Why Should God (Nature) Choose the Normal ν Mass Hierarchy?

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2019-10-14

SFG and Manfred Lindner, PRD **95** (2017) No.3, 033003 [arXiv:1608.01618]

SFG and Jing-Yu Zhu [arXiv:1910.02666]

The Normal ν Mass Hierarchy is Exactly What We Need!

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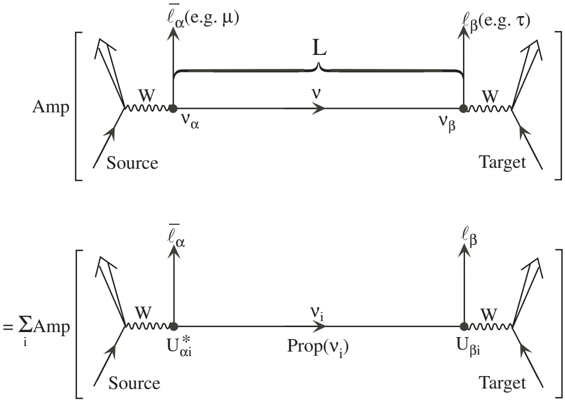
ν Oscillation



[Kayser, hep-ph/0506165]

$$\nu_\alpha = \sum_i \mathbf{V}_{\alpha \mathbf{i}} \nu_i \rightarrow \boxed{\sum_i \mathbf{V}_{\alpha \mathbf{i}} \mathbf{e}^{\mathbf{i}(\mathbf{E_i t} - \vec{\mathbf{P}}_\mathbf{i} \cdot \vec{\mathbf{x}})} \nu_i} = \boxed{\sum_i \mathbf{V}_{\alpha \mathbf{i}} \mathbf{P_i} \mathbf{V}^\dagger_{\mathbf{i}\beta} \nu_\beta} \equiv \sum_\beta \mathbf{A}_{\alpha\beta} \nu_\beta$$

ν Mass & Mixing

- **Mass** & **Mixing** $\Rightarrow$ **Oscillation:**

$$\nu_\alpha(t, L) = \sum_{i\beta} \mathbf{V}_{\alpha \mathbf{i}} e^{-i(E_i t - p_i L)} \mathbf{V}^*_{\beta \mathbf{i}} \nu_\beta \equiv \sum_\beta \mathbf{A}_{\alpha\beta} \nu_\beta$$

$$\mathbf{P}_{\alpha\beta}\big|_{\alpha\neq\beta} \equiv \left|\mathbf{A}_{\alpha\beta}\right|^2 = \sin^2 2\theta \sin^2\left(\delta m^2 \frac{\mathbf{L}}{4\mathbf{E}}\right)$$

- **Flavor** **v.s.** **Mass** Eigenstates:

$$\nu_\alpha = \sum_{\mathbf{i}} \mathbf{V}_{\alpha \mathbf{i}} \nu_{\mathbf{i}}$$

$$\mathbf{V} = \mathcal{P} \begin{pmatrix} c_s c_r & s_s c_r & s_r e^{-i\delta_D} \\ -s_s c_a - c_s s_a s_r e^{i\delta_D} & +c_s c_a - s_s s_a s_r e^{i\delta_D} & s_a c_r \\ +s_s s_a - c_s c_a s_r e^{i\delta_D} & -c_s s_a - s_s c_a s_r e^{i\delta_D} & c_a c_r \end{pmatrix} \mathcal{Q}$$

with $\mathcal{P} \equiv \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ & $\mathcal{Q} \equiv \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$

[$(\mathbf{s}, \mathbf{a}, \mathbf{r}) \equiv (\mathbf{12}, \mathbf{23}, \mathbf{13})$ for (**solar, atmospheric, reactor**) angles]

ν Oscillation Data

(for NH)	-1σ	Best Value	$+1\sigma$
$\mathbf{\Delta m^2_s} \equiv \Delta m^2_{12}\,(\mathbf{10^{-5}}\mathrm{eV}^{\mathbf{2}})$	7.37	**7.56**	7.75
$\lvert \mathbf{\Delta m^2_a} \equiv \Delta m^2_{13} \rvert\ (\mathbf{10^{-3}}\mathrm{eV}^{\mathbf{2}})$	2.51	**2.55**	2.59
$\sin^2 \boldsymbol{\theta_s}$ ($\boldsymbol{\theta_s} \equiv \boldsymbol{\theta_{12}}$)	0.305 (33.5°)	0.321 (**34.5°**)	0.339 (35.6°)
$\sin^2 \boldsymbol{\theta_a}$ ($\boldsymbol{\theta_a} \equiv \boldsymbol{\theta_{23}}$)	0.412 (39.9°)	0.430 (**41.0°**)	0.450 (42.1°)
$\sin^2 \boldsymbol{\theta_r}$ ($\boldsymbol{\theta_r} \equiv \boldsymbol{\theta_{13}}$)	0.02080 (8.29°)	0.02155 (**8.44°**)	0.02245 (8.62°)
$\delta_{\mathrm{D}}, \delta_{\mathrm{Mi}}$	?, **??**	?, **??**	?, **??**

Salas, Forero, Ternes, Tortola & Valle, arXiv:1708.01186

Intelligent Design of Neutrino Parameters

- $\Delta m^2_{21} = 7.5 \times 10^{-5}$ eV^2 $\Rightarrow$ resonant MSW effect @ solar ν
- $\theta_{12} = 34.5^\circ$ $\Rightarrow$ big enough effect @ KamLAND
- $\Delta m^2_{32} = 2.6 \times 10^{-3}$ eV^2 $\Rightarrow$ 0$\sim$ full oscillation @ atmospheric ν
- $\theta_{23} \approx 45^\circ$ $\Rightarrow$ dramatically large effects to be easily seen @ atmospheric ν
- θ_{13} is large enough to be seen @ reactor ν
- $\delta_D \approx -90^\circ$ $\Rightarrow$ most quickly determine MO & large CP
- IH? $\Rightarrow$ more readily measure $0\nu\beta\beta$ & beta decay endpoint
- Neutrinos are Majorana type.

Only that IH is not preferred now!

