Residual $\mathbb{Z}_2$ Symmetries & Experimental Test

Shao-Feng Ge

(gesf02@gmail.com)

Max-Planck-Institut für Kernphysik

2014-11-10

SFG, Duane A. Dicus, Wayne W. Repko, arXiv:1104.0602
SFG, Duane A. Dicus, Wayne W. Repko, arXiv:1108.0964
Andrew D. Hanlon, SFG, Wayne W. Repko, arXiv:1308.6522
Jarah Evslin, SFG, Kaoru Hagiwara, in preparation

ν Mass & Mixing [Majorana ν]

- **Correlated Mass Matrix:**

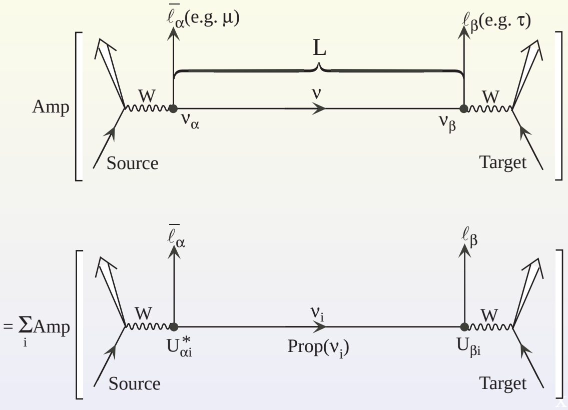
$$\mathcal{L}_\nu = -\frac{1}{2} \begin{pmatrix} \nu_e^T & \nu_\mu^T & \nu_\tau^T \end{pmatrix} \mathcal{C} \begin{pmatrix} A & B_1 & B_2 \\ & C_1 & D \\ & & C_2 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} + \cdots$$

- **Mass Diagonalization:**

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = V^\dagger \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

$$\begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} = V^T \begin{pmatrix} A & B_1 & B_2 \\ & C_1 & D \\ & & C_2 \end{pmatrix} V$$

ν Oscillation



[Kayser, hep-ph/0506165]

$$\nu_\alpha = \sum_i \mathbf{V}_{\alpha \mathbf{i}} \nu_i \rightarrow \boxed{\sum_i \mathbf{V}_{\alpha \mathbf{i}} \mathbf{e}^{\mathbf{i}(\mathbf{E_i t} - \vec{\mathbf{P}}_\mathbf{i} \cdot \vec{\mathbf{x}})} \nu_i} = \boxed{\sum_i \mathbf{V}_{\alpha \mathbf{i}} \mathbf{P_i} \mathbf{V}^\dagger_{\mathbf{i}\beta} \nu_\beta} \equiv \sum_\beta \mathbf{A}_{\alpha\beta} \nu_\beta$$

ν Mass & Mixing

- **Mass & Mixing $\Rightarrow$ Oscillation:**

$$\nu_\alpha(t,L) = \sum_{i\beta} \mathbf{V}_{\alpha i} e^{-i(E_i t - p_i L)} \mathbf{V}^*_{\beta i} \nu_\beta \equiv \sum_\beta \mathbf{A}_{\alpha\beta} \nu_\beta$$

$$\mathbf{P}_{\alpha\beta}\big|_{\alpha\neq\beta} \equiv |\mathbf{A}_{\alpha\beta}|^2 = \sin^2 2\theta \sin^2\left(\delta m^2 \frac{L}{4E}\right)$$

- **Flavor v.s. Mass** Eigenstates:

$$\nu_\alpha = \sum_i \mathbf{V}_{\alpha i}\, \nu_i$$

$$\mathbf{V} = \mathcal{P} \begin{pmatrix} c_s c_r & s_s c_r & s_r e^{-i\delta_D} \\ -s_s c_a - c_s s_a s_r e^{i\delta_D} & +c_s c_a - s_s s_a s_r e^{i\delta_D} & s_a c_r \\ +s_s s_a - c_s c_a s_r e^{i\delta_D} & -c_s s_a - s_s c_a s_r e^{i\delta_D} & c_a c_r \end{pmatrix} \mathcal{Q}$$

with $\mathcal{P} \equiv \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ & $\mathcal{Q} \equiv \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$

[$(s, a, r) \equiv (12, 23, 13)$ for (**solar, atmospheric, reactor**) angles]

ν Oscillation Data

(for NH)	-1σ	Best Value	$+1\sigma$
$\boldsymbol{\delta m^2_s} \equiv \delta m^2_{12}\,(\mathbf{10^{-5}}\mathrm{eV}^{\mathbf{2}})$	7.32	**7.54**	7.80
$\lvert\boldsymbol{\delta m^2_a} \equiv \delta m^2_{13}\rvert\ (\mathbf{10^{-3}}\mathrm{eV}^{\mathbf{2}})$	2.38	**2.44**	2.52
$\sin^2 \boldsymbol{\theta_s}\ (\boldsymbol{\theta_s} \equiv \boldsymbol{\theta_{12}})$	0.291 (32.6°)	0.308 (**33.7°**)	0.325 (34.8°)
$\sin^2 \boldsymbol{\theta_a}\ (\boldsymbol{\theta_a} \equiv \boldsymbol{\theta_{23}})$	0.398 (39.1°)	0.425 (**40.7°**)	0.454 (42.4°)
$\sin^2 \boldsymbol{\theta_r}\ (\boldsymbol{\theta_r} \equiv \boldsymbol{\theta_{13}})$	0.0216 (8.5°)	0.0234 (**8.8°**)	0.0256 (9.2°)
$\boldsymbol{\delta_D}, \boldsymbol{\delta\alpha_i}$	?, **??**	?, **??**	?, **??**

Capozzi, Fogli, Lisi, Marrone, Montanino & Plazzo, arXiv:1312.2878

Evidence of μ–τ Symmetry

- **Two small deviations (1σ level):**

$$-5.9^\circ < \theta_a - 45^\circ < 2.6^\circ \qquad 8.5^\circ < \theta_r < 9.2^\circ$$

with **Best Fit Value**: $\boldsymbol{\theta_a - 45^\circ = -4.3^\circ}$ & $\boldsymbol{\theta_r = 8.8^\circ}$.

