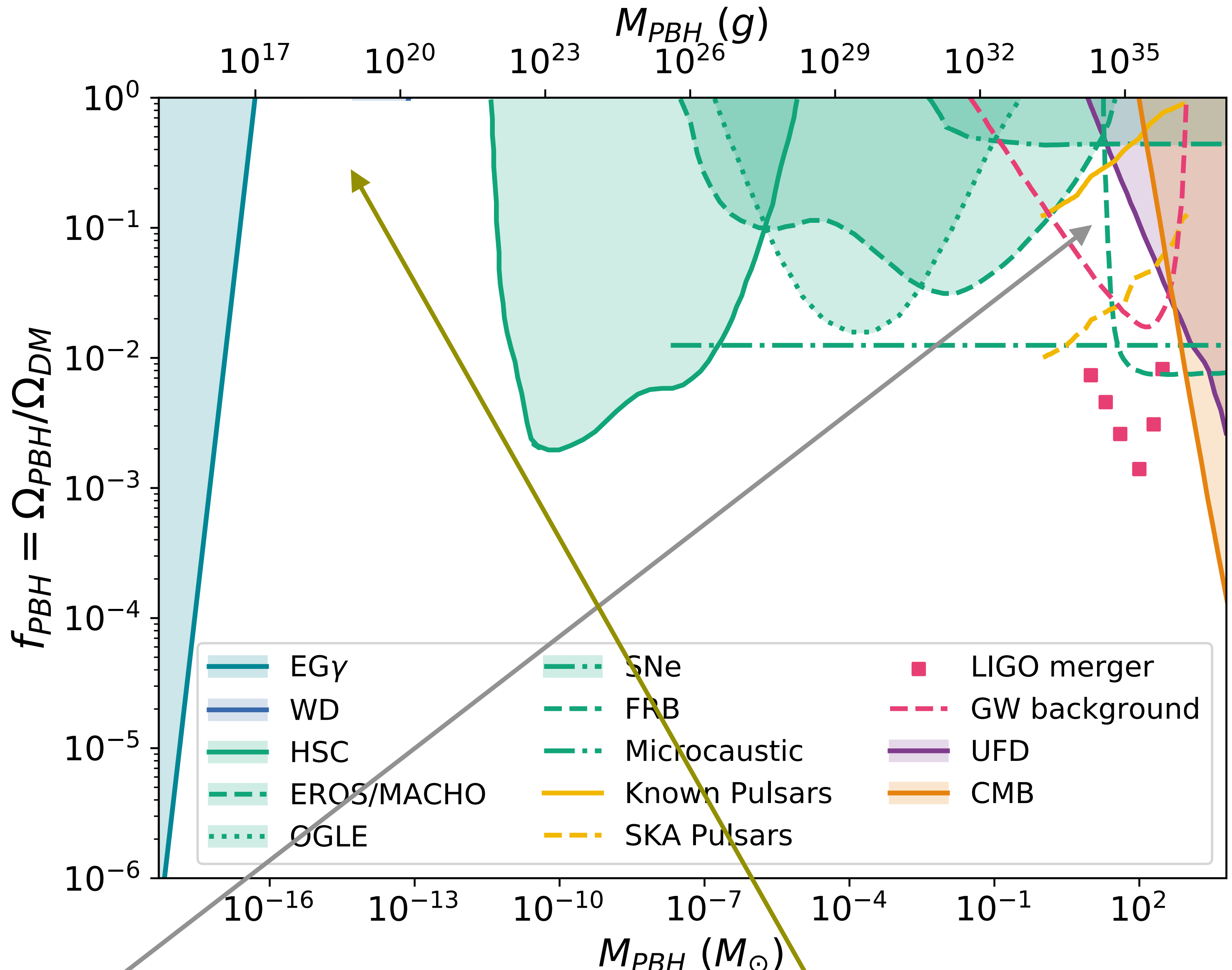
mass of PBH with lifetime equal to age of Universe



Tighter constraints could be obtained by subtracting off known contributions e.g. blazars

c.f. Barrau et al.

Compilation of constraints



unshaded lines (GW background, microcaustic and FRB are potential future constraints)

[edited from Sato-Polito, Kovetz & Kamionkowski]

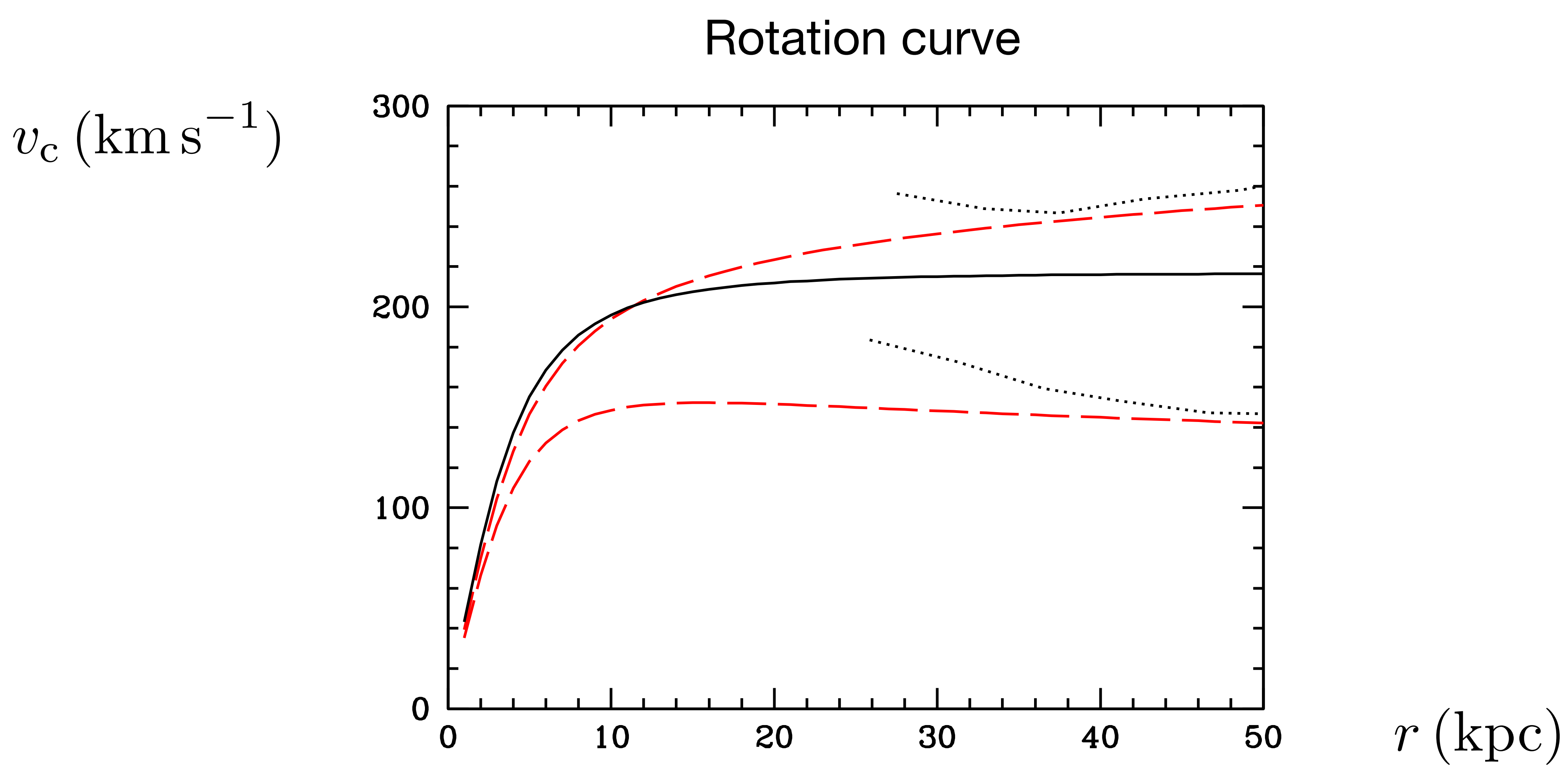
multi-Solar mass Primordial Black Holes making up all of the DM appears to be excluded (possible caveat: clustering).

However there is a hard to probe, open window for very light PBHs.

Astrophysical uncertainties on microlensing constraints

Evans power law halo models: self-consistent halo models, which allow for non-flat rotation curves.

Traditionally used in microlensing studies [Alcock et al. MACHO collab.; Hawkins] since there are analytic expressions for velocity distribution.



standard halo (SH)

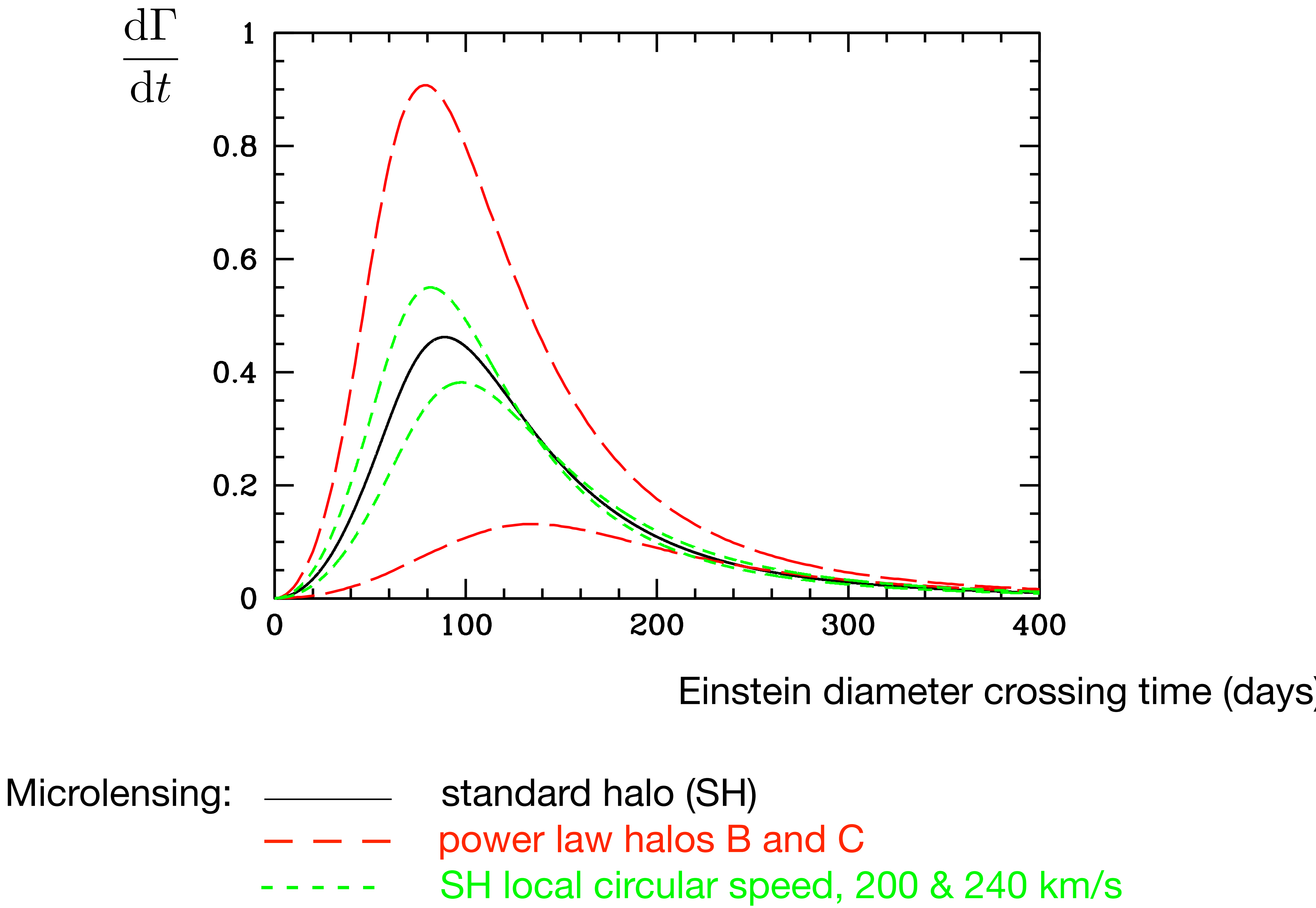
top: power law halo B (massive halo, rising rotation curve)

bottom: power law halo C (light halo falling rotation curve)

