Neutrinos and Lorentz Invariance Violation

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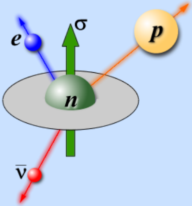
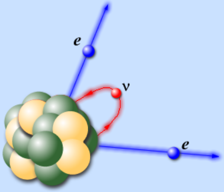
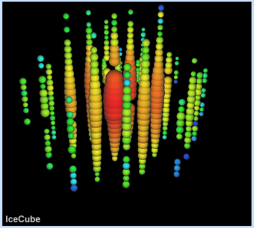
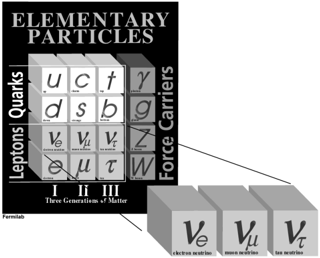
MAX PLANCK INSTITUTE FOR NUCLEAR PHYSICS - PARTICLE AND ASTROPARTICLE THEORY SEMINAR, 27.10.2014

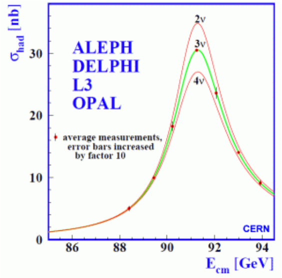






Neutrinos





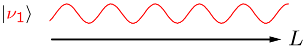
Properties

- Fundamental particles in the Standard Model (SM)
- They carry no electric charge
- They interact only via the weak interaction
- They come in three active flavors: ν_e, ν_μ, ν_τ
- In the SM, they are massless

Neutrinos: propagation of two flavors

- Neutrino eigenstates have well-defined energy
- Propagation controlled by the hamiltonian $\boldsymbol{H}$

$$\boldsymbol{H}\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix}=\begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}\begin{pmatrix}|\nu_1\rangle\\|\nu_2\rangle\end{pmatrix}$$



If a neutrino state $|\nu_1\rangle$ is created at $t=0$, after some time the state is

$$|\psi(t)\rangle = e^{-i\boldsymbol{H}t}|\nu_1\rangle = e^{-iE_1 t}|\nu_1\rangle$$

The probability of measuring the state $|\nu_1\rangle$ after some time $t\simeq L$ is

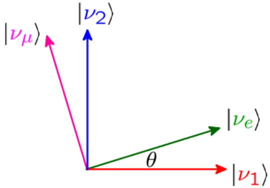
$$P_{\nu_1\rightarrow\nu_1} = |\langle\nu_1|\psi(t)\rangle|^2 = 1$$

A pure ν_1 beam propagates unaltered.

Mixing and oscillations of two flavors

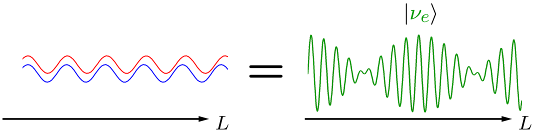
Weakly interacting states $\neq$ eigenstates

$$|\nu_e\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$
$$|\nu_\mu\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$



Evolution of a neutrino state $|\nu_e\rangle$ is created at $t = 0$

$$|\psi(t)\rangle = e^{-i\boldsymbol{H}t}|\nu_e\rangle = e^{-iE_1 t}\cos\theta|\nu_1\rangle + e^{-iE_2 t}\sin\theta|\nu_2\rangle$$



pure ν_e beam may evolve a ν_μ component with time!

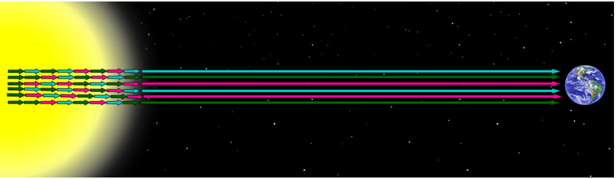
Probability of measuring the state $|\nu_e\rangle$

$$P_{\nu_e\to\nu_e} = 1 - \sin^2 2\theta \, \sin^2\left(\tfrac{1}{2}\,\Delta E_{21}\,L\right)$$

$\Delta E_{21} \neq 0$ indicates physics beyond the SM!

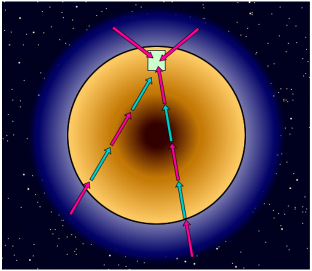
Neutrino Oscillations

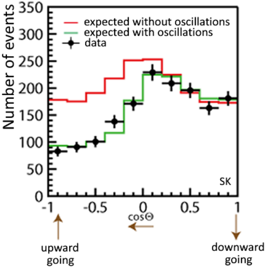
Solar neutrino problem: solved



$$\frac{\left(\Phi_{\nu_e}\right)_{\text{exp}}}{\left(\Phi_{\nu_e}\right)_{\text{th}}} \simeq 0.33$$

Atmospheric neutrino problem: solved





$$\frac{\left(\Phi_{\nu_\mu}/\Phi_{\nu_e}\right)_{\text{exp}}}{\left(\Phi_{\nu_\mu}/\Phi_{\nu_e}\right)_{\text{th}}} \simeq 0.60$$

Three-neutrino massive model

Phenomenological extension of the SM:

- three massive neutrinos

$$h = U^\dagger \begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix} U$$

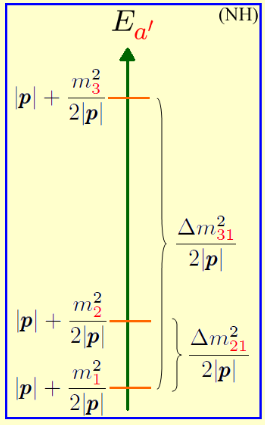
- energy-independent mixing: $U(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$
- neutrinos and antineutrinos are decoupled

6×6 matrix $\rightarrow$ $$\boldsymbol{H}^{3\nu\mathrm{SM}} = \left(\begin{array}{c|c} h & 0 \\ \hline 0 & h^* \end{array}\right)$$

- oscillation probability

$$P_{\nu_a \rightarrow \nu_b}(L) = \sum_{a',b'} U^*_{a'a} U_{a'b} U_{b'a} U^*_{b'b} \, e^{i(E_{a'} - E_{b'})L}$$

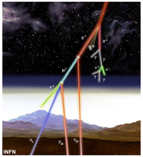
$$E_{a'} = \sqrt{|\boldsymbol{p}|^2 + m^2_{a'}} \approx |\boldsymbol{p}| + m^2_{a'}/2|\boldsymbol{p}|$$

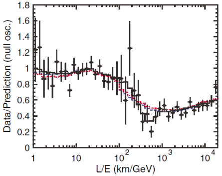


Three-neutrino massive model

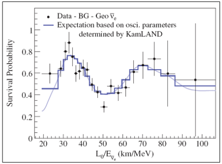
This model successfully describes all established oscillation data

Atmospheric neutrinos

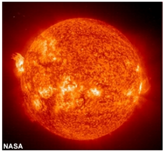


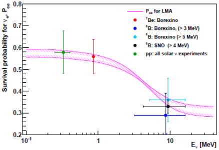


Reactor antineutrinos

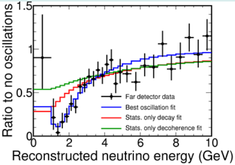


Solar neutrinos





Accelerator neutrinos



Lorentz and CPT violation

Lorentz invariance

- Cornerstone of modern physics.
- Symmetry that underlies Special Relativity.
- Laws of physics are independent of speed and direction of propagation.
- Linked to CPT symmetry (relating properties of matter and antimatter).
- Established experiments indicate that nature is Lorentz invariant (so far).

