

# Can the Neutrino tell us its Mass by Electron Capture?

Amand Faessler

University of Tuebingen;

Heidelberg, MPI 20. May 2015

1)Faessler, Gastaldo, Simkovic, J. Phys. G 42, 015108 (2015)

2)Faessler, Simkovic, Phys. Rev C91, 045505 (2015)

3)Faessler, Enss, Gastaldo, Simkovic, arXiv: 1503.2282

Not about Experiment, the Calorimetric Detector.

# Popular Public Resonance of Paper 2:

Die Chemie-News hat eine Meldung mit Artikel auf Ihre Internetseite gesetzt:

<http://www.chemie.de/news/152561/wird-das-neutrino-beim-einfang-eines-elektrons-sein-gewicht-verraten.html>

Das Physikportal Pro-Physik der DPG hat einen Artikel unter:

[http://www.pro-physik.de/details/news/7865871/  
Verraten Neutrinos beim Einfang ihre Masse.html](http://www.pro-physik.de/details/news/7865871/Verraten_Neutrinos_beim_Einfang_ihre_Masse.html)

# Popular Public Response for second Paper:

Chemie-News:

**Wird das Neutrino beim Einfang eines Elektrons sein Gewicht verraten?**

Forscher halten nach neuen Berechnungen die Lösung einer großen Frage der Elementarteilchenphysik für möglich

22.04.2015

Physik-Portal:

**Verraten Neutrinos beim Einfang ihre Masse?**

21. April 2015

*Neuen Berechnungen zufolge kann Projekt ECHo eine der großen Fragen der Elementar-teilchen-physik klären.*

# Some open problems for neutrinos:

- 1) The absolute value of the anti-neutrino mass?
- 2) The absolute value of the neutrino mass?  
(Topic of this talk)
- 3) Dirac or Majorana particle?
- 4) The Neutrino Hierarchy problem?

2) Determination of the electron neutrino mass by electron capture in



Gamow-Teller:

bound electron(s<sub>1/2</sub>,p<sub>1/2</sub>) + proton [523] [h<sub>11/2</sub>]7/2  
→ neutron [523] [h<sub>9/2</sub>+f<sub>7/2</sub>] 5/2<sup>-</sup> + neutrino

2) Determination of the electron neutrino mass by electron capture in



proposed by De Rujula and Lusignoli, Phys. Lett. 118B, 429 (1982).

### Competing Experiments with 163 Holmium in preparation:

- ) ECHo: Heidelberg, Mainz, Tübingen, ... ;  
Christian Enss, Loredana Gastaldo, Blaum, Düllmann, ...
- ) HOLMES: Milano Biccoca; Stefano Ragazzi, Angelo Nucciotti, ...
- ) NuMECS: Los Alamos, Michigan State Univ. (NSCL), Central Michigan University, Stanford Univ. ; Gerd Kunde, ...

Determination of the electron neutrino mass by electron capture in



$$Q(\text{ECHO}) = (2.843 \pm 0.009^{\text{stat}} \pm 0.06^{\text{syst}}) \text{ [keV]} \approx 2.8 \text{ [keV]}$$

Loredana Gastaldo et al.

$$\begin{aligned} Q &= E^*(\text{excited Dy-Atom}) + E_\nu \\ &= E^*(\text{Dy}) + \sqrt{T\nu}^2 + m\nu^2 \end{aligned}$$

Upper end of spectrum:

$$E^*(\text{Dy, max}) = Q(\text{value}) - \textcolor{magenta}{m_\nu}$$

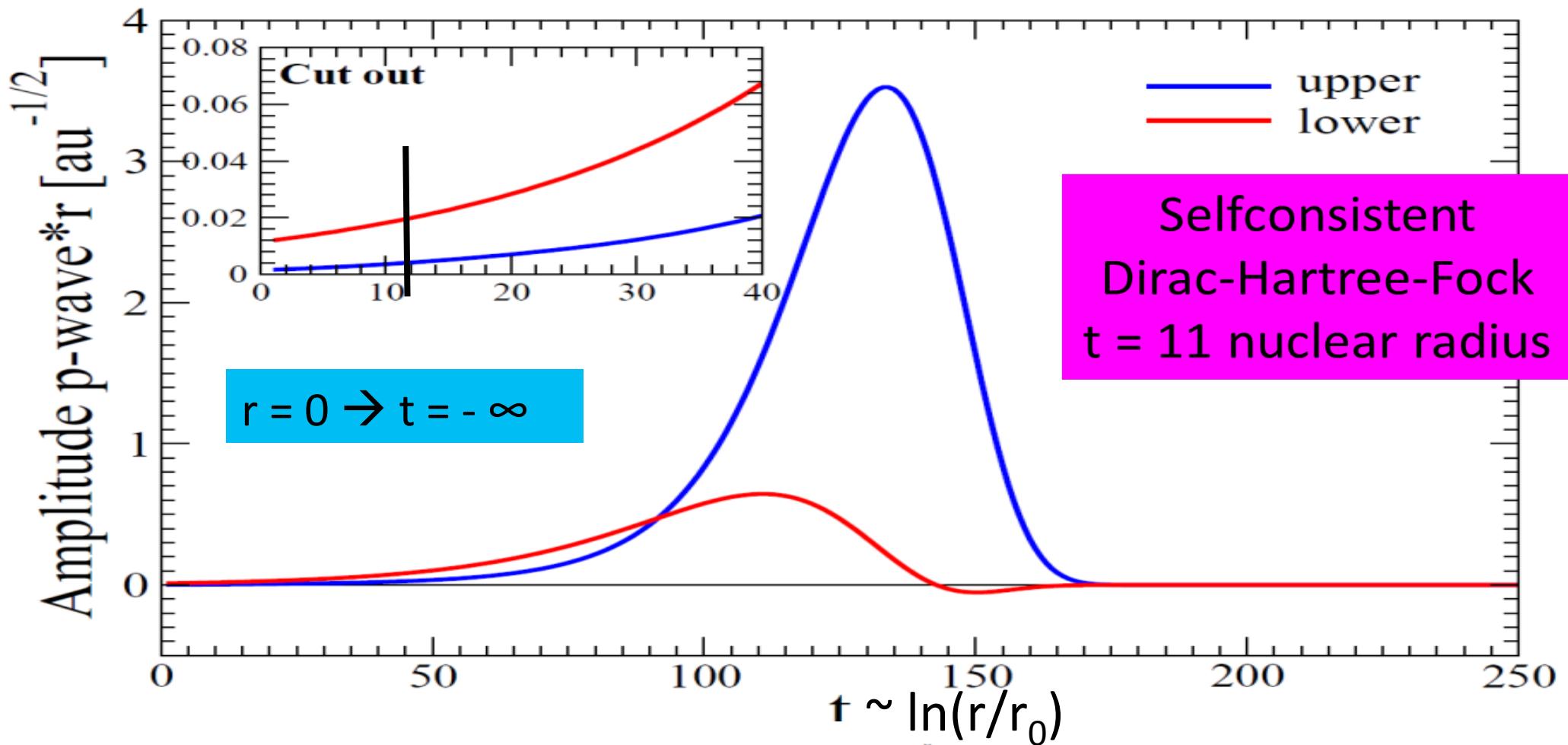
# For Electron Capture (in Holmium 163)

- 1) Electron at nucleus → s1/2 and p1/2
- 2) Electron binding energy < Q-value ≈ 2.8 [keV]

( $1s_{1/2}$ , K,Ho) = 55.6 keV  
( $2s_{1/2}$ , L1,Ho) = 9.4 keV  
( $2p_{1/2}$ ,L2,Ho) = 8.9 keV  
( $2p_{3/2}$ ,L3,Ho) = 8.1 keV