- **Zeroth Order Approximation:**

$$\boxed{\theta_a \approx 45^\circ, \qquad \theta_r \approx 0^\circ.}$$

$\Rightarrow$ **CP** & **μ–τ Symmetric** Mass Matrix:

$$M_\nu^{(0)} = \begin{pmatrix} A & \mathbf{B} & \mathbf{B} \\ & \mathbf{C} & D \\ & & \mathbf{C} \end{pmatrix}$$

Mohapatra & Nussinov [hep-ph/9809415], Lam [hep-ph/0104116]

Horizontal Symmetry

[Lam, PRL**101**:121602(2008), PRD**78**:073015(2008)]

- **Mass Matrix** $\mathbf{M}_\nu$ invariant under **Transformation**:

$$\mathbf{G}_\nu^T \mathbf{M}_\nu \mathbf{G}_\nu = \mathbf{M}_\nu$$

- **Diagonalization**:

$$\mathbf{V}_\nu^T \mathbf{M}_\nu \mathbf{V}_\nu = \mathbf{D}_\nu$$

- **Rephasing**:

$$\mathbf{D}_\nu = \mathbf{d}_\nu^T \mathbf{D}_\nu \mathbf{d}_\nu$$

with $\mathbf{d}_\nu^2 = \mathbf{I}_3$ which constrains $\mathbf{d}_\nu = \mathrm{diag}(\pm, \pm, \pm)$.

- Together

$$\begin{aligned} \mathbf{M}_\nu &= \mathbf{G}_\nu^T \mathbf{M}_\nu \mathbf{G}_\nu = \underline{\mathbf{G}_\nu^T \mathbf{V}_\nu^* \mathbf{D}_\nu \mathbf{V}_\nu^\dagger \mathbf{G}_\nu} \\ &= \mathbf{V}_\nu^* \mathbf{D}_\nu \mathbf{V}_\nu^\dagger = \underline{\mathbf{V}_\nu^* \mathbf{d}_\nu^T \mathbf{D}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger} \end{aligned}$$

- **Consequence**: $\mathbf{V}_\nu^\dagger \mathbf{G}_\nu = \mathbf{d}_\nu \mathbf{V}_\nu^\dagger \Leftrightarrow \boxed{\mathbf{G}_\nu = \mathbf{V}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger}$

- **for Leptons:** $\underline{\mathbf{F}_\ell = \mathbf{V}_\ell \mathbf{d}_\ell \mathbf{V}_\ell^\dagger}$ with $\mathbf{d}_\ell = \mathrm{diag}(\mathbf{e}^{\mathbf{i}\beta_1}, \mathbf{e}^{\mathbf{i}\beta_2}, \mathbf{e}^{\mathbf{i}\beta_3})$.

Symmetry v.s. Mixing

[Lam, PRL**101**:121602(2008), PRD**78**:073015(2008)]

- **Two Nontrivial Independent** possibilities of $\mathbf{d}_\nu$:

$$\mathbf{d}_\nu^{(1)} = \mathrm{diag}(-1,1,1), \quad \mathbf{d}_\nu^{(2)} = \mathrm{diag}(1,-1,1), \qquad \mathbf{d}_\nu^{(3)} = -d_\nu^{(1)} d_\nu^{(2)}\,.$$

- θ_s parameterized in terms of $\mathbf{k}$: $\boxed{\tan\theta_s = \sqrt{2}/k}$

$$V_\nu(k) = \begin{pmatrix} \frac{k}{\sqrt{2+k^2}} & \frac{\sqrt{2}}{\sqrt{2+k^2}} & 0 \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2+k^2}} & \frac{k}{\sqrt{2(2+k^2)}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\mathbf{k} = 2 \quad \theta_s = 35.3^\circ$ [TBM]

$\mathbf{k} = \frac{3}{\sqrt{2}} \quad \theta_s = 33.7^\circ$

$\mathbf{k} = \sqrt{6} \quad \theta_s = 30.0^\circ$

- **Two Independent** *Symmetry Transformations* $\mathbf{G_i} = \mathbf{V}_\nu \mathbf{d}_\nu^{(i)} \mathbf{V}_\nu^\dagger$

$$G_1 = \frac{1}{2+k^2}\begin{pmatrix} 2-k^2 & 2k & 2k \\ & k^2 & -2 \\ & & k^2 \end{pmatrix}, \qquad G_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- $\mathbb{Z}_2^s(\times\overline{\mathbb{Z}}_2^s) \times \mathbb{Z}_2^{\mu\tau} \equiv \mathcal{G} = \{\mathbf{E}, \mathbf{G_1}, \mathbf{G_2}(\equiv G_1 G_3), \mathbf{G_3}\}$

Full v.s. Residual

[Lam, PRL**101**:121602(2008), PRD**78**:073015(2008)]

- **Full Symmetries:**

$\mathcal{H} \equiv \mathcal{G} \times \mathcal{F}$	$\mathcal{G}$	$\mathcal{F}$
S_4	$\mathbb{Z}_2^s \times \mathbb{Z}_2^{\mu\tau}$	$\mathbb{Z}_3 \equiv \{I, F, F^2\}$
$\{G_1, G_3, F\}$	$G_1(G_2), G_3$	$F \equiv \text{diag}\,(1, \omega, \omega^2)$