Einstein & Lorentz (1921)

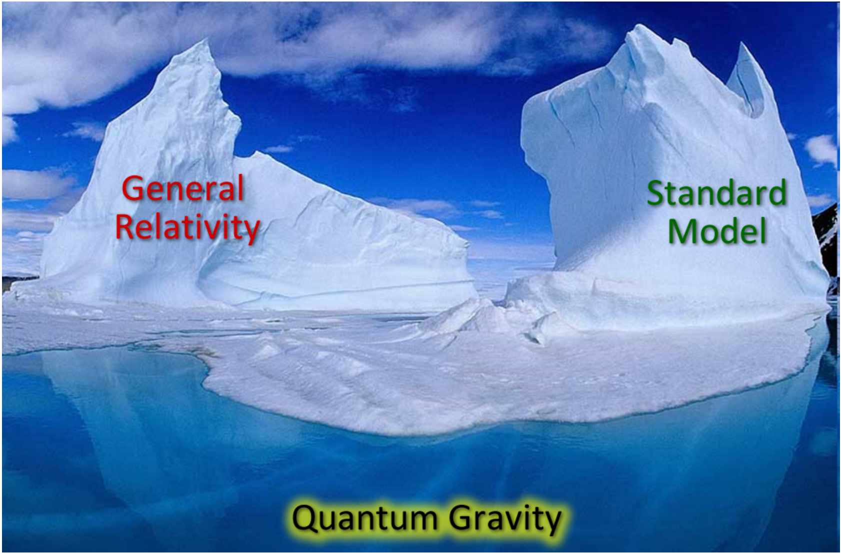
Lorentz violation

- Last 20 years, growing interest in the possibility that Lorentz symmetry may not be exact.
- Quantum gravity candidates involve the breaking of Lorentz symmetry.
- Lorentz symmetry is a basic building block of GR and the SM. Anything this fundamental should be tested.
- New era of high-precision measurements.

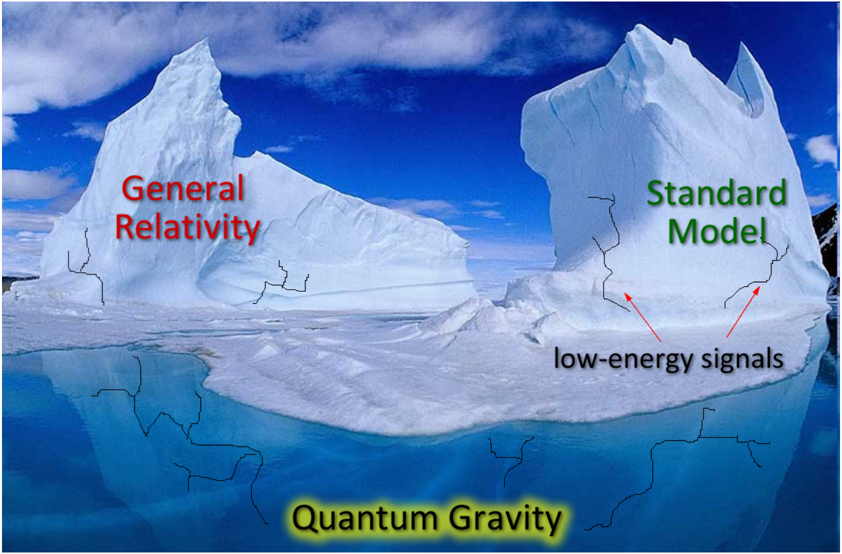
Lorentz violation

GR and the SM are expected to merge
at the Planck scale

Lorentz violation

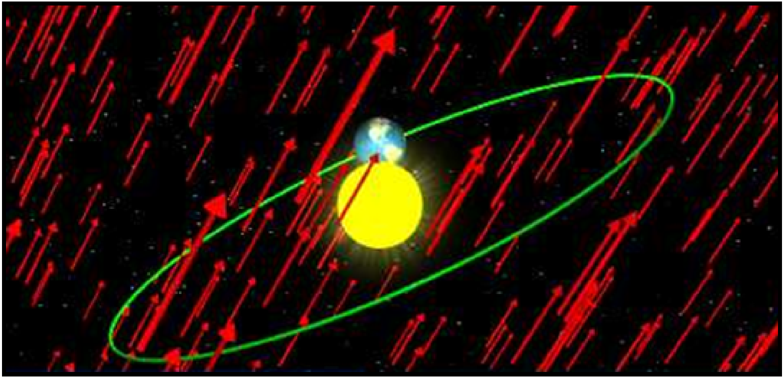


Lorentz violation



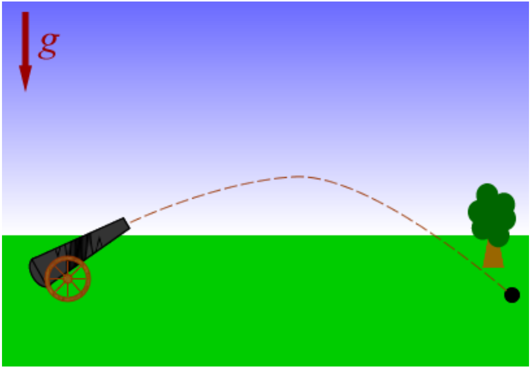
Lorentz violation

Does the universe have a preferred direction?