E( $3s_{1/2}$ ,M1,Ho) = 2.0 keV  
E( $3p_{1/2}$ ,M2,Ho) = 1.8 keV  
E( $4s_{1/2}$ , N1,Ho) = 0.4 keV  
E( $4p_{1/2}$ , N2,Ho) = 0.3 keV  
E( $5s_{1/2}$ ,O1,Ho) = 0.05 keV

The upper and lower relativistic amplitude of the 2p1/2 wave function in Ho multiplied by r



$$t = \ln(r/r_0)/h; \quad h = 0.05; \quad r_0 = 7.1469 \cdot 10^{-5} [\text{au}]. \quad t = 1, 2, \dots, 251$$

Selfconsistent Dirac-Hartree-Fock  
[Grant, Desclaux and Ankudinov et al.  
Comp. Phys. Com. 98 (1996) 359] for  $^{67}\text{Ho}$  and  $^{66}\text{Dy}^*$

$$|G\rangle = a_1^\dagger a_2^\dagger a_3^\dagger \dots a_Z^\dagger |0\rangle$$

Holmium

$$|A_f\rangle = a_1^{\dagger f} a_2^{\dagger f} \dots a_{f-1}^{\dagger f} a_{f+1}^{\dagger f} \dots a_Z^{\dagger f} |0\rangle$$

Dysprosium

A. Faessler, E. Huster, O. Krafft, F. Krahn, Z. Phys. 238 (1970)

# Reexcitation Spectrum (X-rays, Auger- electron by a Calorimeter) of excited Dy\* (Atom)

$$\frac{d\Gamma}{dE_c} \propto (Q - E_c) \sqrt{(Q - E_c)^2 - m_\nu^2} \sum_{f=f'} \lambda_0 B_f \frac{\Gamma_{f'}}{2\pi} \frac{1}{(E_c - E_{f'})^2 + \Gamma_{f'}^2/4}$$

$$\lambda_0 \backslash \text{proto} \mathbf{G}_{\text{weak}}^2 \xi$$

$$\psi_A(r)=\frac{1}{r}\left(\begin{array}{c}P_A\\Q_A\end{array}\right)$$

$$B_f \approx |\psi_f(R)|^2 / |\psi_{3s1/2}(R)|^2$$

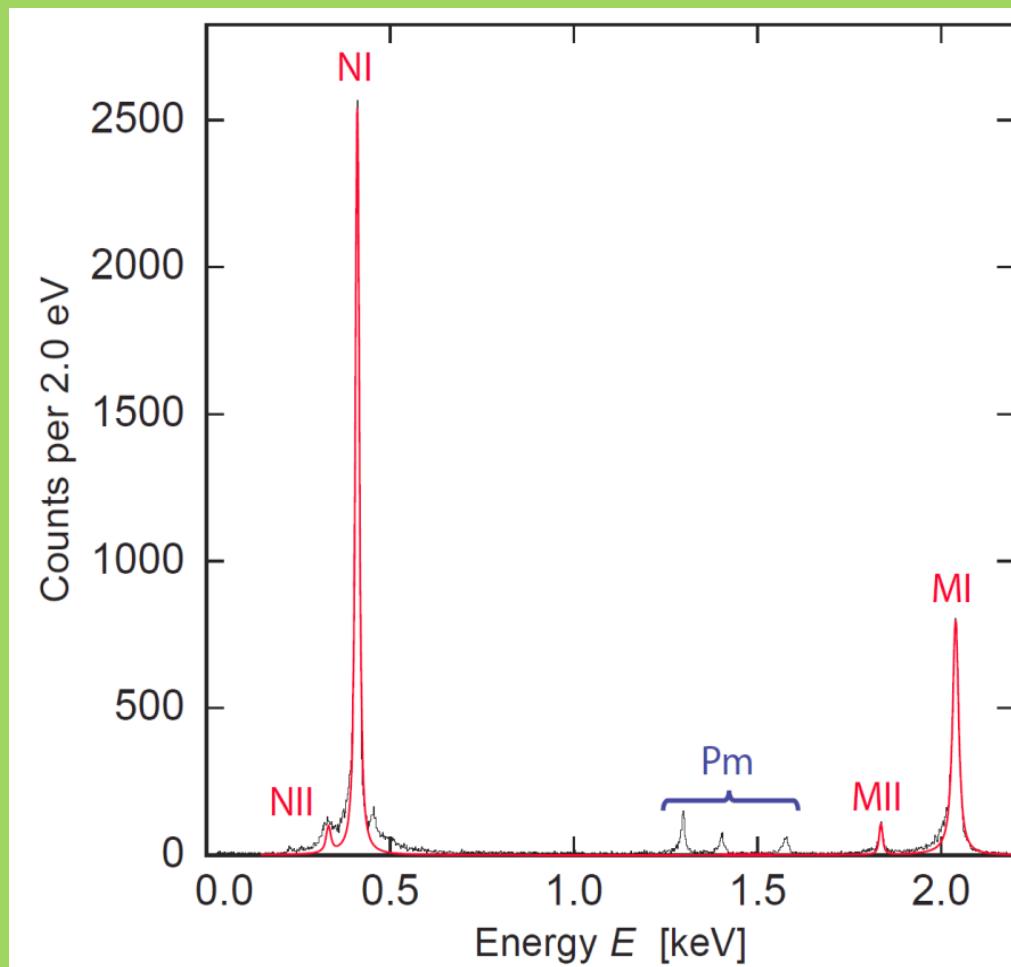
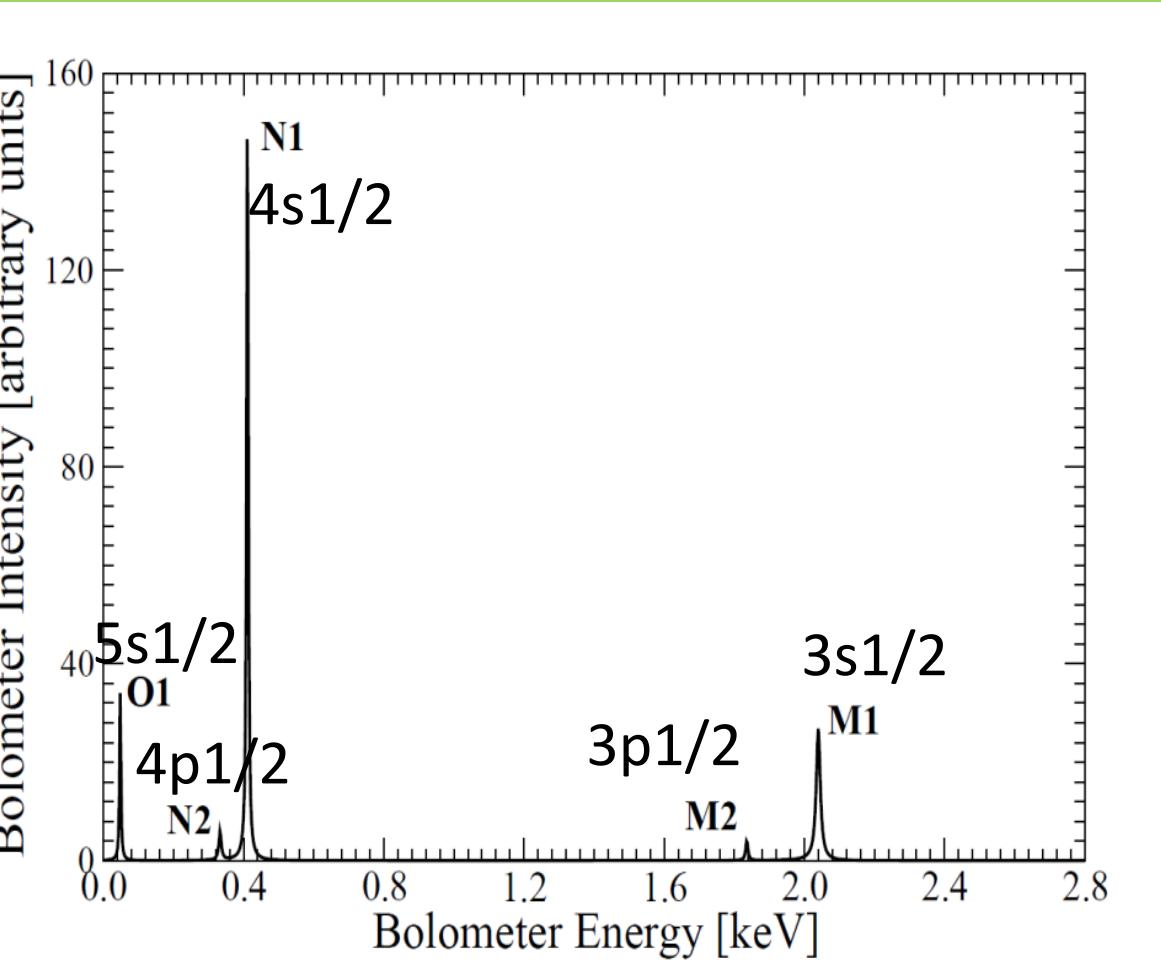
$$\langle A | B' \rangle = \int_{0,\infty} \left( P_A(r) \cdot P_{B'}(r) + Q_A(r) \cdot Q_{B'}(r) \right) \cdot dr = overlap(A,B')$$