Bottom-Up $\Uparrow$ $\Downarrow$ **Top-Down**

See also Hernandez & Smirnov, 1204.0445, 1212.2149

- **Residual Symmetries:**

$$\boldsymbol{\nu}_\mathbf{i}: \quad \mathcal{G} \equiv \mathbb{Z}_2^s(\overline{\mathbb{Z}}_2^s) \times \mathbb{Z}_2^{\mu\tau} \quad \text{for} \quad d_\nu^i = \text{diag}\,(\pm 1, \pm 1, \pm 1)$$

$$\boldsymbol{\ell}_\mathbf{i}: \quad \mathcal{F} \in U(1) \times U(1) \quad \text{for} \quad d_\ell^i = \text{diag}\,(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$$

Residual Symmetry as Effective Theory

- **Full symmetry HAS TO be Broken!**
  - Fermion needs to acquire mass.
  - Non-trivial mixing $\boxed{V_{\mathrm{PMNS}} = V_\ell^\dagger V_\nu}$
- If mixing is **TRUELY determined by symmetry**, it has to be **residual symmetry**
  - VEVs
  - Yukawa couplings
- **Residual Symmetry** as **Custodial Symmetry**
  - **Gauge symmetry has to be broken**. Otherwise, no mixing.
  - **Weak mixing angle** is a function of gauge couplings, which **cannot be dictated by gauge symmetry** (and VEV).
  - **Weak mixing angle is related to** the physical observables, the **gauge boson masses, by custodial symmetry**.

- **Lepton's Representation:**

$$\begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix} \sim \mathbf{3}\,, \quad \begin{matrix} e_R \sim \mathbf{1} \\ \mu_R \sim \mathbf{1}'\,, \\ \tau_R \sim \mathbf{1}'' \end{matrix} \quad \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \sim \mathbf{3}\,.$$

- **A_4 invariant Lagrangian:**

$$\begin{aligned} \mathcal{L}_\ell &= \mathbf{y_1} \overline{e}_R (\mathbf{1} \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \mathbf{1} \varphi_3^\dagger \tau_L) \\ &+ \mathbf{y_2} \overline{\mu}_R (\omega \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \omega^2 \varphi_3^\dagger \tau_L) \\ &+ \mathbf{y_3} \overline{\tau}_R (\omega^2 \varphi_1^\dagger e_L + \mathbf{1} \varphi_2^\dagger \tau_L + \omega \varphi_3^\dagger \tau_L)\,. \end{aligned}$$

- **Mass term** with $\langle \varphi_i \rangle = v_i$:

$$\mathcal{L}_\ell = \begin{pmatrix} \overline{e}_R & \overline{\mu}_R & \overline{\tau}_R \end{pmatrix} \begin{pmatrix} \mathbf{y_1} & & \\ & \mathbf{y_2} & \\ & & \mathbf{y_3} \end{pmatrix} \begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \omega & \mathbf{1} & \omega^2 \\ \omega^2 & \mathbf{1} & \omega \end{pmatrix} \begin{pmatrix} \mathbf{v_1} & & \\ & \mathbf{v_2} & \\ & & \mathbf{v_3} \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$$

$$||$$

$$\mathbf{v_1} = \mathbf{v_2} = \mathbf{v_2} = \mathbf{v} \Rightarrow U_{\ell,R} = I,\ \mathbf{U_{\ell,L}}(\omega),\ m_{\ell,i} = \mathbf{y_i v}.$$

$$\mathbf{y_1} = \mathbf{y_2} = \mathbf{y_2} = \mathbf{y} \Rightarrow U_{\ell,L} = I,\ \mathbf{U_{\ell,R}}(\omega),\ m_{\ell,i} = \mathbf{y v_i}.$$

Partial Residual Symmetry $\mathbb{Z}_2^s$ or $\overline{\mathbb{Z}}_2^s$

- Although $\mathbb{Z}_2^{\mu\tau}$, represented by $\mathbf{G_3}$, is Broken!
- **No particular reason** for $\mathbb{Z}_2^s$ or $\overline{\mathbb{Z}}_2^s$ to be Broken!

$$\begin{aligned}
\mathbb{Z}_2^s \quad &: \quad \mathbf{G_1(k)} \quad = \quad \frac{1}{2+k^2}\begin{pmatrix} 2-k^2 & 2k & 2k \\ & k^2 & -2 \\ & & k^2 \end{pmatrix}, \\
\overline{\mathbb{Z}}_2^s \quad &: \quad \mathbf{G_2(k)} \quad = \quad \frac{1}{2+k^2}\begin{pmatrix} 2-k^2 & 2k & 2k \\ & -2 & k^2 \\ & & -2 \end{pmatrix}.
\end{aligned}$$

- $\mathbb{Z}_2^s$ & $\overline{\mathbb{Z}}_2^s$ are **Dependent**

$$\mathbf{G_1(k) = G_2(k) G_3}$$

- **DIFFERENT Consequences!!!**

Mixing Dictated by $\mathbb{Z}_2^s$

- Mixing matrix,

$$\mathbf{V}_\nu \equiv \mathcal{P}_\nu \mathcal{U}_\nu \mathcal{Q}_\nu$$

- $\mathbf{G}_\nu = \mathbf{V}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger$ $\left(v_\nu \equiv \mathcal{P}_\nu \mathcal{U}_\nu \mathcal{Q}_\nu \right) \Rightarrow \widetilde{G} \equiv \mathcal{P}_\nu^\dagger \mathbf{G_1} \mathcal{P}_\nu = \mathcal{U}_\nu \mathbf{d}_\nu^{(1)} \mathcal{U}_\nu^\dagger$

$$\widetilde{G}_1 = \frac{1}{2+k^2} \begin{pmatrix} \mathbf{2-k^2} & 2ke^{-i(\phi_1-\phi_2)} & 2ke^{-i(\phi_1-\phi_3)} \\ 2ke^{i(\phi_1-\phi_2)} & \mathbf{k^2} & -2e^{-i(\phi_2-\phi_3)} \\ 2ke^{i(\phi_1-\phi_3)} & -2e^{i(\phi_2-\phi_3)} & \mathbf{k^2} \end{pmatrix}$$