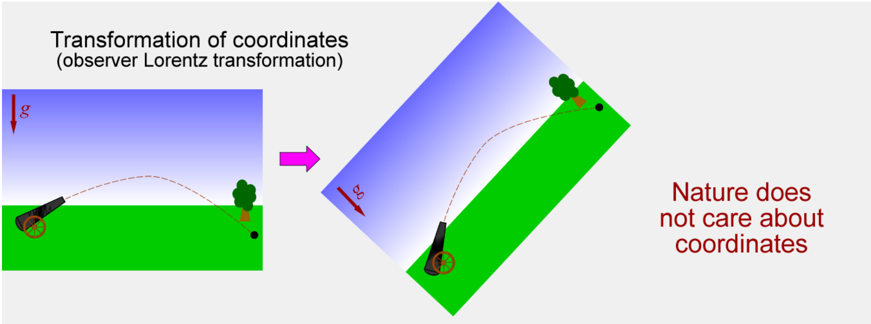


Physics and background fields

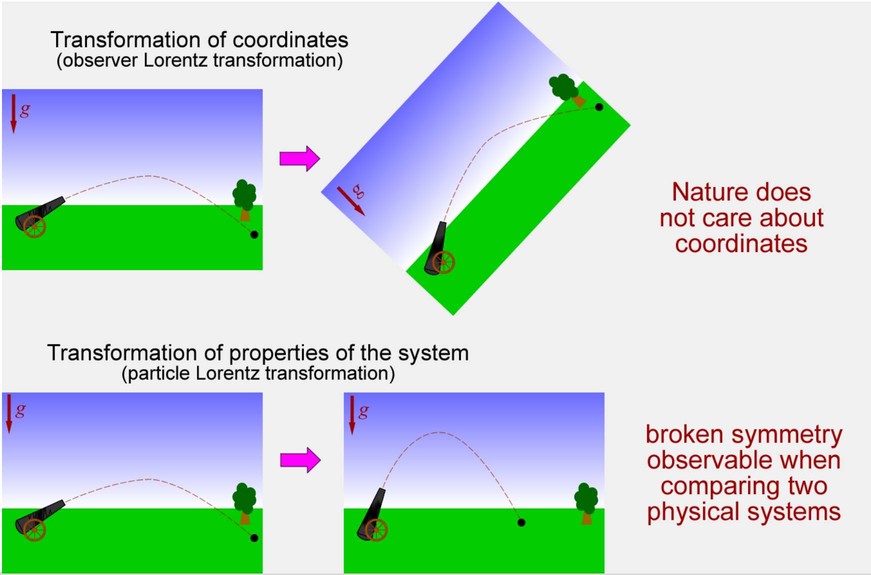
Searching for a broken symmetry



Physics and background fields

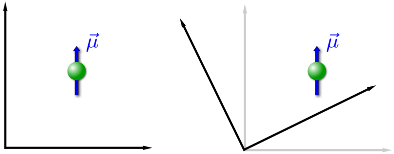


Physics and background fields



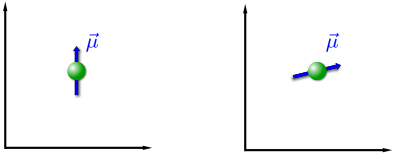
Lorentz transformations

Observer transformation



coordinate invariance

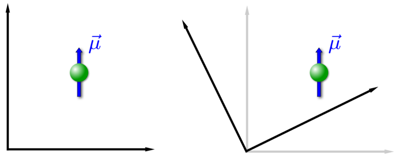
Particle transformation



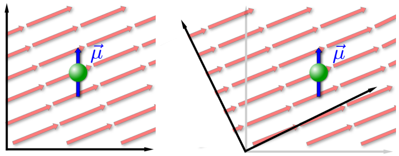
symmetry

Lorentz transformations

Observer transformation

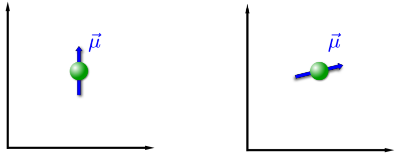


coordinate invariance



coordinate invariance

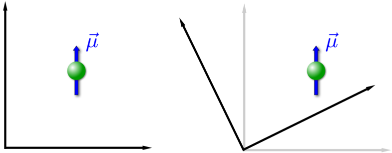
Particle transformation



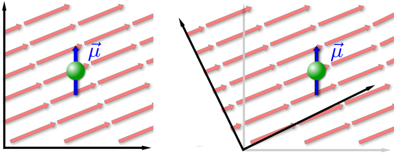
symmetry

Lorentz transformations

Observer transformation

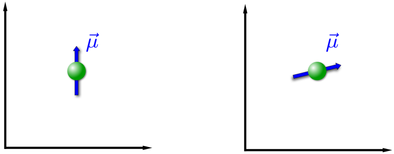


coordinate invariance

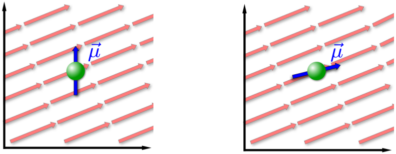


coordinate invariance

Particle transformation



symmetry



broken symmetry

Standard-Model Extension (SME)

SME $=$ Standard Model coupled to General Relativity $+$ all possible terms that break Lorentz symmetry

Colladay & Kostelecký, PRD 55, 6760 (1997)
Colladay & Kostelecký, PRD 58, 116002 (1998)
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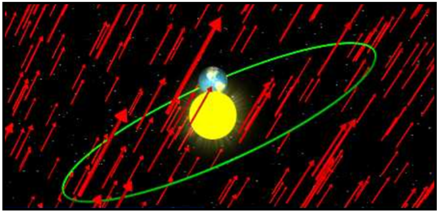

- general framework to search for Lorentz violation
- defined experimental signatures

example (from fermion sector):

$$\mathcal{L}_{\text{LV}} \supset a_\mu \left(\overline{\psi} \gamma^\mu \psi \right)$$

- Standard fields
- Controlling coefficients
- Observer scalars
- CPT violation included (no $m \neq \bar{m}$ terms)

SME: theory & experiment playground

Studies of CPT and Lorentz violation involve:

- neutrino oscillations
- beta decay
- oscillations and decays of K, B, D mesons
- particle-antiparticle comparisons
- matter interferometry
- birefringence and dispersion from cosmological sources
- clock-comparison measurements
- CMB polarization
- collider experiments
- electromagnetic resonant cavities
- equivalence principle
- gauge and Higgs particles
- high-energy astrophysical observations
- laboratory and gravimetric tests of gravity
- post-newtonian gravity in the solar system and beyond
- second- and third-generation particles
- space-based missions
- spectroscopy of hydrogen and antihydrogen
- spin-polarized matter

SME: worldwide searches

Karlsruhe Institute of Technology

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Neutrinos in the SME

Searching for Lorentz-violating neutrinos

Neutrinos in the SME

effective hamiltonian

Kostelecký & Mewes, PRD **69**, 016005 (2004)

$$\boldsymbol{H}_{\text{eff}} = \left(\begin{array}{c|c} h_0 & 0 \\ \hline 0 & h_0^* \end{array}\right) + \left(\begin{array}{c|c} \delta h_{\nu\nu} & \delta h_{\nu\bar{\nu}} \\ \hline \delta h_{\bar{\nu}\nu} & \delta h_{\bar{\nu}\bar{\nu}} \end{array}\right) \qquad \leftarrow 6 \times 6 \text{ matrix}$$

Neutrino 3×3 block:

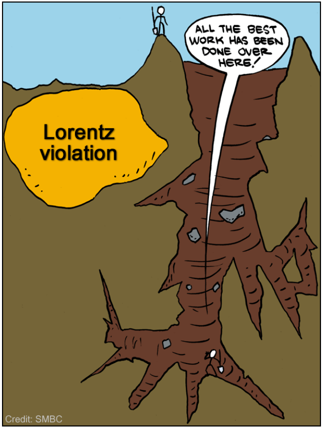
$$\boldsymbol{H}^{\nu}_{ab} = \underbrace{|\boldsymbol{p}|\delta_{ab} + \frac{\boldsymbol{m}^2_{ab}}{2|\boldsymbol{p}|}}_{h_0} + (a_L)^{\alpha}_{ab}\,\hat{p}_\alpha - (c_L)^{\alpha\beta}_{ab}\,\hat{p}_\alpha\,\hat{p}_\beta\,|\boldsymbol{p}|, \qquad a,b{=}e,\mu,\tau;\ \hat{p}^\alpha{=}(1;\hat{\boldsymbol{p}})$$

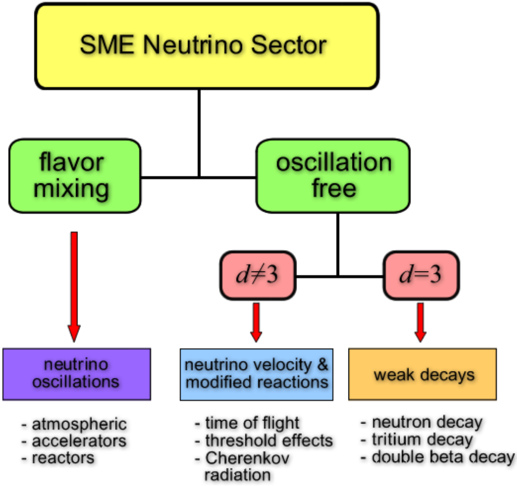
Novel effects

- **unconventional energy dependence**
- **direction dependence**
- **sidereal time dependence**
- **CPT violation**
- ν-$\bar{\nu}$ **mixing**