# Spectrum of the deexcitation of one-, two- and three-hole states in $^{163}\text{Dysprosium}^*$

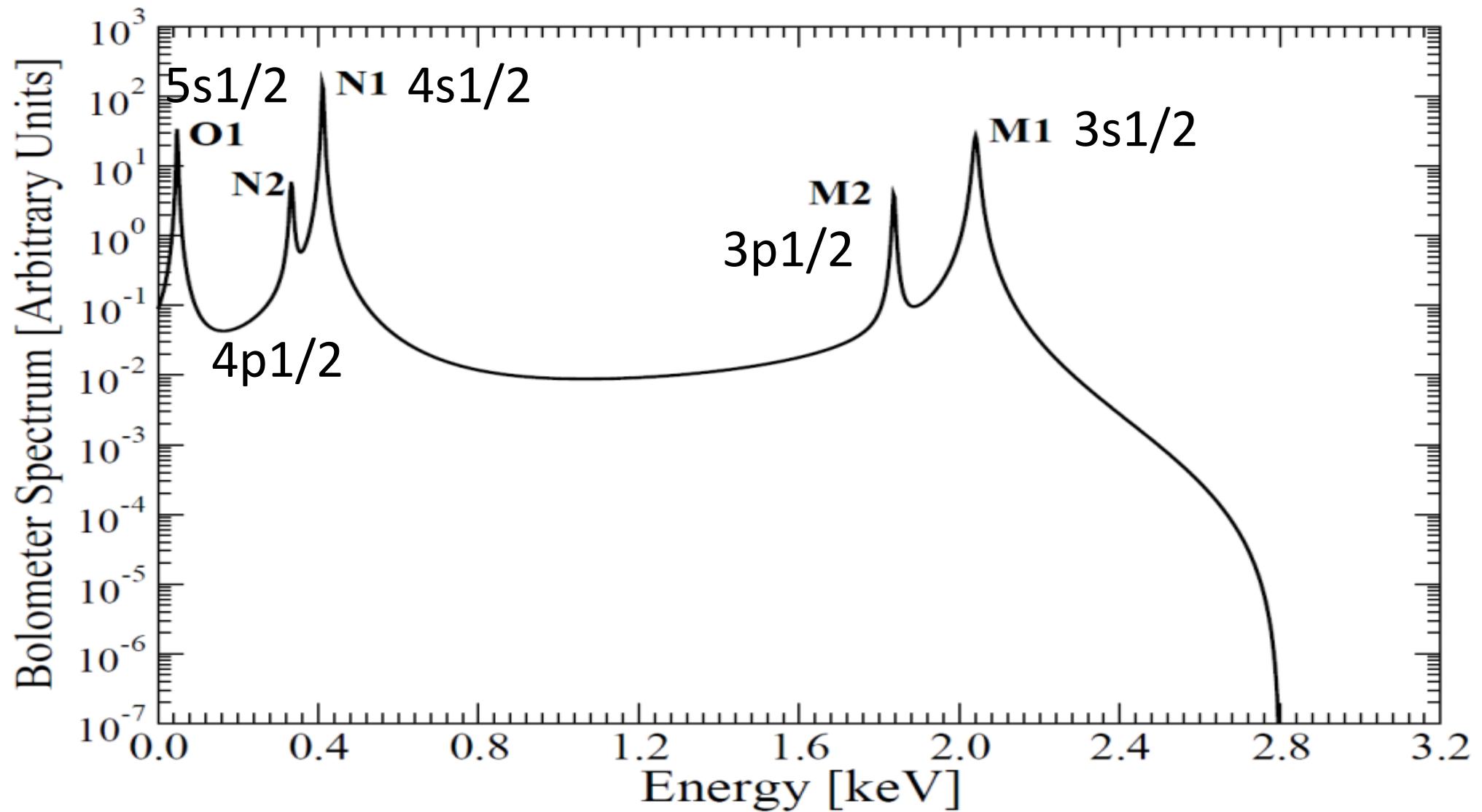
$$\frac{d\Gamma}{dE_c} \propto \sum_{i=1,\dots,N_\nu} (Q - E_c) \cdot U_{e,i}^2 \cdot \sqrt{(Q - E_c)^2 - m_{\nu,i}^2}$$
$$\sum_{h'=b',b'(p^{-1'}q),b'(p_1^{-1}q_1),(p_2^{-1}q_2)} \lambda_0 B_h \frac{\Gamma_{h'}}{2\pi} \frac{1}{(E_c - E_{h'})^2 + \Gamma_{h'}^2/4}$$