$$\begin{aligned}
\widetilde{\mathbf{G}}_{11} &= s_r^2 - c_r^2 (c_s^2 - s_s^2)\,, \\
\widetilde{G}_{12} = \widetilde{G}_{21}^* &= 2 c_r c_s \left(c_s s_a s_r e^{i\delta_D} - c_a s_s \right)\,, \\
\widetilde{G}_{13} = \widetilde{G}_{31}^* &= -2 c_r c_s \left(c_s c_a s_r e^{i\delta_D} + s_a s_s \right)\,, \\
\widetilde{\mathbf{G}}_{22} &= (c_s^2 - s_s^2) c_a^2 + 4 c_a s_a c_s s_s s_r \cos\delta_D + \left[c_r^2 - (c_s^2 - s_s^2) s_r^2 \right] s_a^2\,, \\
\widetilde{G}_{23} = \widetilde{G}_{32}^* &= \left[3c_s^2 - 2 - c_s^2 (c_r^2 - s_r^2) \right] c_a s_a - 2 \left[(c_a^2 - s_a^2) \cos\delta_D + i \sin\delta_D \right] c_s s_s s_r\,, \\
\widetilde{\mathbf{G}}_{33} &= (c_s^2 - s_s^2) s_a^2 - 4 c_a s_a c_s s_s s_r \cos\delta_D + \left[c_r^2 - (c_s^2 - s_s^2) s_r^2 \right] c_a^2\,.
\end{aligned}$$

Correlation between Physical Observables

$$\mathbf{G}_\nu = \mathbf{V}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger$$

$\mathbb{Z}_2^s$ (G_1)	$\overline{\mathbb{Z}}_2^s$ (G_2)
$\cos \delta_D = \dfrac{(s_s^2 - c_s^2 s_r^2)(\mathbf{c_a^2} - \mathbf{s_a^2})}{4 c_a s_a c_s s_s \mathbf{s_r}}$	$\cos \delta_D = \dfrac{(s_s^2 s_r^2 - c_s^2)(\mathbf{c_a^2} - \mathbf{s_a^2})}{4 c_a s_a c_s s_s \mathbf{s_r}}$
$\sin \theta_r = \pm \left[\pm \sqrt{\mathbf{c_D^2} + \cot^2 2\theta_a} - \mathbf{c_D} \right] \tan 2\theta_a (\tan \theta_s)^{\pm 1}$	
$\dfrac{\delta_r}{\delta_a} = -\dfrac{\tan \theta_s}{\cos \delta_D}$	$\dfrac{\delta_r}{\delta_a} = +\dfrac{\cot \theta_s}{\cos \delta_D}$

Model-Independence

$$\frac{\delta_r}{\delta_a} = -\frac{\tan\theta_s}{\cos\delta_D} \qquad \cos\delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_r},$$

- **Minimal Seesaw** [SFG, He, Yin, JCAP2010]

$$\frac{\delta_r}{\delta_a} = -\frac{\tan\theta_s}{\cos\delta_D}$$

Common origin of μ–τ & CP Breaking

- **Trimaximal Mixing** $(k=2)$ @ A_4/S_4 [King, Luhn, 1107.5332]

$$\sqrt{2}s_a - 1 = -\frac{1}{\sqrt{2}} s_r \cos\delta_D$$

- **Unrealistic Bimaximal Mixing** [Lam, 1105.5166]

$$\left\langle s_s, s_a, s_r e^{i\delta_D} \right\rangle = \left\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}} e^{-\frac{i}{2}\pi} \right\rangle$$

Data $\Rightarrow$ Prediction

	-1σ	Best Value	$+1\sigma$
$\sin^2\theta_s$ (θ_s)	0.291 (32.6°)	0.306 (33.6°)	0.324 (34.7°)
$\sin^2\boldsymbol{\theta_a}$ ($\boldsymbol{\theta_a}$)	0.39 (38.6°)	0.42 (**40.4°**)	0.50 (45.0°)
$\sin^2\boldsymbol{\theta_r}$ ($\boldsymbol{\theta_r}$)	0.013 (6.5°)	0.021 (**8.3°**)	0.028 (9.6°)

Fogli et.al. arXiv:1106.6028

Asymmetric Gaussian Distribution $\mathbb{P}_a$:

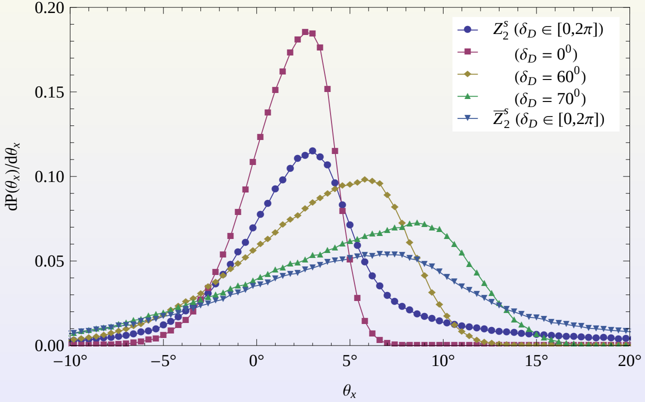
$$\boxed{\mathbb{P}_{\mathbf{a}}(\mathbf{x}-\mathbf{x_0},\boldsymbol{\sigma_l},\boldsymbol{\sigma_u}) = \frac{2\boldsymbol{\sigma_u}}{\boldsymbol{\sigma_l}+\boldsymbol{\sigma_u}}\boldsymbol{\theta}(\mathbf{x}-\mathbf{x_0})\mathbb{P}_{\mathbf{n}}(\mathbf{x}-\mathbf{x_0},\boldsymbol{\sigma_u}) + \frac{2\boldsymbol{\sigma_l}}{\boldsymbol{\sigma_l}+\boldsymbol{\sigma_u}}\boldsymbol{\theta}(\mathbf{x_0}-\mathbf{x})\mathbb{P}_{\mathbf{n}}(\mathbf{x}-\mathbf{x_0},\boldsymbol{\sigma_l})}$$