Experimental searches

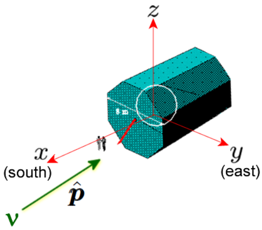
Complementarity between experiments

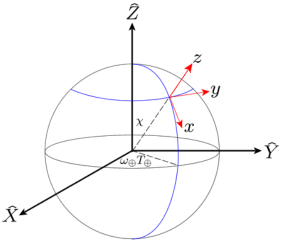


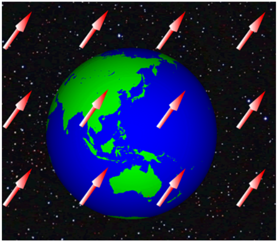


LV neutrino oscillations

Kostelecký & Mewes, PRD **70**, 076002 (2004)
JSD, Kostelecký & Mewes, PRD **80**, 076007 (2009)







Sidereal variation of the oscillation probability:

$$
\begin{aligned}
P_{\nu_b \to \nu_a} = {} & (P_{\mathcal{C}})_{ab} + (P_{\mathcal{A}_s})_{ab} \sin \omega_\oplus T_\oplus + (P_{\mathcal{A}_c})_{ab} \cos \omega_\oplus T_\oplus \\
& + (P_{\mathcal{B}_s})_{ab} \sin 2\omega_\oplus T_\oplus + (P_{\mathcal{B}_c})_{ab} \cos 2\omega_\oplus T_\oplus \\
& + \ldots
\end{aligned}
$$

LV neutrino oscillations

LV as a perturbation over mass-driven oscillations

- characterize effective hamiltonian

JSD, Kostelecký & Mewes, PRD **80**, 076007 (2009)

$$\boldsymbol{H}_{\text{eff}} = \boldsymbol{H}_0 + \delta\boldsymbol{H}$$

- perturbation theory → construct 6×6 time-evolution operator

$$\boldsymbol{S}(t) = e^{-i\boldsymbol{H}_{\text{eff}}t} = \boldsymbol{S}^{(0)}(t) + \boldsymbol{S}^{(1)}(t) + \boldsymbol{S}^{(2)}(t) + \cdots$$

- derive oscillation probabilities ($A, B = e, \mu, \tau, \bar{e}, \bar{\mu}, \bar{\tau}$)

$$P_{\nu_B \to \nu_A}(t) = \left|\boldsymbol{S}^{(0)}(t) + \boldsymbol{S}^{(1)}(t) + \boldsymbol{S}^{(2)}(t) + \cdots\right|^2$$

LV neutrino oscillations

LV as a perturbation over mass-driven oscillations

- characterize effective hamiltonian

JSD, Kostelecký & Mewes, PRD **80**, 076007 (2009)

$$\boldsymbol{H}_{\text{eff}} = \boldsymbol{H}_0 + \delta\boldsymbol{H}$$

- perturbation theory $\rightarrow$ construct 6×6 time-evolution operator

$$\boldsymbol{S}(t) = e^{-i\boldsymbol{H}_{\text{eff}}t} = \boldsymbol{S}^{(0)}(t) + \boldsymbol{S}^{(1)}(t) + \boldsymbol{S}^{(2)}(t) + \cdots$$

- derive oscillation probabilities ($A, B = e, \mu, \tau, \bar{e}, \bar{\mu}, \bar{\tau}$)

$$P_{\nu_B \rightarrow \nu_A}(t) = \left|\boldsymbol{S}^{(0)}(t) + \boldsymbol{S}^{(1)}(t) + \boldsymbol{S}^{(2)}(t) + \cdots\right|^2$$

Neutrino Osc.	Antineutrino Osc.	Neutrino-antineutrino Osc.
$P^{(0)}_{\nu_b \rightarrow \nu_a}$	$P^{(0)}_{\bar{\nu}_b \rightarrow \bar{\nu}_a}$	–
$P^{(1)}_{\nu_b \rightarrow \nu_a}$	$P^{(1)}_{\bar{\nu}_b \rightarrow \bar{\nu}_a}$	–
$P^{(2)}_{\nu_b \rightarrow \nu_a}$	$P^{(2)}_{\bar{\nu}_b \rightarrow \bar{\nu}_a}$	$P^{(2)}_{\nu_b \rightarrow \bar{\nu}_a}$

LV neutrino oscillations

Example: ν_μ disappearance

JSD, Kostelecký & Mewes, PRD **80**, 076007 (2009)

$$P_{\nu_\mu\to\nu_\tau} \approx P^{(0)}_{\nu_\mu\to\nu_\tau} + P^{(1)}_{\nu_\mu\to\nu_\tau}$$

$$\begin{aligned}
P^{(0)}_{\nu_\mu\to\nu_\tau} &= \sin^2 2\theta_{23} \sin^2\left(1.27\Delta m^2_{\text{atm}} L/E\right) \\
P^{(1)}_{\nu_\mu\to\nu_\tau} &= 2L\Big\{(P_{\mathcal{C}})_{\tau\mu} + (P_{\mathcal{A}_s})_{\tau\mu}\sin\omega_\oplus T_\oplus + (P_{\mathcal{A}_c})_{\tau\mu}\cos\omega_\oplus T_\oplus \\
&\quad +(P_{\mathcal{B}_s})_{\tau\mu}\sin 2\omega_\oplus T_\oplus + (P_{\mathcal{B}_c})_{\tau\mu}\cos 2\omega_\oplus T_\oplus\Big\}
\end{aligned}$$

LV neutrino oscillations

Karlsruhe Institute of Technology

Example: ν_μ disappearance

JSD, Kostelecký & Mewes, PRD **80**, 076007 (2009)

$$P_{\nu_\mu\to\nu_\tau} \approx P^{(0)}_{\nu_\mu\to\nu_\tau} + P^{(1)}_{\nu_\mu\to\nu_\tau}$$

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$$(P_{\mathcal{B}_s})_{\tau\mu} = \tfrac{1}{2}\,\text{Re}(\mathcal{B}^{(1)}_s)_{\mu\tau}\sin\left(2.54\Delta m^2_{\text{atm}} L/E\right)$$

LV neutrino oscillations

Example: ν_μ disappearance

JSD, Kostelecký & Mewes, PRD **80**, 076007 (2009)

$$P_{\nu_\mu\to\nu_\tau} \approx P^{(0)}_{\nu_\mu\to\nu_\tau} + P^{(1)}_{\nu_\mu\to\nu_\tau}$$

$$P^{(0)}_{\nu_\mu\to\nu_\tau} = \sin^2 2\theta_{23} \sin^2\left(1.27\Delta m^2_{\text{atm}} L/E\right)$$

$$P^{(1)}_{\nu_\mu\to\nu_\tau} = 2L\big\{(P_{\mathcal{C}})_{\tau\mu} + (P_{\mathcal{A}_s})_{\tau\mu}\sin\omega_\oplus T_\oplus + (P_{\mathcal{A}_c})_{\tau\mu}\cos\omega_\oplus T_\oplus + (P_{\mathcal{B}_s})_{\tau\mu}\sin 2\omega_\oplus T_\oplus + (P_{\mathcal{B}_c})_{\tau\mu}\cos 2\omega_\oplus T_\oplus\big\}$$

$$(P_{\mathcal{B}_s})_{\tau\mu} = \tfrac{1}{2}\,\text{Re}(\mathcal{B}_s^{(1)})_{\mu\tau}\sin\left(2.54\Delta m^2_{\text{atm}} L/E\right)$$

$$(\mathcal{B}_s^{(1)})_{\mu\tau} = N^X N^Y E\big((c_L)^{XX}_{\mu\tau} - (c_L)^{YY}_{\mu\tau}\big) - \big(N^X N^X - N^Y N^Y\big) E\,(c_L)^{XY}_{\mu\tau}$$

