Dark matter self-interactions in the matter power spectrum

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Based on
2201.06551 and 2105.03429 (JHEP 12 (2021) 139)
in collaboration with Michele Redi and Andrea Tesi

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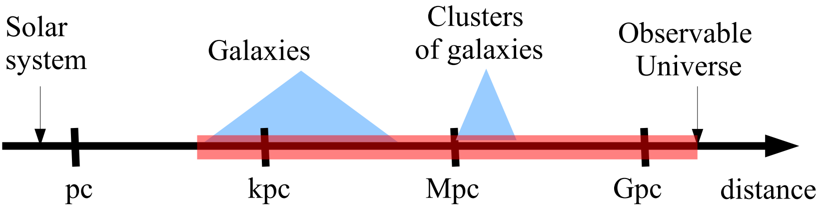
Evidences for Dark Matter



What do we know about Dark Matter ?

DARK MATTER

$J = ?$

Mass $m = ?$
Mean life $\tau = ?$

DECAY MODES	Fraction (Γ_i/Γ)	Confidence level	p (MeV/c)
?	?	?	?

- No electric charge, no color charge (Smith et al. '79, Perl et al. '01).
- Non-relativistic at the time of formation of the first structures (White, Frenk, Davis '83).
- Life time longer than the age of the Universe.

$\Longrightarrow$ Evidence for physics beyond the SM.
$\Longrightarrow$ Could be a nightmare scenario?

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Outline

- Introduction: Matter power spectrum
- Perturbation equations
- Background distribution function
- Results and Bounds
- Example nightmare scenario dark-QCD
- Conclusions & Outlook

Introduction: Matter power spectrum

- Consider density contrast

$$\delta(\vec{x}, t) = \frac{\rho(\vec{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)}$$

- Time evolution of the density contrast governed by perturbation equations. Predict statistical properties of the fluctuation field. e.g. waves on a lake.
- Two-point correlation function of density contrast

$$\langle \delta(\vec{k}, t_0)\delta(\vec{k}', t_0)\rangle = (2\pi)^3 \delta^{(3)}(\vec{k} + \vec{k}')P_m(k)\,,$$

- Useful! Can be computed and measured at some scales!

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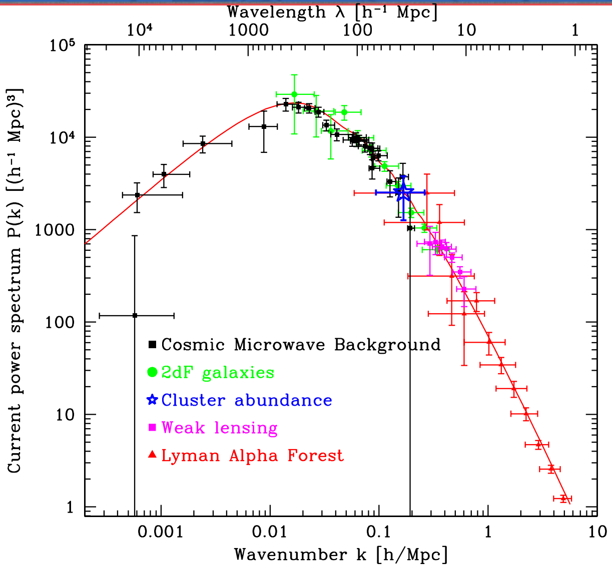
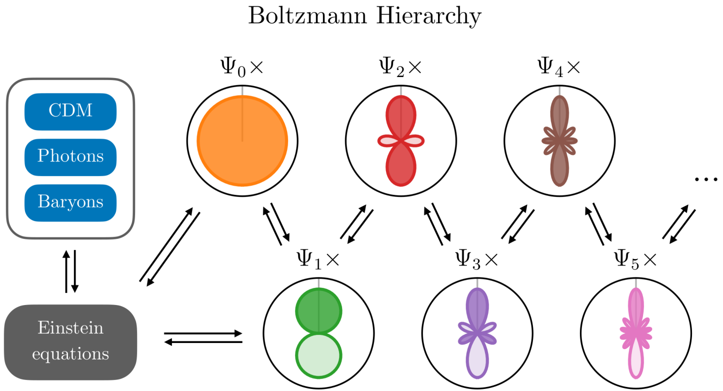
Matter power spectrum from data Tegmark & Zaldarriaga '02



Linear perturbation equations



Ji, Kamionkowski, Bernal ’22

$\rightarrow$ Use Boltzmann solvers like CLASS.