S. Wojcicki (1995) & M. Goodman (2012)

NH is Preferred by Global Fit

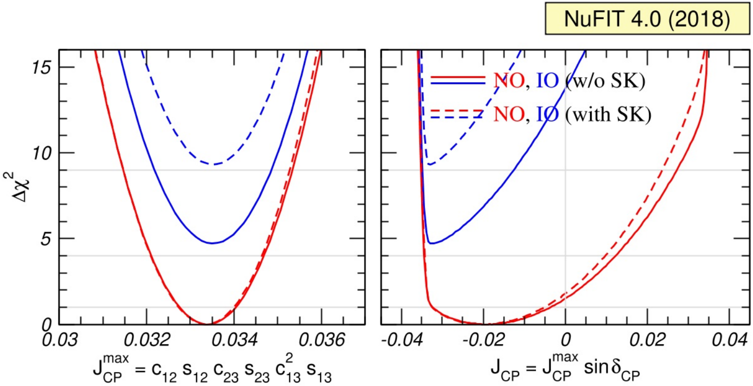
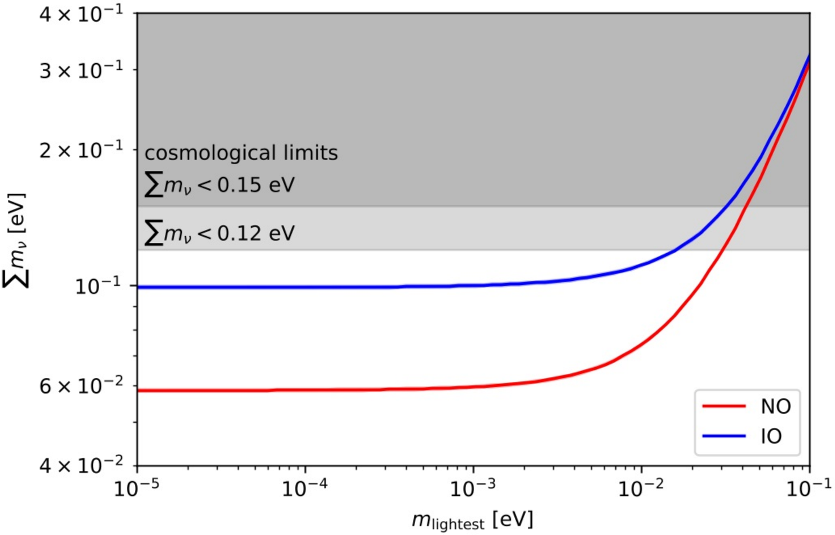


Figure 3. Dependence of the global $\Delta\chi^2$ function on the Jarlskog invariant. The red (blue) curves are for NO (IO). Solid (dashed) curves are without (with) adding the tabulated SK-atm $\Delta\chi^2$.

Esteban, Gonzalez-Garcia, Hernandez-Cabezudo, Maltoni & Schwetz [arXiv:1811.05487]

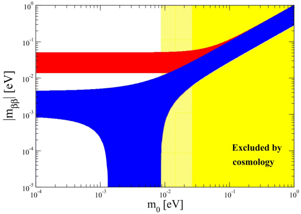
NH is Preferred by Cosmological Data

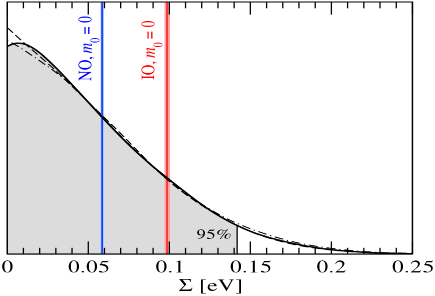


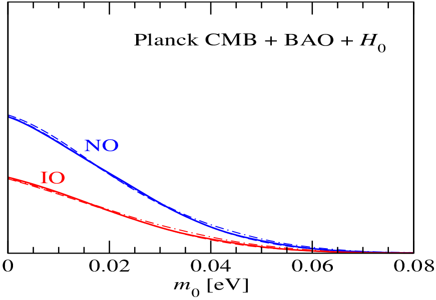
Salas, Gariazzo, Mena, Ternes & Tortola [arXiv:1806.11051]

Cosmological Data on Mass Sum

$$\Sigma \equiv m_1 + m_2 + m_3$$



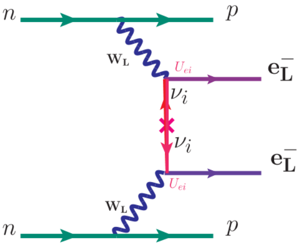


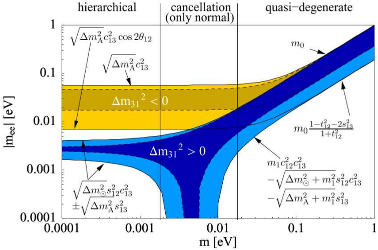


Hannestad & Schwetz [arXiv:1606.04691]

$0\nu 2\beta$ Decay

- Mediated by **Majorana Neutrino** + **Lepton # Violation**





- **Helicity Suppression → Mass Suppression**

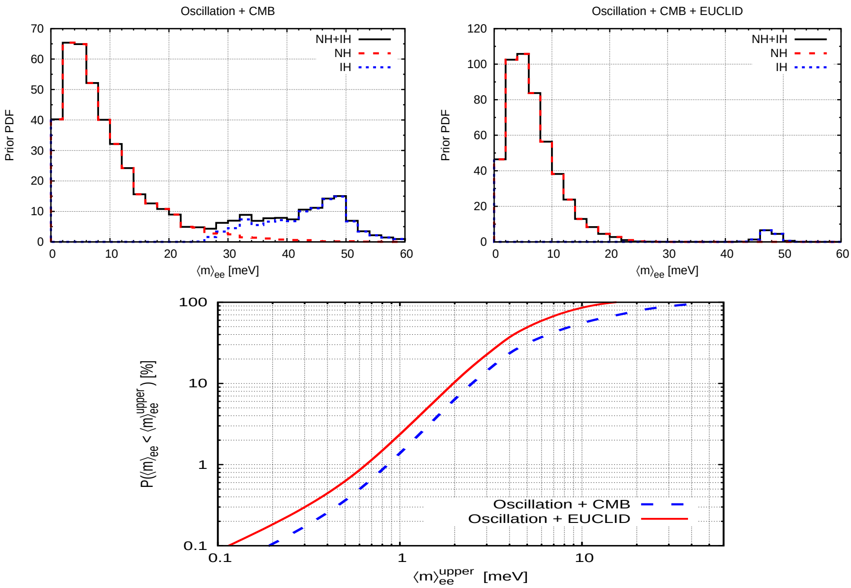
$$\mathcal{M} \propto \sum_i U_{ei} \frac{i}{\not{p} - m_i} U_{ei} \approx \sum_i U_{ei} \frac{m_i}{p^2} U_{ei}$$

- **Effective Electron Neutrino Mass**

$$\langle m \rangle_{ee} \equiv \left| \sum_i m_i U_{ei}^2 \right| = \left| c_s^2 c_r^2 m_1 e^{i\delta_{M1}} + s_s^2 c_r^2 m_2 + s_r^2 m_3 e^{i\delta_{M3}} \right|$$

see also SFG & Werner Rodejohann [1507.05514]

Preference of NH ⇒ Non-Observation of $0\nu2\beta$?



God's Mistake?

Neutrino Mistakes: Wrong tracks and Hints, Hopes and Failures

1901.07068

Maury C. Goodman

8 God's mistake

So in 2012 I extrapolated the intelligent design concept to the still unanswered questions about neutrinos. This implied (1) the CP violation parameter $\delta \sim 3\pi/2$ to most quickly determine the mass ordering and to get large CP violation; (2) the inverted mass order so that we can more readily measure $0\nu\beta\beta$ to distinguish Dirac and Majorana neutrinos, and perhaps measure the beta decay endpoint, and (3) neutrinos should be Majorana which seems to be the more interesting case for theorists, and we want our theorists to be happy.

Question 3 hasn't been answered yet, but early comparisons of T2K, NOvA and reactor data suggest $\delta \sim 3\pi/2$ may be close to the answer. However there is increasing evidence that the mass order is normal, in contradiction to the apparent "Intelligent Design" answer. Did god make a mistake? The more likely answer is that the normal mass order is just what we want and we aren't intelligent enough to realize why yet.