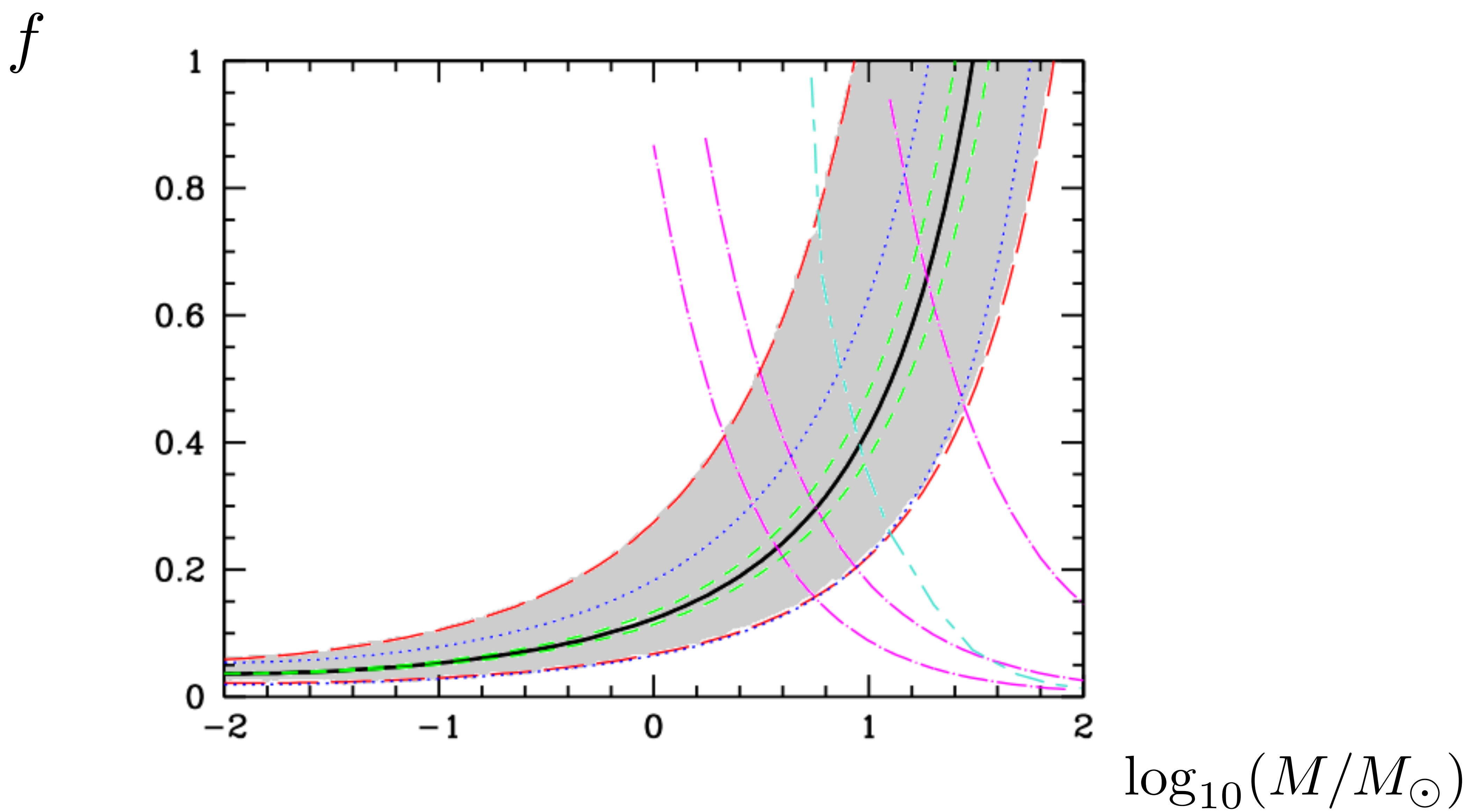
envelope of MW rotation curve data [Bhattacharjee et al.]

Microlensing differential event rate

(f=1 M= 1 $M_\odot$, and perfect detection efficiency)



Constraints on halo fraction for delta-function MF:



standard halo (SH)

power law halos C and B

SH local density, 0.005 and 0.015 $M_{\odot}\,\mathrm{pc}^{-3}$

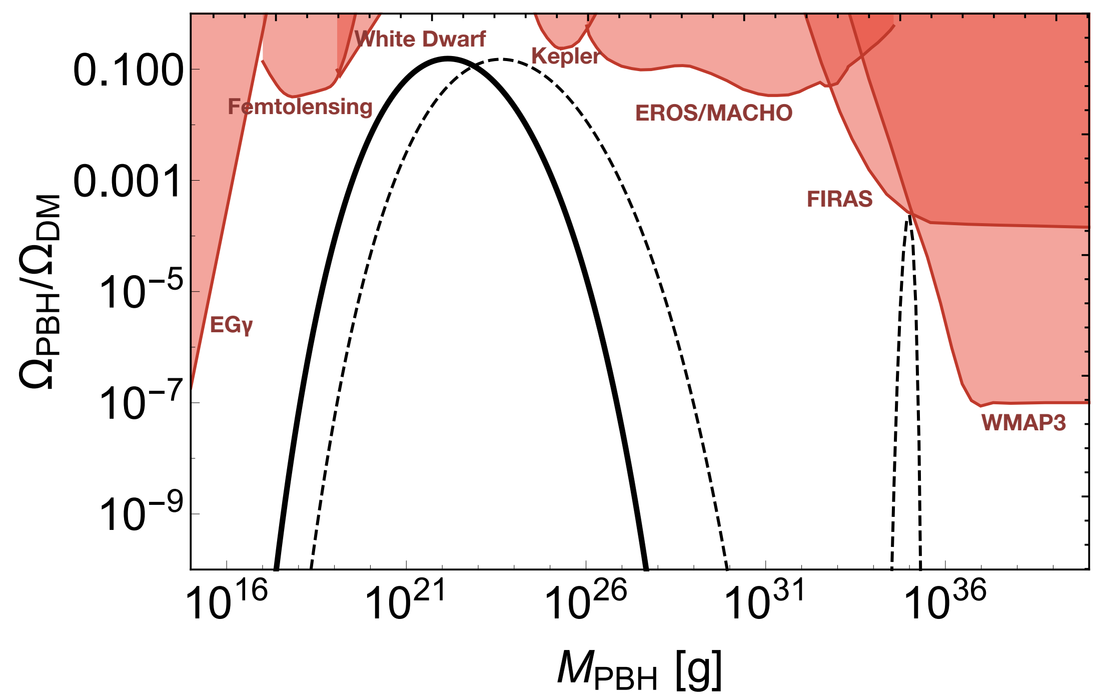
SH local circular speed, 200 & 240 km/s

Brandt dwarf galaxy constraints

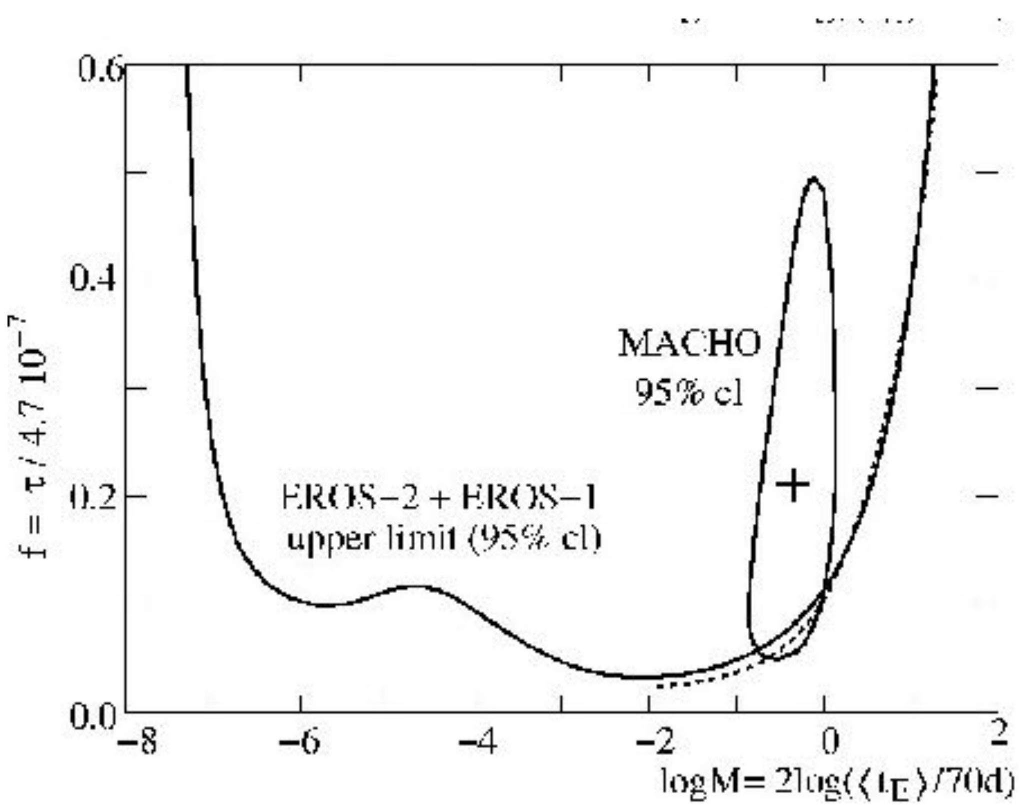
Application to (realistic) extended mass functions

Is subtle....

Can't just compare df/dM to constraints on f as a function of M:

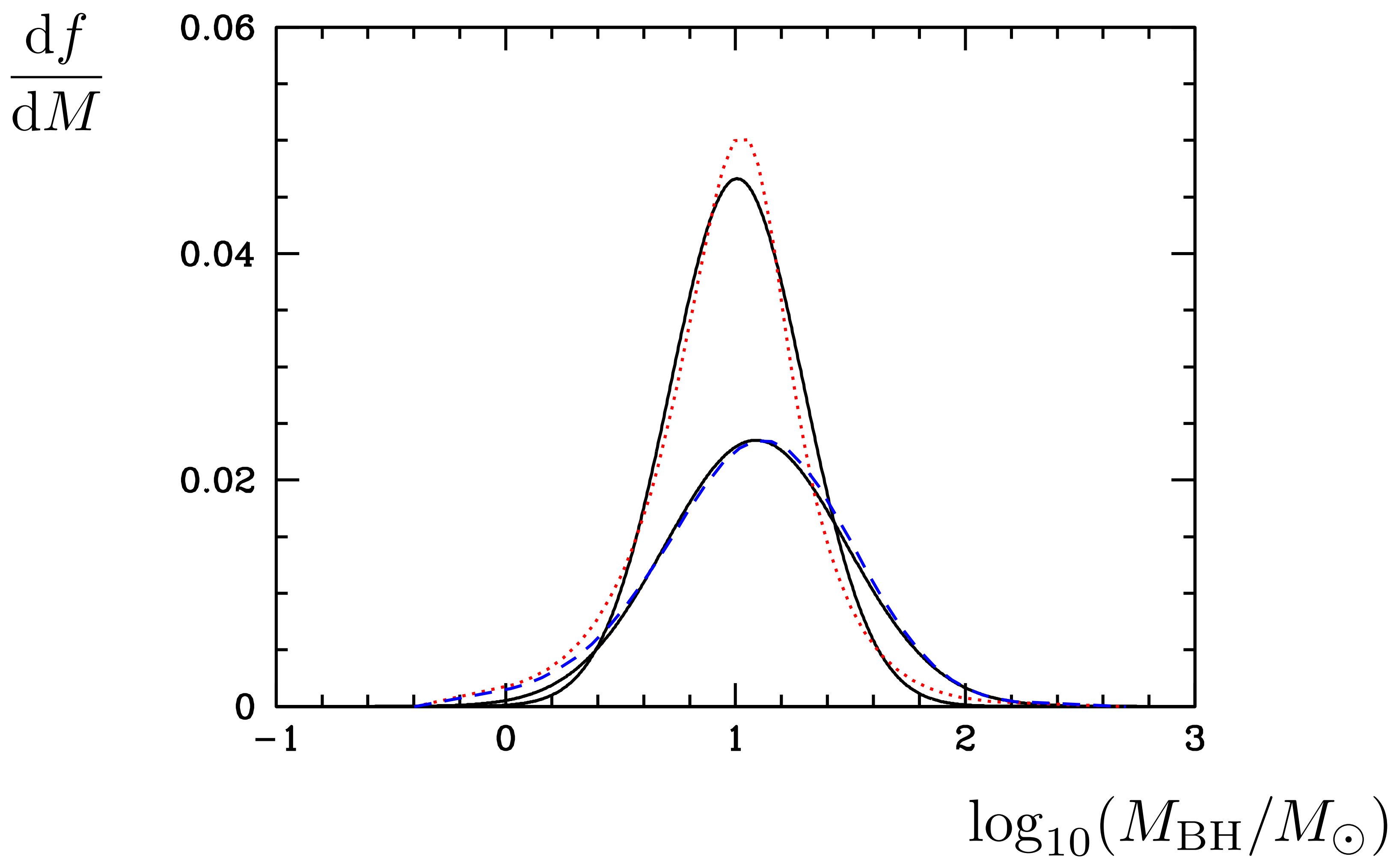


Beware 'double-counting':
for instance EROS microlensing constraints, allow f~0.2 for M~5 M_{sun} or f~0.4 for M~10 M_{sun}, **but NOT BOTH.**