$$\frac{d\mathbb{P}(\boldsymbol{\theta_r})}{d\theta_r} = \int \delta\left[\theta_r \mp \left(\pm\sqrt{c_D^2+\cot^2 2\theta_a} - c_D\right)\tan 2\theta_a(\tan\theta_s)^{\pm 1}\right]\mathbb{P}_{\mathbf{a}}(\mathbf{s_a^2})\mathbb{P}_{\mathbf{a}}(\mathbf{s_s^2})\,ds_a^2\,ds_s^2\,\frac{d\delta_D}{2\pi}$$

$$\frac{d\mathbb{P}(\boldsymbol{\delta_D})}{d\delta_D} = \int \delta\left\{\delta_D - \arccos\left[\frac{(s_s^2-c_s^2 s_r^2)(c_a^2-s_a^2)}{4c_a s_a c_s s_s s_r}\right]\right\}\mathbb{P}_{\mathbf{a}}(\mathbf{s_a^2})\mathbb{P}_{\mathbf{a}}(\mathbf{s_s^2})\mathbb{P}_{\mathbf{a}}(\mathbf{s_r^2})\,ds_a^2\,ds_s^2\,ds_r^2$$

Prediction of Large θ_r (θ_x)

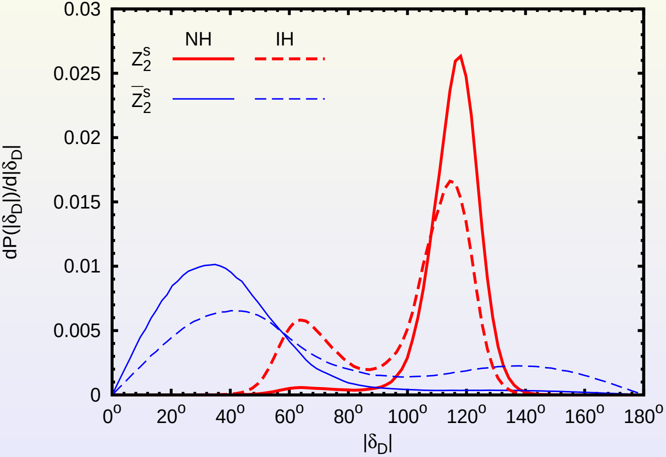
$$\sin\theta_r = \pm\left[\pm\sqrt{\cos^2\delta_D + \cot^2 2\theta_a} - \cos\delta_D\right]\tan 2\theta_a \tan^{\pm}\theta_s$$



MINOS ($\theta_r \in [3^\circ, 9^\circ]/[5^\circ, 11^\circ]$) & **T2K** ($\theta_r \in [5^\circ, 16^\circ]/[6^\circ, 18^\circ]$) @ 90% C.L.

Prediction of Large δ_D

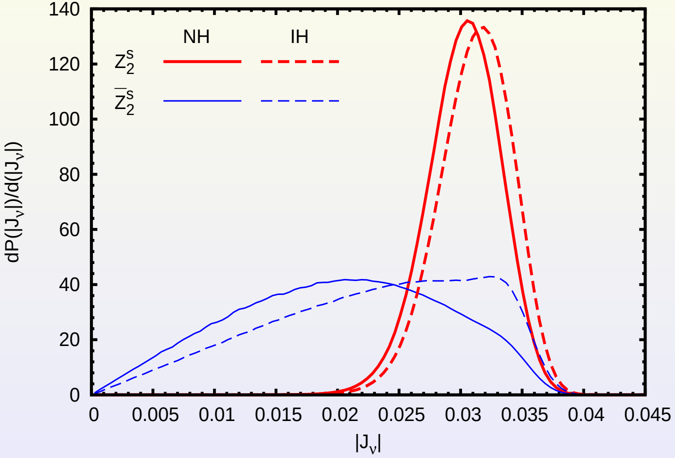
$$\cos\delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_r} \qquad \cos\delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_r}$$



1 σ Indication for $\delta_D = -74^\circ(-110^\circ)$ [Schwetz et.al. 1108.1376]

Prediction of To-Be-Measured J_ν

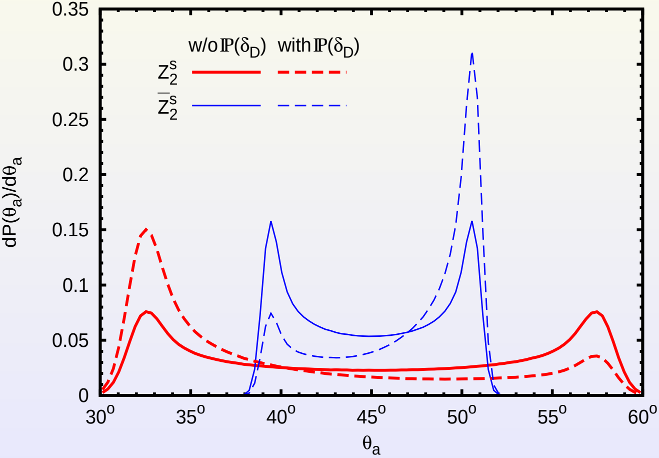
$$J_\nu \equiv c_a s_a c_s s_s c_r^2 s_r \sin\delta_D$$