**Importance =**  $Bf\Gamma f'/(Q-Ef')^2 + \Gamma f' \tau'/2/4 \approx Bf\Gamma f' \tau'/(Q-Ef' \tau')^2$

# Theoretical and experimental (ECHO) one-hole deexcitation of Dy.



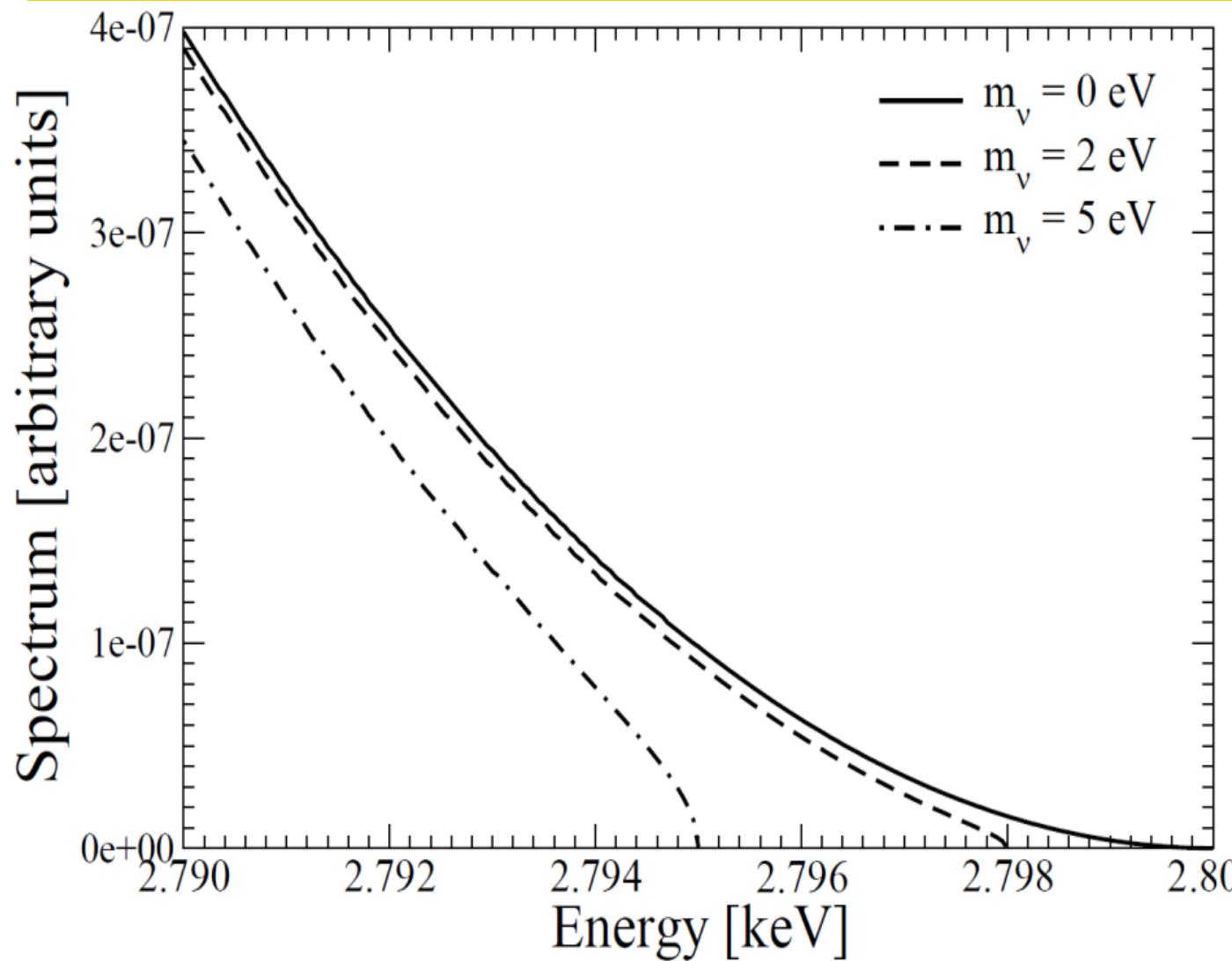
# Logarithmic10 one-hole Deexcitation Spectrum



the –resonance only  
important at  $Q$  after folding  
in detector response  
theory as function of  
parameters:  
 $m_\nu$  (neutrino mass)  
 $Q - E_f'$  (Q-value – reson.)  
 $\Gamma_{f'}$  (width of resonance)  
 $I_f$  (intensity of resonan.)

Two Resonances :  
fit 7 parameters

Last 10 eV below the Q-value



Robertson [Phys. Rev. C91 (2015) 03504] takes the Two-Hole probability from Carlson and Nestor, Phys. Rev. A 8, 2887 (1973)

calculated for  $^{54}\text{Xe}$  and not for  $^{67}\text{Ho} \rightarrow ^{66}\text{Dy}$

- Sudden approximation; probability for an empty (hole ) state in Dy:

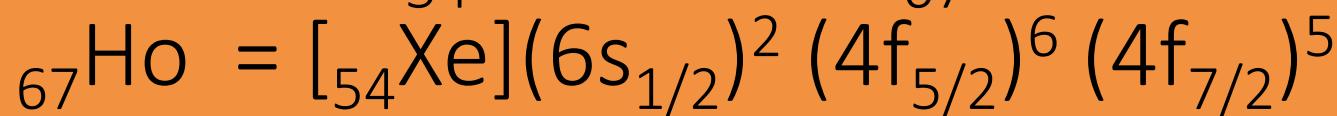
$$\langle \Psi_{(nlj)}(\text{Ho}) | \Psi_{(nlj)}^*(\text{Dy}^*) \rangle \approx 0.999 \quad \text{Always less than 1; } j = \frac{1}{2}; 2j+1 = 2$$

$$\text{Hole-probability: } P_{1-h} \approx 1 - [\langle \Psi_{(nlj)}(\text{Ho}) | \Psi_{(nlj)}^*(\text{Dy}^*) \rangle]^{2(2j+1)} = \\ 1 - 0.999^4 = 0.004 \rightarrow 0.4\%; \quad 1 - 0.98^4 = 0.077 \rightarrow 7.7\% \text{ (19 *larger)}$$

[Overlap  $|Z\rangle$  with  $|Z-1\rangle$ ] largest difference for outside electrons.

In addition to  $^{54}\text{Xe}$  in  $^{66}\text{Dy}$ :  $(6s_{1/2})^2, (4f_{5/2})^6, (4f_{7/2})^4,$

Difference between  $^{54}\text{Xenon}$  and  $^{67}\text{Holmium}$  (Literatur)



-	$E \text{ [eV]} \text{ Xenon}$	$E \text{ [eV]} \text{ Holmium}$
$5s_{1/2}$	23.3	49.9
$5p_{1/2}$	13.4	26.3 (30.8)
$5p_{3/2}$	12.1	26.3 (24.1)
<i>Overlap</i>	<i>for (-2 %) 0.979; P(2-hole)</i>	<i>for 0.999; P(2-hole)</i>
$j = 1/2$	8.1 %	0.4 %
$j = 3/2$	15.6 %	0.8 %

ur and the Robertson (arXiv: 1411.2906v1) 2-hole probabilities in D

<i>1. hole</i>	<i>2. hole</i>	$E_c[eV]$	$\Gamma[eV]$	$P - Fae[\%]$	$P - Rob[\%]$
4s1/2	—	409.0	5.4	24.40	23.29
4s1/2	4s1/2	841.4	5.4	0.021	0.001
4s1/2	4p1/2	752.5	5.4	0.052	0.004
4s1/2	4p3/2	717.2	5.4	0.091	0.01
4s1/2	4d3/2	569.0	5.4	0.088	0.077
4s1/2	4d5/2	569.0	5.4	0.125	0.123
4s1/2	4f5/2	417.6	5.4	0.027	0.0
4s1/2	4f7/2	414.2	5.4	0.023	0.0
4s1/2	5s1/2	458.3	5.4	0.066	0.254
4s1/2	5p1/2	439.8	5.4	0.039	0.629
4s1/2	5p3/2	433.1	5.4	0.058	1.502

Numerical values of Ho-Dy overlaps with holes in the selfconsistent Dirac-Hartree-Fock.  
A. Faessler, E. Huster, O. Krafft, F. Krahn, Z. Phys. 238 (1970) 352.

hole in  $3s1/2$ :  $\langle Ho, 3s1/2 | Dy, 3s1/2 \rangle = 0.999390;$

$\boxed{\langle Ho, 4s1/2 | Dy, 4s1/2 \rangle = 0.999332;}$

Probability(Hole  $4s1/2$ ) = 0.27 %

hole in  $4s1/2$ :  $\langle Ho, 3s1/2 | Dy, 3s1/2 \rangle = 0.999377;$

$\boxed{\langle Ho, 4s1/2 | Dy, 4s1/2 \rangle = 0.998870;}$

Probability(Hole  $4s1/2$ ) = 0.45 %; 66% larger

Improvement in our work over Robertson:  
Faessler + Simkovic, Phys. Rev. C91, 045505 (2015)

-hole probabilities in  $^{67}\text{Ho}$  and  $^{66}\text{Dy}$  and not in  $^{54}\text{Xe}$ ; Carlson+Nestor 1973  
the electron holes  $\text{ns}_{1/2}$  and  $\text{np}_{1/2}$  selfconsistently in Dy (Dirac-HF).

