New 5D perspectives on the QCD axion

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Strong CP problem

$$\mathcal{L}_{QCD} \supset \theta G\tilde{G}$$

$\boxed{\text{CP-odd term}}$

Basis independent: $\bar{\theta} = \theta + \arg(\det \mathcal{M}_q)$

Observable effect:

Neutron electric dipole moment $d_N \simeq (5 \times 10^{-16} e \cdot \mathrm{cm})\bar{\theta}$

$$|d_N| \lesssim 3 \times 10^{-26} e \cdot \mathrm{cm} \quad \Rightarrow \quad \bar{\theta} \lesssim 10^{-10}$$

Why is $\bar{\theta}$ so small?

Possible solutions:

- massless up quark

  $\arg(\det \mathcal{M}_q) = 0$ ✗ $m_u/m_d \simeq 0.5$

- spontaneous CP violation

  CP = exact symmetry

  $\bar{\theta}$ generated at higher order maybe

- Peccei-Quinn mechanism and axion

  (anomalous) global $\mathrm{U}(1)_{PQ}$, spontaneously broken at f_a

  $\longrightarrow$ *axion*

  axion potential minimum cancels $\bar{\theta}$ ✓ 

Features of the Axion Solution

Axion Lagrangian: $$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \underbrace{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G}}_{\text{dim 5 term}} + \frac{1}{4}a\underbrace{g_{a\gamma\gamma}}_{\text{diphoton coupling}}F\tilde{F} + \frac{1}{f_a}\underbrace{J^\mu\partial_\mu a}_{\text{axial current coupling}}$$

Axion mass: $$m_a^2 = \frac{\mathcal{T}}{f_a^2} \qquad \mathcal{T} \equiv -i\int d^4x\, \langle 0|\mathrm{T}\left[\frac{1}{32\pi^2}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}(x),\ \frac{1}{32\pi^2}G^b_{\rho\sigma}\widetilde{G}^{b\rho\sigma}(0)\right]|0\rangle$$ topological susceptibility

$$\Rightarrow \quad m_a^2 f_a^2 \sim \underbrace{\tfrac{1}{8}}_{\text{light-quark contribution}}\Lambda^4_{\mathrm{QCD}}$$

[or precisely, $m_a = 5.70(7)\ \mu\mathrm{eV}\left(\frac{10^{12}\ \mathrm{GeV}}{f_a}\right)$]

[Cortona, Hardy, Pardy Vega, Villadoro, 1511.02867]

Axion quality: Gravitational violation of $\mathrm{U}(1)_{PQ}$ $\qquad \frac{c_n}{M_P^{n-4}}\phi^n + h.c.$

Terms must be suppressed to very high order! $(c_n \sim 1, n \geq 10)$

[however, if only gravitational instantons $c_n \sim e^{-S} \longrightarrow \quad S \geq 200$]

Where does dim-5 axion coupling come from?

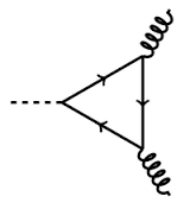
Example: KSVZ [Kim '79; Shifman, Vainshtein, Zakharov '80]

$U(1)_{PQ}$ Dirac fermion: $\Psi \to e^{iq_\Psi \alpha\gamma_5}\Psi$ complex scalar: $\Phi \to e^{iq_\Phi \alpha}\Phi$

Scalar potential: $V(\Phi) = -m_{PQ}^2|\Phi|^2 + \lambda_{PQ}|\Phi|^4$ $\Rightarrow$ $\Phi = \frac{1}{\sqrt{2}}(f_a + \underbrace{\rho}_{\text{radial mode}})e^{i\overbrace{\frac{a}{f_a}}^{\text{axion}}}$ $f_a = \sqrt{\frac{m_{PQ}^2}{\lambda_{PQ}}}$

Yukawa coupling:
$(2q_\Psi + q_\Phi = 0)$ $\Delta\mathcal{L} = h_{ij}\Phi\bar{\Psi}_{Ri}\Psi_{Lj} + \text{h.c.}$ $\Rightarrow$ $m_\Psi \sim h\frac{f_a}{\sqrt{2}}$

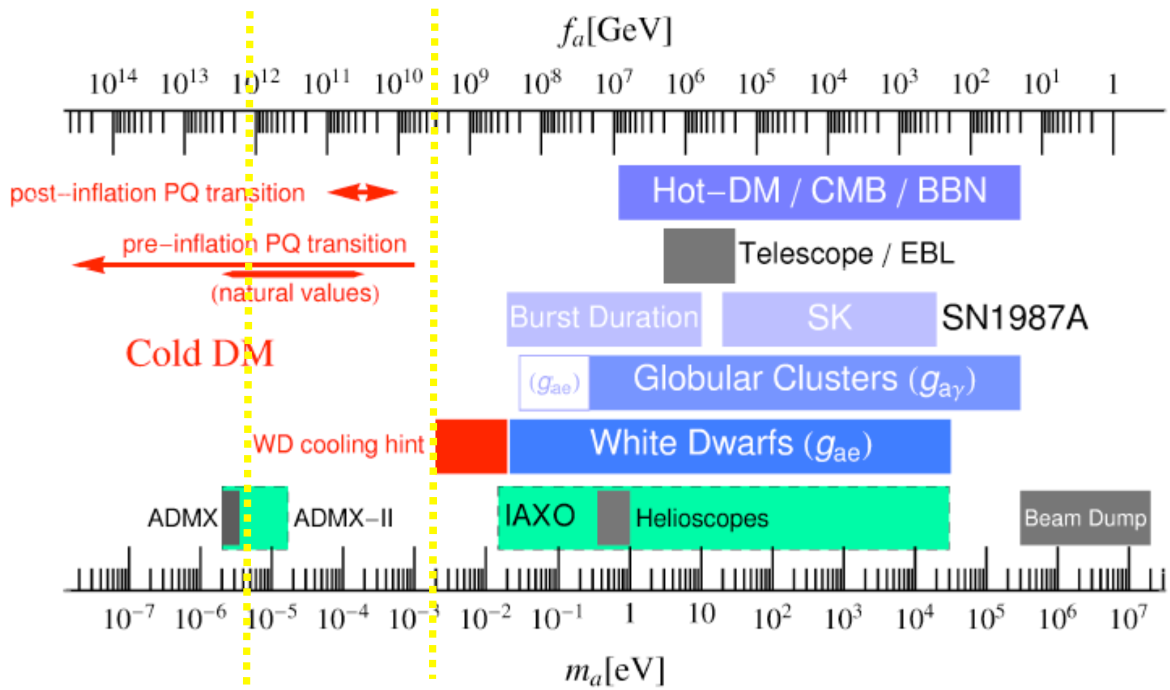
Integrate out Dirac fermions:



$\Rightarrow$ $\frac{g_s^2}{32\pi^2}\frac{a}{f_a}G\tilde{G}$

(Also DFSZ [Dine, Fischler, Srednicki '81; Zhitnitsky '80])

Limits on axion searches



$$\boxed{10^9\ \mathrm{GeV} \lesssim f_a \lesssim 10^{12}\ \mathrm{GeV}}$$

$\frac{1}{f_a} J^\mu \partial_\mu a$ ⇨ axion weakly-coupled - “invisible”

$m_a^2 f_a^2 \sim \frac{1}{8}\Lambda_{QCD}^4$ ⇨ $10^{-5}\ \mathrm{eV} \lesssim m_a \lesssim 10^{-3}\ \mathrm{eV}$

Questions

- What is the origin of the axion potential and spontaneous breaking scale f_a ?
- How to solve the axion quality problem?
- Can the QCD axion mass be different?