Maury Goodman's talk @ Neutrino History WS (2018) in Paris

God Didn't Make a Mistake!

www.hep.anl.gov/ndk/longbnews/1901.html

Long-Baseline news, January 2019

*** Neutrino mistakes

In my article on neutrino mistakes for the nu history workshop, arXiv:1901.07068, I playfully attributed the increasing evidence for the normal mass order as god's mistake. New experiments on neutrinoless double beta decay are targeting the region suggested by the inverted mass order. But I added, "The more likely answer is that the normal mass order is just what we want and we aren't intelligent enough to realize why yet." I have been pointed to arXiv:1608.01618 which points out that if the order is normal, we can have a chance to simultaneously pin down the two Majorana CP phases which is not possible if it's inverted.

High Energy Physics - Phenomenology

The Normal Neutrino Mass Hierarchy is Exactly What We Need!

Shao-Feng Ge, Jing-Yu Zhu

(Submitted on 7 Oct 2019)

The preference of the normal neutrino mass hierarchy from the recent cosmological constraints and the global fits of neutrino oscillation experiments does not seem like a wise choice at first glance since it obscures the neutrinoless double beta decay and hence the Majorana nature of neutrinos. Contrary to this naive expectation, we point out that the actual situation is the opposite. Choosing the normal mass hierarchy opens up the possibility of determining the solar octant and simultaneously measuring the two Majorana CP phases. If the neutrino mass hierarchy is normal, the funnel region would completely disappear if the solar mixing angle lives in the higher octant. With a typical $\mathcal{O}(\mathrm{meV})$ sensitivity on the effective mass $|m_{ee}|$, the neutrinoless double beta decay experiment can tell if the funnel region really exists and hence if the solar octant takes the higher octant. With the sensitivity further improved to sub-meV, the two Majorana CP phases can be simultaneously determined. We need the normal neutrino mass hierarchy with very good reasons.

LMA-dark & Degeneracies

- Parametrize the ν mixing as $V_\nu = U_{23}(\theta_a)U_{13}(\theta_r)U_{12}(\theta_s,\delta_D)$:

$$\begin{pmatrix} e^{i\delta_D} & & \\ & 1 & \\ & & 1 \end{pmatrix} \begin{pmatrix} c_r c_s & c_r s_s & s_r e^{-i\delta_D} \\ -c_a s_s - c_s s_a s_r e^{i\delta_D} & +c_a c_s - s_a s_r s_s e^{i\delta_D} & c_r s_a \\ +s_a s_s - c_s c_a s_r e^{i\delta_D} & -s_a c_s - c_a s_r s_s e^{i\delta_D} & c_r c_a \end{pmatrix} \begin{pmatrix} e^{-i\delta_D} & & \\ & 1 & \\ & & 1 \end{pmatrix}$$

- Hamiltonian in vacuum:

$$H_{vac} = O_{23} O_{13} \begin{pmatrix} H^{(2)} & 0 \\ 0 & \Delta_a - \frac{\Delta_s}{2} \end{pmatrix} O_{13}^T O_{23}^T \,, \quad \text{with} \quad H^{(2)} = \frac{\Delta_s}{2} \begin{pmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12} e^{i\delta_D} \\ \sin 2\theta_{12} e^{-\delta_D} & \cos 2\theta_{12} \end{pmatrix}$$

- Then H_{vac} reverses its sign under

$$\begin{aligned} \sin\theta_s &\leftrightarrow \cos\theta_s \,, \\ \delta_D &\rightarrow \pi - \delta_D \,, \\ \Delta m_a^2 &\rightarrow -\Delta m_a^2 + \Delta m_s^2 \end{aligned}$$

- The matter term $H_{mat} = V_{cc}\text{diag}\{1+\epsilon_{ee}, 0, 0\}$ reverses its sign under

$$\epsilon_{ee} \rightarrow -2 - \epsilon_{ee}$$

Degeneracies in MH, Solar Octant & CP

- **Hamiltonian reverses sign under combined transformations**

$$
\begin{aligned}
\sin\theta_s &\leftrightarrow \cos\theta_s\,, \\
\delta_D &\rightarrow \pi - \delta_D\,, \\
\Delta m_a^2 &\rightarrow -\Delta m_a^2 + \Delta m_s^2\,, \\
\epsilon_{ee} &\rightarrow -2 - \epsilon_{ee}\,.
\end{aligned}
$$

- **Degeneracies**
  - **Mass Hierarchy**: NH $\leftrightarrow$ IH
  - **Solar Octant**: $\theta_s \leftrightarrow \frac{\pi}{2} - \theta_s$
  - **CP Octant**: $\delta_D \leftrightarrow \pi - \delta_D$

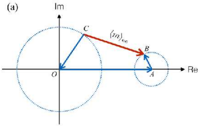
  Since this combined transformation is **universal for all neutrino oscillation experiments**, there is **no way for oscillation experiments to resolve the degeneracies**.

- **Any solutions from other type of experiments?**

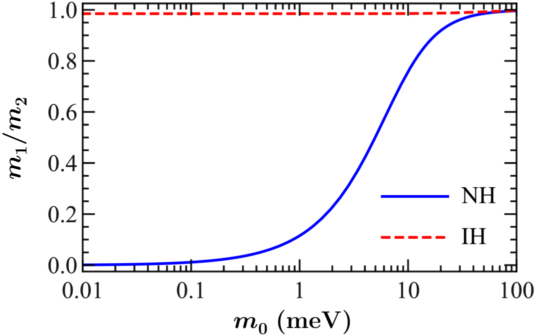
Solar Octant & Mass Degeneracy

The **octant transformation**, $\mathbf{c_s \leftrightarrow s_s}$ is **equivalent** to $\mathbf{m_1 \leftrightarrow m_2}$,

$$m_{ee} = c_r^2 c_s^2 m_1 e^{i\tilde{\delta}_{\rm M1}} + c_r^2 s_s^2 m_2 + s_r^2 m_3 e^{i\tilde{\delta}_{\rm M3}}$$



where $\tilde{\delta}_{\rm Mi} \equiv \delta_{\rm Mi} - \delta_D$.

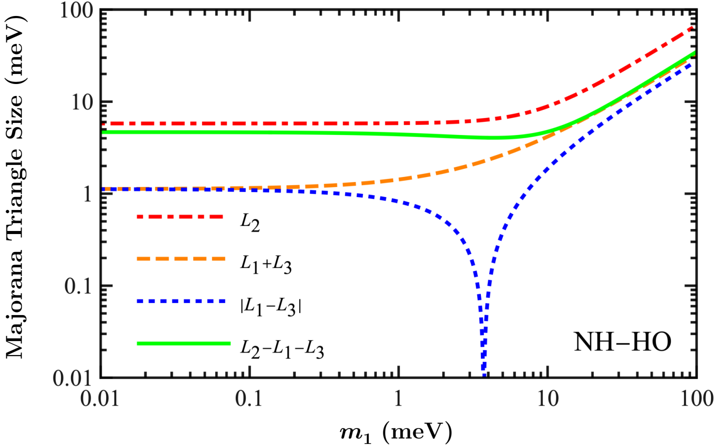


$\mathbf{c_s^2 m_1 \leftrightarrow s_s^2 m_2}$

SFG & Jing-Yu Zhu [arXiv:1910.02666]