Linear perturbation equations

$$f(\vec{x},\vec{q},\eta) = \underbrace{f_0(q,\eta)}_{\text{background distribution}} \underbrace{[1+\Psi(\vec{x},\vec{q},\eta)]}_{\text{linear perturbation}}$$

Phase space distribution obeys Boltzmann equations Ma and Bertschinger '95

$$\dot{\Psi}+i\frac{q}{\epsilon}(\vec{k}\cdot\vec{n})\Psi+\frac{d\log f_0}{d\log q}\left[\dot{\phi}-i\frac{\epsilon}{q}(\vec{k}\cdot\vec{n})\psi\right] = \underbrace{\frac{1}{f_0}\left(\frac{\partial f}{\partial\eta}\right)_{\text{coll}}}_{\text{microphysics: interactions}} - \frac{d\log f_0}{d\eta}\Psi\,.$$

ϕ,ψ scalar metric perturbations in Newtonian gauge. In spherical harmonics

$$\begin{aligned}
\dot{\Psi}_0+\frac{q}{\epsilon}k\Psi_1+\dot{\phi}\frac{d\log f_0}{d\log q} &= -\frac{d\log f_0}{d\eta}\Psi_0\,,\\
\dot{\Psi}_1-\frac{q}{3\epsilon}k\left(\Psi_0-2\Psi_2\right)+\frac{\epsilon}{3q}k\psi\frac{d\log f_0}{d\log q} &= -\frac{d\log f_0}{d\eta}\Psi_1\,,\\
\dot{\Psi}_\ell-\frac{q}{(2\ell+1)\epsilon}k\left[\ell\Psi_{\ell-1}-(\ell+1)\Psi_{\ell+1}\right] &= \frac{1}{f_0}\left(\frac{\partial f}{\partial\eta}\right)^{(\ell)}_{\text{coll}}-\frac{d\log f_0}{d\eta}\Psi_\ell\,,\ell\geq 2\,.
\end{aligned}$$

Linear perturbation equations: perfect fluid

- $\tau_{\rm coll} \to 0$ higher multipoles relaxes quickly to zero, i.e. $\Psi_{\ell\geq 2} = 0$.
- Only relevant quantities: density perturbation (δ), velocity divergence (θ).
- Independently of the explicit form of f_0

$$
\begin{aligned}
\dot{\delta} + (1+w)\theta + 3H_\eta\delta\left(c_s^2 - w\right) &= 3(1+w)\dot{\phi}\,, \\
\dot{\theta} + H_\eta(1-3w)\theta + \frac{\dot{w}}{1+w}\theta - \frac{c_s^2}{1+w}k^2\delta &= k^2\psi\,,
\end{aligned}
$$

$w = p/\rho$ is the equation of state of the fluid. $w = p/\rho$ and the sound speed c_s^2 are related by the differential equation $c_s^2 = \dot{p}/\dot{\rho}$.

$$
\mathrm{rel}: \quad c_s^2 = w = \frac{1}{3}, \qquad \mathrm{non-rel}: \quad c_s^2 = \frac{5}{3}w = \frac{1}{3}\frac{a_*^2}{a^2}\,,
$$

where a_* can be obtained, for example, by matching to the non-relativistic regime of f_0.

The role of interactions and approximations

- We are interested in DM candidates which are warm and $\mathcal{O}$(KeV). Relaxation time approximation:

$$\frac{1}{f_0}\left(\frac{\partial f}{\partial \eta}\right)^{(\ell)}_{\mathrm{coll}} \approx -\frac{1}{\tau_{\mathrm{coll}}}\Psi_\ell \,,$$

$$\tau_{\mathrm{coll}} \equiv \frac{a^{-1}}{n(a)\sigma_{\mathrm{el}} v} \approx 1.6\times 10^5 \mathrm{Mpc}\left(\frac{0.12}{\Omega h^2}\right)\left(\frac{1\,\mathrm{cm}^2/\mathrm{g}}{\sigma_{\mathrm{el}}/M}\right)\times\frac{a^2}{v(a)}\,.$$

$v(a) = 1/\sqrt{3}\, a_\star/a$

- $\tau_{\mathrm{coll}} \to \infty$ (collisionless)
  $\tau_{\mathrm{coll}} \to 0$ (perfect fluid)

Distribution functions and matching

- When relativistic and in kinetic eqbn. $T_D = T_{0,\mathrm{rel}}/a$ and $T_{0,\mathrm{rel}}$ is the temperature of DM today if it self-decoupled while relativistic

$$f_0(q,\eta) = g_s \left[\exp\left(\frac{E(q)-\mu}{T_D}\right) \pm 1\right]^{-1} \equiv f_0^{\mathrm{rel}} = \left[\exp\left(\mathrm{q}-\mu_0\right) \pm 1\right]^{-1}$$

and $\mathrm{q} \equiv q/T_{0,\mathrm{rel}}$.