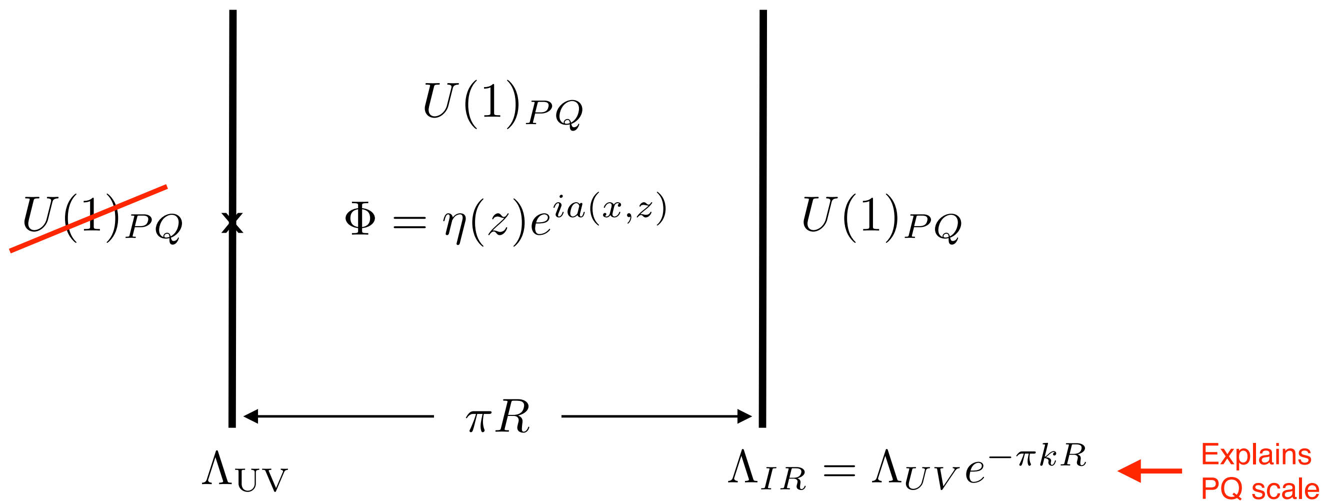
⇨ Use 5th dimension to address these questions!

I. Axion Quality Problem [Cox, TG, Nguyen 1911.09385]

5D metric: $$ds^2 = A^2(z)(dx^2 + dz^2) \equiv g_{MN}dx^M dx^N \qquad A(z) = \frac{1}{kz}$$

AdS curvature scale

"slice of AdS"



5D action: $$S = 2\int_{z_{UV}}^{z_{IR}} d^5x\,\sqrt{-g}\left(-\frac{1}{4g_5^2}F^{MN}F_{MN} - \frac{1}{2}(\mathcal{D}^M\Phi)^\dagger(\mathcal{D}_M\Phi) - \frac{1}{2}m_\Phi^2\Phi^\dagger\Phi \right.$$
$$\left. -\frac{1}{2g_5^2\xi_{PQ}}\left(g^{\mu\nu}\partial_\mu V_\nu + \underbrace{\xi_{PQ}}_{\text{Gauge fixing parameter}}A^{-3}\partial_z(AV_z) - \xi_{PQ}g_5^2\underbrace{X_\Phi}_{\text{PQ charge}}\eta^2 a\right)^2\right)$$

PQ symmetry breaking

$$U_{IR}(\Phi) = \frac{\lambda_{IR}}{k^2}\left(\Phi^\dagger\Phi - k^3 v_{IR}^2\right)^2$$

$$U_{UV}(\Phi) = (-\ell_{UV}\, k^{5/2}\,\Phi + \text{h.c.}) + b_{UV}\, k\, \Phi^\dagger\Phi$$

explicit violation

$$\boxed{\eta(z) = k^{3/2}\left(\lambda\,(kz)^{4-\Delta} + \sigma\,(kz)^{\Delta}\right)}$$

"Bulk VEV"

$$\lambda = \frac{\ell_{UV}}{\Delta - 4 + b_{UV}}(kz_{UV})^{\Delta-4},$$

explicit breaking

$$\sigma = \sqrt{v_{IR}^2 - \frac{\Delta}{2\lambda_{IR}}}\,(kz_{IR})^{-\Delta} \equiv \sigma_0\,(kz_{IR})^{-\Delta}$$

spontaneous breaking

Holographic (AdS/CFT) interpretation:

5D		4D
local $U(1)_{PQ}$ symmetry	$\leftrightarrow$	global $U(1)_{PQ}$ symmetry
$\Phi,\ m_\Phi^2 = \Delta(\Delta-4)k^2$	$\leftrightarrow$	$\mathcal{O}$, dimension Δ
$\Lambda_{\rm UV} e^{-\pi k R} = \Lambda_{\rm IR}$	$\leftrightarrow$	$\Lambda_{\rm IR} = \Lambda_{\rm UV} e^{-\frac{8\pi^2}{b_{CFT} g^2}}$
σ	$\leftrightarrow$	$\langle \mathcal{O} \rangle$
λ	$\leftrightarrow$	$\lambda \Phi \mathcal{O}$

Bulk equations of motion

$$A\,\Box V_z + g_5^2 A^3\eta^2\left(\partial_z a - V_z\right) + \xi A\partial_z\left(A^{-1}\partial_z\left(AV_z\right) - g_5^2 A^2\eta^2 a\right) = 0$$
$$A^3\eta^2\,\Box a + \partial_z\left(A^3\eta^2\left(\partial_z a - V_z\right)\right) + \xi A^2\eta^2\left(\partial_z\left(AV_z\right) - g_5^2 A^3\eta^2 a\right) = 0$$

Kaluza-Klein expansion

$$a(x^\mu, z) = \sum_{n=0}^{\infty} f_a^{(n)}(z)a^{(n)}(x^\mu)$$
$$V_z(x^\mu, z) = \sum_{n=0}^{\infty} f_{V_z}^{(n)}(z)a^{(n)}(x^\mu)$$

Boundary conditions

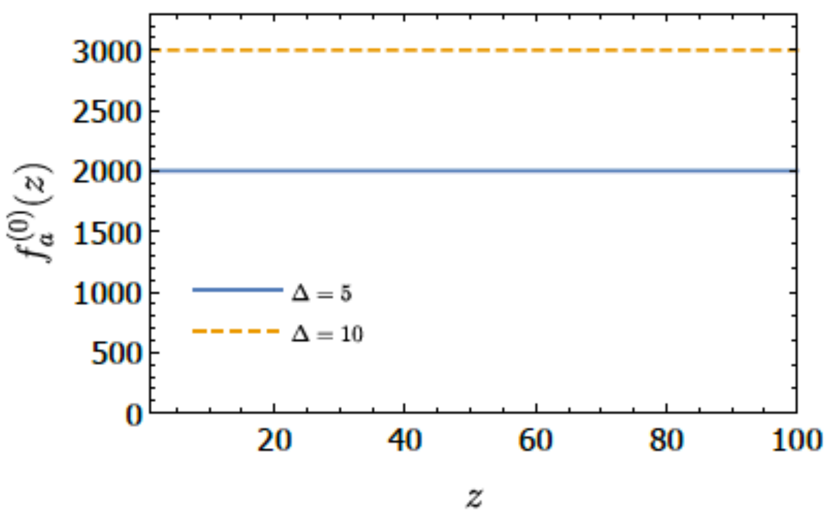
$$V_\mu\Big|_{z_{UV}} = 0\,, \qquad \partial_z V_\mu\Big|_{z_{IR}} = 0\,,$$
$$\xi\left(\partial_z\left(AV_z\right) - g_5^2 A^3\eta^2 a\right)\Big|_{z_{UV}} = 0\,, \qquad V_z\Big|_{z_{IR}} = 0\,,$$
$$\pm 2A^3\eta^2\left(\partial_z a - V_z\right) - A^4\frac{\delta U}{\delta a}\Bigg|_{z_{UV}, z_{IR}} = 0\,.$$