LV neutrino oscillations

PRL 105, 151601 (2010) PHYSICAL REVIEW LETTERS week ending 8 OCTOBER 2010

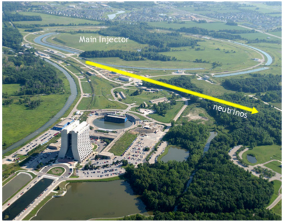
Search for Lorentz Invariance and CPT Violation with the MINOS Far Detector

(MINOS Collaboration)

In the SME, $P^{(1)}_{\mu\tau}$ is given by [8]

$$P^{(1)}_{\mu\tau} = 2L\{(P^{(1)}_{\mathcal{C}})_{\tau\mu} + (P^{(1)}_{\mathcal{A}_s})_{\tau\mu}\sin\omega_\oplus T_\oplus + (P^{(1)}_{\mathcal{A}_c})_{\tau\mu}\cos\omega_\oplus T_\oplus + (P^{(1)}_{\mathcal{B}_s})_{\tau\mu}\sin 2\omega_\oplus T_\oplus + (P^{(1)}_{\mathcal{B}_c})_{\tau\mu}\cos 2\omega_\oplus T_\oplus\}, \quad (1)$$

where $L = 735$ km is the distance from neutrino production in the NuMI beam to the MINOS FD [2], $T_\oplus$ is the local sidereal time (LST) at neutrino detection, and the coefficients $(P^{(1)}_{\mathcal{C}})_{\tau\mu}$, $(P^{(1)}_{\mathcal{A}_s})_{\tau\mu}$, $(P^{(1)}_{\mathcal{A}_c})_{\tau\mu}$, $(P^{(1)}_{\mathcal{B}_s})_{\tau\mu}$, and $(P^{(1)}_{\mathcal{B}_c})_{\tau\mu}$ contain the LV and CPTV information.



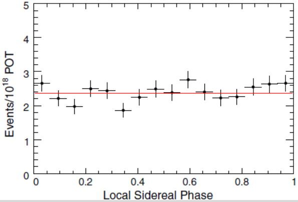


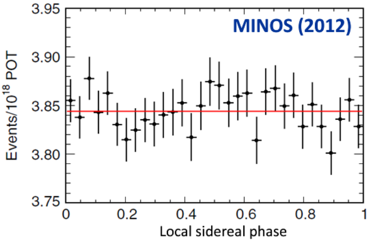
TABLE III. 99.7% C.L. limits on SME coefficients for $\nu_\mu \to \nu_\tau$; $(a_L)^\alpha_{\mu\tau}$ have units [GeV]; $(c_L)^{\alpha\beta}_{\mu\tau}$ are unitless.

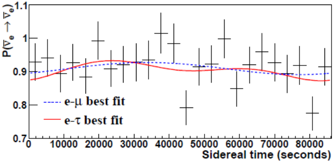
Coeff.	Limit	Coeff.	Limit
$(a_L)^X_{\mu\tau}$	5.9×10^{-23}	$(a_L)^Y_{\mu\tau}$	6.1×10^{-23}
$(c_L)^{TX}_{\mu\tau}$	0.5×10^{-23}	$(c_L)^{TY}_{\mu\tau}$	0.5×10^{-23}
$(c_L)^{XX}_{\mu\tau}$	2.5×10^{-23}	$(c_L)^{YY}_{\mu\tau}$	2.4×10^{-23}
$(c_L)^{XY}_{\mu\tau}$	1.2×10^{-23}	$(c_L)^{YZ}_{\mu\tau}$	0.7×10^{-23}
$(c_L)^{XZ}_{\mu\tau}$	0.7×10^{-23}	...	...

LV neutrino oscillations

Experimental searches

- **LSND** PRD **72**, 076004 (2005)
- **MINOS** PRL **101**, 151601 (2008)
- **IceCube** PRD **82**, 112003 (2010)
- **MINOS** PRL **105**, 151601 (2010)
- **MINOS** PRD **85**, 031101 (2012)
- **Double Chooz** PRD **86**, 112009 (2012)
- **MiniBooNE** PLB **718**, 1303 (2013)
- Rebel & Mufson AP **48** 78 (2013)
- Conrad, JSD, Katori, Spitz PLB **727**, 412 (2013)
- **Super-Kamiokande** arXiv:1410.4267
- ...

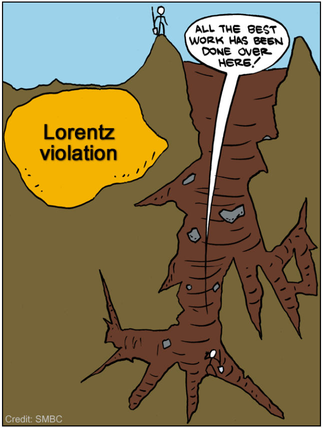


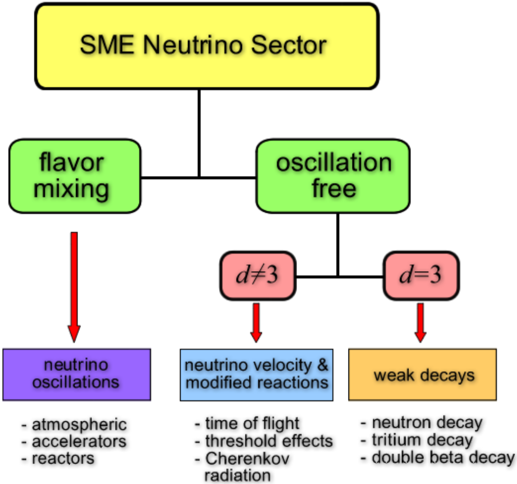


Double Chooz (2012)

Experimental searches

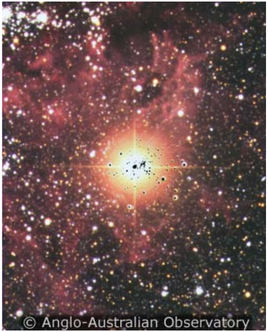
Complementarity between experiments





LV neutrino velocity

Kostelecký & Mewes, PRD **85**, 096005 (2012)



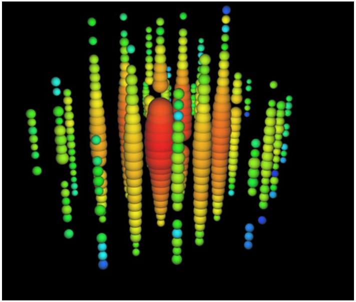
- Sensitive to **oscillation-free** effects
- Neutrino velocity can depend on:
  - **energy:** E
  - **sidereal time:** $\omega_\oplus T_\oplus$
  - **direction of propagation:** ${}_0\mathcal{N}_{jm}$
  - **particle or antiparticles**
- Physical effects
  - $v \neq 1 \to$ unconventional reactions
  - dispersion
- For beam experiments:

$$v \approx 1 - \frac{m^2}{2E^2} + \sum_{djm} (d-3) E^{d-4} e^{im\omega_\oplus T_\oplus} {}_0\mathcal{N}_{jm} \left[(a_{\text{of}}^{(d)})_{jm} - (c_{\text{of}}^{(d)})_{jm} \right]$$