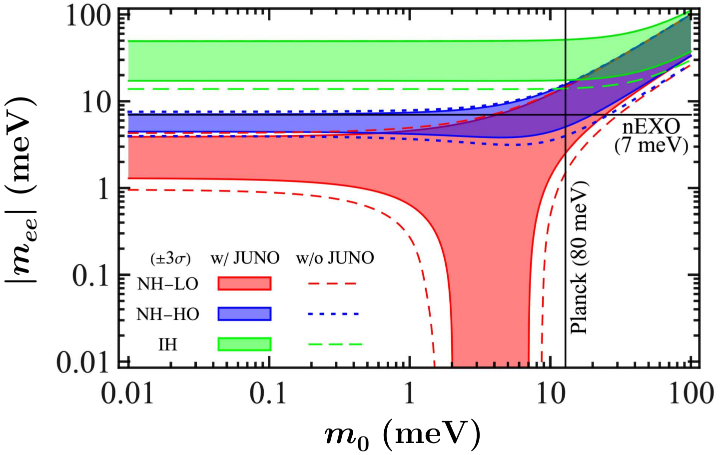
Boundary Parameters

$$L_2^{\mathrm{HO}}(c_r^2 c_s^2 m_2) > L_1^{\mathrm{HO}}(c_r^2 s_s^2 m_1) + L_3(s_r^2 m_3)$$



SFG & Jing-Yu Zhu [arXiv:1910.02666]

Distinguishing Solar Octant in the $0\nu2\beta$ Decay



w/o JUNO: $|m_{ee}| \geq 3.2\,\text{meV}$ @ $m_1 = 5.3\,\text{meV}$
with JUNO: $|m_{ee}| \geq 3.8\,\text{meV}$ @ $m_1 = 4.5\,\text{meV}$

SFG & Jing-Yu Zhu [arXiv:1910.02666]

Uncertainty Reduction with Reactor Neutrino

- **Neutrinoless Double Beta Decay**

$$m^{\nu}_{ee} = \left| c_s^2 c_r^2 m_1 e^{i\delta_{M1}} + s_s^2 c_r^2 m_2 + s_r^2 m_3 e^{i\delta_{M3}} \right|$$

- **Reactor Neutrino**

$$P_{ee} = 1 - 4 c_r^4 c_s^2 s_s^2 \sin^2 \Delta_{21} - 4 c_s^2 c_r^2 s_r^2 \sin^2 \Delta_{31} - 4 s_s^2 c_r^2 s_r^2 \sin^2 \Delta_{32}$$

  - **Short Baseline – Daya Bay**

$$P_{ee} \approx 1 - 4 \mathbf{c_r^2 s_r^2} \sin^2 \boldsymbol{\Delta_{31}}$$

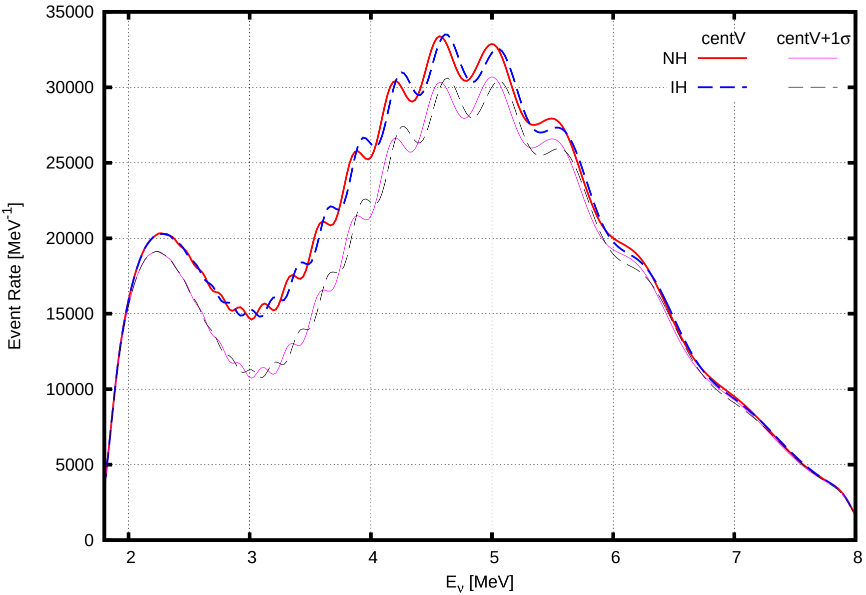
  - **Medium Baseline – JUNO**

$$\begin{aligned} P_{ee} \quad = \quad & 1 - 4 c_r^4 \mathbf{c_s^2 s_s^2} \sin^2 \boldsymbol{\Delta_{21}} - 4 c_r^2 s_r^2 \sin^2 |\boldsymbol{\Delta_{31}}| \\ & - 4 s_s^2 c_r^2 s_r^2 \sin^2 \boldsymbol{\Delta_{21}} \cos(2|\boldsymbol{\Delta_{31}}|) \\ & \pm 2 s_s^2 c_r^2 s_s^2 \sin(2\boldsymbol{\Delta_{21}}) \sin(2|\boldsymbol{\Delta_{31}}|), \end{aligned}$$

SFG, Kaoru Hagiwara, Naotoshi Okamura, Yoshitaro Takaesu [arXiv:1210.8141]