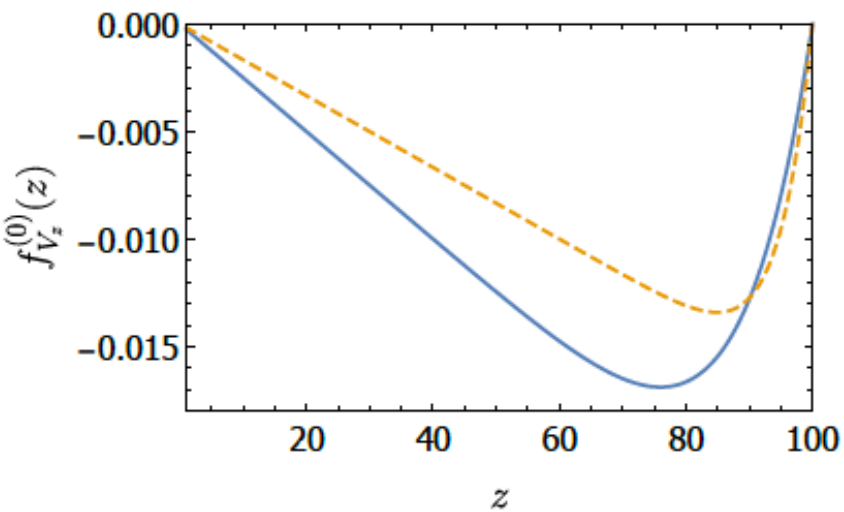
Massless axion $(\lambda = 0)$

$(z_{IR} \gg z_{UV})$

$$f_{V_z}^{(0)}(z) \simeq \frac{-1}{2\sigma_0\sqrt{\Delta-1}}\frac{z}{z_{IR}}\left(g_5^2 k\sigma_0^2\left(1-\left(\frac{z}{z_{IR}}\right)^{2(\Delta-1)}\right)+\mathcal{O}(\sigma_0^4)\right),$$

$$f_a^{(0)}(z) \simeq \frac{z_{IR}}{\sigma_0}\sqrt{\Delta-1}\left(1+\frac{g_5^2 k\sigma_0^2}{4\Delta(\Delta-1)}\left(\frac{(\Delta-1)^2}{2\Delta-1}+\frac{z^2}{z_{IR}^2}\left(\left(\frac{z}{z_{IR}}\right)^{2(\Delta-1)}-\Delta\right)\right)+\mathcal{O}(\sigma_0^4)\right)$$





Global $U(1)_{PQ}$ symmetry: $\boxed{a^{(0)}(x^\mu) \to a^{(0)}(x^\mu) + \alpha_0}$

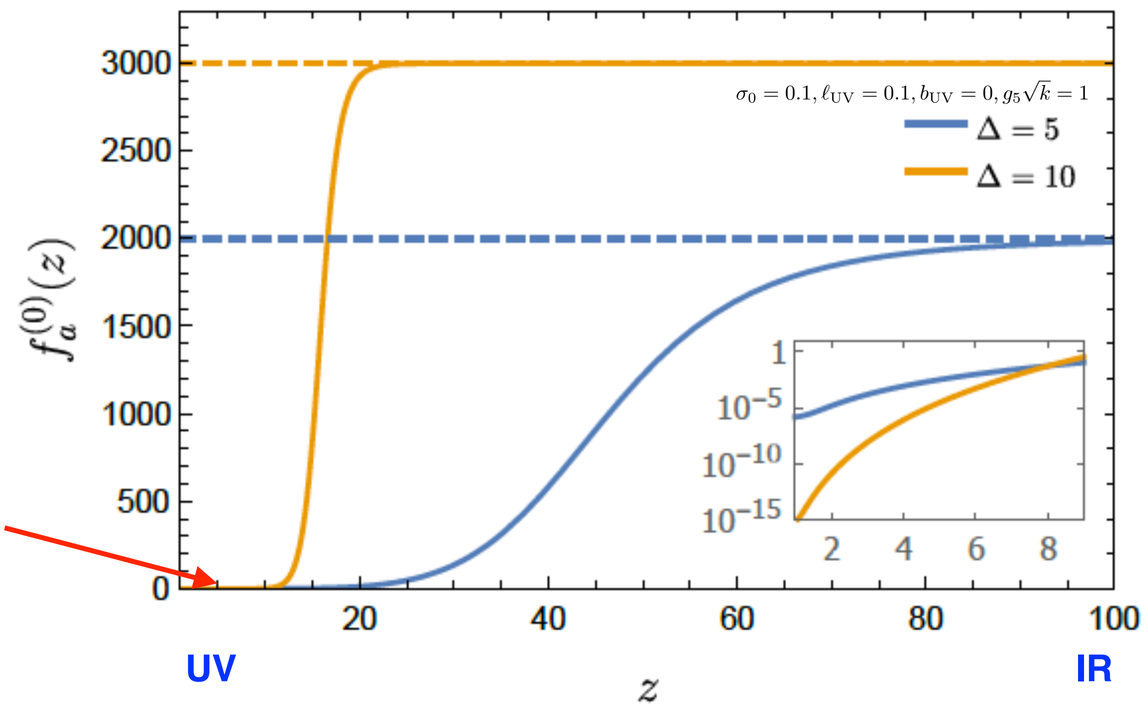
Massive axion $(\lambda \neq 0)$

$$U_{UV}(\Phi) \supset -2\ell_{UV} k^{5/2} \eta \cos(a) = -2\ell_{UV} k^{5/2} \eta \left(1 - \frac{1}{2}a^2 + \dots\right)$$

$(z_{IR} \gg z_{UV})$

$$f_a^{(0)}(z) \simeq z_{IR} \frac{k^{3/2}}{\eta(z)} \sqrt{\Delta - 1} \left(\frac{z}{z_{IR}}\right)^{\Delta} \left[1 + \frac{2\lambda(\Delta-2)(kz_{UV})^{\Delta}(kz)^{2(2-\Delta)}}{\ell_{UV} + 2\sigma_0(\Delta-2)(z_{UV}/z_{IR})^{\Delta}}\right]$$

[Cox, TG, Nguyen 1911.09385]



UV

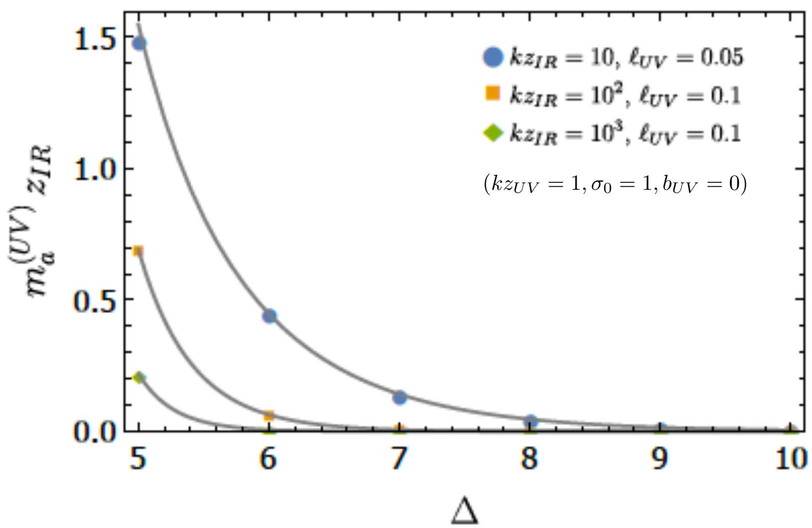
IR

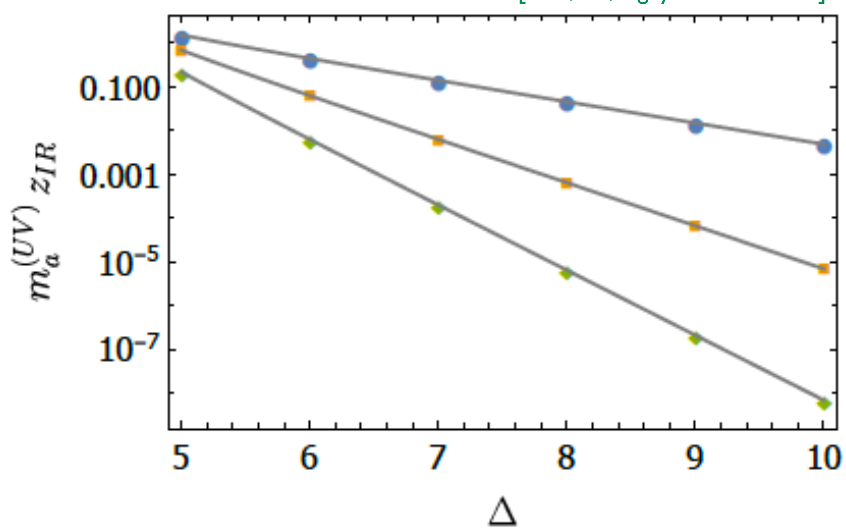
Axion profile suppressed!