$^{163}\text{Dy}$  has more than 8 additional 2-hole states compared to  $^{54}\text{Xe}$  and  
not in Carlson, Nestor.

For the 1-hole states the exchange and overlap corrections are included.  
(Bahcal 1965, Faessler 1970; not included in Vatai approximation.)

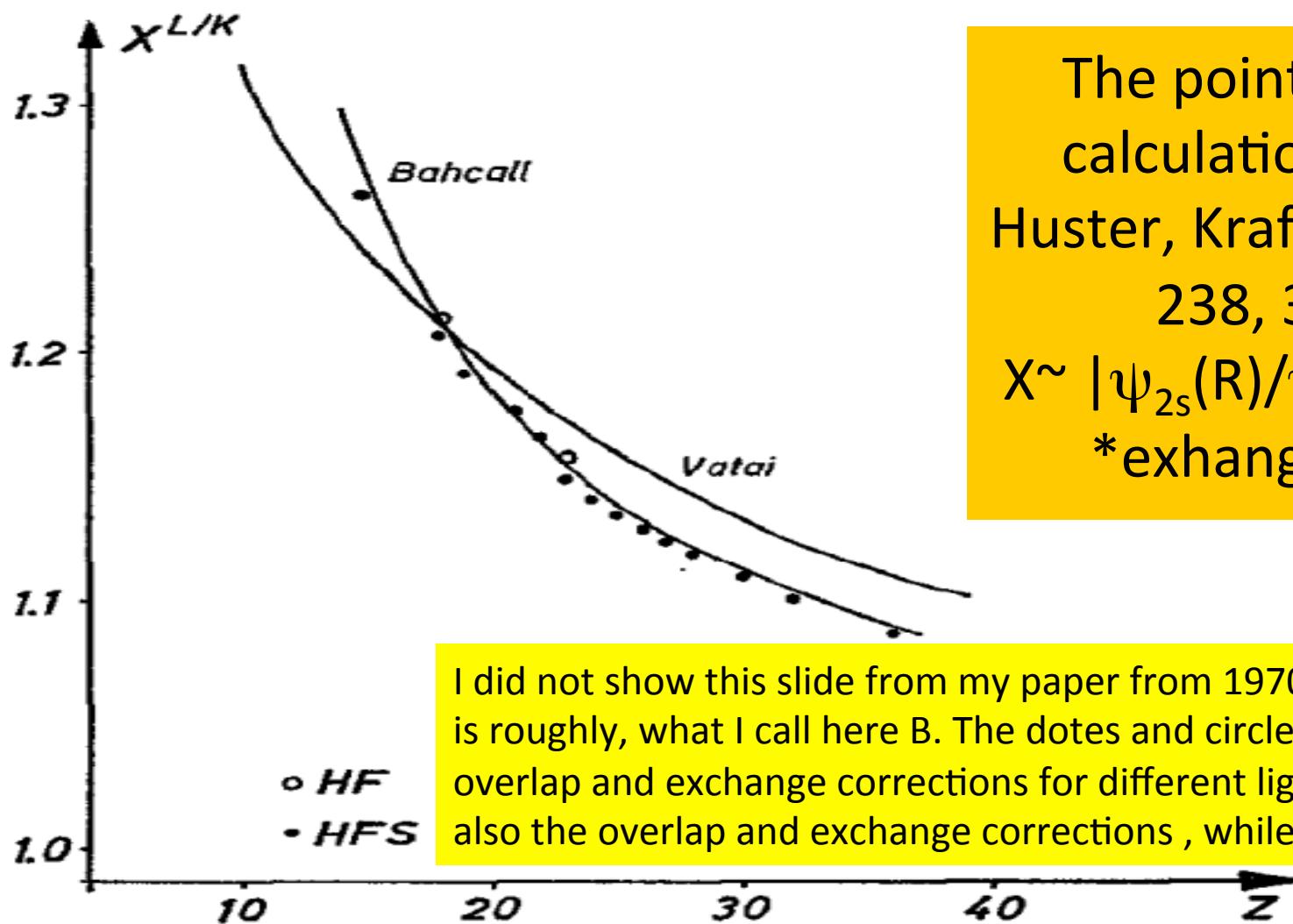
Dirac-Hartree-Fock includes finite charge distribution of the nucleus.

Probability for hole states  $\sim \Psi_{(n,l,j)}(R) * R^2$  and not at  $r = 0.0$ .

Derived in second quantization: automatic antisymmetrization.

New Q-value:  $Q = 2.8$  keV.

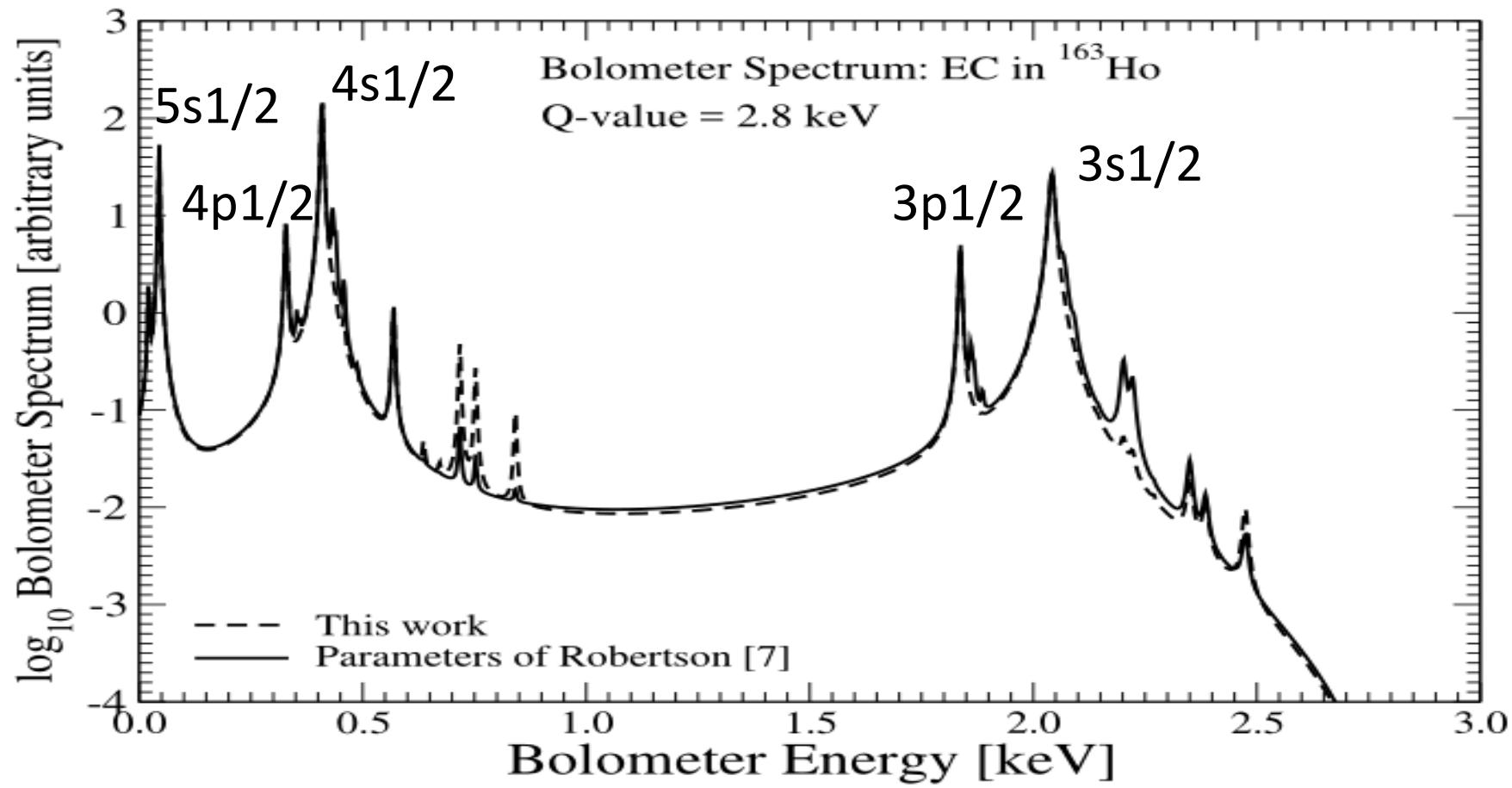
**A. Faessler, E. Huster, O. Krafft and F. Krahn:**