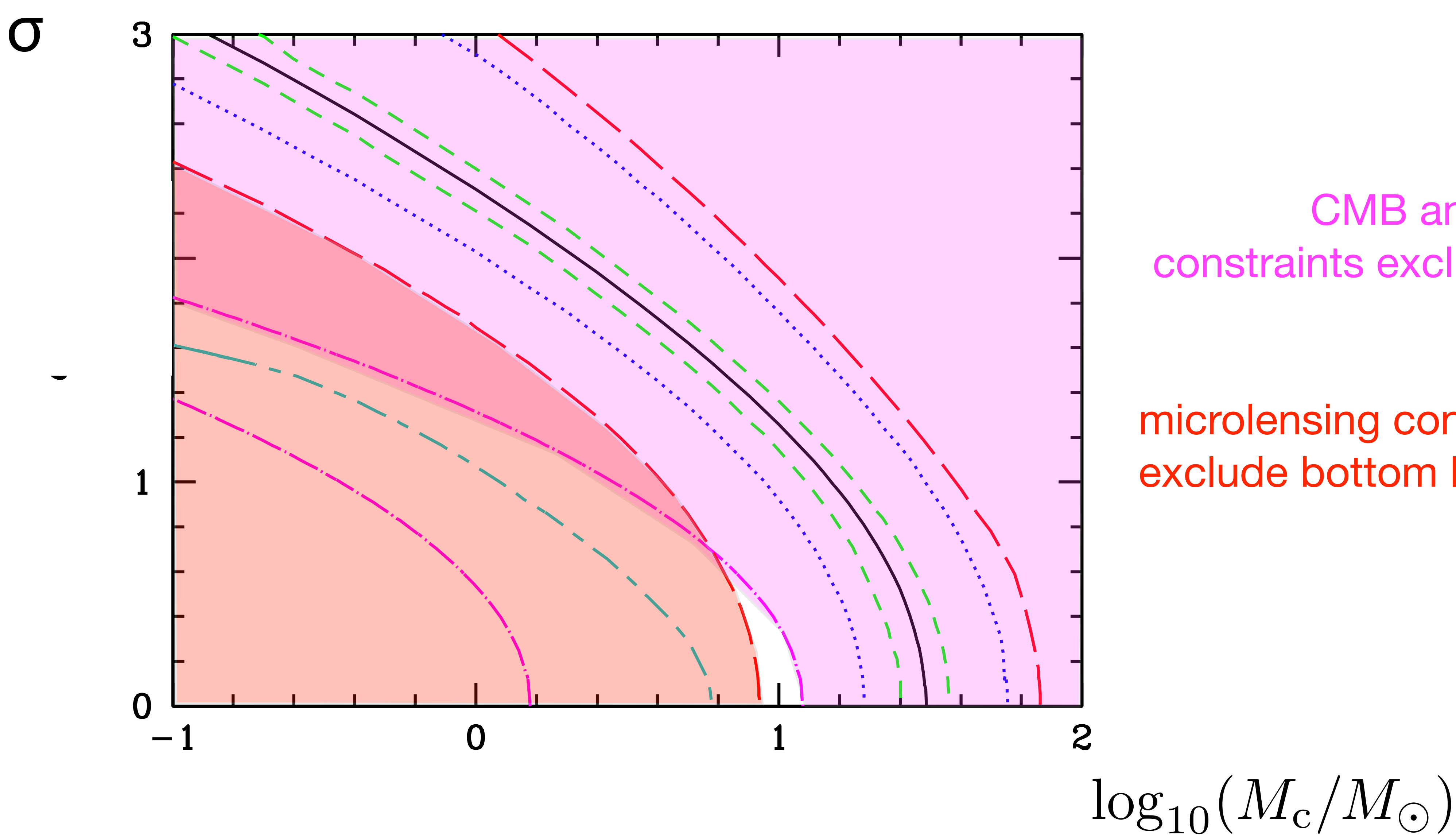


The extended mass functions found by Carr et al. for the axion-curvaton and running mass inflation models, including critical collapse, are well approximated by a log-normal distribution:

$$\psi(M) \equiv \frac{\mathrm{d}f}{\mathrm{d}M} \propto \exp\left[-\frac{(\log M - \log M_{\mathrm{c}})^2}{2\sigma^2}\right]$$



Magellanic Clouds microlensing constraints on
width of log-normal MF with f=1



CMB and dynamical constraints exclude top right

microlensing constraints exclude bottom left

standard halo (SH)

power law halos C and B

SH local density, 0.005 and 0.015

SH local circular speed, 200 & 240 km/s

Brandt dwarf galaxy constraints

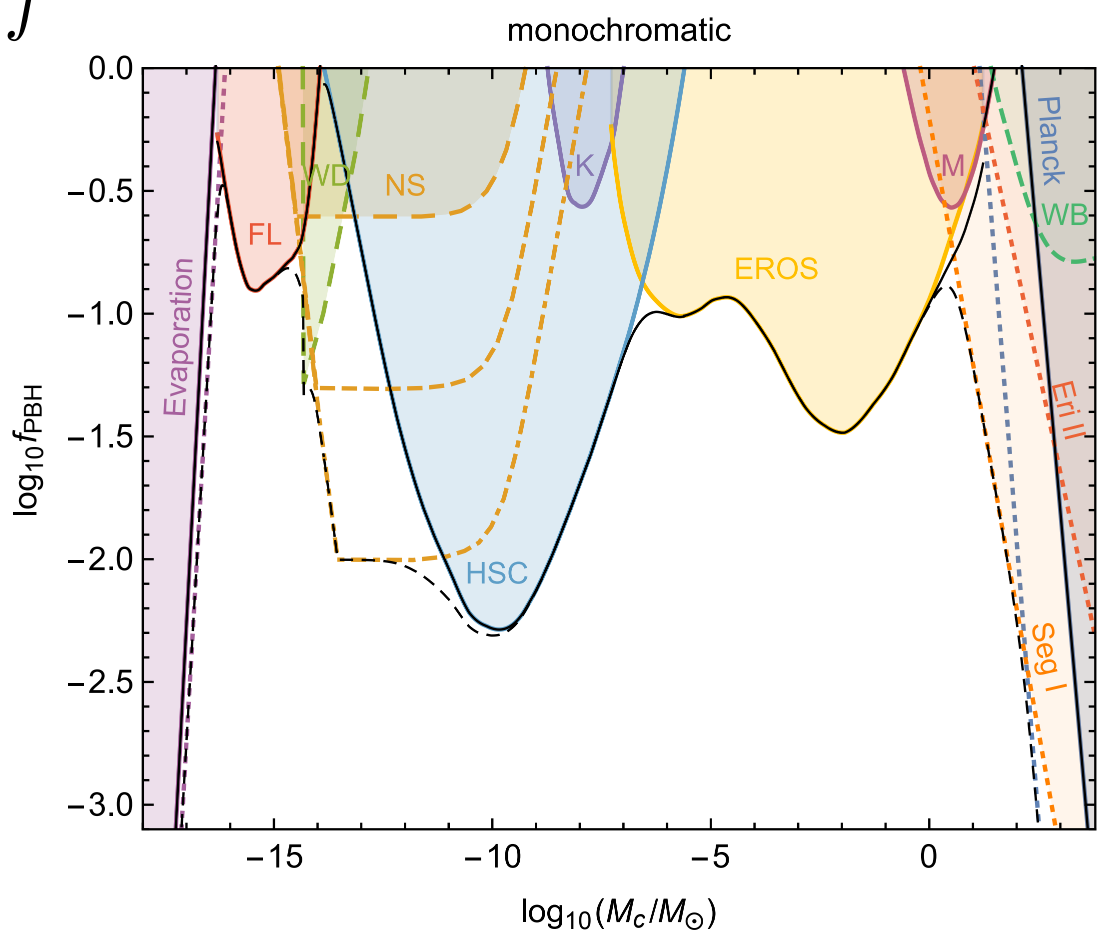
For extended mass functions, constraints on f are smeared out, and gaps between constraints are 'filled in':

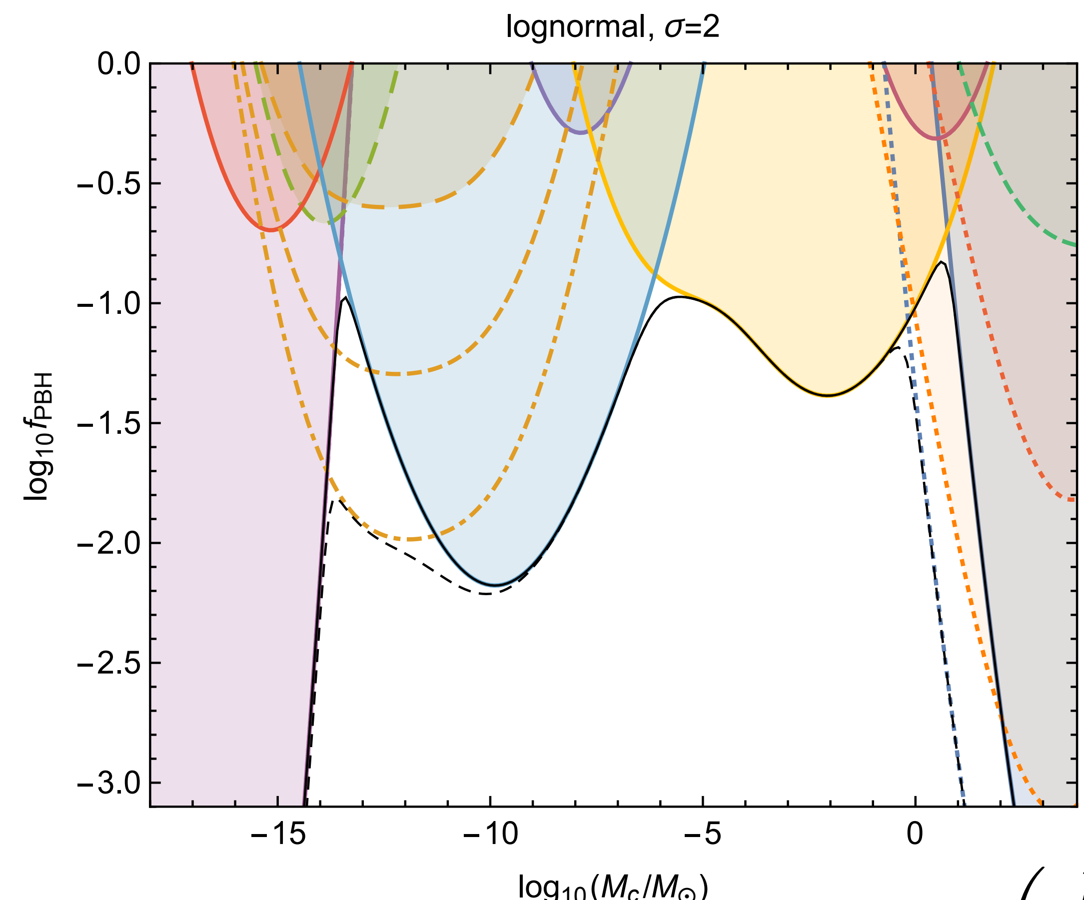
Green; Carr et al., see also Bellomo et al.

monochromatic

log-normal
(fixed width)

$\log_{10} f$





Carr et al.

$\log_{10}\left(\frac{M}{M_\odot}\right)$

n.b. some of these constraints have been revisited and either substantially modified (e.g. HSC microlensing) or removed (white dwarfs and femtolensing of GRBs).

Summary

Primordial Black Holes can form in the early Universe, for instance from the collapse of large density perturbations during radiation domination.

A non-negligible number of PBHs will only be produced if the amplitude of the fluctuations is ~3 orders of magnitude larger on small scales than on cosmological scales.

This can be achieved in inflation models (e.g. with a feature in the potential or multiple fields). However this is not natural/generic.

PBHs are expected to have an extended mass function (due to critical collapse and also width of primordial power spectrum).

There are numerous constraints on the abundance of PBHs from gravitational lensing, their dynamical effects, accretion and other astrophysical processes.

It appears that Solar mass PBHs can't make up all of the dark matter, but lighter PBHs could.

Limits are collectively tighter for (realistic) extended mass functions than for the delta-function which is usually assumed when calculating constraints.