- non-relativistic: $T_D \sim 1/a^2$. Define $T_{0,\mathrm{NR}}$ to be the temperature that DM has today if it self-decoupled in this regime

$$f_0^{\mathrm{NR}} = e^{\mu_0^{\mathrm{NR}}} \exp\left(-\alpha \mathrm{q}^2\right), \qquad \alpha \equiv \frac{(T_{0,\mathrm{rel}})^2}{2MT_{0,\mathrm{NR}}}.$$

Matching

- **Number conservation**.

$$na^3 = \frac{1}{(2\pi)^3} \int d^3 q f_0(q,t)$$

- **Entropy conservation**.

$$sa^3 \equiv \frac{(\rho + p - \mu n)}{T} a^3 = -\frac{1}{(2\pi)^3} \int d^3 q \left[f_0 \log f_0 \pm (1 \mp f_0) \log(1 \mp f_0) \right]$$

$\rightarrow \alpha$ uniquely determined! Results are exact if the system evolves from relativistic to non- relativistic in kinetic equilibrium.

Matching

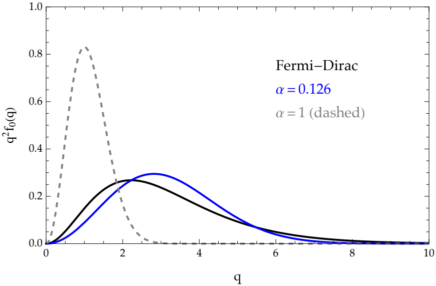
For fermions we find

$$\alpha\Big|_{\text{fermion}} = \frac{\pi^{1/3} e^{\frac{5}{3} - \frac{14\pi^4}{405\zeta(3)}}}{(6\zeta(3))^{2/3}} = 0.126164\,,$$

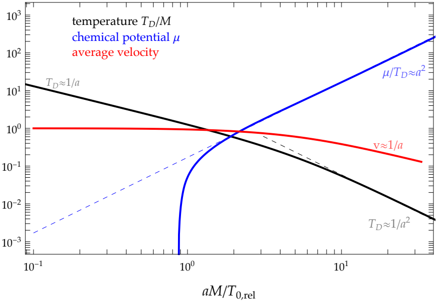
while for bosons,

$$\alpha\Big|_{\text{boson}} = \frac{\pi^{1/3} e^{\frac{5}{3} - \frac{4\pi^4}{135\zeta(3)}}}{4\zeta(3)^{2/3}} = 0.155395\,.$$

Matching

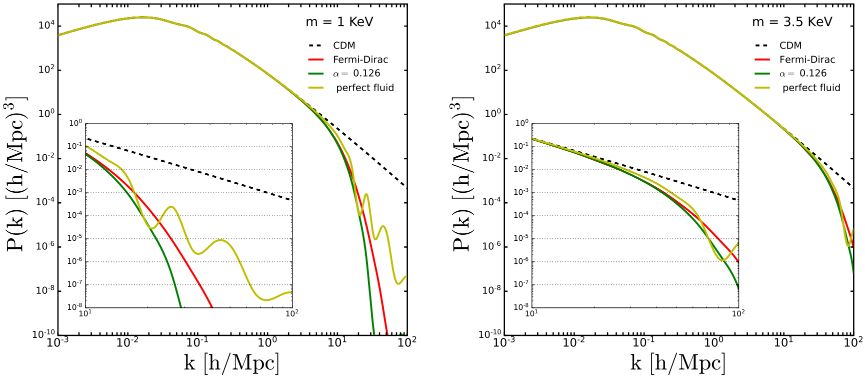


Fermi–Dirac
$\alpha = 0.126$
$\alpha = 1$ (dashed)
$q^2 f_0(q)$
q

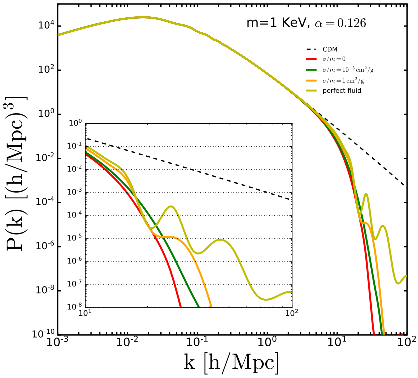


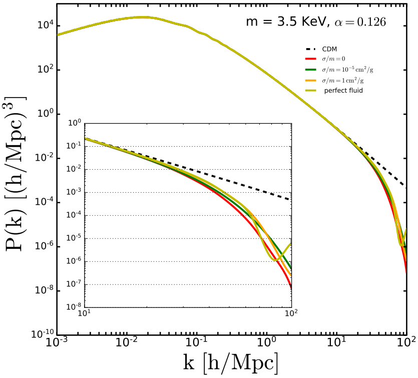
temperature T_D/M
chemical potential μ
average velocity
$T_D \approx 1/a$
$\mu/T_D \approx a^2$
$v \approx 1/a$
$T_D \approx 1/a^2$
$aM/T_{0,\mathrm{rel}}$