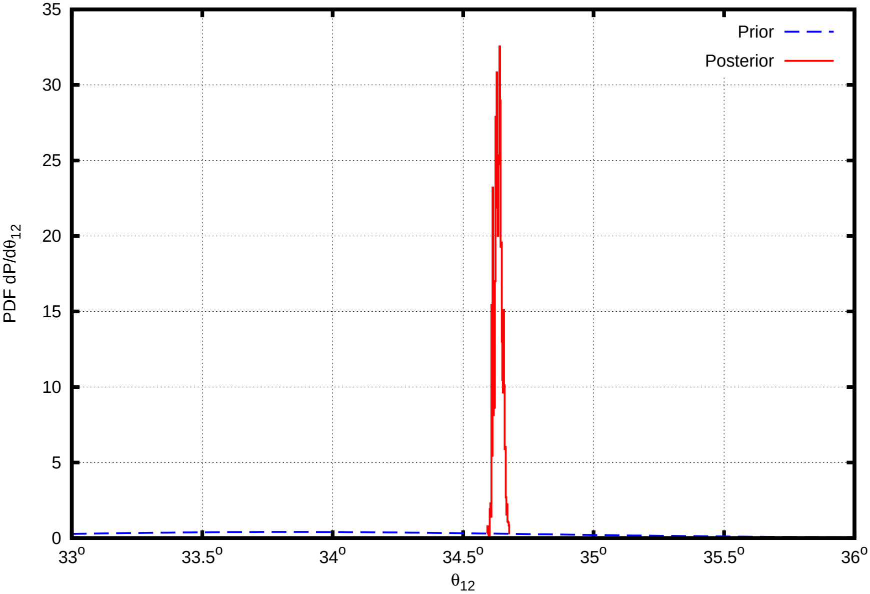
Precision Measurement @ JUNO

SFG & Rodejohann, arXiv:1507.05514

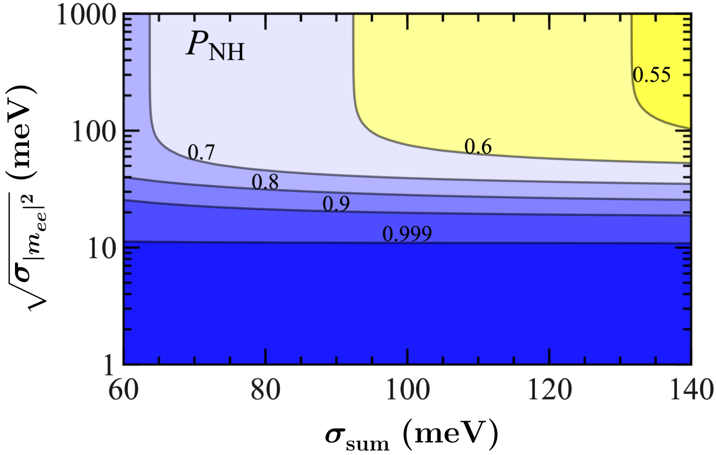


Precision Measurement @ JUNO

SFG & Rodejohann, arXiv:1507.05514

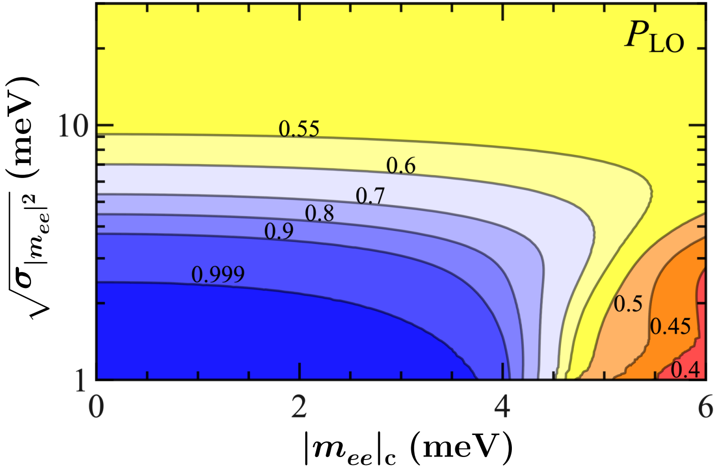


Probability of NH



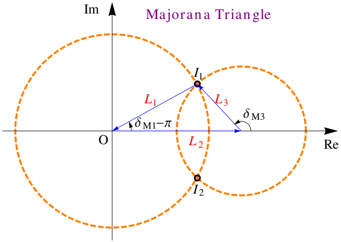
SFG & Jing-Yu Zhu [arXiv:1910.02666]

Probability of LO



SFG & Jing-Yu Zhu [arXiv:1910.02666]

Any chance of obtaining some information?



$$\langle m \rangle_{ee} \equiv \overrightarrow{L_1} + \overrightarrow{L_2} + \overrightarrow{L_3} \,,$$

with

$$\overrightarrow{L_1} \equiv m_1 U_{e1}^2 = m_1 c_r^2 c_s^2 e^{i\boldsymbol{\delta_{\mathrm{M1}}}} \,,$$

$$\overrightarrow{L_2} \equiv m_2 U_{e2}^2 = \sqrt{m_1^2 + \Delta m_s^2} c_r^2 s_s^2 \,,$$

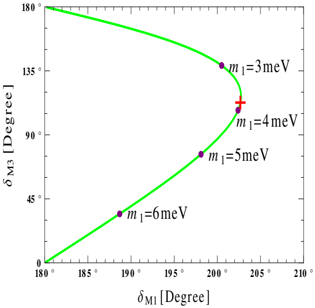
$$\overrightarrow{L_3} \equiv m_3 U_{e3}^2 = \sqrt{m_1^2 + \Delta m_a^2} s_r^2 e^{i\boldsymbol{\delta_{\mathrm{M3}}}} \,.$$

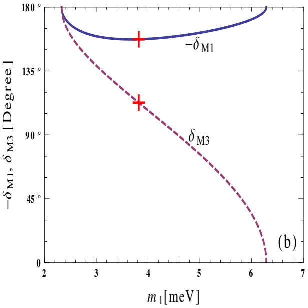
Determine 2 Majorana Phases Simultaneously

$$|L_1 - L_3| \le L_2 \le L_1 + L_3 \,.$$

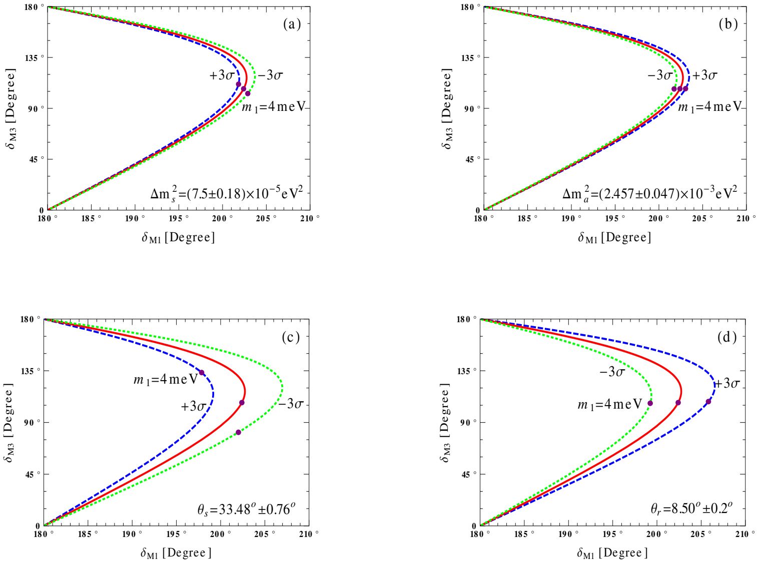
$$\cos \delta_{M1} = -\frac{L_1^2 + L_2^2 - L_3^2}{2 L_1 L_2} = -\frac{m_1^2 c_r^4 c_s^4 + m_2^2 c_r^4 s_s^4 - m_3^2 s_r^4}{2 m_1 m_2 c_r^4 c_s^2 s_s^2} \,,$$

$$\cos \delta_{M3} = +\frac{L_1^2 - L_2^2 - L_3^2}{2 L_2 L_3} = +\frac{m_1^2 c_r^4 c_s^4 - m_2^2 c_r^4 s_s^4 - m_3^2 s_r^4}{2 m_2 m_3 c_r^2 s_r^2 s_s^2} \,.$$