The points are different calculations of Faessler, Huster, Krafft, Krahn: Z. Phys. 238, 352 (1970);  
 $X \sim |\psi_{2s}(R)/\psi_{1s}(R)|^2 * \text{overlap} * \text{exchange corrections}$

I did not show this slide from my paper from 1970 in the talk. The quantity is roughly, what I call here B. The dots and circles are my calculation of overlap and exchange corrections for different light atoms. Bahcall includes also the overlap and exchange corrections , while Vatai neglected it.

- and 2-hole excitations of our work;  $Q = 2.8$  keV



Probabilities for the formation of 1- and 2-hole states in  $^{66}\text{Dy}$  after electron capture in  $^{67}\text{Ho}$ .

$$P_f = | \langle A'_f | a_i | G \rangle |^2 = | \langle 0 | a'_Z a'_{Z-1} \dots a'_{f+1} a'_{f-1} \dots a'_1 \cdot a_f \cdot a_1^\dagger a_2^\dagger a_3^\dagger \dots a_Z^\dagger | 0 \rangle |^2$$

$$P_{p/f} = | \langle A'_{p', f'} | a_f | G \rangle |^2 = \sum_{q' > F} | \langle 0 | a'_{q'} a'_{Z} \dots a'_{p'+1} a'_{p'-1} \dots a'_{f+1} a'_{f-1} \dots a'_1 \cdot a_f \cdot a_1^\dagger a_2^\dagger a_3^\dagger \dots a_Z^\dagger | 0 \rangle |^2$$

Faessler, Huster, Krafft, Krahn, Z. Phys. 238 1, 352 (1970)

# Probability for two-hole States:

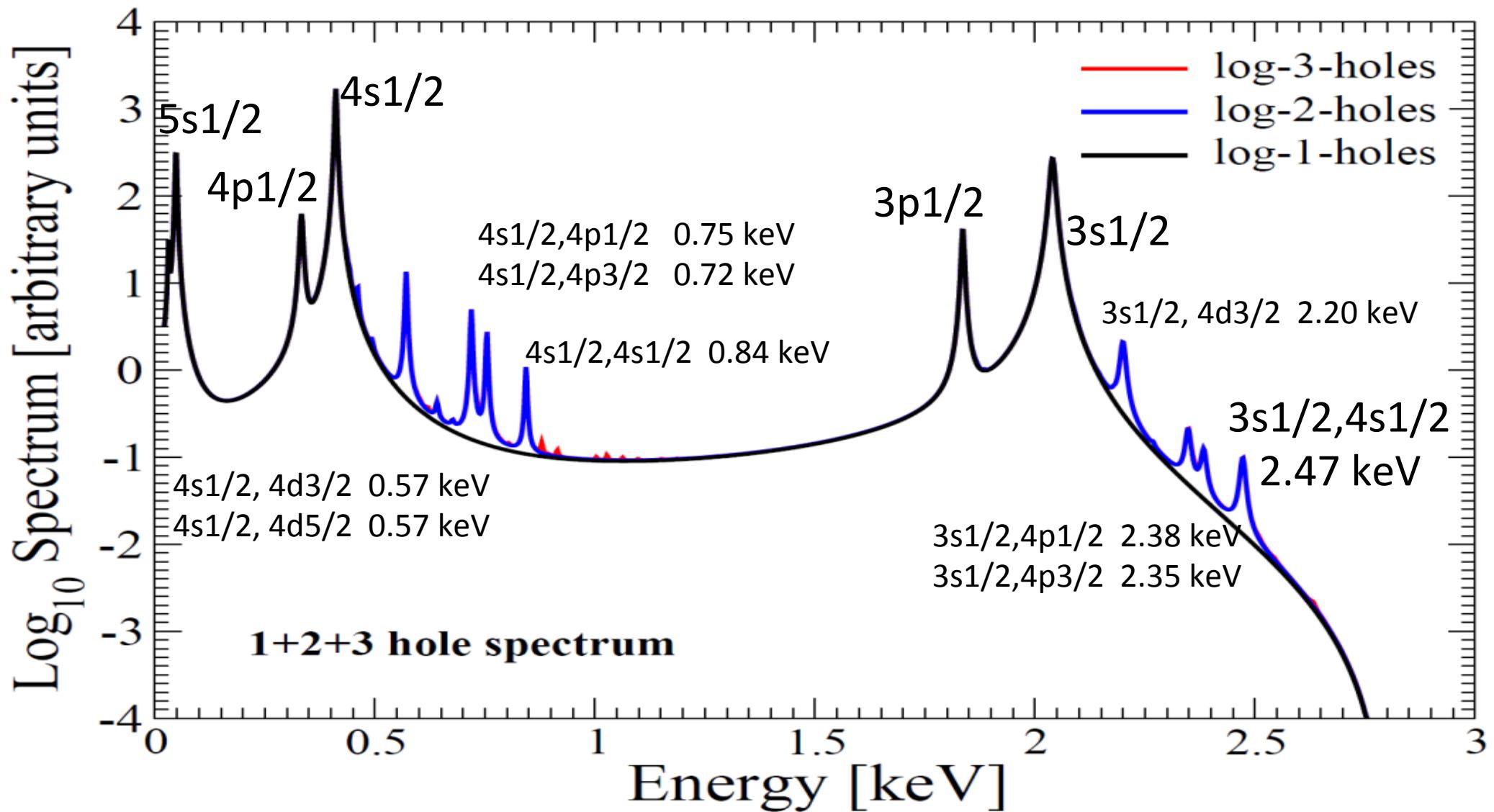
## Completeness relation:

$$1 = \sum_{q' < F} \langle p | q' \rangle \langle q' | p \rangle + \sum_{q' > F} \langle p | q' \rangle \langle q' | p \rangle$$