Axion mass

$$(m_a^{(UV)} z_{IR})^2 \simeq \frac{4\ell_{UV}}{\sigma_0} \frac{(\Delta-1)(\Delta-2)}{\Delta-4+b_{UV}} \left(\frac{z_{IR}}{z_{UV}}\right)^{4-\Delta} \qquad (z_{IR} \gg z_{UV})$$

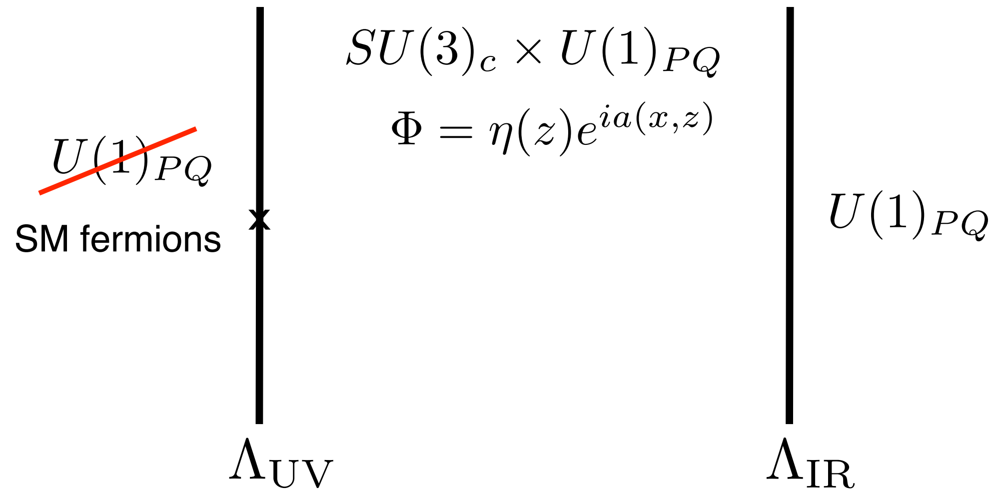
[Cox, TG, Nguyen 1911.09385]





UV axion mass suppressed for large Δ

Axion Quality Problem



Bulk Chern-Simons term: $-\frac{\kappa}{32\pi^2}\int_{z_{UV}}^{z_{IR}} d^5x\, \epsilon^{MNPQR} V_M G^a_{NP} G^a_{QR}$ ⟵ generates axion-gluon coupling

$$\Rightarrow \quad \boxed{(m_a^{(UV)})^2 = \frac{4\ell_{UV}\sigma_0(\Delta-2)}{\kappa^2(\Delta-4+b_{UV})}\left(\frac{\kappa\sqrt{\Delta-1}}{\sigma_0}\right)^{\Delta}\left(\frac{F_a}{\Lambda_{UV}}\right)^{\Delta-4} F_a^2}$$

where

$$F_a \simeq \frac{1}{\kappa}\frac{\sigma_0}{\sqrt{\Delta-1}} z_{IR}^{-1}$$

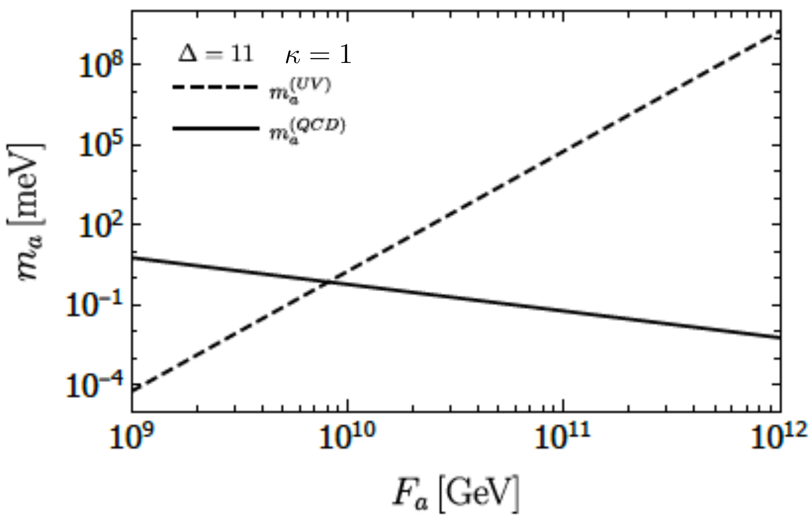
Axion potential:

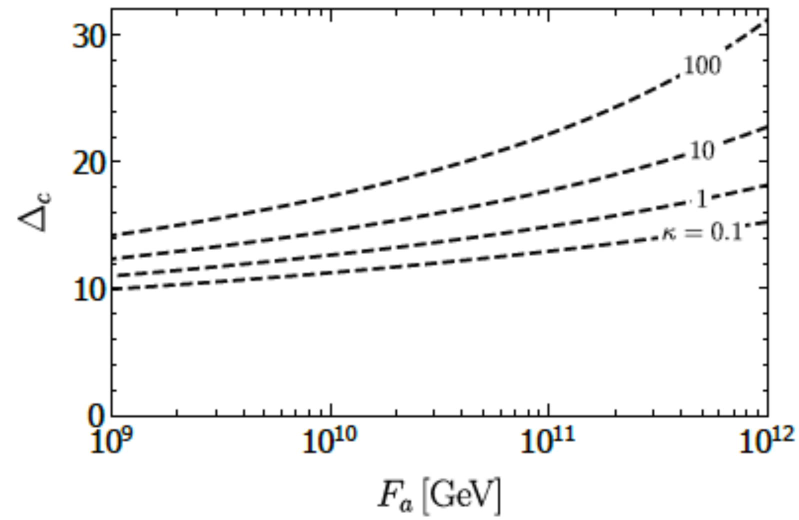
$$V(a^{(0)}) \simeq -(m_a^{(QCD)})^2 F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \bar{\theta}\right) - (m_a^{(UV)})^2 F_a^2 \cos\left(\frac{a^{(0)}}{F_a} + \delta\right)$$

relative phase

Require: $(m_a^{(UV)})^2 \lesssim 10^{-10}\,(m_a^{(QCD)})^2$

[Cox, TG, Nguyen 1911.09385]





10^9 GeV $\lesssim F_a \lesssim 10^{12}$ GeV $\Rightarrow$ $\Delta_c \gtrsim 10$

Holographic interpretation:

5D axion,
local U(1) PQ symmetry

$\Longleftrightarrow$

4D composite axion, accidental
global U(1) PQ symmetry

Example: [Gavela, Ibe, Quilez, Yanagida: 1812.08174]

New strong gauge group

	$SU(5)$	$SU(3)_c$
$\psi_{\bar{5}}$	$\bar{\mathbf{5}}$	$\mathbf{R}$
ψ_{10}	$\mathbf{10}$	$\mathbf{R}$

Global symmetries

	$SU(n)_{\bar{5}}$	$SU(n)_{10}$	$U(1)_{B-L} \equiv U(1)_{PQ}$
$\psi_{\bar{5}}$	$\square$	$\mathbf{1}$	-3
ψ_{10}	$\mathbf{1}$	$\square$	1