Results I: Power spectrum fermion WDM



Results II: Power spectrum fermion WDM with self interactions





Setting bounds: Area criteria

- High-resolution quasars (at redshifts $z \approx 2-6$) data-set of MIKE/HIRES+XQ-100 Irsic et al. '17
- Sets bounds on the linear matter power spectrum for $k \in [0.5, 20]h/\mathrm{Mpc}$. Little dependence on DM models, excludes at 95% CL $\mathrm{m_{WDM}} > 3.5\mathrm{KeV}$ Murgia et al. '17
- A measure to quantify the deviation of the area under the curve with respect to cdm case Viel et al. '15, DEramo et al. '20,Egana-Ugrinovic et al. '21,Yunis et al. '21
- $$P_{1D}(k) \equiv \int_k^{k_\infty} \frac{dK}{2\pi} K P_m(K)\,,$$

$$r(k) \equiv \frac{P_{1D}(k)}{P_{1D,\mathrm{cdm}}(k)}\,, \qquad \delta A \equiv 1 - \frac{1}{k_{\max} - k_{\min}} \int_{k_{\min}}^{k_{\min}} dk\, r(k)\,.$$

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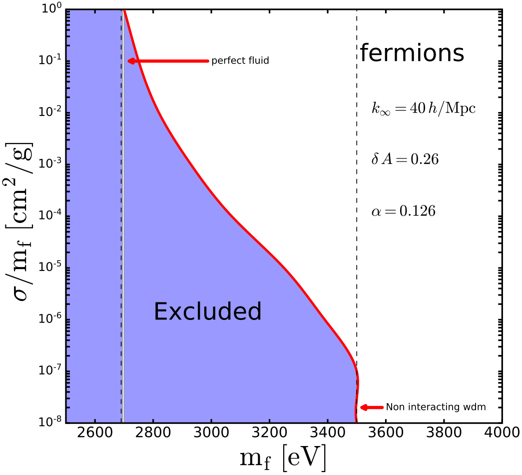
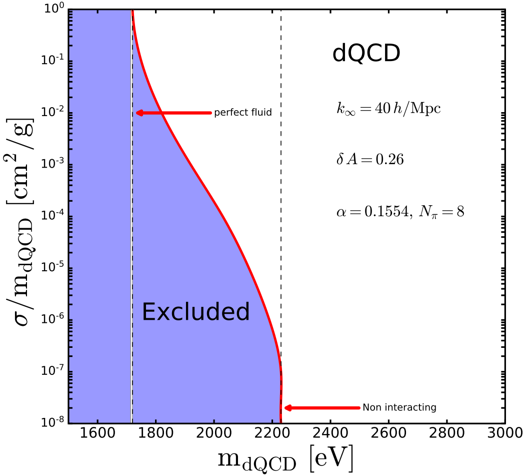
Bounds on fermion WDM

$$M_{\rm WDM}^{\rm int} > 2.7(3.5)\,\mathrm{KeV}$$



Bounds on Bosonic WDM



$$\boxed{M_{\rm boson} > 1.7(2.2)\,\mathrm{KeV}}$$

Summary

- Matter power spectrum can be used to set constraints on DM particles $\mathcal{O}$(KeV).
- Determined DM background distribution in the non-relativistic regime using general entropy and number conservation. Captures all the essential physics roughly independent of the initial relativistic dist!
- Decrease in mass suppresses the matter power spectrum, while increase in $\sigma_{\rm el}$ increases power at small scales!
- In the limit of large DM self interactions, DM can be thought of as a perfect cosmological fluid. Very easy to solve.

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Realization of a nightmare scenario: Dark QCD

RG, Redi and Tesi 2105.03429 (JHEP 12 (2021) 139)

Confining dark sectors and cosmological history

- Can dark matter (DM) be a baryon/pion of new confining dark sectors? $\Longrightarrow$ composite DM Bai, Hill '10 + Boddy et.al. '14 +Gresham, Lou, Zurek '17 + Bai, Long, Lu '18 + many more
- Cosmologically accidentally stable, like protons $\Longrightarrow$ dark baryon number Antipin et al. '15 + Niel et al. '16 + Mitradate et al '17 + Contino et al. '18 + Redi et al '18
- Here we focus on

$$\int d^4x \sqrt{-g} \left[\mathcal{L}_{\rm SM} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \bar{\psi}_i \left(\not{D} - m_i \right) \psi_i + \sum \frac{\mathcal{O}_{\rm SM} \mathcal{O}_{\rm dark}}{M_{\rm Pl}^{\#}} \right].$$

with possibility of both dark-pion and -baryon DM!