Back-up slides

Probing origin of BH binaries using their spins

Farr, Holtz & Farr;… Fernandez & Profumo

Dimensionless spin of individual BH: $\chi = \frac{|\mathbf{S}|}{GM^2}$

Effective spin parameter: $\chi_{\mathrm{eff}} = \frac{M_1 \chi_1 \cos\theta_1 + M_2 \chi_2 \cos\theta_2}{M_1 + M_2}$

θ_i=tilt angle between $\mathbf{S}_i$ and orbital AM $\mathbf{L}$

Astrophysical BH binaries:

i) formed in dense stellar environments, spins uncorrelated with orbit: $\chi_{\mathrm{eff}} \approx 0$

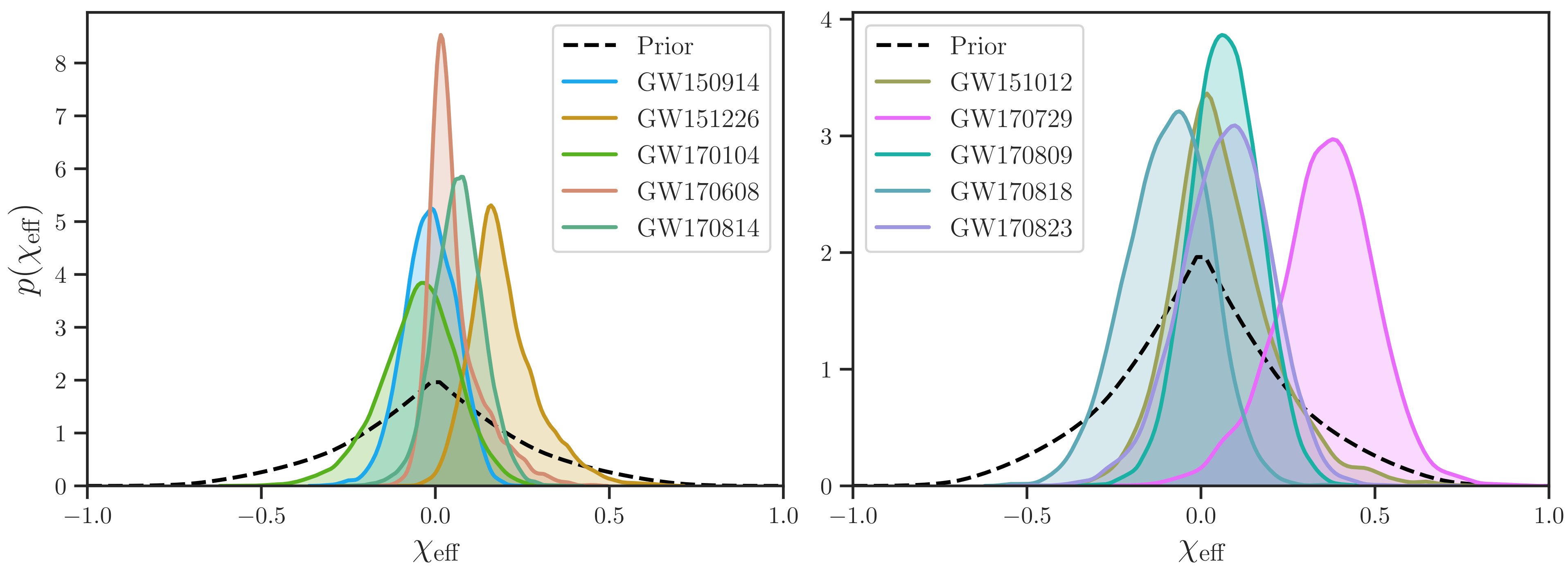
ii) formed in isolation, spins generally aligned with orbital AM: $\chi_{\mathrm{eff}} \approx 1$

Primordial BH binaries:

small intrinsic spins, $\chi_i \approx 0 \quad \rightarrow \quad \chi_{\mathrm{eff}} \approx 0$

de Luca et al.

Effective spin parameter probability distributions of 10 BH-BH events observed in LIGO-Virgo runs O1 and O2



Fernandez & Profumo

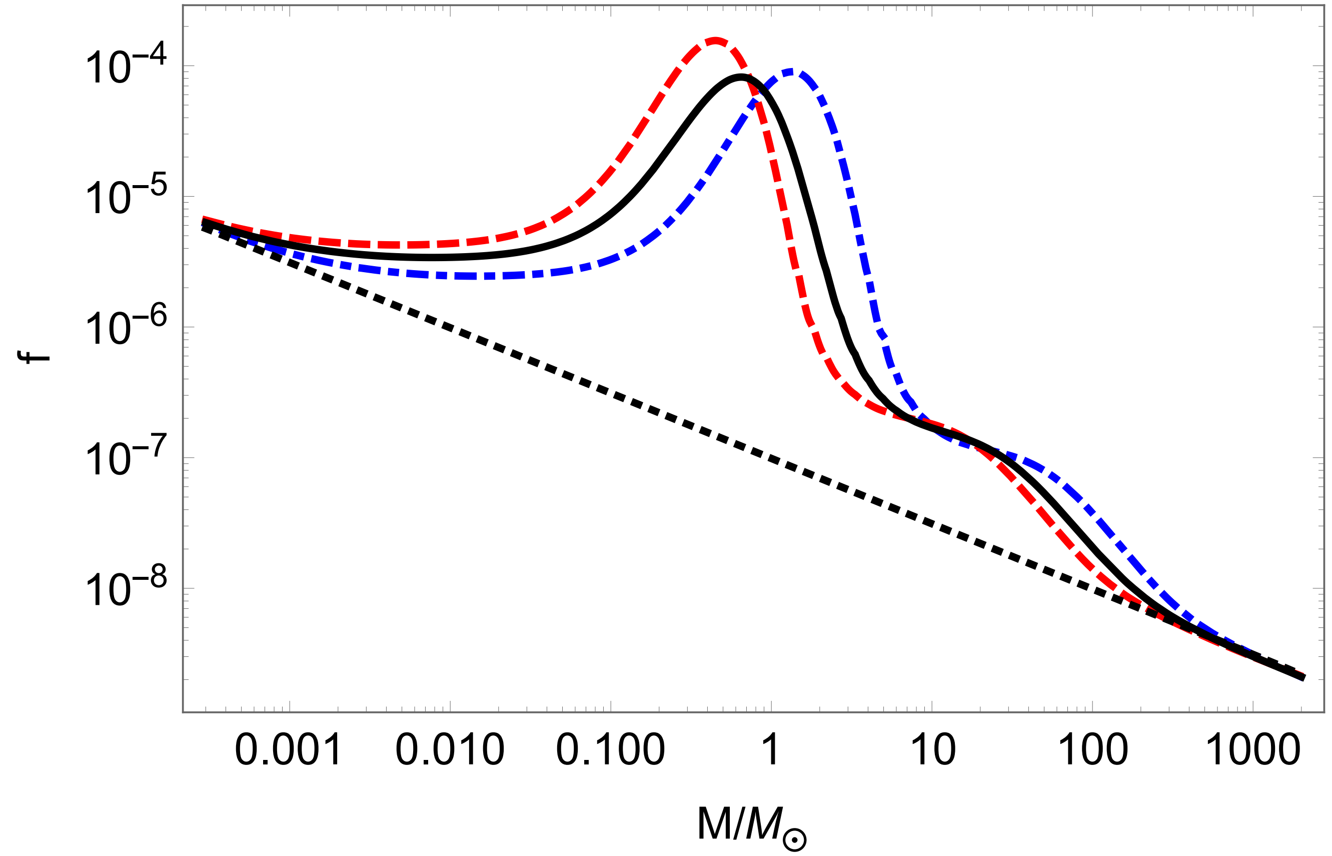
Entire population having large $\chi_{\rm eff} \approx 1$ already disfavoured.

With O(100) events (~1 year of O3) will be able to distinguish low intrinsic spin ($\chi_i \approx 0$) and spins uncorrelated with orbit.

iii) phase transitions

Reduction in the equation of state parameter ($w=p/\rho$) at phase transitions decreases the threshold for PBH formation δ_c and enhance the abundance of PBHs formed on this scale. (Horizon mass at QCD phase transition is of order a solar mass.) Jedamzik

Using new lattice calculation of QCD phase transition Byrnes et al. transition find a 2 order of magnitude enhancement in β (but still need a mechanism for amplifying the primordial perturbations):



Byrnes et al.

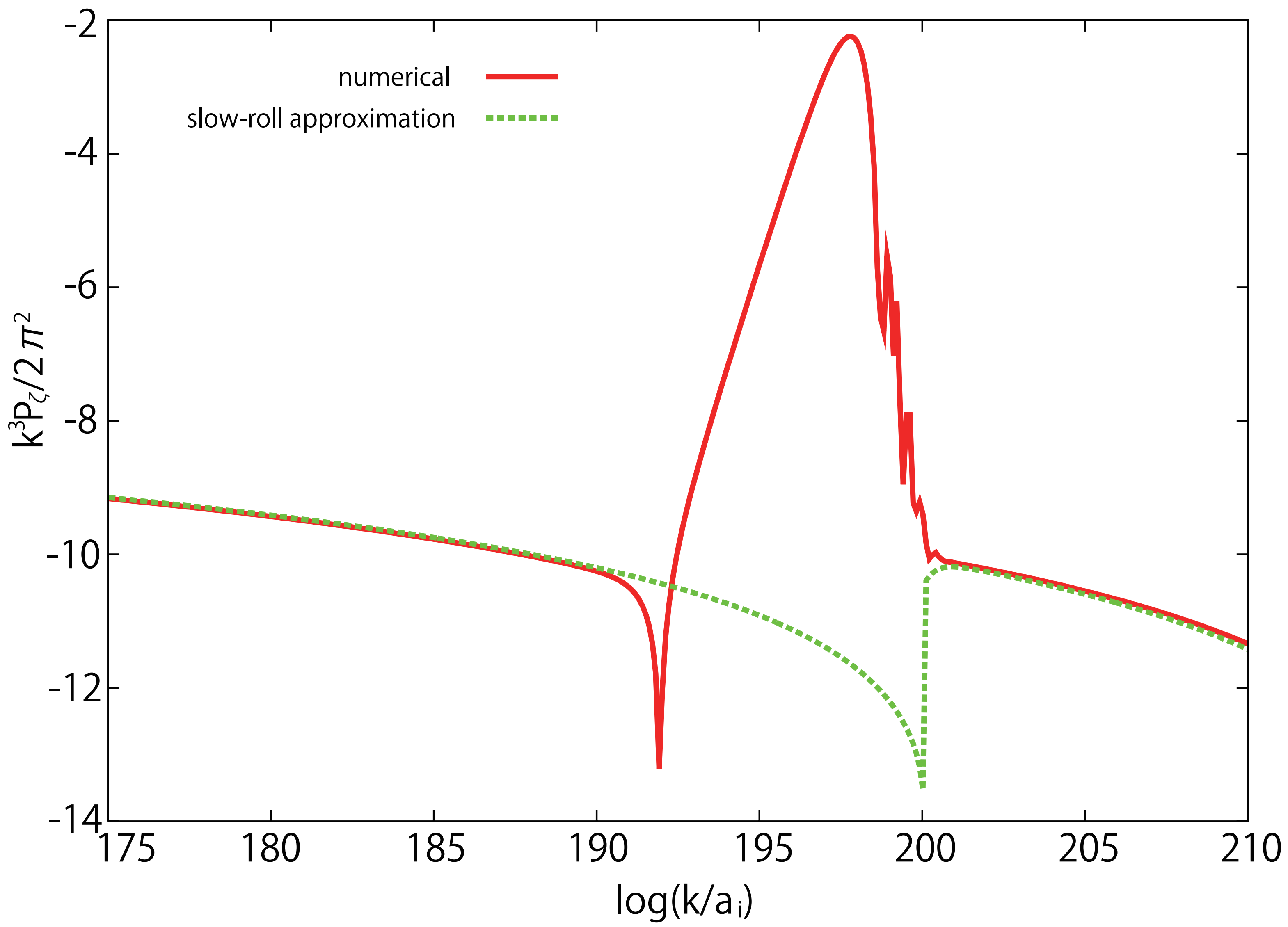
In single field models need to violate slow roll (and hence standard expressions for amplitude of fluctuations aren’t valid).

Models which might naively be expected to produce large perturbations (e.g. potentials with an inflection point, $V'(\phi) \to 0$ ‘ultra-slow-roll’) don’t. Kannike et al.; Germani & Prokopec; Motohashi & Hu; Ballesteros & Taoso

b) double inflation

Saito, Yokoyama & Nagata; Kannike et al.

Perturbations on scales which leave the horizon close to the end of the 1st period, of inflation get amplified during the 2nd period.