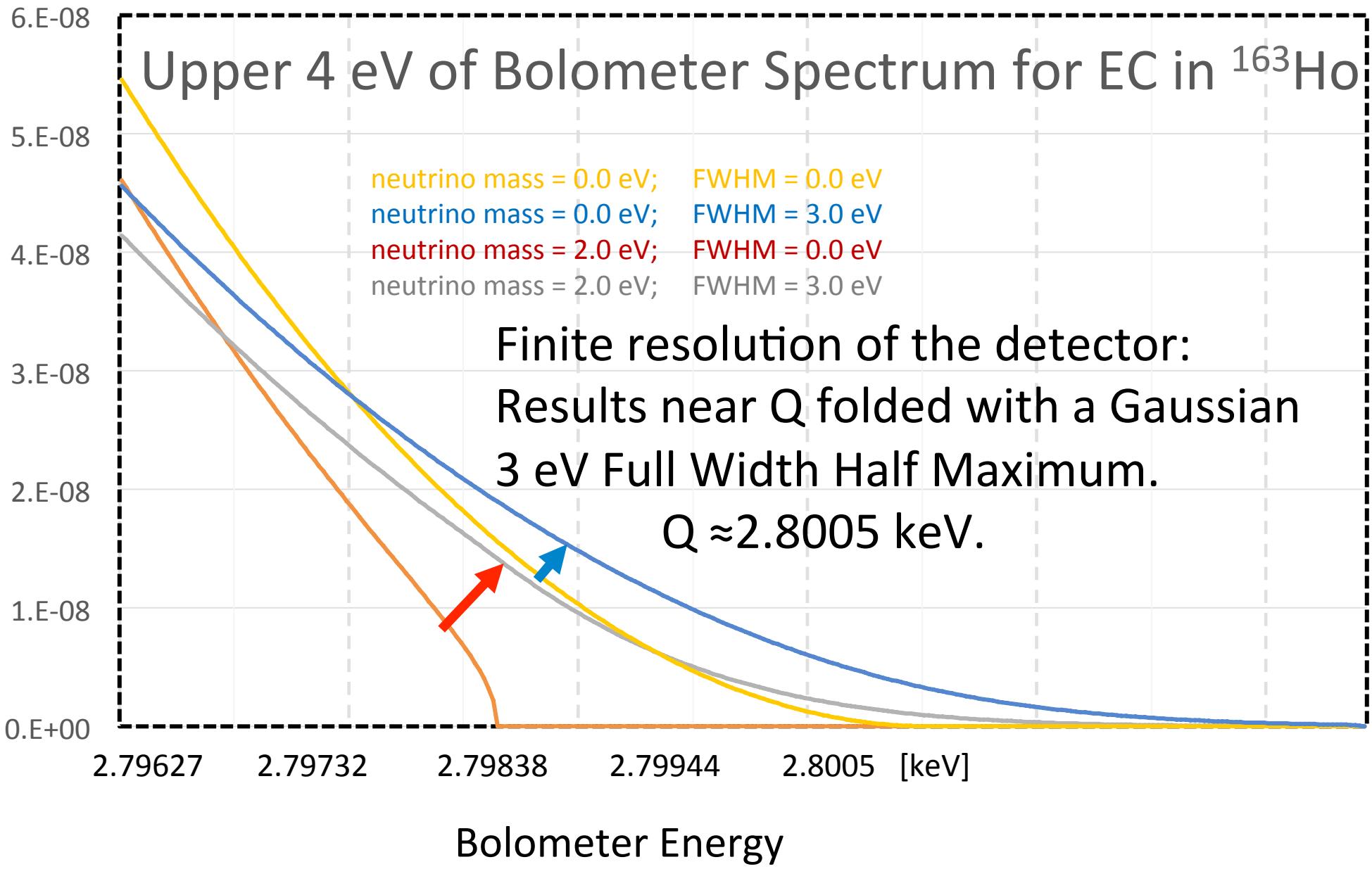
$$\begin{aligned} P_{p/f} &= \left( 1 - \sum_{q' < F} \langle p_{Ho} | q'_{Dy} \rangle \langle q'_{Dy} | p_{Ho} \rangle \right) = \\ &\quad \left( 1 - \langle p_{Ho} | p'_{Dy} \rangle \langle p'_{Dy} | p_{Ho} \rangle - \sum_{q' < F, \neq p'} \langle p_{Ho} | q'_{Dy} \rangle \langle q'_{Dy} | p_{Ho} \rangle \right) \end{aligned}$$

apply the Vatai approximation, we obtain the same formulas for  $^{54}\text{Xe}$  as Carlson and  
or.

# One-, two- and three-hole state deexcitation in Dy\*



Bolometer Spectrum [arb. units]



Importance of different resonance deexcitations of Dy\* at the Q value for the determination of the neutrino mass.

$$\frac{d\Gamma}{dE_c} \propto \sum_{i=1,\dots,N_\nu} (Q - E_c) \cdot U_{e,i}^2 \cdot \sqrt{(Q - E_c)^2 - m_{\nu,i}^2}$$
$$\sum_{h'=b', b'(p^{-1'} q), b'(p_1^{-1} q_1'), (p_2^{-1} q_2')} \lambda_0 B_{h'} \frac{\Gamma_{h'}}{2\pi} \frac{1}{(E_c - E_{h'})^2 + \Gamma_{h'}^2/4}$$

Importance  $\approx B h \Gamma h \tau / (Q - E h \tau)^2$

Importance of highest two-hole state  $(3s_{1/2}, 4s_{1/2})^{-1}$  at 2.47 keV relative to the highest one-hole state at 2.04 keV

$$E_C = Q - E_c;$$

$$E_{f'} = Q - E_{f'}$$

For folding theory with the central function of the detector we have the following 4 parameters, if one resonance is important:

neutrino mass

Q value:  $\Delta_{f'} = Q - E_{f'}$

Width of resonance  $\Gamma_{f'}$

strength  $S \sim B_f$

$$\begin{aligned} \frac{d\Gamma}{dE_c} &\propto (Q - E_c) \cdot \sqrt{(Q - E_c)^2 - m_\nu^2} \cdot \frac{S}{(E_c - E_{f'})^2 + \Gamma_{f'}^2/4} \\ &= \Delta E_C \cdot \sqrt{\Delta E_C^2 - m_\nu^2} \cdot \frac{S}{(\Delta_{f'} - \Delta E_C)^2 + \Gamma_{f'}^2/4} \end{aligned}$$

$$\text{Relative weight} \propto \frac{P_{1\text{-hole}} \%}{(Q - E_{f'})^2} = \frac{100 \%}{(2.80 - 2.04)^2} = 174$$