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Dark QCD Model

- Asymptotically free non-abelian theories, e.g. SO(N), Sp(N), SU(N). Here we consider SU(N) with N_F light flavours.
- Dark Baryons Antipin et al '15: Similar to SM we choose $M_B \sim 10f$
  - for $N_F = 1$, baryons are spin $N/2$ antisymmetric combinations of N quarks.
  - For N_F even, baryons are spin 0 particles in the symmetric representation of the flavor group.
  - For N_F odd, baryons are spin 1/2 particles in the octet-like representation of flavor.
- Stability: Accidental dark-baryon number $\mathcal{Q}_i \to e^{i\alpha}\mathcal{Q}_i \implies B = \epsilon^{i_1\cdots i_n}\mathcal{Q}_{i_1}^{\{\alpha_1} \cdots \mathcal{Q}_{i_n}^{\alpha_n\}}$

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- Asymptotically free non-abelian theories, e.g. SO(N), Sp(N), SU(N). Here we consider SU(N) with N_F light flavours.
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  - for $N_F = 1$, baryons are spin $N/2$ antisymmetric combinations of N quarks.
  - For N_F even, baryons are spin 0 particles in the symmetric representation of the flavor group.
  - For N_F odd, baryons are spin $1/2$ particles in the octet-like representation of flavor.
- Stability: Accidental dark-baryon number $\mathcal{Q}_i \to e^{i\alpha}\mathcal{Q}_i \implies B = \epsilon^{i_1\cdots i_n}\mathcal{Q}_{i_1}^{\{\alpha_1} \cdots \mathcal{Q}_{i_n}^{\alpha_n\}}$

Dark QCD Model II

- Global chiral symmetry SU(N_F)$\times$ SU(N_F) $\to$ SU(N_F)

$$\mathcal{L}_\pi = \frac{f^2}{4}\mathrm{Tr}(\partial_\mu U)^2 + b\mathrm{Tr}[MU + h.c.] + \mathrm{WZW}, \qquad U = \exp[i\pi/f]$$

and $\quad M_{ij} = m_i\delta_{ij}$. Resulting in $N_F^2 - 1$ goldstone bosons in the adjoint

- Dark pions RG, Michele Redi, Andrea Tesi: Similar to SM we choose $M_\pi < 5f$
- Stability: not absolute. Violated by

$$\frac{1}{\Lambda_5}\bar{\Psi}^i\gamma^5\Psi^j|H|^2 + \frac{1}{\Lambda_6^2}\bar{\Psi}^i\gamma^\mu\gamma^5\Psi^j\bar{f}\sigma^\mu f\,.$$

$\langle 0|\bar{\Psi}\gamma^5\Psi|\pi\rangle = c\,4\pi f^2 \implies$ mixing with higgs $\frac{4\pi f^2}{\Lambda_5}|H^2|\pi$

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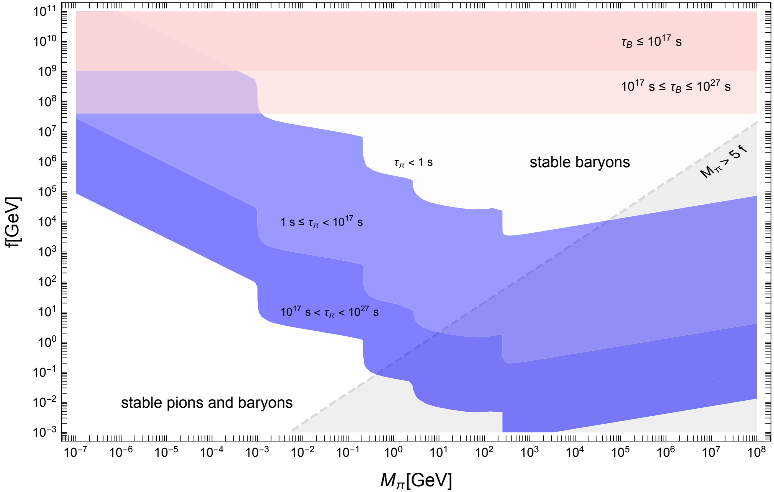
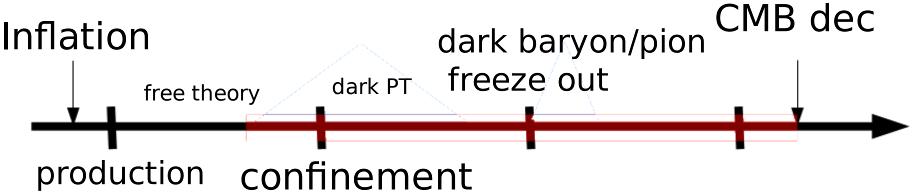
DM stability



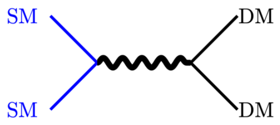
Dark QCD cosmology



Tree level gravitational production

Our dQCD sector is completely secluded from SM. Only feeble interactions!