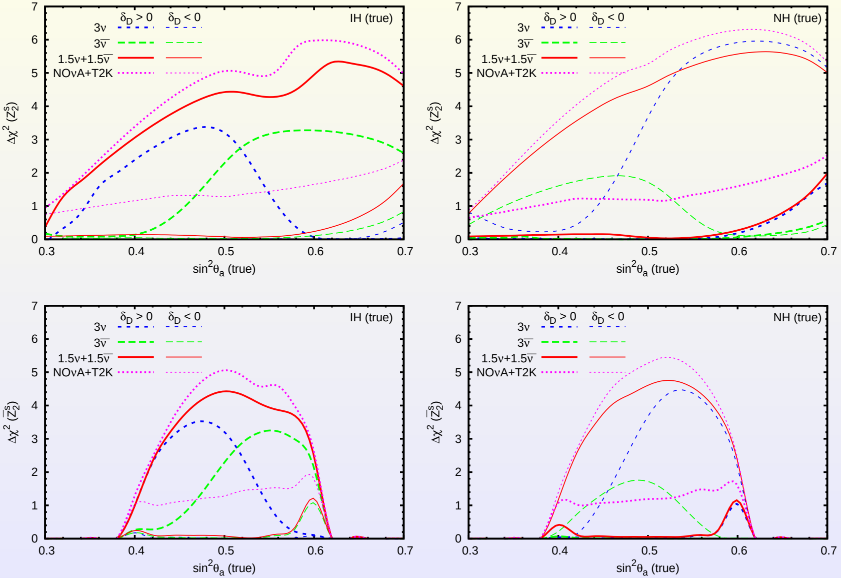
To be further Tested by **T2K** & **NOνA** $\cdots$

Prediction of To-Be-Measured θ_a

$$\tan 2\theta_a = \frac{s_s^2 - c_s^2 s_r^2}{2 c_s s_s s_r \cos \delta_D} \qquad \tan 2\theta_a = \frac{s_s^2 s_r^2 - c_s^2}{2 c_s s_s s_r \cos \delta_D}$$

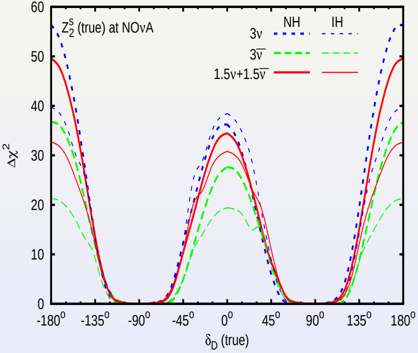


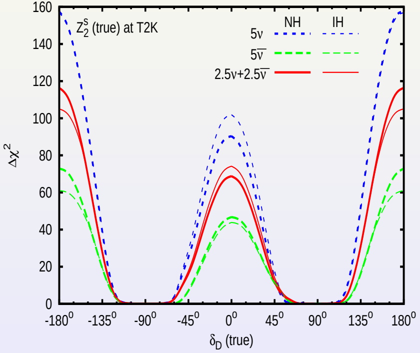
Hierarchy sensitivity (NOvA+T2K)



Distinguishability between $\mathbb{Z}_2^s$ and $\overline{\mathbb{Z}}_2^s$ (NOvA+T2K)

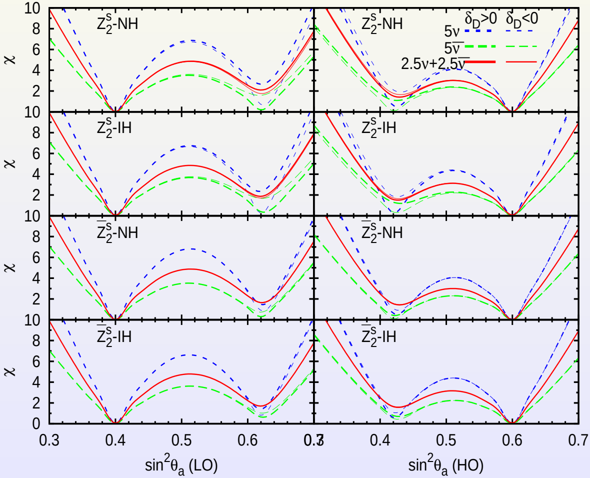
$$\cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_r} \qquad \cos \delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_r}$$





Octant Sensitivity with $\mathbb{Z}_2^s$ or $\overline{\mathbb{Z}}_2^s$ (T2K)

$$\cos \delta_D = \frac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_r} \qquad \cos \delta_D = \frac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_r}$$



Precision Measurement of CP δ_D?

$$\mathbf{G}_\nu = \mathbf{V}_\nu \mathbf{d}_\nu \mathbf{V}_\nu^\dagger$$

$\mathbb{Z}_2^s$ (G_1)	$\overline{\mathbb{Z}}_2^s$ (G_2)
$\cos\delta_D = \dfrac{(s_s^2 - c_s^2 s_r^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_r}$	$\cos\delta_D = \dfrac{(s_s^2 s_r^2 - c_s^2)(c_a^2 - s_a^2)}{4 c_a s_a c_s s_s s_r}$

(for NH)	-1σ	Best Value	$+1\sigma$
$\sin^2\theta_s$ $(\theta_s \equiv \theta_{12})$	0.291 (32.6°)	0.308 $(\mathbf{33.7^\circ})$	0.325 (34.8°)
$\sin^2\theta_a$ $(\theta_a \equiv \theta_{23})$	0.398 (39.1°)	0.425 $(\mathbf{40.7^\circ})$	0.454 (42.4°)
$\sin^2\theta_r$ $(\theta_r \equiv \theta_{13})$	0.0216 (8.5°)	0.0234 $(\mathbf{8.8^\circ})$	0.0256 (9.2°)
δ_D	**?**	**?**	**?**