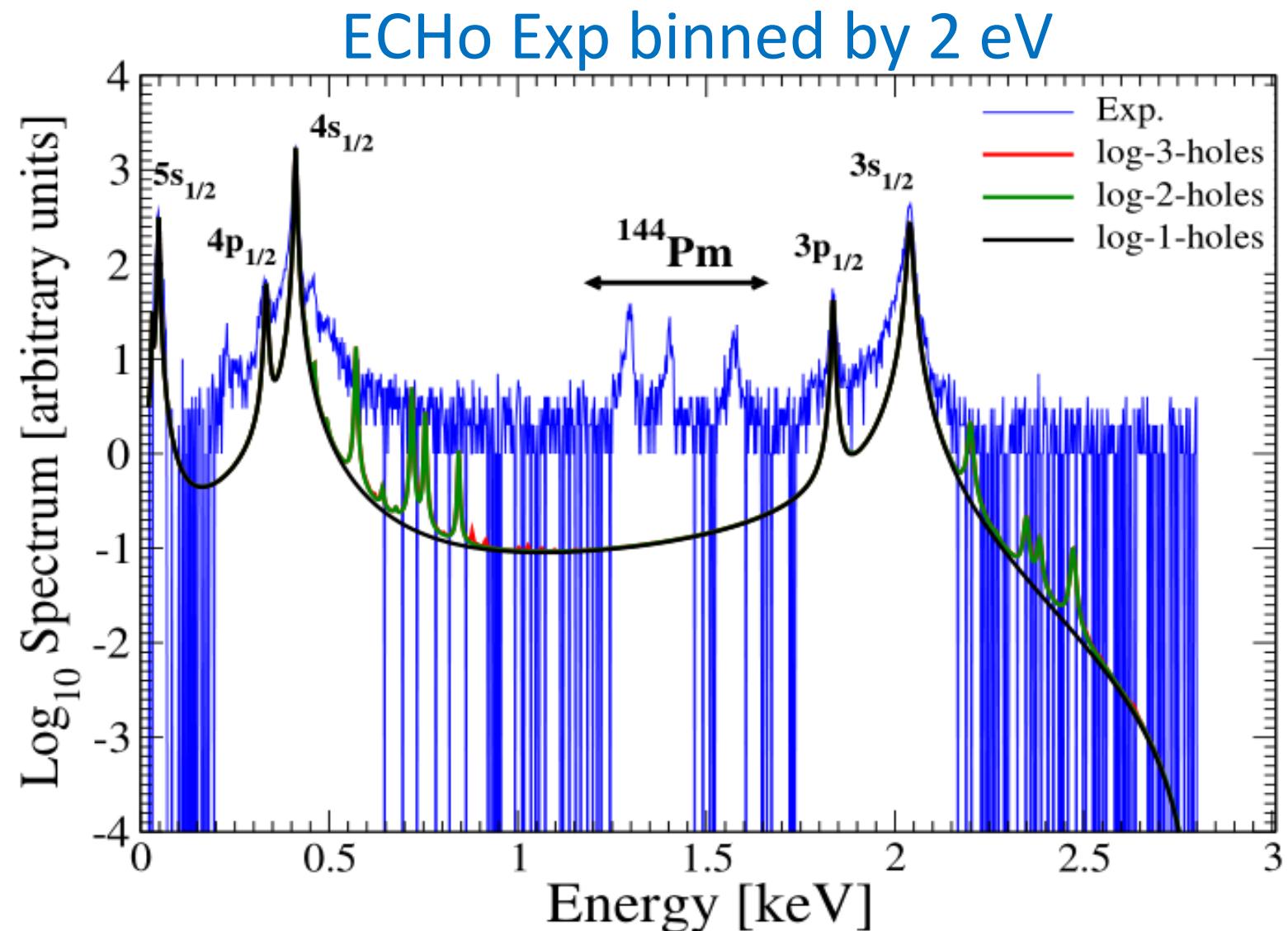
$$(3s_{1/2})^{-1}$$

$$\text{Relative weight} \propto \frac{P_{2\text{-hole}} \%}{(Q - E_{f'})^2} = \frac{0.167 \%}{(2.80 - 2.47)^2} = 1.6$$

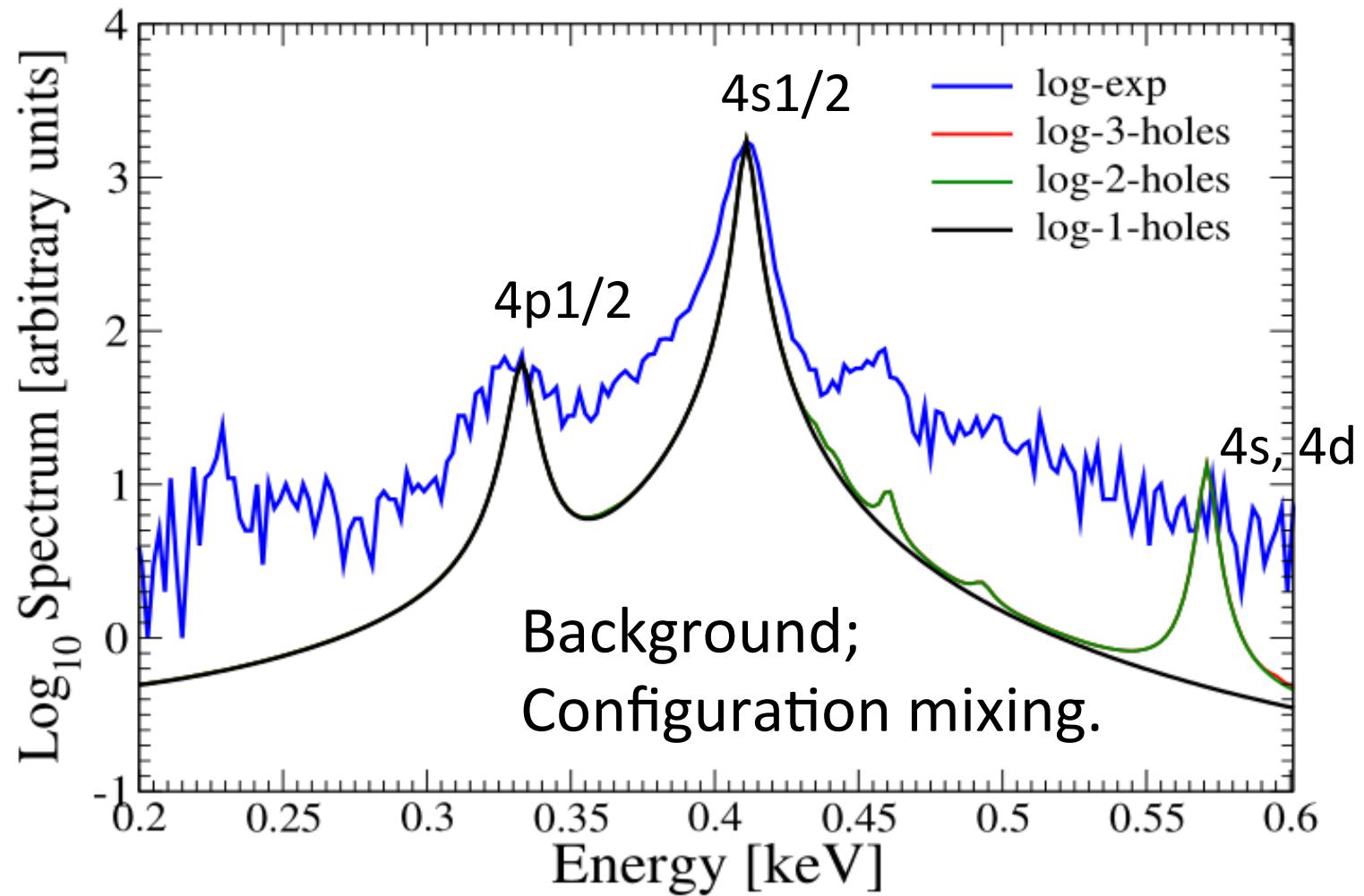
$$(3s_{1/2}, 4s_{1/2})^{-1}$$

comparison  
with the  
CHo data.

background  
must be  
included into  
theoretical  
treatment.



detailed comparison around the  $p_{1/2}$  and  $s_{1/2}$  one-hole excitations.



## Summary:

THE END

Determination of the electron neutrino mass by electron capture in  
 $^{163}\text{Holmium}$ ?

One-hole, the two-hole (for the first time correctly) and the three-hole (for the first time) deexcitation of  $^{163}\text{Dysprosium}$ .

Measured by the Bolometer (X-rays, Auger-electrons)

Mold with Spectral Function of detector and fit 4 parameters to data

One resonance:  $m_\nu$ ,  $Q$ ,  $\Gamma_f$ ,  $B_f$ . More resonances important?

Background? Conf. mixing?

$m_\nu$  determination seems to be (very) difficult but (perhaps) not impossible.