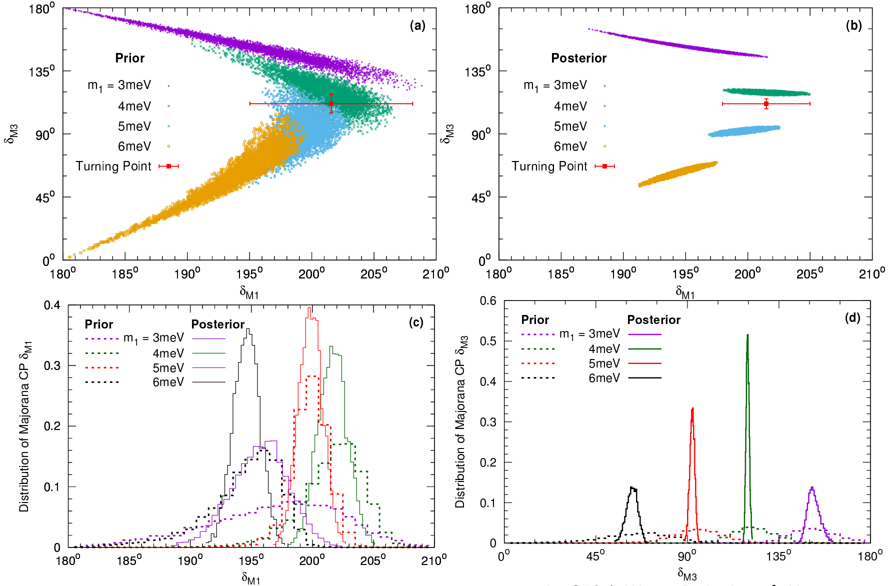


Uncertainties from Oscillation Parameters



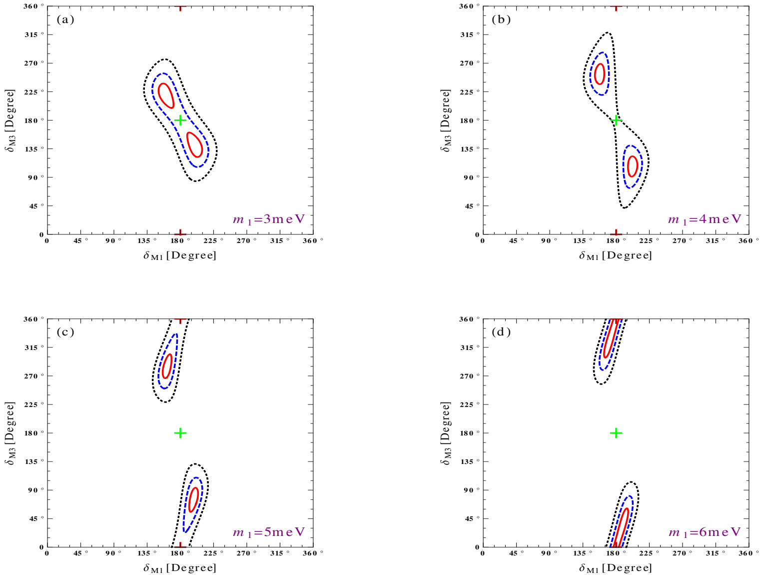
see also SFG & Werner Rodejohann [arXiv:1507.05514]

Uncertainties from Oscillation Parameters

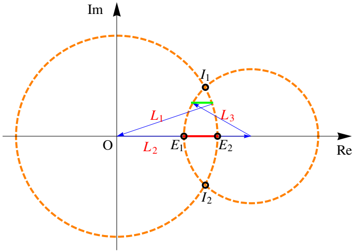


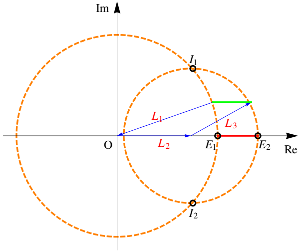
see also SFG & Werner Rodejohann [arXiv:1507.05514]

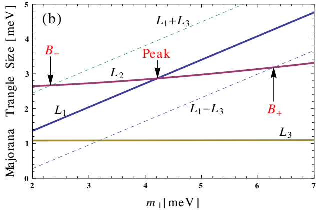
Uncertainties from $\langle m \rangle_{ee}$

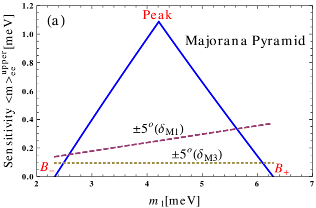


Majorana Pyramid & Projected Uncertainty









Extracting Majorana CP Phases from Nothing

- Null observation seems to be very unfortunate!
- But **not bad at all!**
- Vanishing m_{ee} $\Rightarrow$ Determine the **2** Majorana CP Phases Simultaneously!

$$|\mathbf{m_{ee}}| = \mathbf{0} \quad \Rightarrow \quad \mathbb{R}(\mathbf{m_{ee}}) = \mathbb{I}(\mathbf{m_{ee}}) = \mathbf{0}$$

$$|\mathbf{m_{ee}}| < \mathbf{f} \quad \Rightarrow \quad \mathbb{R}(\mathbf{m_{ee}}) < \mathbf{f} \quad \& \quad \mathbb{I}(\mathbf{m_{ee}}) < \mathbf{f}$$

- Non-zero m_{ee} can only determine a **single** degree of freedom

$$|\mathbf{m_{ee}}| = \mathbf{f}$$

- **Missing Piece**
  - Null observation of $0\nu2\beta \not\Rightarrow$ 2 Majorana CP phases;
  - Neutrinos have to be Majorana type in the first place!
  - Either assumption or independent measurement.

Prey of Leptonic CP Phases

TNT2K for better Dirac CP measurement [1506.05023, 1605.01670, 1607.08513, 1704.08518]

Summary

Thank You!

Minimal Neutrinos

Georg G. Raffelt

Stars as Laboratories for Fundamental Physics

The Astrophysics of Neutrinos, Axions, and Other Weakly Interacting Particles

In the standard model, neutrinos have been assigned the most minimal properties compatible with experimental data: zero mass, zero charge, zero dipole moments, zero decay rate, zero almost everything.

Neutrinos are not just invisible but very boring!

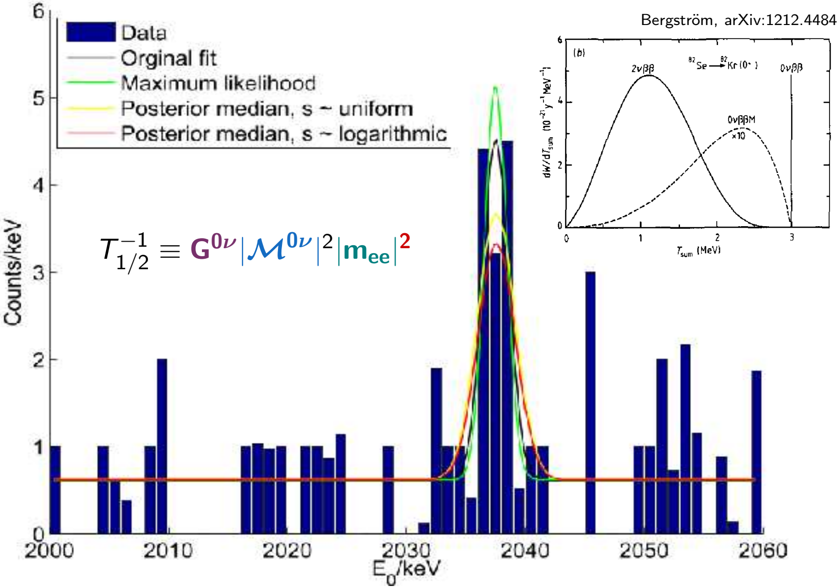
Lazy Neutrino

Nothing can interest me!!!