Capozzi, Fogli, Lisi, Marrone, Montanino & Plazzo, arXiv:1312.2878

The Dirac CP Phase δ_D @ Accelerator Exp

- To leading order in $\alpha = \frac{\delta M^2_{21}}{|\delta M^2_{31}|} \sim 3\%$, the oscillation probability relevant to measuring δ_D @ **T2(H)K**,

$$
\begin{aligned}
P_{\substack{\nu_\mu \to \nu_e \\ \overline{\nu}_\mu \to \overline{\nu}_e}} \approx & \, 4 s_a^2 c_r^2 s_r^2 \sin^2 \phi_{31} \\
& - 8 c_a s_a c_r^2 s_r c_s s_s \sin\phi_{21} \sin\phi_{31} \left[\cos\delta_D \cos\phi_{31} \pm \sin\delta_D \sin\phi_{31}\right]
\end{aligned}
$$

for ν & $\overline{\nu}$, respectively. $[\phi_{ij} \equiv \frac{\delta m^2_{ij} L}{4 E_\nu}]$

- $\nu_\mu \to \nu_\mu$ Exps measure $\sin^2(2\theta_a)$ precisely, but not $\sin^2\theta_a$.

- Run both ν & $\overline{\nu}$ modes @ first peak $[\phi_{31} = \frac{\pi}{2}, \phi_{21} = \alpha\frac{\pi}{2}]$,

$$
\begin{aligned}
P_{\overline{\nu}_\mu \to \overline{\nu}_e} + P_{\nu_\mu \to \nu_e} &= 2 s_a^2 c_r^2 s_r^2 \,, \\
P_{\overline{\nu}_\mu \to \overline{\nu}_e} - P_{\nu_\mu \to \nu_e} &= \alpha\pi \sin(2\theta_s) \sin(2\theta_r) \sin(2\theta_a) \cos\theta_r \sin\delta_D \,.
\end{aligned}
$$

The Dirac CP Phase δ_D @ Accelerator Exp

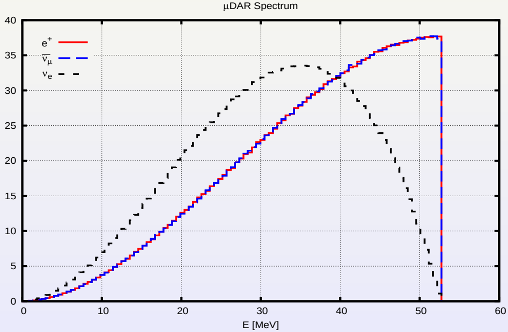
Accelerator experiment, such as **T2(H)K**, uses off-axis beam to compare ν_e & $\overline{\nu}_e$ appearance @ the oscillation maximum.

- **Disadvantages**:
  - **Efficiency**:
    - Proton accelerators produce ν more efficiently than $\overline{\nu}$ ($\sigma_\nu > \sigma_{\overline{\nu}}$).
    - The $\overline{\nu}$ mode needs more beam time [$T_{\overline{\nu}} : T_\nu = 2 : 1$].
    - Undercut statistics $\Rightarrow$ Difficult to reduce the uncertainty.
  - **Degeneracy**:
    - Only $\sin\delta_D$ appears in $P_{\nu_\mu \to \nu_e}$ & $P_{\overline{\nu}_\mu \to \overline{\nu}_e}$.
    - Cannot distinguish δ_D from $\pi - \delta_D$.
- **Solution**:

  Measure $\overline{\nu}$ mode with μ^+ decay @ rest (μDAR)

μDAR ν Oscillation Experiments

- A cyclotron produces 800 MeV proton beam @ fixed target.
- Produce $\pi^\pm$ which stops &
  - π^- is absorbed,
  - π^+ decays @ rest: $\pi^+ \to \mu^+ + \nu_\mu$.
- μ^+ stops & decays @ rest: $\boldsymbol{\mu}^+ \to \mathbf{e}^+ + \overline{\boldsymbol{\nu}}_{\boldsymbol{\mu}} + \boldsymbol{\nu}_{\mathbf{e}}$.

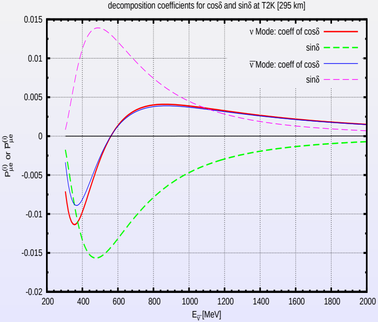


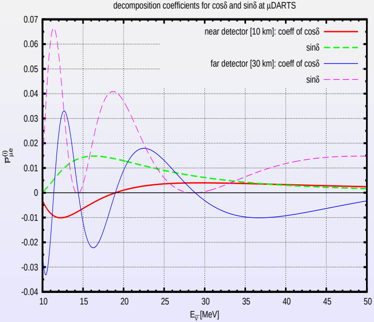
- $\overline{\nu}_\mu$ travel in all directions, oscillating as they go.
- A detector measures the $\overline{\nu}_e$ from $\overline{\boldsymbol{\nu}}_{\boldsymbol{\mu}} \to \overline{\boldsymbol{\nu}}_{\mathbf{e}}$ **oscillation**.

Accelerator + μDAR Experiments

Combining $\nu_\mu \to \nu_e$ **@ accelerator** [narrow **peak @ 500 MeV**] & $\overline{\nu}_\mu \to \overline{\nu}_e$ **@ μDAR** [wide **peak $\sim$ 45 MeV**] solves the 2 problems:

- **Efficiency**:
  - $\overline{\nu}$ @ high intensity μDAR is plentiful enough.
  - Accelerator Exps can devote all run time to the ν mode. With same run time, the statistical uncertainty drops by $\sqrt{3}$.
- **Degeneracy**: (**decomposition in propagation basis** [1309.3176])





DAEδALUS Project

- It's the **FIRST** proposal along this line:
  - **3** μDAR with **3** high-intensity cyclotron complexes.
  - **1** detector.
  - Different baselines: **1.5**, **8** & **20** km to break degeneracies.
- **Disadvantages**:
  - The scattering lepton from IBD @ low energy is **isotropic**.
  - **Cannot** distinguish $\overline{\nu}_e$ from different sources
  - Baseline **cannot be measured**.
  - Cyclotrons **cannot** run simultaneously (20$\sim$25% duty factor).
  - **Large** statistical uncertainty.
  - **Higher intensity** is necessary.
  - **Expensive** & Technically **challenging**.