- From SM plasma: tree level gravitational production Garny et al '16, '17 + Ema et al. '18 + Tang et al. '16 and from higher dimensional operators



$$\mathcal{A} = \frac{1}{M_p^2 s}\left(T^{\rm SM}_{\mu\nu}T^{\rm DM}_{\alpha\beta}\eta^{\mu\alpha}\eta^{\nu\beta} - \frac{1}{2}T^{\rm SM}T^{\rm DM}\right)$$

yield is controlled by reheating temperature:

$$Y_{\rm D} = 6\times 10^{-6}\, c_D \left(\frac{T_R}{M_{\rm Pl}}\right)^3 .$$

and

$$f_D(T,p) \approx \frac{2\pi^4 g_*}{135} Y_D \frac{p\, e^{-p/T}}{T} ,$$

- Higher dimensional operators

Planck suppressed operators

Can be more important than gravitational production depending on dimensionality

- phenomenologically relevant operators:

$$\frac{1}{\Lambda_{\mathrm{UV}}^{d-2}}|H|^2\mathcal{O}\,, \qquad\qquad [\mathcal{O}] = d\,,$$

- yield is controlled by reheating temperature and the effective scale:

$$Y_D = \int_0^{T_R} \frac{dT}{T}\frac{\langle\sigma v\rangle s}{H}Y_{\mathrm{eq}}^2 = a_{\mathcal{O}}\frac{135\sqrt{5/2}}{4(2d-5)g_*^{3/2}\pi^7}\left(\frac{T_R}{\Lambda_{\mathrm{UV}}}\right)^{2d-5}\frac{M_{\mathrm{Pl}}}{T_R}\,.$$

with

$$\langle\sigma v\rangle = \frac{1}{g_i^2}\frac{a_{\mathcal{O}}}{4\pi}\frac{T^{2d-6}}{\Lambda_{\mathrm{UV}}^{2d-4}}\,.$$

Dark phase transition I

- First order or cross over? depends on N and N_F
- pure gluonic theories: first order Paneoro '09 + Lucini et al. '12 and Brambilla et al. '14
- with light fermions: weakly first order for $3 \leq N_F \lesssim 4N$ for $N > 3$ Brambilla et al. '14, which is expected to be adiabatic with the critical temperature $T_c \simeq \mathcal{O}(1)\, f$ Borsanyi '12

Dark phase transition II

- The system reorganizes in color neutral states. SM temperature when this happens is T_Λ
- In confined phase baryons and pions are the physical degrees of freedom. As pions are light $M_\pi < 5f$, they are relativistic at production.
- Baryons are heavy $M_B \sim 10f$. Both pions and baryons are in thermal eqbn with the dark plasma
- Ratio of temperatures $\xi \equiv T_D/T|_{T_\Lambda}$ right after the phase transition

$$\frac{\xi}{\xi_0} \approx \left(\frac{2(N^2-1)+4NN_F}{N_F^2-1}\right)^{\frac{1}{3}} .$$

(assuming cross-over and adiabatic)

Dark sector temperature evolution

- Temperature of i-th species

$$T_i \equiv \frac{P_i(T_i)}{n_i(T_i)} = \frac{g_i}{n_i(T_i)} \int \frac{d^3 p_i}{(2\pi)^3} \frac{p_i^2}{3E_i} f_i(T_i)\,.$$

- Solve Boltzmann equations for the 2nd-moment Bai et al. '19 + Mondino et al. '20

$$n\frac{\dot{T}_D}{T_D} + nHT_D + \sum_i n_i(\delta_i - 1)HT_D \approx -(\dot{n} + 3Hn)\,.$$

- Analytical solution in limiting cases:

$$T_D(T) = \begin{cases} \left(\frac{g_*^s(T)}{g_*^s(T_\Lambda)}\right)^{\frac{1}{3}} \xi\, T, & T_D > M_\pi \quad \text{equivalent to } T_\Lambda \geq T > M_\pi/\xi \\ \left(\frac{g_*^s(T)}{g_*^s(T_\Lambda)}\right)^{\frac{2}{3}} \xi^2 \frac{T^2}{M_\pi}, & T_D < M_\pi \quad \text{equivalent to } T < M_\pi/\xi \end{cases}$$

Phenomenology

Sector completely secluded!