Effects of **dimension-three operators** ($d = 3$) not observable

LV modified reactions

- observation of TeV-PeV neutrinos


IceCube Collaboration

- dispersion relation for high-energy neutrinos (neglecting CPT-odd terms)

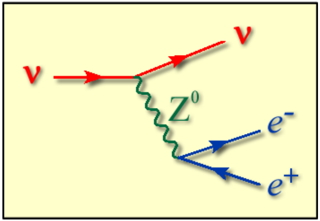
$$E(\boldsymbol{p}) = |\boldsymbol{p}| - \sum_{djm} |\boldsymbol{p}|^{d-3}\, Y_{jm}(\hat{\boldsymbol{p}})\,(c^{(d)}_{\text{of}})_{jm}$$

JSD, Kostelecký & Mewes, PRD 89, 043005 (2014)

- **energy loss as Cherenkov radiation**

$$\nu \to \nu + e^- + e^+$$

$$i\mathcal{M} = \frac{-i\sqrt{2}G_F M_Z^2}{(k+k')^2 - M_Z^2}\overline{\nu}(p')\gamma^\alpha \nu(p) \times \overline{u}(k)\gamma_\alpha(2\sin^2\theta_W - P_L)v(k')$$



LV modified reactions

JSD, Kostelecký & Mewes, PRD 89, 043005 (2014)

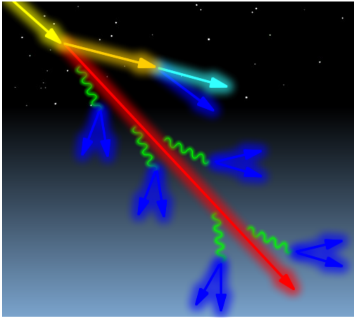
IceCube has observed PeV neutrinos

Aartsen et al., PRL 111, 021103 (2013), 113, 101101 (2014)

good: very high energy
bad: not many events
approach: consider isotropic LV

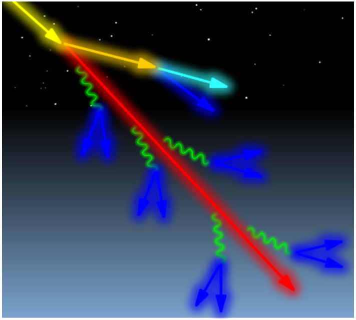
Energy loss per distance

$$\frac{dE}{dx} \propto -\int \frac{d^3p'}{2E_{p'}} \frac{d^3k}{2E_k} \frac{d^3k'}{2E_{k'}} \sum_{\text{spin}} |\mathcal{M}|^2 \frac{(E_k + E_{k'})}{E_p} \delta^4(p - p' - k - k')$$



LV modified reactions

JSD, Kostelecký & Mewes, PRD 89, 043005 (2014)



since we *do* observe PeV neutrinos

propagation distance		distortion distance
L	$<$	$D(E)$

characteristic distortion distance:

$$D(E) = -\frac{E}{(dE/dx)}$$

Conservative approach:

suppose PeV events are atmospheric

$$L \approx 1000\,\text{km} < D(E)$$

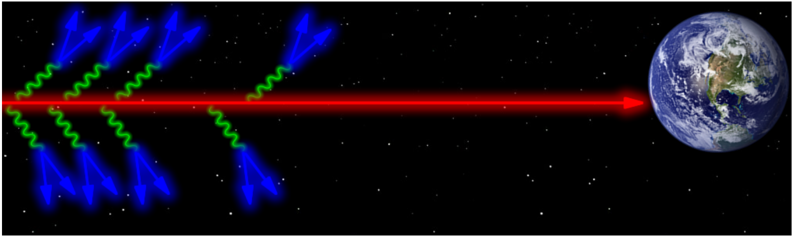
Lower bounds:

Coefficient	Atmospheric Čerenkov
$\mathring{c}^{(4)}$	$> -3 \times 10^{-13}$
$\mathring{c}^{(6)}$	$> -3 \times 10^{-25}$ GeV^{-2}
$\mathring{c}^{(8)}$	$> -2 \times 10^{-37}$ GeV^{-4}
$\mathring{c}^{(10)}$	$> -2 \times 10^{-49}$ GeV^{-6}

LV modified reactions

if PeV events are astrophysical:

JSD, Kostelecký & Mewes, PRD 89, 043005 (2014)



$\rightarrow$ neutrinos will lose energy falling below threshold

threshold condition:

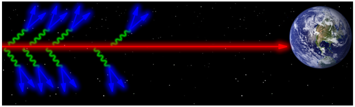
$$-\sum_{djm} |\boldsymbol{p}|^{d-2}\, Y_{jm}(\boldsymbol{p})\, (c_{\text{of}}^{(d)}) \lesssim 2m_e^2$$

Lower bounds:

Coefficient	Astrophysical Čerenkov	
$\mathring{c}^{(4)}$	$> -5 \times 10^{-19}$	
$\mathring{c}^{(6)}$	$> -5 \times 10^{-31}$	GeV^{-2}
$\mathring{c}^{(8)}$	$> -5 \times 10^{-43}$	GeV^{-4}
$\mathring{c}^{(10)}$	$> -5 \times 10^{-55}$	GeV^{-6}

LV modified reactions

JSD, Kostelecký & Mewes, PRD 89, 043005 (2014)