New Proposals

1 μDAR source $+$ **2** detectors

Advantages:

- Full (**100%**) duty factor!
- **Lower** intensity: $\sim$ **9mA** [$\sim$ **4**$\times$ lower than DAEδALUS]
- Not far beyond the current state-of-art technology of cyclotron [**2.2mA** @ Paul Scherrer Institute]
- MUCH **cheaper** & technically **easier**.
  - Only one cyclotron.
  - Lower intensity.

Disadvantage?

- A second detector!
  - **μDAR** with **T**wo **S**cintillators (**μDARTS**) [1401.3977]
  - **T**okai '**N** **T**oyama to(**2**) **K**amioka (**TNT2K**) [1412.xxxx]

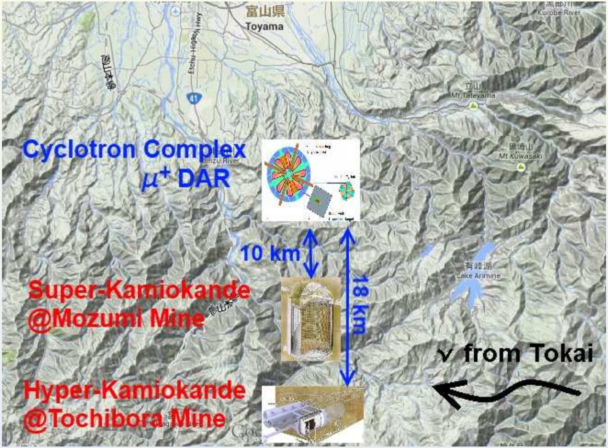
μDARTS – JUNO & RENO50

- **Two detectors** are suggested to overcome the **unknown energy response**. [Ciuffoli et al., PRD 2014; 1307.7419]

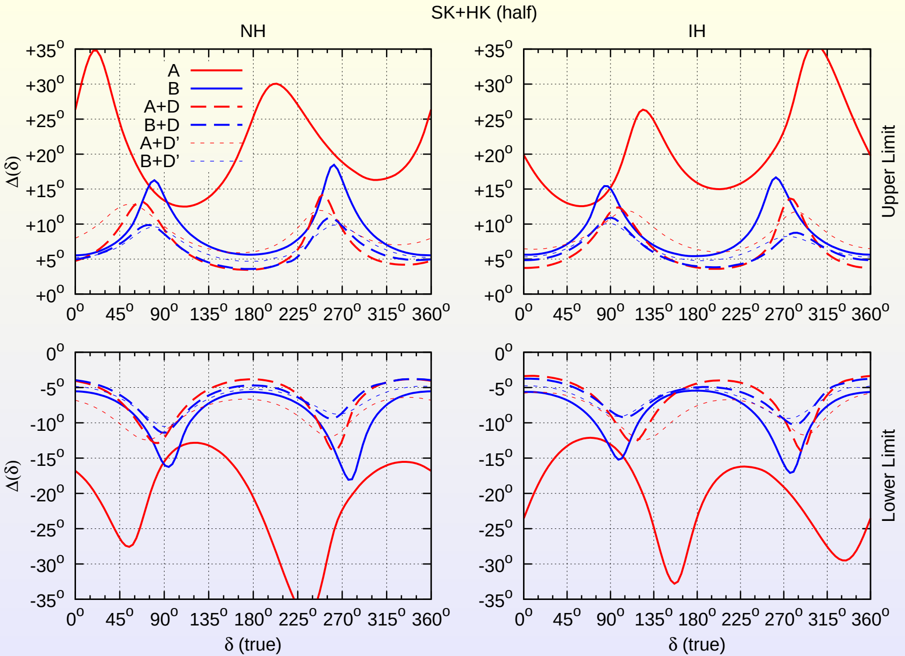
- China Atomic Energy Center has a proposal for cyclotron

TNT2K

- T2(H)K + μSK + μHK



- μDAR is also useful for **material**, **medicine** industries in Toyama



A: ν only, B: $\nu + \overline{\nu}$, D: $+\mu$DAR, D':$+\mu$DAR+Gd

Simulated with NuPro

Summary

- **Horizontal Symmetry**
  - $\mathbb{Z}_2^{\mu\tau}$ $\left[G_3 \text{ v.s. } d_\nu^{(3)}\right]$
  - $\mathbb{Z}_2^s$ $\left[G_1 \text{ v.s. } d_\nu^{(1)}\right]$ & $\overline{\mathbb{Z}}_2^s$ $\left[G_2 \text{ v.s. } d_\nu^{(2)}\right]$
- **Residual Symmetry** as **Custodial Symmetry**
  - Full symmetry has to be broken. Otherwise, no mixing.
  - Mixing angles dictated by residual symmetry.
  - Physical observables, mixing angles & CP phase, are correlated.
- **Phenomenological consequences**
  - Nonzero θ_r
  - Large δ_D
  - Non-maximal θ_a
  - Distinguishing $\mathbb{Z}_2^s$ & $\overline{\mathbb{Z}}_2^s$ with NOνA & T2K
- **μDAR Experiment**
  - Full (100%) duty factor
  - Low intensity, technically easier
  - Cheaper
  - JUNO, RENO50, SK+HK

Thank You!