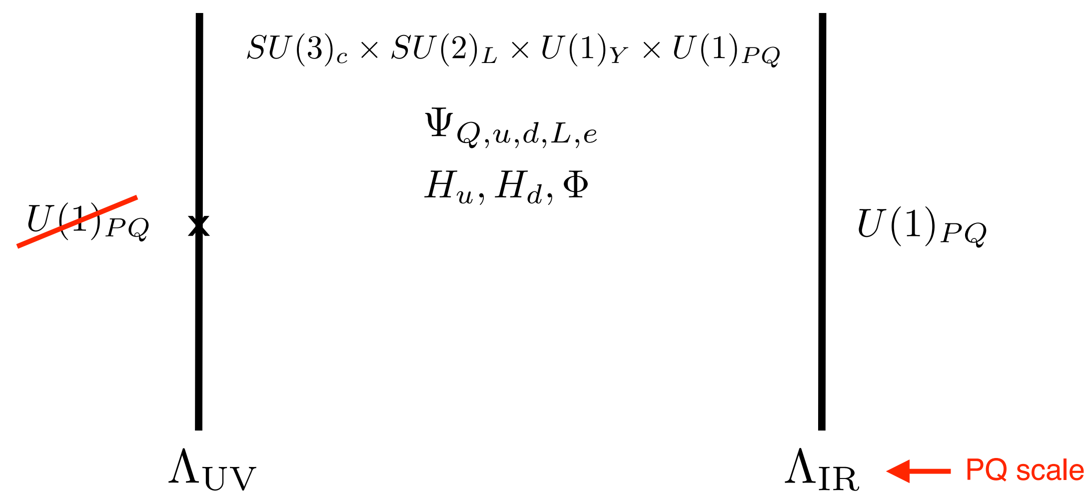
Chiral condensate: $\langle \mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}} \rangle \sim \Lambda_5^6 \quad \Longrightarrow \quad SU(n)_{\bar{5}} \times SU(n)_{10} \longrightarrow G \supset SU(3)_c$

PQ condensate: $\langle \bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\mathbf{10}\,\bar{\mathbf{5}}\,\bar{\mathbf{5}}\,\mathbf{10} \rangle \sim \Lambda_5^9$ $\longleftarrow$ dimension 9! $\Rightarrow$ $\mathcal{L}_{PQ} = c\,\frac{(4\pi)^2}{2!\,4!}\left(\frac{N}{5}\right)^9 \frac{f_a^9}{M_{\rm Pl}^5} e^{-i\frac{10}{N}a/f_a} + \text{h.c.}$

Flavored Warped Axion

[Bonnefoy, Cox, Dudas, TG, Nguyen 2012.09728]

Consider DFSZ-like axion model with bulk Standard Model fermions:



UV boundary:

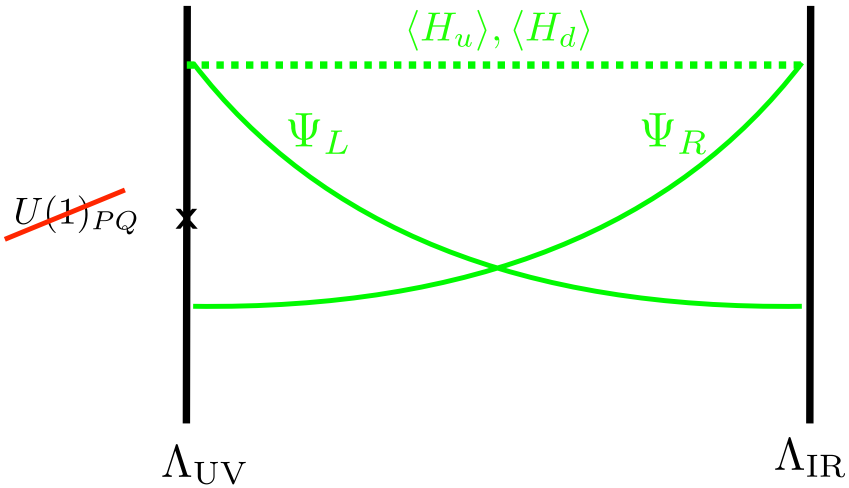
$$
\begin{aligned}
U_{UV}(\Phi, H_u, H_d) = {} & \lambda_u(|H_u|^2 - v_u^2)^2 + \lambda_d(|H_d|^2 - v_d^2)^2 + b_{UV}\, k|\Phi|^2 \\
& + (a|H_u|^2 + b|H_d|^2)|\Phi|^2 + c(H_u H_d \Phi^2 + h.c.) \\
& + d|H_u H_d|^2 + e|H_u^\dagger H_d|^2 \,,
\end{aligned}
$$

constant bulk Higgs vevs

Bulk VEVs:

$$
H_u = \frac{v_u}{\sqrt{2}} e^{i\frac{a_u(x,z)}{v_u}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad H_d = \frac{v_d}{\sqrt{2}} e^{i\frac{a_d(x,z)}{v_d}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \Phi = \eta(z) e^{ia(x,z)}
$$

Bonus feature: explains fermion mass hierarchy [TG, Pomarol '00]



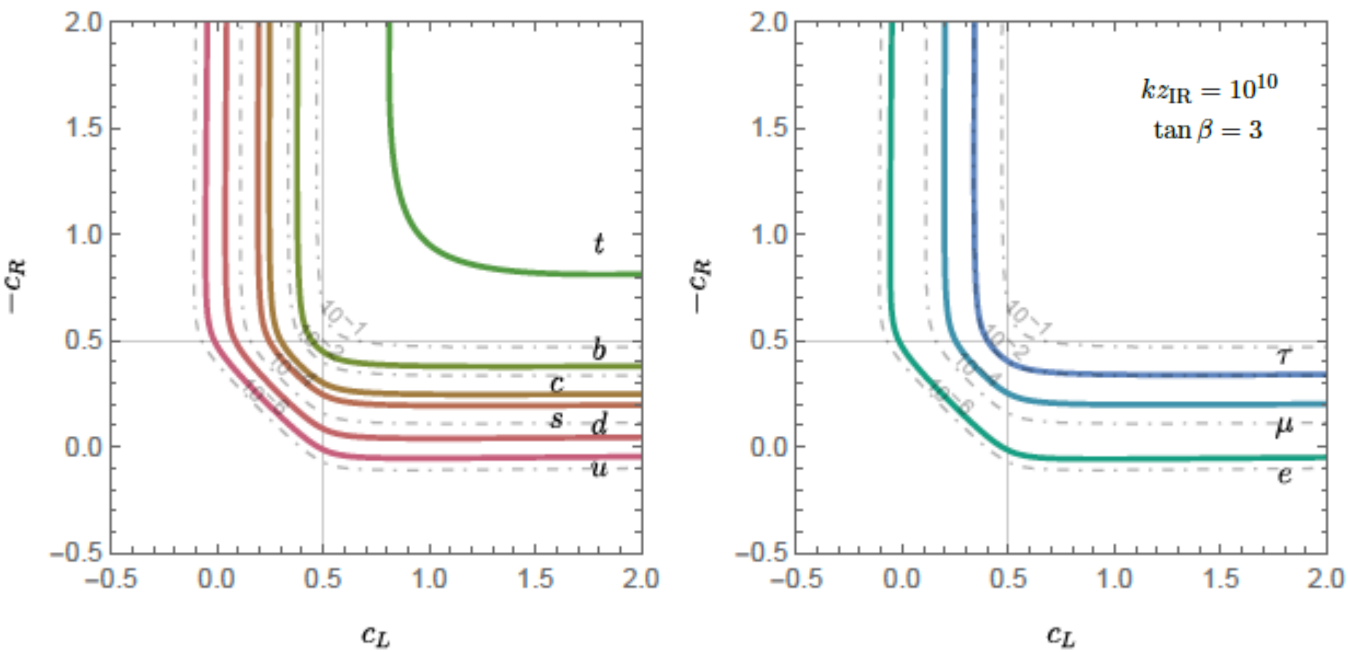
PQ scale

$$m_u^{ij} = y_{u,ij}^{(5)} \frac{\sqrt{2} v_u}{\sqrt{k}} \int_{z_{UV}}^{z_{IR}} \frac{dz}{(kz)^5} \, f_{Q_{iL}}^0(z) f_{U_{jR}}^0(z)$$

$$f_{Q_{iL}}^0(z) = \mathcal{N}_{Q_i} (kz)^{2-\boxed{c_{Q_i}}}$$
$$f_{U_{iR}}^0(z) = \mathcal{N}_{U_i} (kz)^{2+\boxed{c_{U_i}}}$$

bulk fermion mass parameters

(similarly $m_{d,e}^{ij}$)