- No direct/indirect/collider detection constraints $\Longrightarrow$ nightmare scenario
- Possible constraints from cosmology: Bullet cluster, structure formation and CMB !
- Pheno governed by only 3 parameters

$$f\,, \quad M_\pi\,, \quad \xi$$

  with dark pion decay constant f and $M_B \sim 10\,f$
- Bonus: possible stochastic gravitational wave signals

Phenomenology: constraints

- Bullet cluster: limit on DM self-interaction Spergel et al. '99 $\sigma_{\rm el}^{DM}/M_{DM} < \mathrm{cm}^2/\mathrm{g}$.
  Easily evaded for dark baryons as $\sigma_{\rm el}^{B} \approx \frac{4\pi}{M_B^2}$
  Constrains pions Hochberg et al. '14:

$$\sigma_{\rm el}^{\pi} \simeq \frac{1}{64\,\pi}\frac{(3N_F^4 - 2N_F^2 + 6)}{N_F^2(N_F^2-1)}\frac{M_\pi^2}{f^4} \quad \overset{N_F\to 3}{=} \quad \frac{77}{1536\,\pi}\frac{M_\pi^2}{f^4}\,.$$

- CMB: If pion mass is vanishingly small $\implies$ dark radiation

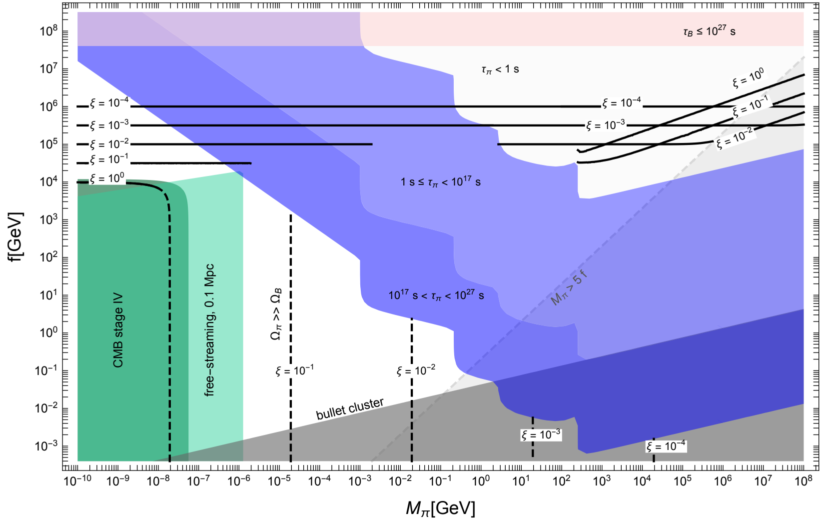
$$\Delta N_{\rm eff}\big|_{\rm CMB} = \frac{4}{7}\left(\frac{g_\nu}{g_{\rm eff}}\right)^{4/3}(N_F^2-1)\xi^4 = 0.027(N_F^2-1)\xi^4\,,$$

  exclusion obtained by taking $\Delta N_{\rm eff} \lesssim 0.25$ Fields et al. '19

- Structure formation:

$$\lambda_{\rm FS}\big|_{\Omega_{\rm DM}} \approx 5.2\,\mathrm{Mpc}\,\xi^4\left(\frac{N_F^2-1}{8}\right)\left(\frac{106.75}{g_*^s(T_\Lambda)}\right)^{\frac{1}{3}}\,.$$

Phenomenology: Results



Conclusions and Outlook

- New confining gauge theories $\Longrightarrow$ accidental stable DM, if the dark sector is neutral under SM.
- In dark SU(N) theories with light fermions both the lightest pion and baryon are DM candidates. Dark baryons typically heavy $\sim$ 100 TeV and dark pion are light $\sim$ keV.
- Constraints on light pion scenario from cosmology. Dark sectors as hot or hotter than the SM plasma is excluded!

Thank You !