Daya Bay & LHC changed Physics in 2012

- **Higgs boson** $\Rightarrow$ electroweak symmetry breaking & mass.
- **Chiral symmetry breaking** $\Rightarrow$ majority of mass.
- **The world seems not affected by the tiny neutrino mass?**
  - Neutrino mass $\Rightarrow$ Mixing
  - 3 Neutrino $\Rightarrow$ possible **CP violation**
  - CP violation $\Rightarrow$ **Leptogenesis**
  - **Leptogenesis** $\Rightarrow$ **Matter-Antimatter Asymmetry**
  - There is something left in the Universe.
  - Baryogenesis from quark mixing is not enough.
- Majorana $\nu \Leftrightarrow$ **Lepton Number Violation**
- **Residual $\mathbb{Z}_2$ Symmetries:** $\cos \delta_{\mathbf{D}} = \frac{(s_s^2 - c_s^2 s_r^2)(\mathbf{c_a^2} - \mathbf{s_a^2})}{4 c_a s_a c_s s_s \mathbf{s_r}}$ 1108.0964 1104.0602

Experimental Measurement



$$T_{1/2}^{-1} \equiv G^{0\nu} |\mathcal{M}^{0\nu}|^2 |m_{ee}|^2$$

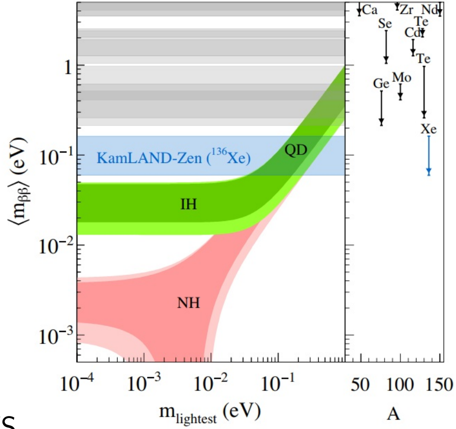
Bergström, arXiv:1212.4484

The Current $0\nu 2\beta$ Experiments are Approaching IH

KamLAND-Zen 400 Phase 1+2 combined

$$T^{0\nu}_{1/2} > 1.07 \times 10^{26}\ \mathrm{yr}$$

(sensitivity 5.6×10^{25} yr)



It also provides upper limit of m_{lightest} at 180-480 meV.

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$$\langle m_{\beta\beta} \rangle < (61 - 165)\ \mathrm{meV}$$

PRL117, 082503 (2016)

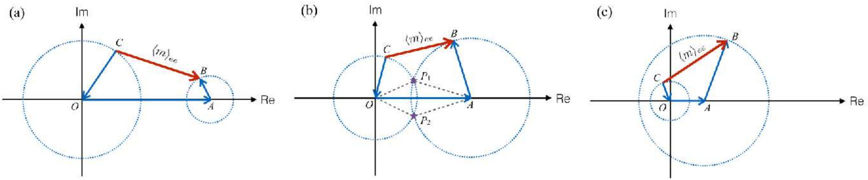
Intrinsic Variation of Majorana CP Phases

- Majorana CP Phases $\delta_{\rm M1}$ & $\delta_{\rm M3}$ are Unknown   Xing & Zhou, arXiv:1404.7001

$$m_{ee} = c_s^2 c_r^2 m_1 e^{i\delta_{M1}} + s_s^2 c_r^2 m_2 + s_r^2 m_3 e^{i\delta_{M3}}$$

- Geometric Interpretation:

$$\overrightarrow{OA} \equiv c_s^2 c_r^2 m_1 e^{i\delta_{M1}}, \qquad \overrightarrow{AB} \equiv s_s^2 c_r^2 m_2, \qquad \overrightarrow{CO} \equiv s_r^2 m_3 e^{i\delta_{M3}}.$$



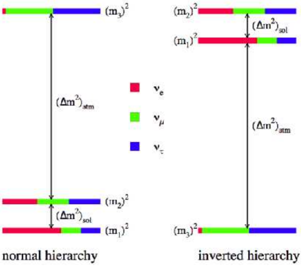
$OA \geq AB + CO$	$OA - AB - CO \leq \lvert m_{ee} \rvert \leq CO + OA + AB$
$\lvert AB - OC \rvert \leq OA \leq AB + CO$	$0 \leq \lvert m_{ee} \rvert \leq CO + OA + AB$
$OA \leq \lvert AB - CO \rvert$	$AB - CO - OA \leq \lvert m_{ee} \rvert \leq AB + OA - CO$

Dependence on Mass Hierarchy

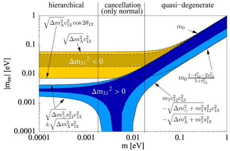
- **Mass Hierarchy**

$$\text{NH} \quad : \quad m_1 < m_2 < m_3$$

$$\text{IH} \quad : \quad m_3 < m_1 < m_2$$



- **Effect on m_{ee}**



$$\text{NH} \quad : \quad |m_{ee}| \approx \left| s_s^2 c_r^2 \sqrt{\Delta m_s^2} + s_r^2 \sqrt{\Delta m_a^2} e^{i\delta_{M3}} \right|$$

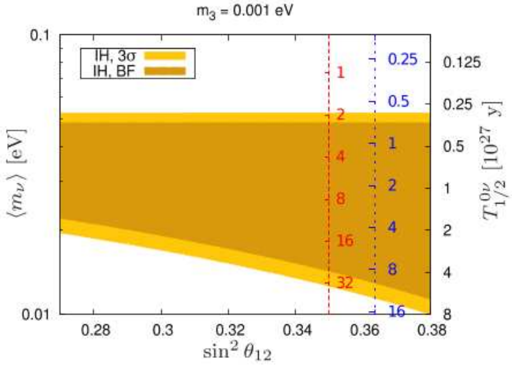
$$\text{IH} \quad : \quad |m_{ee}| \approx \left| c_s^2 c_r^2 \sqrt{\Delta m_a^2} e^{i\delta_{M1}} + s_s^2 c_r^2 \sqrt{\Delta m_a^2 + \Delta m_s^2} \right|$$

@ **vanishing smallest mass**: $m_1 \approx 0$ (**NH**) or $m_3 \approx 0$ (**IH**)

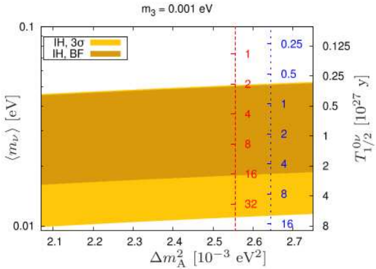
Uncertainty from Neutrino Mixing

- **Effective Mass** – **Large Uncertainty**

$$\text{IH} \quad : \quad c_r^2(1-2s_s^2)\sqrt{\Delta m_a^2} \leq m_{ee}^{\nu} \leq c_r^2\sqrt{\Delta m_a^2}$$

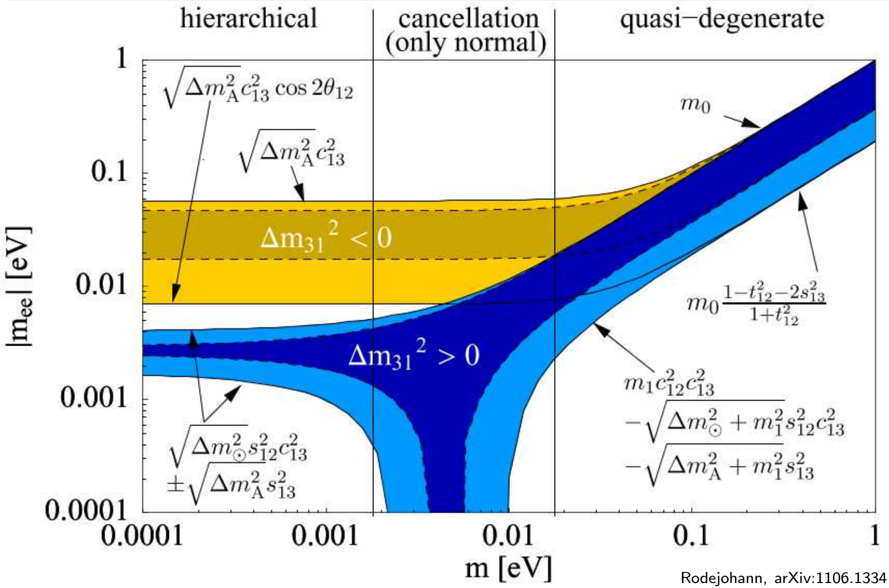


	$\langle m_\nu \rangle^{\rm IH}_{\rm min}$ [eV]	
$\sin^2\theta_{12}$	minimal	maximal
0.270	0.0196	0.0240
0.318	0.0154	0.0189
0.380	0.0100	0.0123