Axion-fermion couplings:

$$-2\int_{z_{UV}}^{z_{IR}} d^5x\,\sqrt{-g}\,\frac{1}{\sqrt{k}}\left(y^{(5)}_{u,ij}\frac{v_u}{\sqrt{2}}\bar{Q}_{u_i}U_j e^{i\frac{a_u(x,z)}{v_u}} + y^{(5)}_{d,ij}\frac{v_d}{\sqrt{2}}\bar{Q}_{d_i}D_j e^{i\frac{a_d(x,z)}{v_d}} + \text{h.c.}\right)$$

where $\frac{a_{u,d}}{v_{u,d}} = X_{H_{u,d}}\widehat{f}^0_{aX}(z)\,a^0(x^\mu)+\ldots$ $\qquad \widehat{f}^0_{aX}(z) = \begin{cases} f^0_a(z)\,, & m_0 = 0 \\ f^0_a(z) - f^0_a(z_{UV}) + f^0_{a,\cancel{PQ}}(z_{UV})\,, & m_0 \neq 0 \end{cases}$

5D field redefinition: $Q_{u_i}(x,z) \to e^{i\beta\frac{a_u(x,z)}{v_u}}Q_{u_i}(x,z), \quad U_i(x,z) \to e^{i(\beta-1)\frac{a_u(x,z)}{v_u}}U_i(x,z)$

$$\Rightarrow \quad i\int d^4x\,\frac{\partial_\mu a^0}{2F_a}\left(\bar{u}_i\gamma^\mu\left((c^V_u)_{ij} - (c^A_u)_{ij}\gamma^5\right)u_j\right)$$

where $\frac{1}{(F^{V,A}_u)_{ij}} \equiv \frac{(c^{V,A}_u)_{ij}}{F_a} = X_{H_u}\int_{z_{UV}}^{z_{IR}}\frac{dz}{(kz)^4}\,\widehat{f}^0_{aX}\left((A^u_R)_{ik}(f^0_{U_{kR}})^2(A^{u\dagger}_R)_{kj} \mp (A^u_L)_{ik}(f^0_{Q_{kL}})^2(A^{u\dagger}_L)_{kj}\right)$

Overlap between axion and fermion profiles

$A^u_L m^{ij}_u A^{u\dagger}_R = m_{u_i}$

9 $c_{Q_i}, c_{u_i}, c_{d_i}$ parameters - (6 quark masses + 2 CKM) = 1 free parameter $(c_{Q_3}+c_{u_3})$

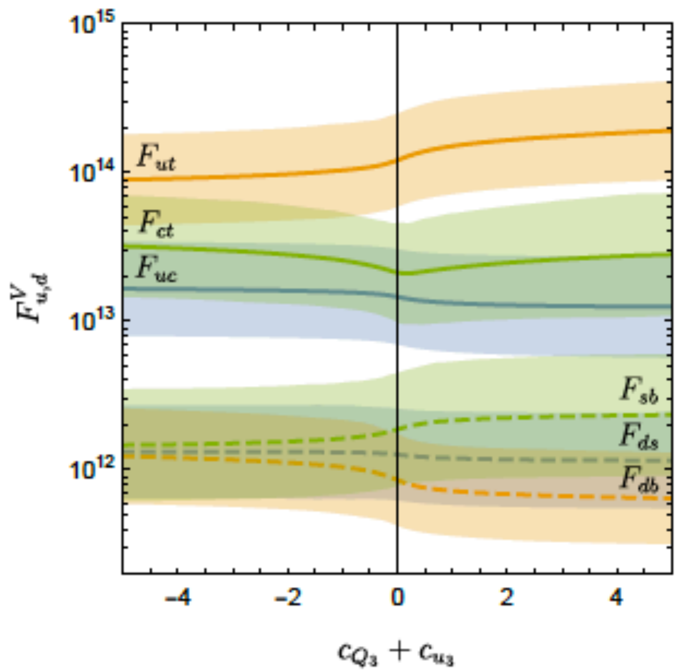
6 c_{L_i}, c_{e_i} parameters - (3 charged lepton masses + 2 PMNS) = 1 free parameter $(c_{L_3}+c_{e_3})$

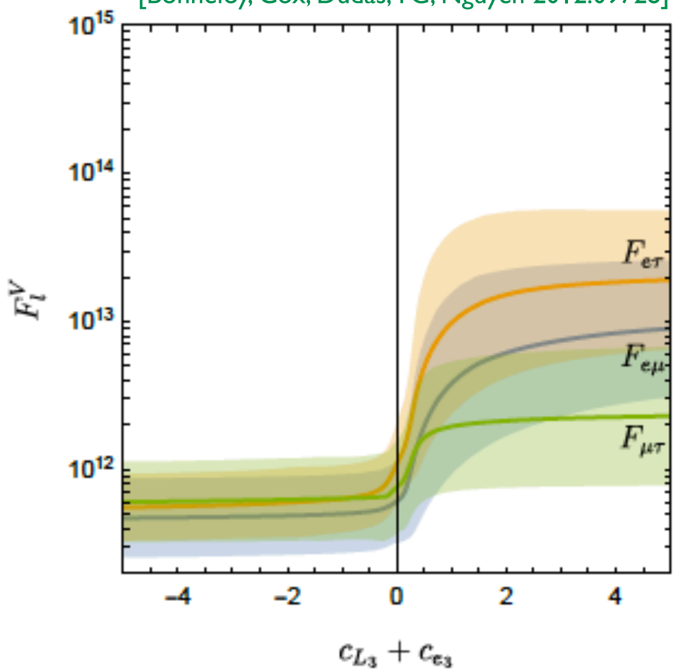
Obtain:

$$f_a^0(z) = \frac{1}{F_a}\left(1 + \underbrace{g_a^0(z)}_{\text{z-dependent part of axion profile}}\right)$$

$$(c_u^{V,A})_{ij} = X_{H_u}\int_{z_{UV}}^{z_{IR}} \frac{dz}{(kz)^4}\, g_a^0(z)\left((A_R^u)_{ik}(f^0_{U_{kR}})^2(A_R^{u\dagger})_{kj} \mp (A_L^u)_{ik}(f^0_{Q_{kL}})^2(A_L^{u\dagger})_{kj}\right)$$

[Bonnefoy, Cox, Dudas, TG, Nguyen 2012.09728]





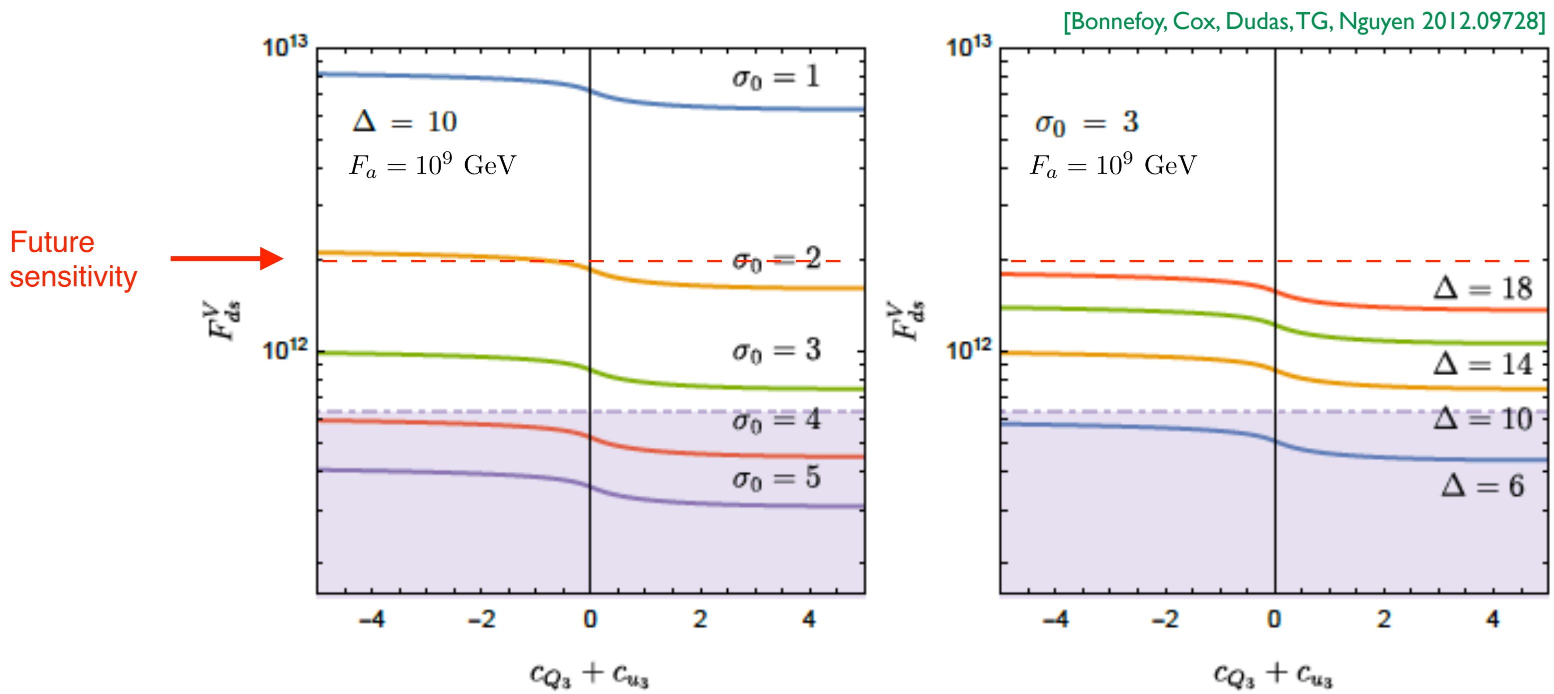
$$\boxed{\frac{1}{(F_u^{V,A})_{ij}} \equiv \frac{(c_u^{V,A})_{ij}}{F_a}}$$