Phase transition

Phase Transition and Thermal History

Thermal history

After production the dark sector is in the deconfined phase, renormalizable interactions such as strong gauge interactions keep the dark sector in thermal equilibrium among themselves Arnold et al. '02 and Kurkela et al. '14

- conserving energy we find initial dark sector temperature

$$\xi^0 \equiv \frac{T_D}{T} = \left(\frac{g_* \rho_D}{g_D \rho_{SM}}\right)^{\frac{1}{4}} ,$$

- graviton exchange

$$\xi^0_{gr} \approx 0.3 \left(\frac{T_R}{M_{\rm Pl}}\right)^{\frac{3}{4}} .$$

- dimension 5 operator

$$\xi^0_{|H|^2} \approx 0.2 \left(\frac{a_{\bar{\Psi}\Psi}}{g_D} \frac{M_{\rm Pl} T_R}{\Lambda_{\rm UV}^2}\right)^{\frac{1}{4}} \sim 0.1 \left(\frac{T_R}{\Lambda_{\rm UV}}\right)^{\frac{1}{4}} .$$

Dark matter from dQCD

We identify four regimes

- Light pion DM
- Baryon DM
- Non-relativistic pions decoupled: B B $\rightarrow n\,\pi$
- Non-relativistic pions in equilibrium: cannibalism

Dark matter from dQCD: dark-pion DM

- Abundance of dark pions is mainly set by the phase transition. If in eqbn

$$Y_\pi = (N_F^2 - 1)\frac{45\zeta(3)}{2\pi^4 g_*}\xi^3 .$$

- For $M_\pi < \mathcal{O}(100)$ GeV, relic density today is

$$\frac{\Omega h^2}{0.12} \approx 0.6(N_F^2 - 1)\frac{M_\pi}{0.1\mathrm{GeV}}\left(\frac{\xi}{10^{-2}}\right)^3\left(\frac{106.75}{g_*}\right)$$

$$\rightarrow \quad M_\pi^{\mathrm{DM}} \approx \frac{0.14\,\mathrm{keV}}{N_F^2\xi^3}\left(\frac{g_*}{106.75}\right) .$$

Dark matter from dQCD: dark-baryon DM I

- Abundance of baryon is set by annihilation into multi-pion final states. Need to solve

$$\frac{dY_B}{dT_D} = \frac{\langle \sigma v \rangle s(T)}{H(T)\, T_D} \left[Y_B^2 - (Y_B^{\mathrm{eq}}(T_D))^2 \right] ,$$

and corresponding abundance in the light pion mass regime is

$$\frac{\Omega_B h^2}{0.12} \approx \xi \left(\frac{g_* + g_D \xi^4}{106.75} \right)^{1/2} \left(\frac{M_B}{100\,\mathrm{TeV}} \right)^2$$

$$\rightarrow \quad M_B^{\mathrm{DM}} \approx \frac{100\,\mathrm{TeV}}{\sqrt{\xi}} \left(\frac{106.75}{g_* + g_D \xi^4} \right)^{\frac{1}{4}} .$$

Dark matter from dQCD: dark-baryon DM II

- In the heavy pion regime, pions can decay via mixing with higgs for $d = 5$ scenario. Leads to early matter dominated era. Need to account for entropy dilution

$$\eta = \frac{s}{s_\Gamma} \approx \min\left[1,\ 0.8\frac{(T_\Gamma/M_\pi)^{3/4}}{Y_\pi^{3/4}}\right] \approx \min\left[1,\ \frac{1.3}{g_*^{1/4}}\left(\frac{M_{\rm Pl}^2\Gamma_\pi^2}{M_\pi^4}\right)^{1/4}\frac{1}{Y_\pi}\right].$$

resulting in baryon abundance:

$$M_B^{\rm DM} \approx \frac{100\,{\rm TeV}}{\sqrt{\xi}}\left(\frac{106.75}{g_* + g_D\xi^4}\right)^{\frac{1}{4}} \max\left[1,\ \ 10N_F\xi\left(\frac{g_* + g_D\xi^4}{106.75}\right)^{\frac{1}{8}}\left(\frac{M_\pi}{10^4\,{\rm GeV}}\right)^{3/8}\right]$$

Dark matter from dQCD: non-rel pion regimes

- Solve

$$\dot{n}_B + 3Hn_B \approx -\langle\sigma v\rangle_{n,\max}\left[n_B^2 - \left(\frac{n_\pi}{n_\pi^{\rm eq}(T_D)}\right)^n (n_B^{\rm eq}(T_D))^2\right] .$$

SM nuclear physics data Amsler et al. '97 suggests that the dominant channel is the one dominated by the largest number of pions allowed kinematically, i.e. $Q_{n,\max} = 2M_B - nM_\pi \to 0$.
Abundance now

$$\frac{\Omega_B^{(n,\max)}}{\Omega_B} \approx 0.1\frac{M_B}{\sqrt{m_\pi Q_{n,\max}}}\frac{\langle\sigma v\rangle}{\langle\sigma v\rangle_{n,\max}} .$$

$Q_{n,\max}/T_D|_{f.o.} \approx 2\log(\langle\sigma v\rangle_{n,\max} M_\pi M_{\rm Pl}\xi^2) \implies$ no substantial deviation from the previous case!