Also double inflation models where large scale perturbations are produced during 1st period, and small scale (PBH forming) perturbations during 2nd (Kawasaki et al.; Kannike et al.; Inomata et al.)

stochastically generated inflation models

Peiris & Easther

Generate inflation models stochastically using slow roll ‘flow equations’.

Get a class of models where inflation can continue indefinitely (and is assumed to be ended via an auxiliary mechanism). In these models the amplitude of fluctuations decreases with increasing k and can be large enough to form PBHs (while still satisfying cosmological constraints).

axion-like curvaton

Kawasaki, Kitajima & Yanagida

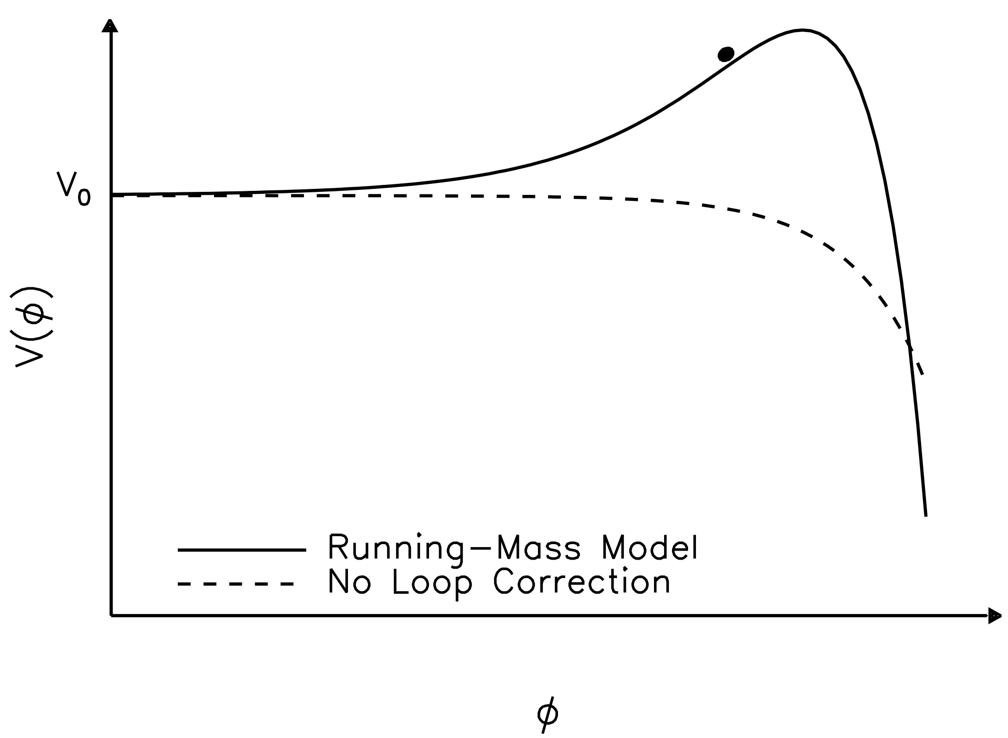
Large scale perturbations generated by inflaton, small scale (PBH forming) perturbations by curvaton (a spectator field during inflation gets fluctuations and decays afterwards producing perturbations Lyth & Wands)

ii) monotonically increasing power spectrum

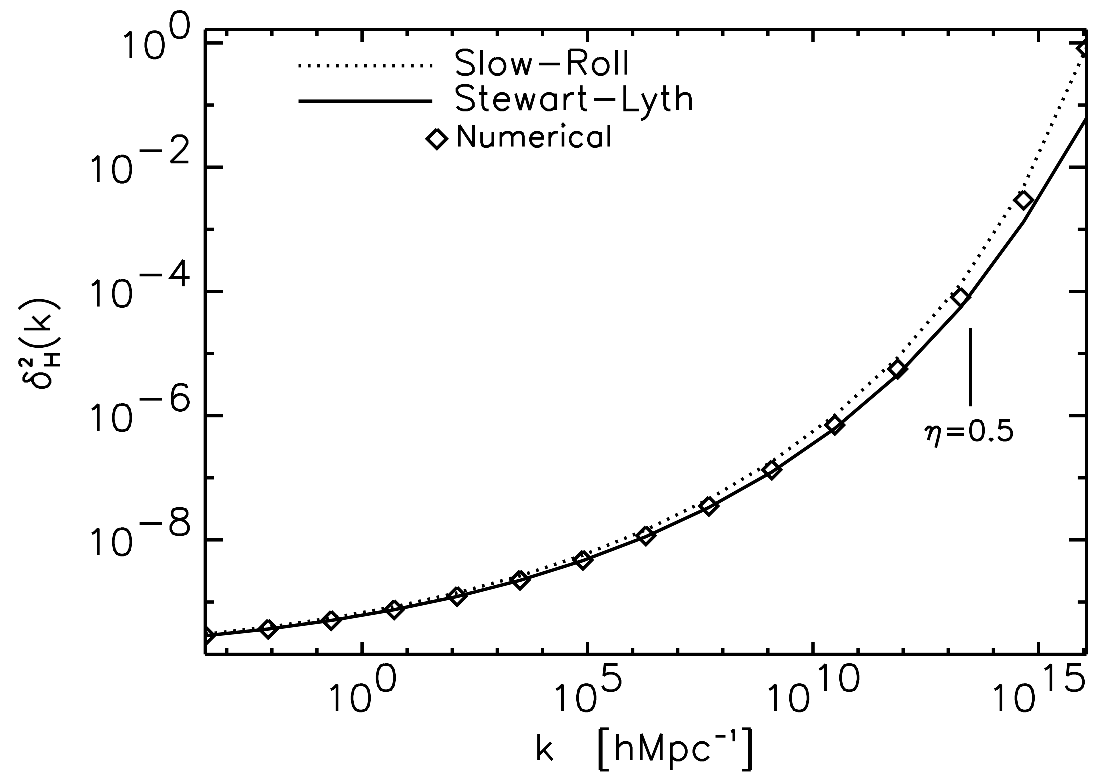
running-mass inflation Stewart

$$V(\phi) = V_0 + \frac{1}{2} m_\phi^2(\phi)\phi^2$$

potential



primordial power spectrum



Leach, Grivell, Liddle

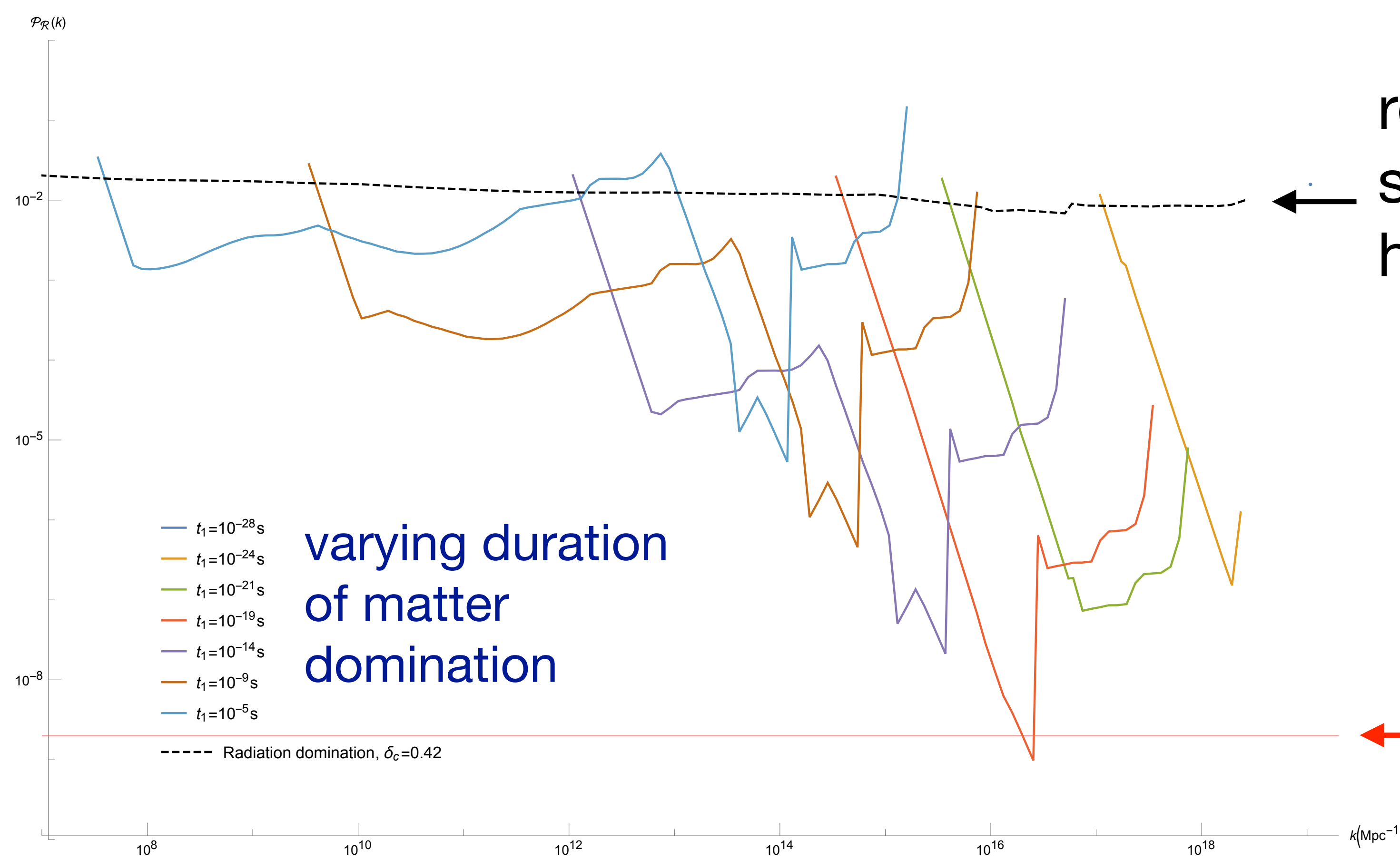
PBH formation during an early (pre nucleosynthesis) period of matter domination

During matter domination PBHs can form from smaller fluctuations (no pressure to resist collapse) in this case fluctuations must be sufficiently spherically symmetric Yu, Khlopov & Polnarev; Harada et al. and

$$\beta(M) \approx 0.056\sigma^{5(+1.5?)}$$

The required increase in the amplitude of the perturbations is reduced Georg, Sengör & Watson; Georg & Watson; Carr, Tenkanen & Vaskonen; Cole & Byrnes:

Primordial curvature perturbation power spectrum



required amplitude for standard cosmological history

amplitude on CMB scales

Cole & Byrnes

k

Microlensing
stars, supernovae, quasars

Dynamical effects
dwarf galaxies, wide binary stars

Evaporation
gamma-rays

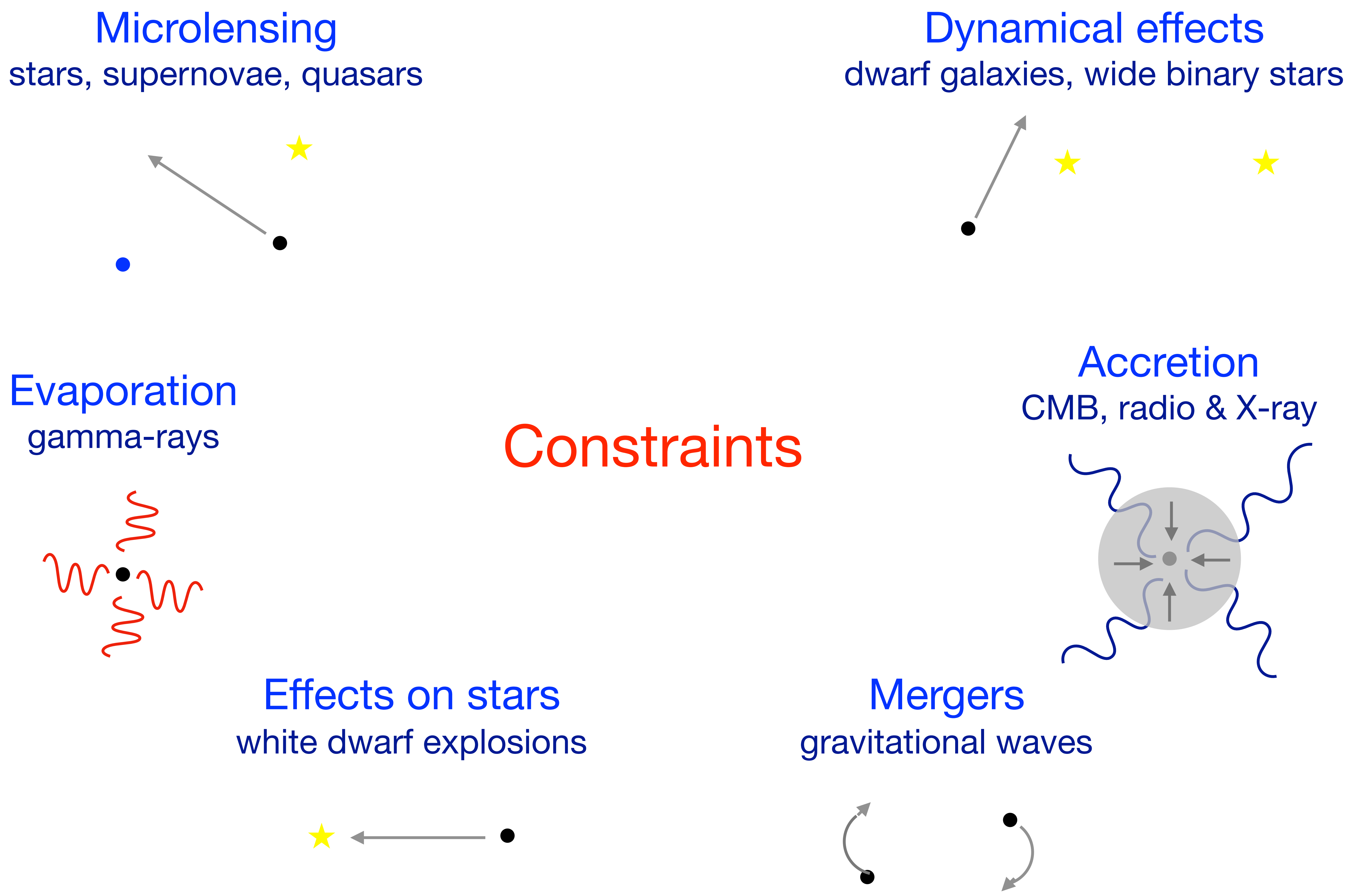
Constraints

Accretion
CMB, radio & X-ray

Effects on stars
white dwarf explosions

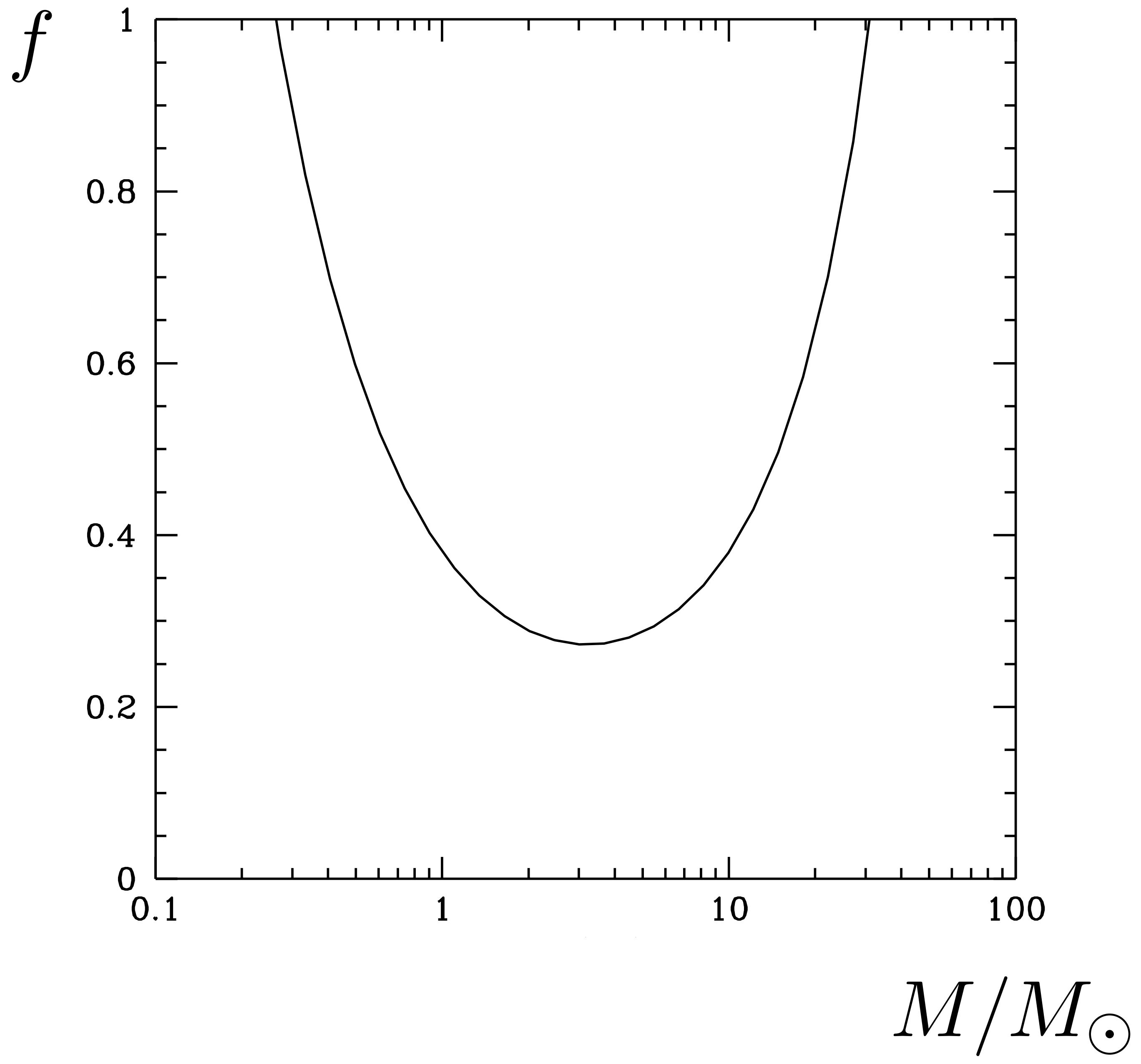
Mergers
gravitational waves

[Initially all constraints assume a delta-function PBH mass function.]



[Initially all constraints assume a delta-function PBH mass function.]

MACHO constraints on fraction of halo in compact objects in the 1-30 $M_\odot$ range:

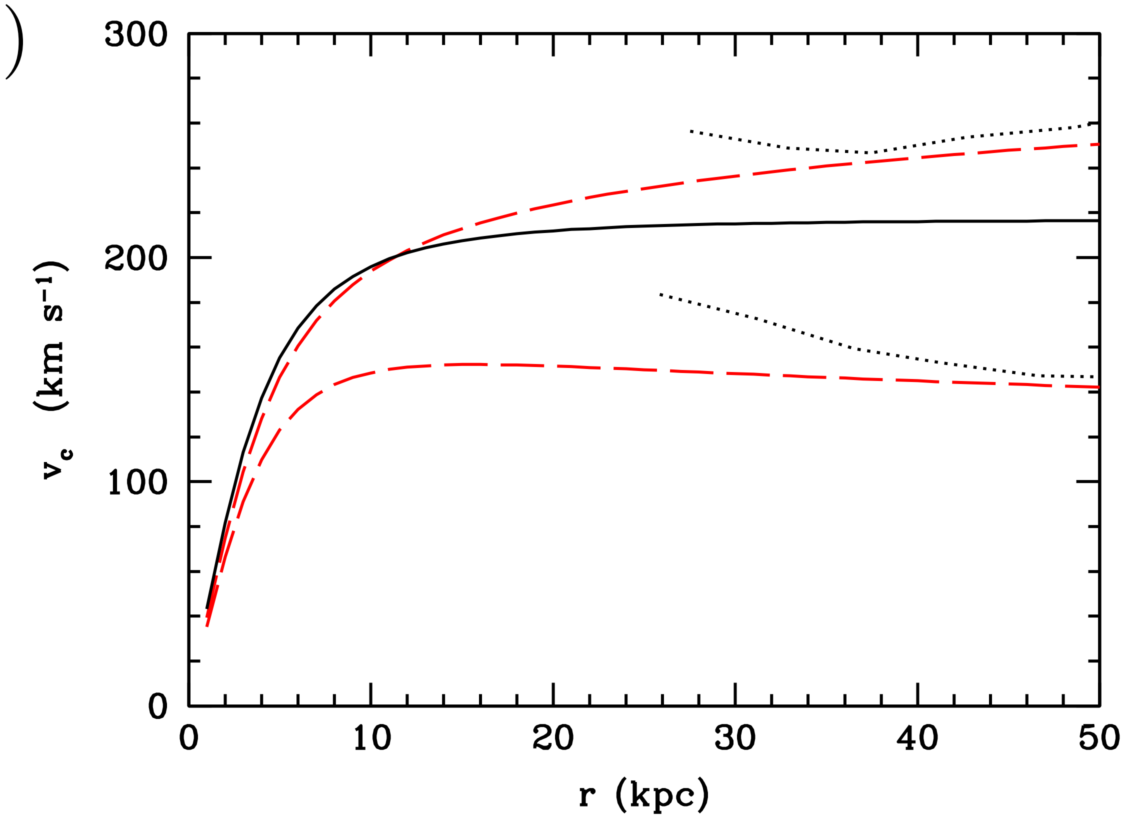


MACHO

Evans power law halo models: self-consistent halo models, which allow for non-flat rotation curves. Traditionally used in microlensing studies since there are analytic expressions for velocity distribution.

Rotation curve

$v_{\rm c}\,({\rm km\,s^{-1}})$



$r\,({\rm kpc})$

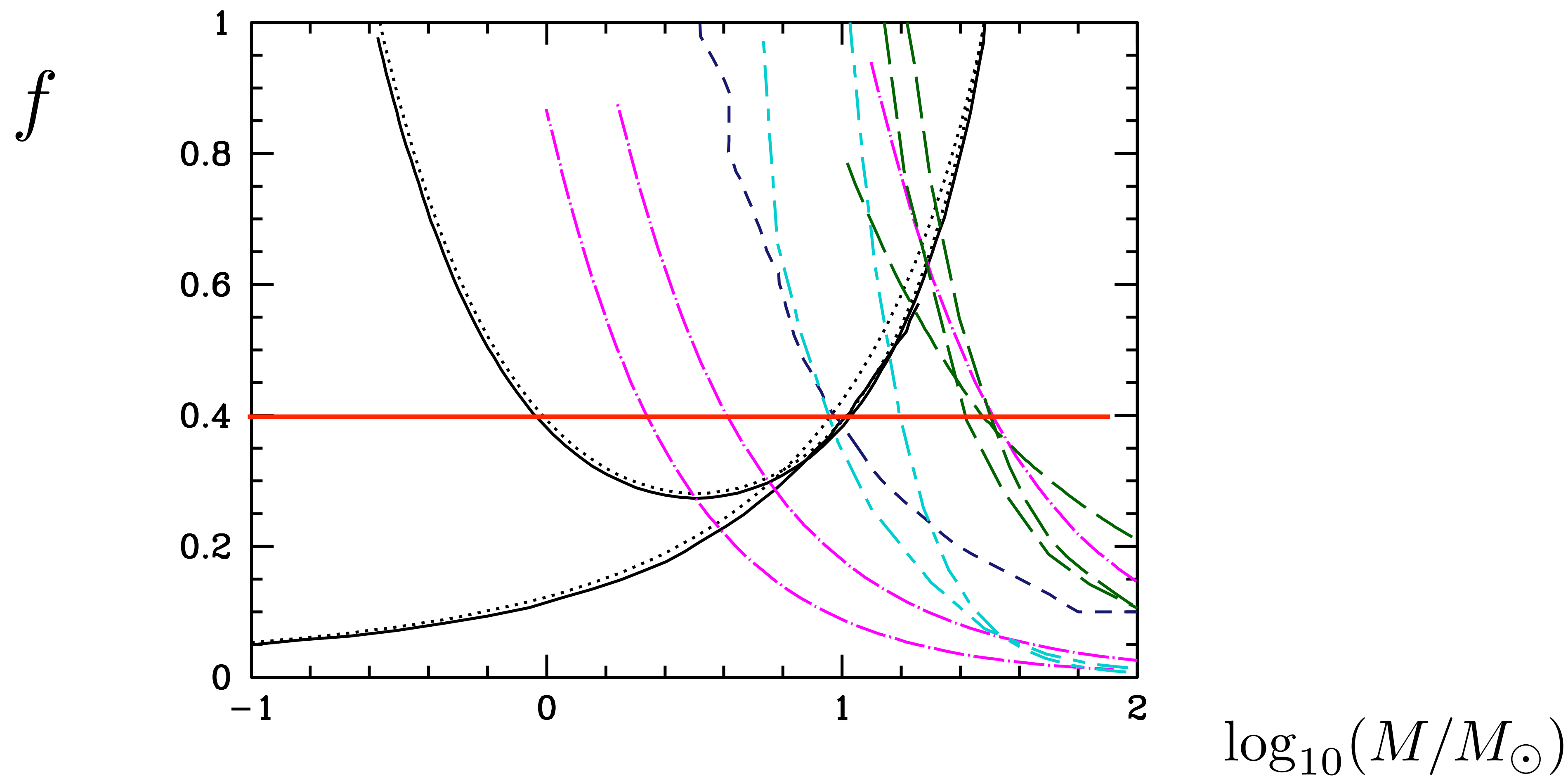
standard halo (SH)

top: power law halo B (massive halo, rising rotation curve)

bottom: power law halo C (light halo falling rotation curve)

……….. envelope of MW rotation curve data Bhattacharjee et al.