$kz_{\rm IR} = 10^{10}, g_5^2 k = 1$

$\Delta = 10 \quad \sigma_0 = 3$

$F_a \simeq 10^9$ GeV.

Scan over $y^{(5)}_{u,d,e} \sim 1$



Experimental limits : $(F_d^V)_{12} \gtrsim 6.8 \times 10^{11}\,\mathrm{GeV}$ $(K^+ \to \pi^+ a \text{ decays})$

[Martin Camalich, Pospelov, Vuong, Ziegler, Zupan 2002.04623]

$$\sigma_0 \gtrsim 4, \quad \Delta \gtrsim 6$$

2. Axion mass from 5D small instantons

[TG, Khoze, Pomarol, Shirman: 2001.05610]

QCD axion mass:

$$m_a^2 = \frac{\mathcal{T}}{f_a^2} \qquad \mathcal{T} \equiv -i \int d^4x \, \langle 0 | \mathrm{T} \left[\frac{1}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}(x), \; \frac{1}{32\pi^2} G^b_{\rho\sigma} \widetilde{G}^{b\rho\sigma}(0) \right] |0\rangle$$

topological susceptibility

Dilute instanton gas approximation

$$\mathcal{T} \propto \int \frac{d\rho}{\rho^5} C[3] \left(\frac{2\pi}{\alpha_s(1/\rho)} \right)^6 e^{-\frac{2\pi}{\alpha_s(1/\rho)}}$$

QCD asymptotically free $\Rightarrow$ $\mathcal{T} \propto \Lambda^4_{QCD}$ "Large instantons" $\rho \sim 1/\Lambda_{QCD}$

Fermion zero modes: $(\rho m_f)^{N_f} \longrightarrow$ suppression $\frac{\prod_f m_f}{\Lambda_{QCD}^{N_f}}$

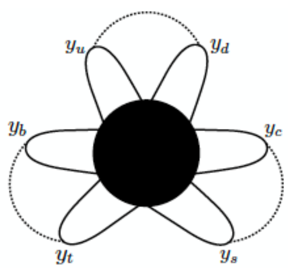
$\Rightarrow$

$$\boxed{m^2_{a,QCD} = \frac{m_u m_d}{(m_u+m_d)^2} \frac{m_\pi^2 f_\pi^2}{f^2}}$$

How to enhance axion mass?

- Change QCD coupling in UV $\alpha_s(1/\rho) \sim 1$ "Small instantons" $\rho \sim 1/\Lambda_{UV}$

- Close fermion loops with Higgs boson



$$\kappa_f = \frac{y_u}{4\pi}\frac{y_d}{4\pi}\frac{y_c}{4\pi}\frac{y_s}{4\pi}\frac{y_t}{4\pi}\frac{y_b}{4\pi} \approx 10^{-23}$$

$\left(\text{otherwise } \frac{m_u m_d m_c m_s m_b m_t}{\Lambda_{UV}^6}\right)$

$$\Rightarrow \quad \boxed{m_a^2 f_a^2 \sim \frac{1}{8}\Lambda_{QCD}^4 + \Lambda_I^4} \qquad \text{where } \Lambda_I \gg \Lambda_{QCD}$$

new contribution

Use 5th dimension to make QCD axion heavy!

QCD in 5D

Flat space 5D metric: $ds^2 = dx^2 + dy^2$

$$S_5 = -\int d^4x \int_0^L dy \left(\frac{1}{4g_5^2} \mathrm{Tr}[G_{MN}^2] + \frac{b_{CS}}{32\pi^2} \varepsilon^{MNRST} B_M \mathrm{Tr}[G_{NR} G_{ST}] + \frac{1}{4g_5^2} F_{MN}^2 + \dots \right),$$

Gluon: $G_{MN}, A_\mu(+,+), A_5(-,-)$ ⇨ $A_\mu^{(0)}(x)$ QCD gluon

Axion: $F_{MN}, B_\mu(-,-), B_5(+,+)$ ⇨ $B_5^{(0)}(x) \equiv a(x)$ QCD axion

5D Chern-Simons: axion- gluon coupling

Orbifold compactification: $L = \pi R$

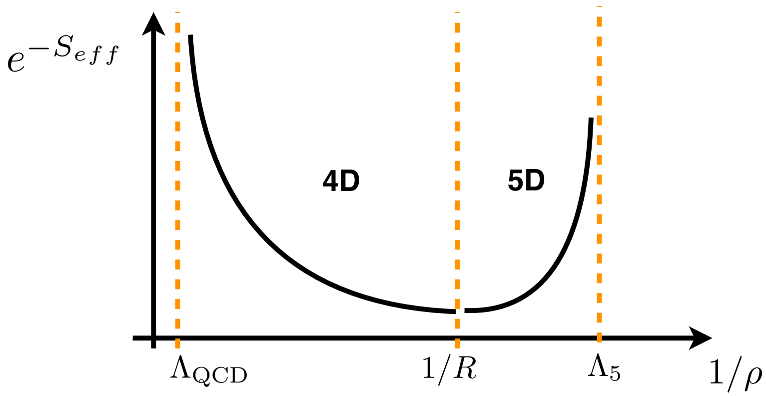
⇨ $$S_4 = \int d^4x \left(\frac{1}{4g_s^2} \mathrm{Tr}[G_{\mu\nu}^2] + \frac{1}{32\pi^2} \frac{a}{f} \mathrm{Tr}[G_{\mu\nu} \widetilde{G}^{\mu\nu}] + \frac{1}{2} (\partial_\mu a)^2 + \dots \right)$$

where $\frac{1}{g_s^2} \equiv \frac{L}{g_5^2}, \quad \frac{1}{f} \equiv b_{CS} g_s L$

5D small instantons

5D instanton: $A_\mu^a(x,y) = A_\mu^{(I)a}(x) = \frac{2\eta_{a\mu\nu}(x-x_0)_\nu}{(x-x_0)^2+\rho^2}, \quad A_5^a(x,y) = 0$ $\quad S_5^{(I)} = \frac{8\pi^3 R}{g_5^2} = \frac{2\pi}{\alpha_s}$ Finite action

Fluctuations + Kaluza-Klein contributions



small instantons!

$$\int_{1/\Lambda_5}^{R} \frac{d\rho}{\rho^5} C[3] \left(\frac{2\pi}{\alpha_s(1/R)}\right)^6 e^{-S_{\rm eff}} \equiv \frac{K}{R^4}$$

$$\boxed{S_{\rm eff} = \frac{2\pi}{\alpha_s(1/R)} - 3\xi(R/\rho)\frac{R}{\rho} + b_0 \ln\frac{R}{\rho}}$$

power law term!