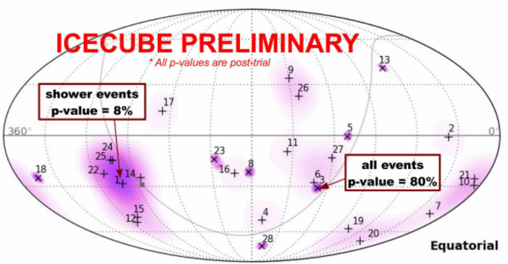


d	j	Lower bound	Coefficient	Upper bound
4	0	$-4\times10^{-19} <$	$(c_{\rm of}^{(4)})_{00}$	
4	1	$-1\times10^{-17} <$	$(c_{\rm of}^{(4)})_{10}$	$< 4\times10^{-17}$
		$-3\times10^{-17} <$	$\mathrm{Re}(c_{\rm of}^{(4)})_{11}$	$< 2\times10^{-17}$
		$-2\times10^{-17} <$	$\mathrm{Im}(c_{\rm of}^{(4)})_{11}$	$< 2\times10^{-17}$
4	2	$-1\times10^{-17} <$	$(c_{\rm of}^{(4)})_{20}$	$< 7\times10^{-17}$
		$-2\times10^{-17} <$	$\mathrm{Re}(c_{\rm of}^{(4)})_{21}$	$< 3\times10^{-17}$
		$-2\times10^{-17} <$	$\mathrm{Im}(c_{\rm of}^{(4)})_{21}$	$< 5\times10^{-17}$
		$-5\times10^{-17} <$	$\mathrm{Re}(c_{\rm of}^{(4)})_{22}$	$< 2\times10^{-17}$
		$-3\times10^{-17} <$	$\mathrm{Im}(c_{\rm of}^{(4)})_{22}$	$< 4\times10^{-17}$
6	0	$-3\times10^{-31} <$	$(c_{\rm of}^{(6)})_{00}$	
6	1	$-2\times10^{-28} <$	$(c_{\rm of}^{(6)})_{10}$	$< 9\times10^{-28}$
		$-6\times10^{-28} <$	$\mathrm{Re}(c_{\rm of}^{(6)})_{11}$	$< 5\times10^{-28}$
		$-3\times10^{-28} <$	$\mathrm{Im}(c_{\rm of}^{(6)})_{11}$	$< 3\times10^{-28}$
6	2	$-4\times10^{-28} <$	$(c_{\rm of}^{(6)})_{20}$	$< 7\times10^{-27}$
		$-1\times10^{-27} <$	$\mathrm{Re}(c_{\rm of}^{(6)})_{21}$	$< 2\times10^{-27}$
		$-1\times10^{-27} <$	$\mathrm{Im}(c_{\rm of}^{(6)})_{21}$	$< 3\times10^{-27}$
		$-5\times10^{-27} <$	$\mathrm{Re}(c_{\rm of}^{(6)})_{22}$	$< 6\times10^{-28}$
		$-1\times10^{-27} <$	$\mathrm{Im}(c_{\rm of}^{(6)})_{22}$	$< 4\times10^{-27}$
		⋮		

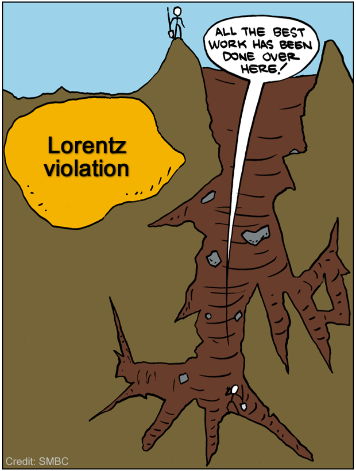
Astrophysical Cherenkov threshold

$$-\sum_{djm} |\boldsymbol{p}|^{d-2}\, Y_{jm}(\boldsymbol{p})\, (c_{\rm of}^{(d)})_{jm} \lesssim 2m_e^2$$

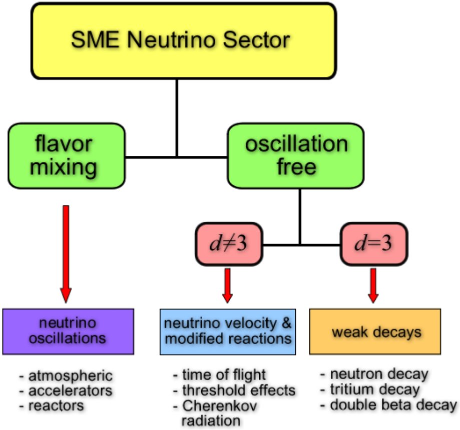
two-sided bounds can be obtained from several events distributed in the sky



Experimental searches



Complementarity between experiments

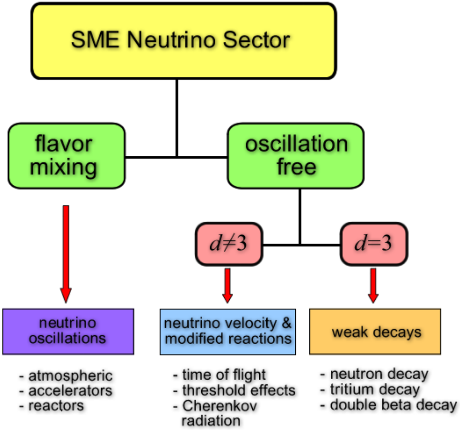


Countershaded relativity violations: comparatively large relativity- violating effects that have escaped detection

Kostelecký & Tasson, PRL **102**, 010402 (2009)

Experimental searches

Complementarity between experiments



Countershaded relativity violations: comparatively large relativity- violating effects that have escaped detection

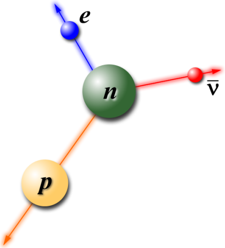
Kostelecký & Tasson, PRL **102**, 010402 (2009)

LV beta decay

Theoretical considerations:

JSD, Kostelecký & Lehnert, PRD **88**, 071902 (2013)
JSD, Adv.HEP **2014**, 305298 (2014)

- antineutrino spinors get modified
- modified antineutrino phase space
- coefficients: $(a_{\text{of}}^{(3)})_{00}$, $(a_{\text{of}}^{(3)})_{10}$, $\text{Re}(a_{\text{of}}^{(3)})_{11}$, $\text{Im}(a_{\text{of}}^{(3)})_{11}$



$$d\Gamma \propto E\omega \left\{ 1 + a\frac{\boldsymbol{p}\cdot\widetilde{\boldsymbol{q}}}{E\omega} + A\frac{\hat{\boldsymbol{n}}\cdot\boldsymbol{p}}{E} + B\frac{\hat{\boldsymbol{n}}\cdot\widetilde{\boldsymbol{q}}}{\omega} + D\frac{\hat{\boldsymbol{n}}\cdot(\boldsymbol{p}\times\widetilde{\boldsymbol{q}})}{E\omega} \right\} \frac{d^3p}{2E}\frac{d^3q}{2\omega}\,\delta(E+\omega-E_0)$$

LV in W boson

J.P. Noordmans, PRC 87, 055502 (2013)
E.A. Dijck et al., PRD88, 07190 (2013)
J.P. Noordmans et al., PRL111, 171601 (2013)
S.E. Müller et al., PRD88, 071901 (2013)
B. Altschul, PRD88, 076015 (2013)
K.K. Vos et al.,PLB729, 112 (2014)

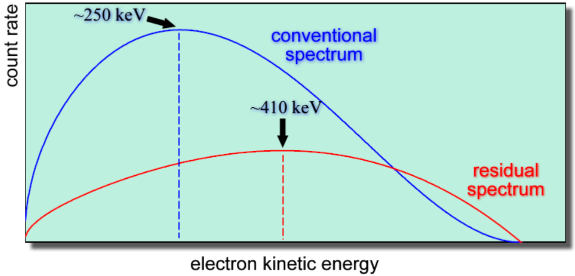
Observable effects

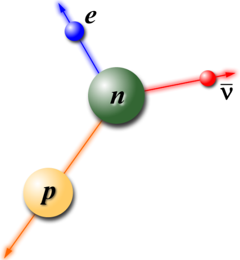
- spectrum distortion
- modified experimental electron-neutrino asymmetry a_{exp}
- modified experimental neutrino asymmetry B_{exp}

LV neutron decay

JSD, Kostelecký & Lehnert, PRD **88**, 071902 (2013)
JSD, Adv.HEP **2014**, 305298 (2014)

1. spectrum distortion





- generated by isotropic Lorentz violation: $(a_{\text{of}}^{(3)})_{00}$
- requires searching for deviations from conventional spectrum
- effect is maximal at a well-defined energy
- $(a_{\text{of}}^{(3)})_{00}$ also controls a **new source of CP violation**