Compilation of ~Solar mass region constraints



LMC microlensing (EROS & MACHO)

dwarf galaxy dynamical constraints

wide binary disruption

(tightest) CMB constraints

X-ray & radio

SNe microlensing

Doesn't include Mediavilla et al. microlensing of quasars (no constraint on f published)
or gravitational waves from mergers.

Method for applying delta-function constraints to extended mass functions:

Carr, Raidal, Tenkanen, Vaskonen& Veermae, see also Bellomo, Bernal, Raccanelli & Verde:

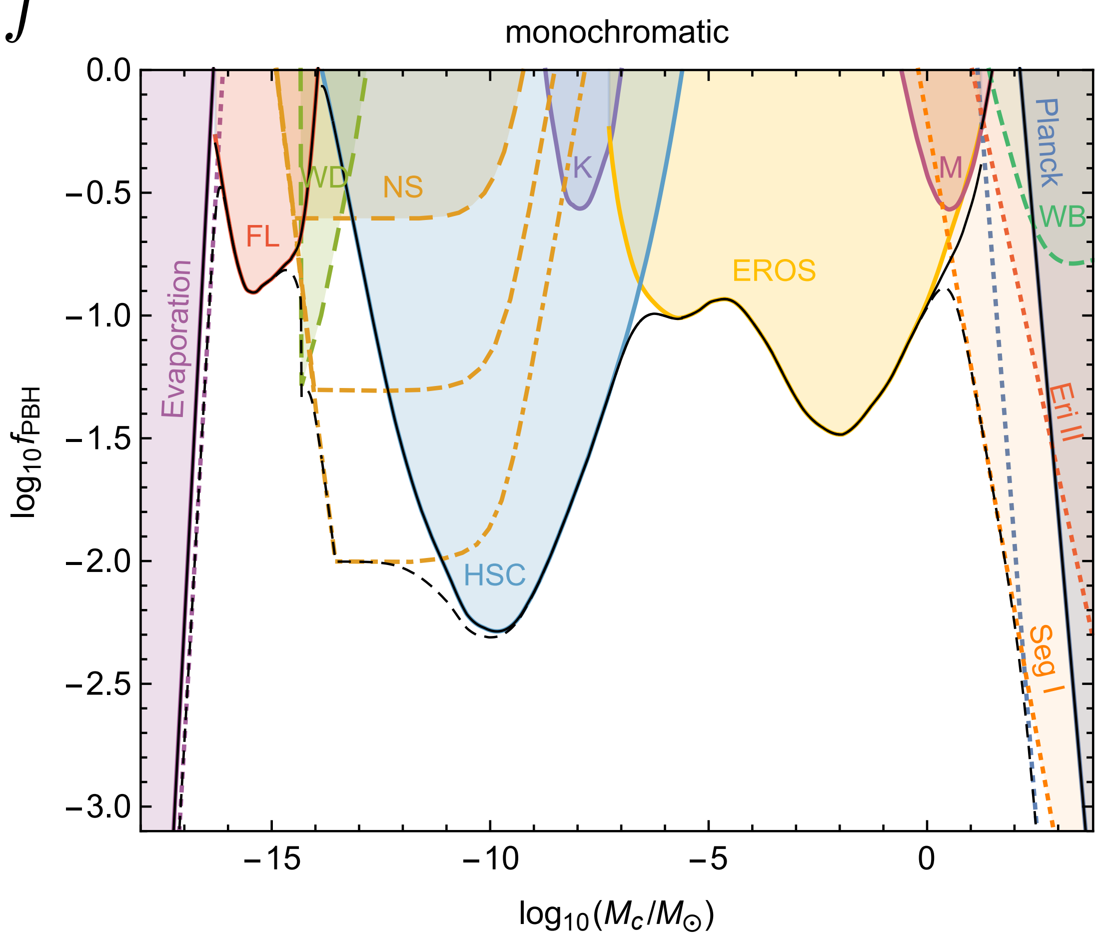
If $f_{max}(M)$ is the maximum allowed PBH fraction for a delta-function MF, an extended mass function $\psi(M)$ has to satisfy:

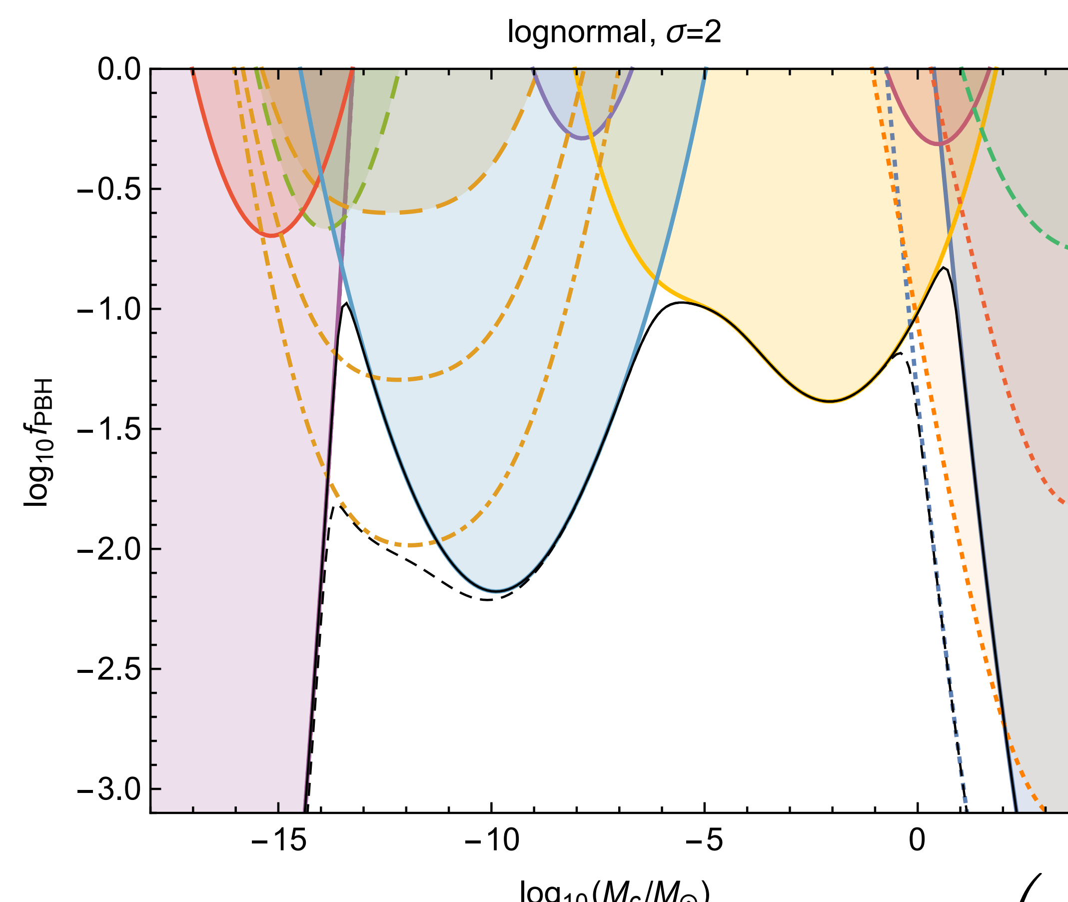
$$\int \mathrm{d}M \frac{\psi(M)}{f_{\max}(M)} \leq 1$$

monochromatic

log-normal
(fixed width)

$\log_{10} f$





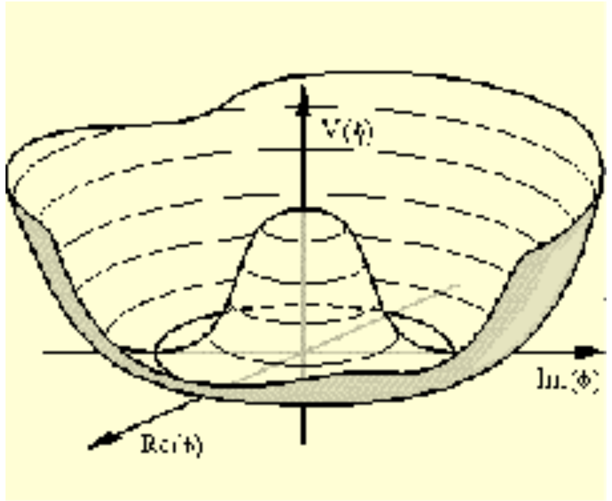
$\log_{10}\left(\frac{M}{M_\odot}\right)$

Formation: other mechanisms

Collapse of cosmic string loops Hawking; Polnarev & Zemboricz;

Cosmic strings are 1d topological defects formed during symmetry breaking phase transition.

String intercommute producing loops.

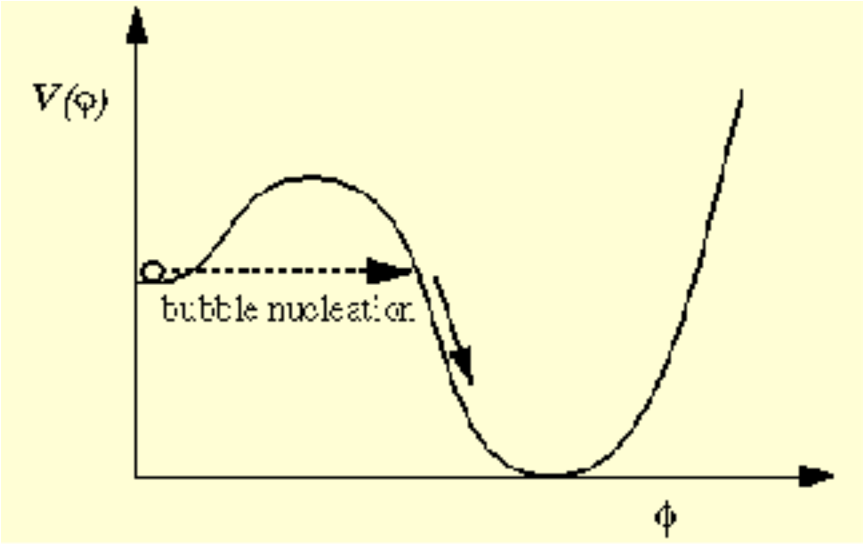


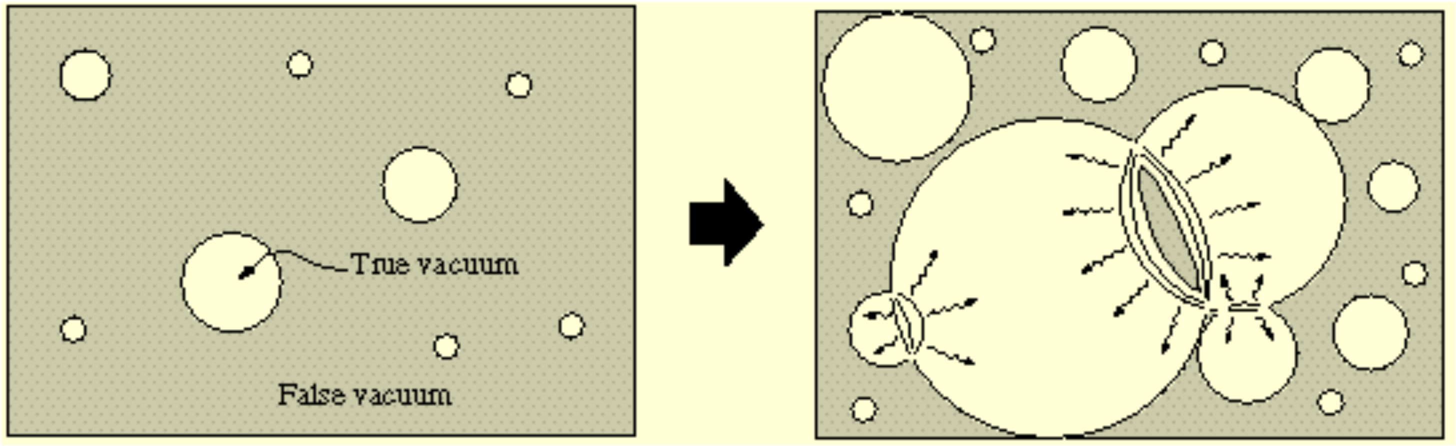
Small probability that loop will get into configuration where all dimensions lie within Schwarzschild radius (and hence collapse to from a PBH with mass of order the horizon mass at that time).