$\xi(R/\rho) \sim 1/3$

$R/\rho \gtrsim 20$

$$K \simeq C[3]\left(\frac{2\pi}{\alpha_s(1/R)}\right)^6 (\Lambda_5 R)^{3-b_0} e^{-\frac{2\pi}{\alpha_s(1/R)} + \Lambda_5 R}$$

power law contribution can overcome suppression

Valid up to $\frac{g_5^2 \Lambda_5}{24\pi^3} \sim 1$ or $\Lambda_5 R \lesssim \frac{6\pi}{\alpha_s}$

Higher dimension terms:

$$\Delta S_5 = -\frac{1}{4g_5^2}\int d^4x \int_0^L dy\, \frac{c_6}{\Lambda_5^2}\,\mathrm{Tr}\, G_{MN}\Box G^{MN}$$

$$\Rightarrow \quad S_{\rm eff} = \frac{2\pi}{\alpha_s} + \boxed{\frac{3\pi}{\alpha_s}\frac{c_6}{(\Lambda_5\rho)^2}} - 3\xi(R/\rho)\frac{R}{\rho} + \dots$$

Higher dimension contribution

Extremum:
$(c_6 > 0)$

$$\frac{1}{\rho_*} \simeq \frac{3}{c_6}\xi(R/\rho)\left(\frac{g_5^2\Lambda_5}{24\pi^3}\right)\Lambda_5$$

Provided $\frac{g_5^2\Lambda_5}{24\pi^3} \ll 1 \quad \Rightarrow \quad \rho_* \gg \frac{1}{\Lambda_5}$

i.e. instantons of size near UV cutoff (Λ_5) are suppressed

Axion mass from 5D small instantons

Assume boundary Standard Model fermions $(b_0 = 7)$ and QCD in bulk

Yukawa coupling suppression from Higgs loops

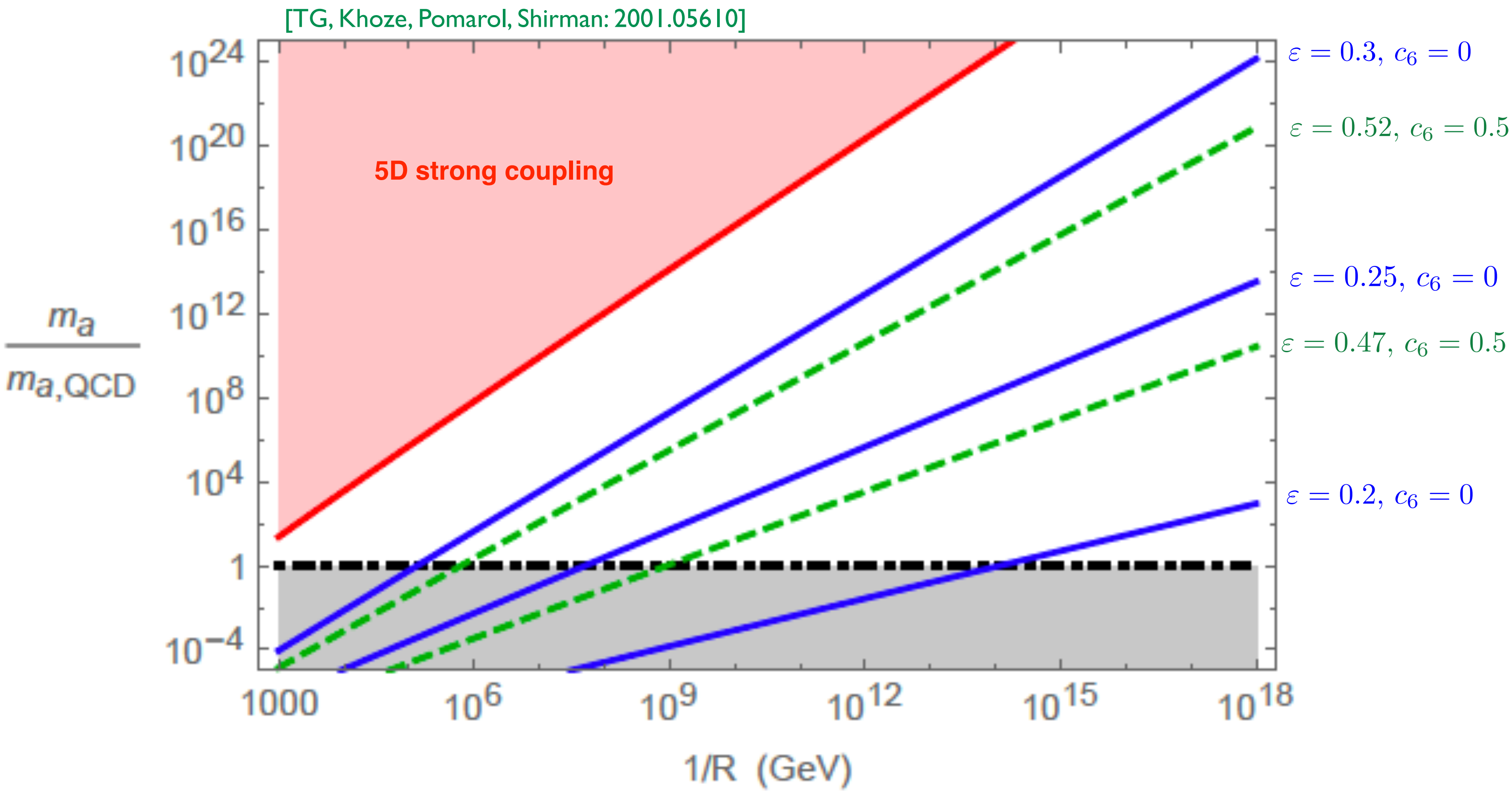
5D enhancement

$$\frac{m_a}{m_{a,QCD}} \simeq \sqrt{2\kappa_f C[3]} \left(\frac{2\pi}{\alpha_s(1/R)}\right)^3 \frac{(m_u + m_d)}{\sqrt{m_u m_d}} \frac{1}{m_\pi f_\pi R^2} \frac{e^{-\frac{1}{2}\left(\frac{2\pi}{\alpha_s(1/R)} - \Lambda_5 R\right)}}{(\Lambda_5 R)^{\frac{1}{2}(b_0-3)}}$$

Write $\quad \Lambda_5 R = \dfrac{6\pi\varepsilon}{\alpha_s(1/R)} \quad$ where $\varepsilon \lesssim 1$ (perturbativity limit)

Positive exponent: $\quad \dfrac{2\pi}{\alpha_s(1/R)} - \Lambda_5 R < 0 \quad \Rightarrow \quad \varepsilon \gtrsim 0.14$

Axion mass can be enhanced up to maximum: $\quad m_{a,5f}^2 \sim \kappa_f \dfrac{\Lambda_5^4}{f^2}$



Small 5D instantons can dominate for $\frac{1}{R} \gtrsim 100\ \mathrm{TeV}$

Other possibilities:

- ***Strong QCD*** [Holdom, Peskin 1982] [Flynn, Randall 1987]

- ***Enlarge QCD color***

$$SU(3+N') \to SU(3)_c \times SU(N')$$ [Dimopoulos, Susskind '79; Dimopoulos '79]

$$SU(3+N) \times SU(N)' \to SU(3)_c \times SU(N)_D$$ [TG, Nagata Shifman: 1604.01127] [Gaillard, Gavela, Houtz, Quilez,del Rey: 1805.06465]

$$SU(3)_1 \times SU(3)_2 \times \cdots \times SU(3)_k \to SU(3)_c$$ [Agrawal, Howe 1710.04213] [Csaki, Ruhdorfer, Shirman 1912.02197]

- ***Mirror QCD*** [Rubakov '97] [Berezhiani, Gianfagna, Gianotti '00] [Dimopoulos, Hook, Huang, Marques-Tavares: 1606.03097]

⇨ Axion mass is sensitive to UV completion!

Conclusion

- Axion quality problem can be solved in 5D warped dimension

  — *dual to 4D dynamical axion with accidental PQ symmetry*

- Flavoured warped axion

  — *solves axion quality and explains fermion mass hierarchy*

  — *predicts off-diagonal axion-fermion couplings*

- 5D small instantons

  — *can enhance axion mass and not spoil strong CP solution*

  — *axion mass could be a sensitive probe of UV physics*