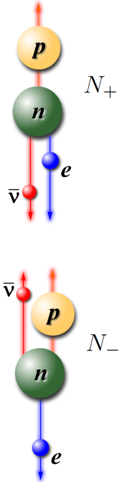
LV neutron decay

2. electron-neutrino asymmetry

JSD, Kostelecký & Lehnert, PRD **88**, 071902 (2013)
JSD, Adv.HEP **2014**, 305298 (2014)

$$a_{\text{exp}} = \frac{N_+ - N_-}{N_+ + N_-}$$

$$= a\beta + \sqrt{\frac{3}{\pi}}\frac{a^2\beta^2 - 1}{T_0 - T}\Big[\cos\chi(a_{\text{of}}^{(3)})_{10} + \sqrt{2}\sin\chi\,\text{Im}(a_{\text{of}}^{(3)})_{11}\sin\omega_\oplus T_\oplus - \sqrt{2}\sin\chi\,\text{Re}(a_{\text{of}}^{(3)})_{11}\cos\omega_\oplus T_\oplus\Big]$$



- sensitivity to anisotropic Lorentz violation
- constant modification: $(a_{\text{of}}^{(3)})_{10}$
- sidereal variation of asymmetry: $\text{Re}(a_{\text{of}}^{(3)})_{11}$, $\text{Im}(a_{\text{of}}^{(3)})_{11}$

LV neutron decay

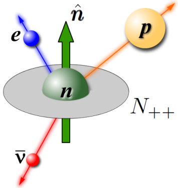
Karlsruhe Institute of Technology

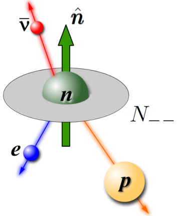
3. neutrino asymmetry

JSD, Kostelecký & Lehnert, PRD **88**, 071902 (2013)
JSD, Adv.HEP **2014**, 305298 (2014)

$$B_{\text{exp}} = \frac{N_{--} - N_{++}}{N_{--} + N_{++}}$$

$$= (B_{\text{exp}})_0 + \delta B_{\mathcal{C}} + \delta B_{\mathcal{A}_s} \sin \omega_\oplus T_\oplus + \delta B_{\mathcal{A}_c} \cos \omega_\oplus T_\oplus$$





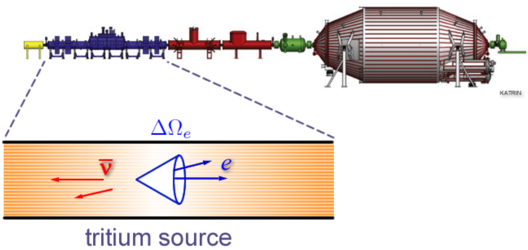
- polarized neutrons
- sensitivity to anisotropic Lorentz violation
- constant modification: $(a_{\text{of}}^{(3)})_{10}$
- sidereal variation of asymmetry: $\text{Re}(a_{\text{of}}^{(3)})_{11}$, $\text{Im}(a_{\text{of}}^{(3)})_{11}$

LV tritium decay

Endpoint measurements

JSD, Kostelecký & Lehnert, PRD **88**, 071902 (2013)
JSD, Adv.HEP **2014**, 305298 (2014)

$$\frac{d\Gamma}{dE} \propto \int_{\Delta\Omega_e} E|\boldsymbol{p}|\, d\Omega_e \, \frac{d^3q}{2\omega} \, \delta(E + \omega - E_0)$$



- Mainz
- Troitsk
- KATRIN

LV tritium decay

Endpoint measurements

JSD, Kostelecký & Lehnert, PRD **88**, 071902 (2013)
JSD, Adv.HEP **2014**, 305298 (2014)

$$\frac{d\Gamma}{dT} = 3C_R\Big[\big(T_{\text{eff}} - T\big)^2 - \tfrac{1}{2}\,m^2\Big]$$

$$T_{\text{eff}} = T_0 + \delta T_{\mathcal{C}} + \delta T_{\mathcal{A}_s}\sin\omega_\oplus T_\oplus + \delta T_{\mathcal{A}_c}\cos\omega_\oplus T_\oplus$$

LV tritium decay

Endpoint measurements

JSD, Kostelecký & Lehnert, PRD **88**, 071902 (2013)
JSD, Adv.HEP **2014**, 305298 (2014)

$$\frac{d\Gamma}{dT} = 3C_R\left[\left(T_{\text{eff}} - T\right)^2 - \tfrac{1}{2}\, m^2\right]$$

$$\begin{aligned} T_{\text{eff}} &= T_0 + \delta T_{\mathcal{C}} \\ &\quad + \delta T_{\mathcal{A}_s} \sin\omega_\oplus T_\oplus + \delta T_{\mathcal{A}_c} \cos\omega_\oplus T_\oplus \end{aligned}$$

Effective dimension-two coefficient $(c^{(2)}_{\text{eff}})$

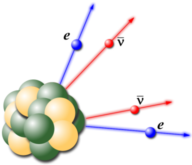
- Lorentz violation, CPT invariance

$$m^2_{\text{eff}} = m^2 + m^2_{\mathcal{C}} + m^2_{\mathcal{A}_s} \sin\omega_\oplus T_\oplus + m^2_{\mathcal{A}_c} \cos\omega_\oplus T_\oplus$$

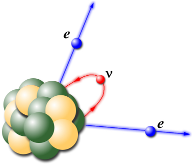
- can mimic behavior of a *tachyonic neutrino*

LV double beta decay

JSD, Kostelecký & Lehnert, PRD **88**, 071902 (2013)
JSD, PRD **89**, 036002 (2014)



$$d\Gamma \propto \int |M^{2\nu}|^2 F(Z,E_1)F(Z,E_2) \times \delta(E_1+E_2+\omega_1+\omega_2-\Delta M) \times \frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3q_1}{2\omega_1}\frac{d^3q_2}{2\omega_2}$$

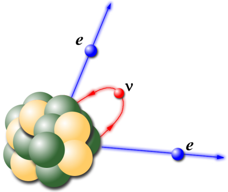


$$d\Gamma \propto \int |M^{0\nu}|^2 F(Z,E_1)F(Z,E_2) \times \delta(E_1+E_2-\Delta M)\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}$$

LV double beta decay

$0\nu\beta\beta$ half-life gets modified

JSD, Kostelecký & Lehnert, PRD **88**, 071902 (2013)
JSD, PRD **89**, 036002 (2014)

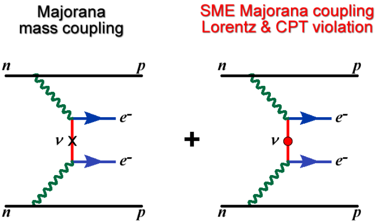


- neutrino propagator modifies the effective mass

$$\frac{1}{T_{1/2}} = G(Z,Q)\,|M^{0\nu}|^2\, m^2$$

$$m^2 \to m^2 + m\frac{g}{R} + \left(\frac{g}{R}\right)^2$$

R: nuclear radius



- **neutrinoless double beta decay can occur for massless neutrinos**

Summary

Karlsruhe Institute of Technology

- Tests of Lorentz invariance constitute a **worldwide effort** across multiple disciplines
- We have determined the **key experimental signatures** of Lorentz and CPT violation in
    - neutrino oscillations
    - ToF & modified reactions
    - single & double beta decays
- Many effects of Lorentz violation **remain unexplored**
- Interesting prospects for **low- and high-energy experiments**
- Rich research area for **theory-experiment collaboration**

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Karlsruhe Institute of Technology

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"Today we say that the law of relativity is supposed to be true at all energies, but someday somebody may come along and say how stupid we were."

R.P. Feynman