Probability is time independent, therefore PBHs have extended mass spectrum.

Bubble collisions Hawking

1st order phase transitions occur via the nucleation of bubbles.





PBHs can form when bubbles collide (but bubble formation rate must be fine tuned).

PBH mass is of order horizon mass at phase transition.

Fragmentation of inflaton scalar condensate into oscillons/Q_balls

Cotner & Kusenko; Cotner, Kusenko & Takhistov

Mass function

scale-invariant primordial density perturbation power spectrum

Carr

For PBH formation during radiation domination:

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto M^{-5/2}$$

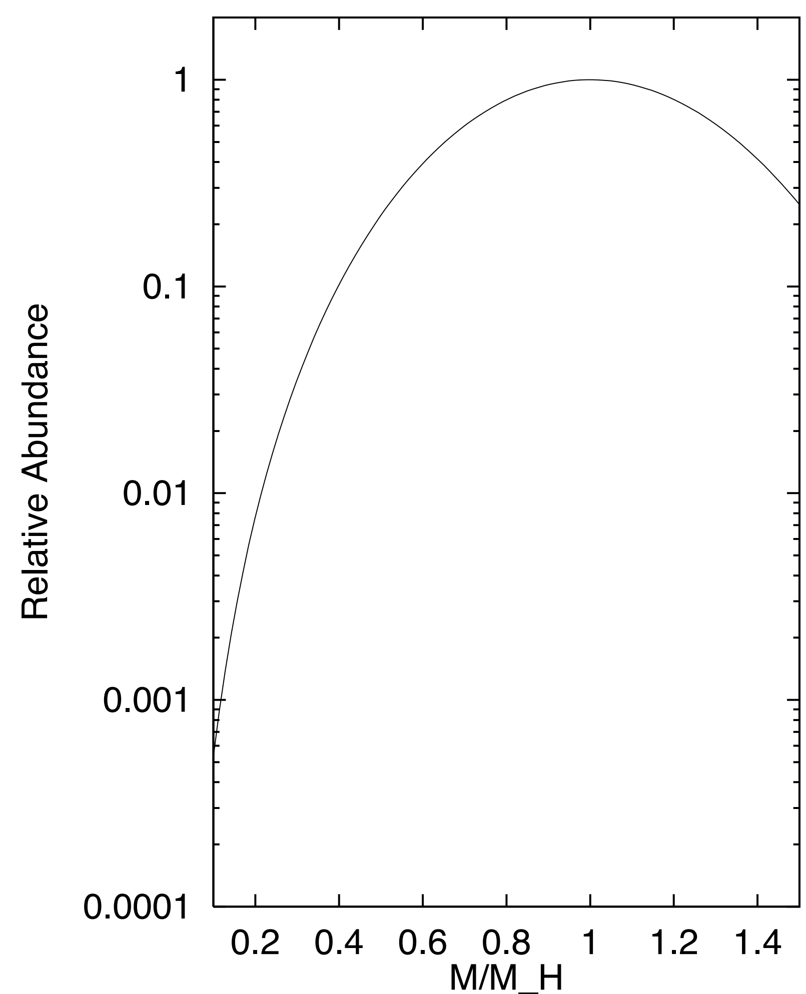
However if power spectrum is completely scale-invariant, then the number of PBHs formed is negligible.

PBHs which form from the collapse of cosmic string loops would also have a power-law mass function.

delta-function primordial density perturbation power spectrum, taking into account critical collapse $M = kM_{\mathrm{H}}(\delta - \delta_{\mathrm{c}})^{\gamma}$

Niemeyer & Jedamzik

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto \left(\frac{M}{M_{\mathrm{H}}}\right)^{(-1+1/\gamma)} \exp\left[-(1+\gamma)\left(\frac{M}{M_{\mathrm{H}}}\right)^{(1/\gamma)}\right]$$

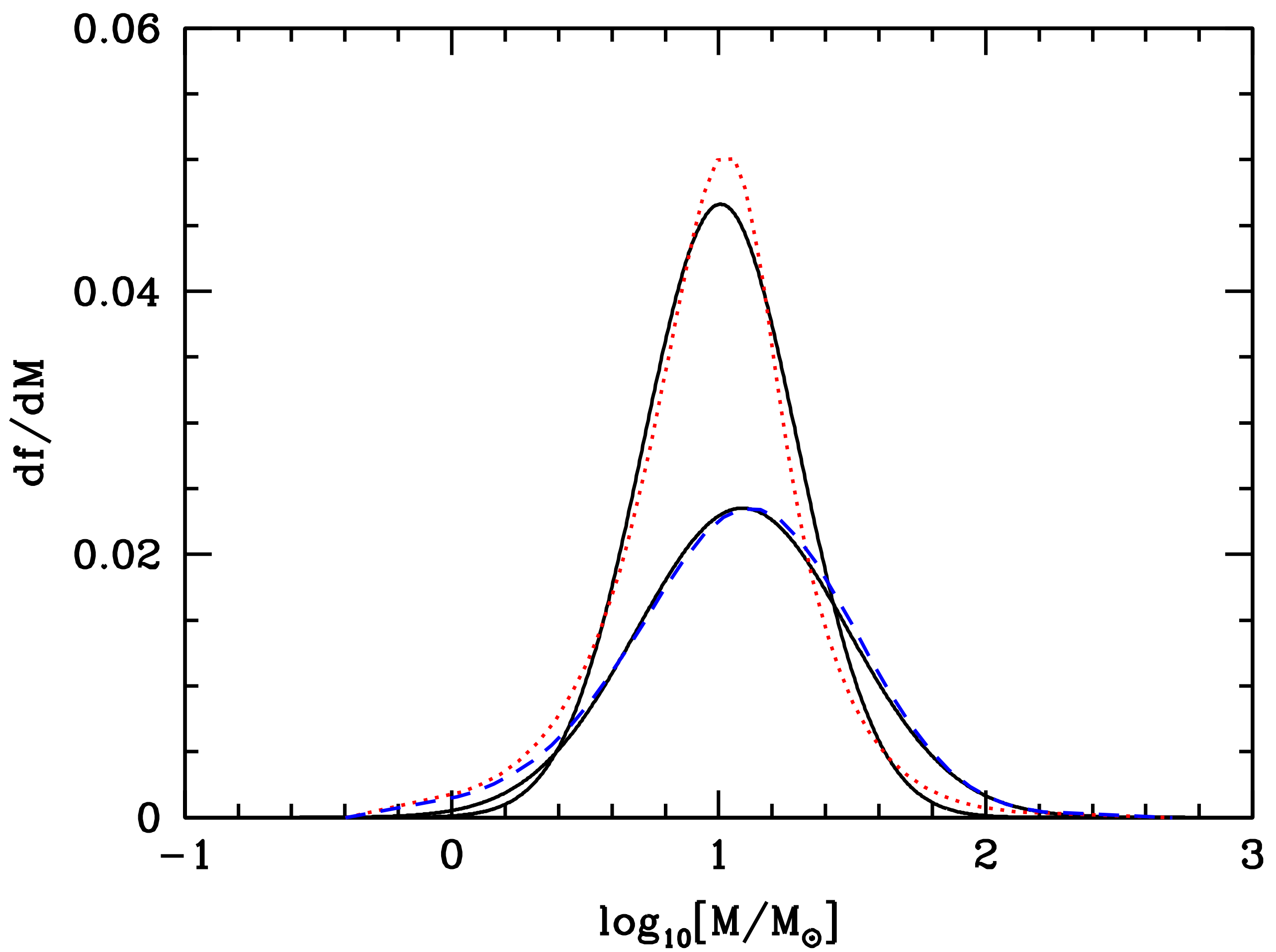


Yokoyama

non delta-function primordial density perturbation power spectrum, taking into account critical collapse

Extended MFs produced by inflation models with finite width peak in power spectrum, often well approximated by a log-normal distribution: Green; Kannike et al.

$$\frac{\mathrm{d}n}{\mathrm{d}M} \propto \exp\left\{-\frac{\left[\log(M/M_{\mathrm{c}})\right]^2}{2\sigma^2}\right\}$$



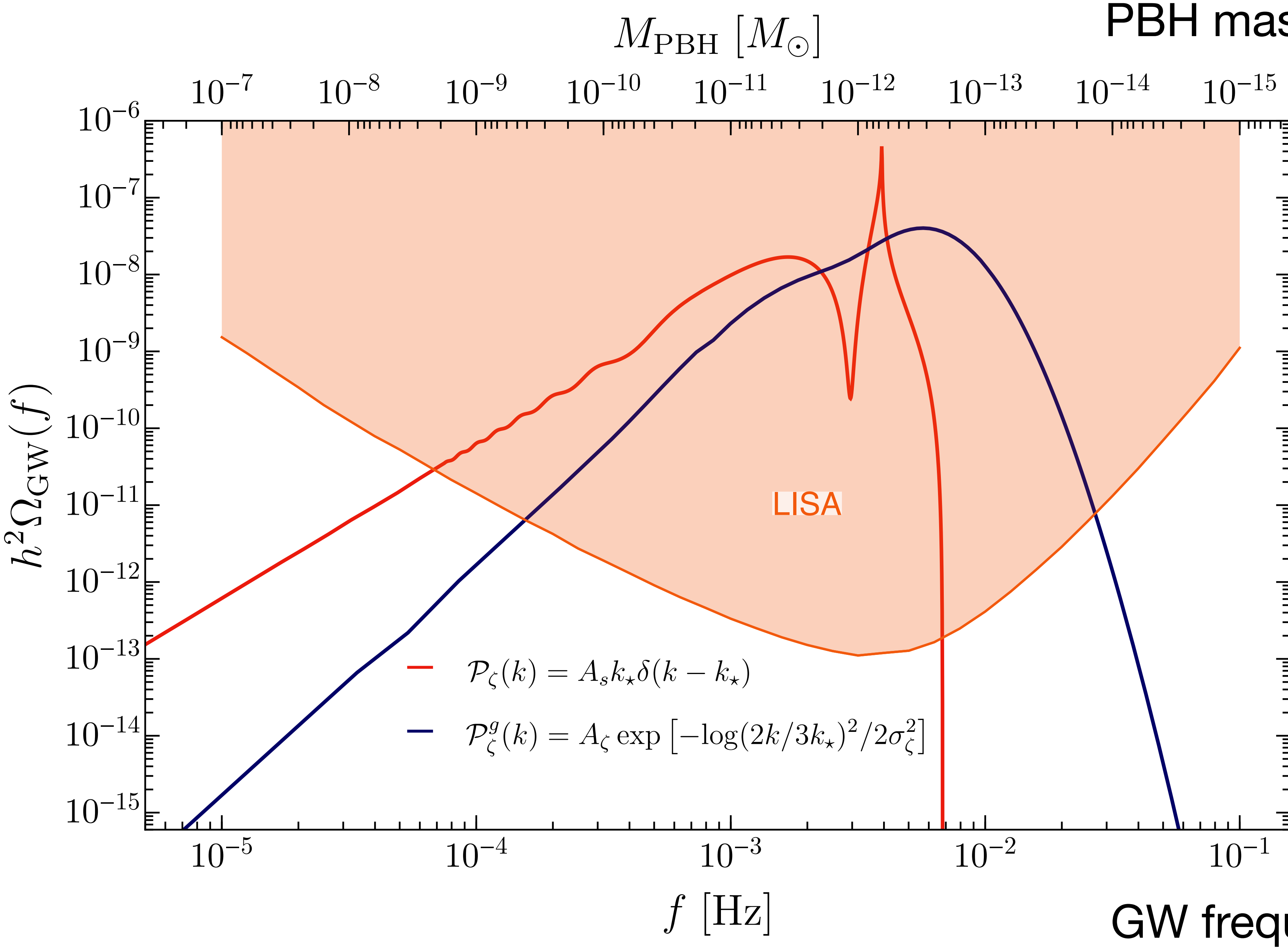
axion-like curvaton

running mass inflation

If PBHs form from collapse of large curvature perturbations, then these perturbations act as 2nd order source of (non-gaussian) gravitational waves, which could be detected by LISA. Saito & Yokoyama; Cai, Pi & Sasaki; Bartolo et al.

PBH mass

GW energy density



GW frequency

Bartolo et al.

delta-function and gaussian primordial curvature power spectrum which give $f_{\rm PBH} = 1$.