Dueck, Rodejohann & Zuber, arXiv:1103.4152

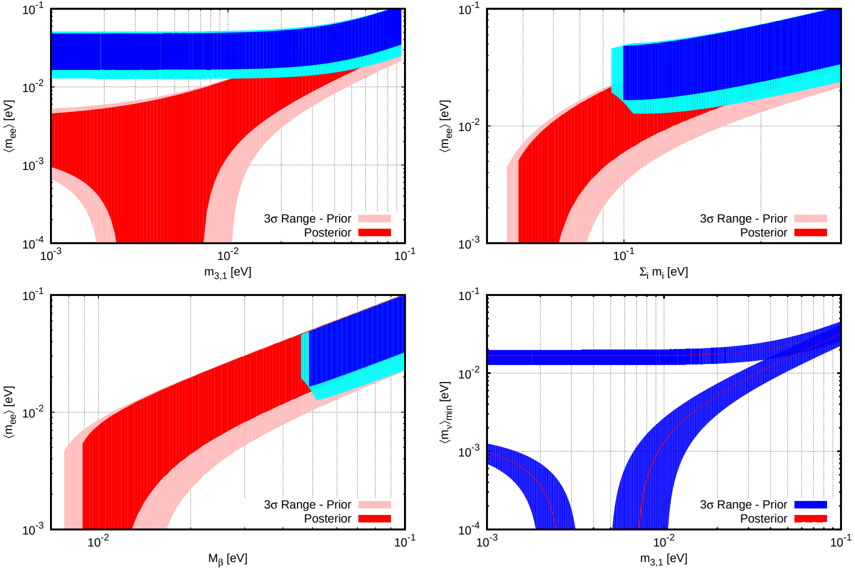
Dependence on Mass Hierarchy



Rodejohann, arXiv:1106.1334

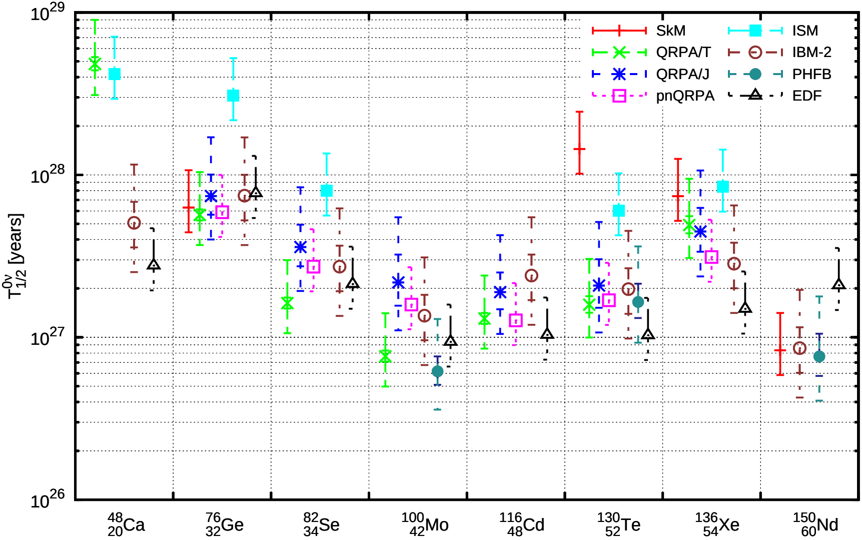
Effect on Effective Mass

SFG & Rodejohann, arXiv:1507.05514



Effect on Lifetime

SFG & Rodejohann, arXiv:1507.05514



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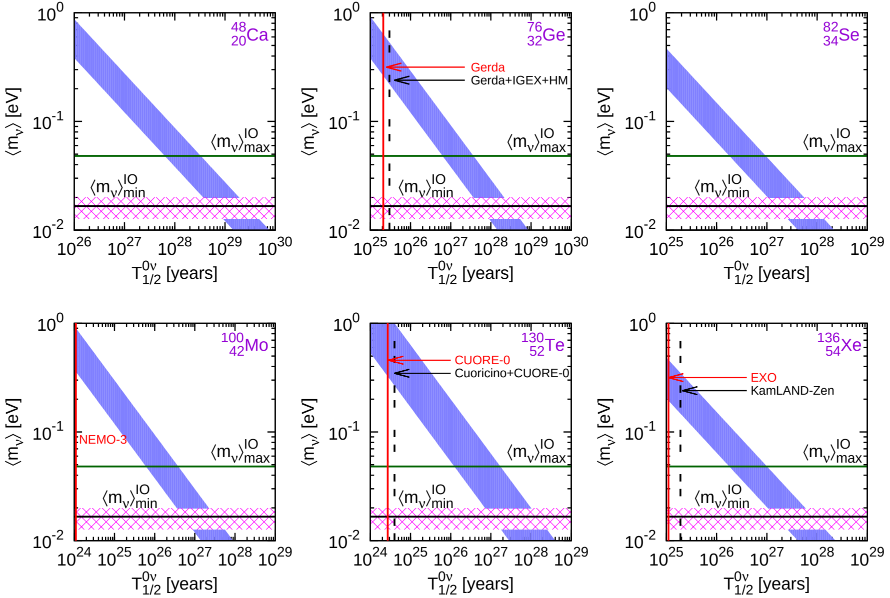

Data Interpretation

- **Half-Life Time**

$$T_{1/2}^{-1} \equiv \mathbf{G^{0\nu}} |\mathcal{M}^{0\nu}|^2 |\mathbf{m_{ee}}|^{\mathbf{2}}$$

Phase Space Factor, **Nuclear Matrix Element** & **Effective Mass**

- **Event Rate**

$$S \equiv st \propto \frac{N_{target}\, t}{T_{1/2}}, \qquad B \equiv bt \propto N_{target}\, t \delta E$$

- **Confidence Level**

$$C \equiv \frac{S}{\sqrt{B+S}} \approx \begin{cases} \sqrt{S} \propto \sqrt{s}\sqrt{t} & S \gg B \\ \frac{S}{\sqrt{B}} \propto \frac{s}{\sqrt{b}}\sqrt{t} & S \ll B \end{cases}$$

- **Lifetime Sensitivity** vs **Required Run Time**

$$T \propto \begin{cases} t & S \gg B \\ \sqrt{\mathbf{t}} & S \ll B \end{cases} \Rightarrow \quad t_{run} \propto \begin{cases} T_{1/2} \\ T_{1/2}^{\mathbf{2}} \end{cases} \propto \begin{cases} |\mathbf{m_{ee}}|^{\mathbf{2}} \\ |\mathbf{m_{ee}}|^{\mathbf{4}} \end{cases}$$