From the lowest energies to the highest: constraints on CP violation from permanent electric dipole moments



MPIK Gentner Colloquium, 30 November 2022 Skyler Degenkolb

Case 1: trapped neutron interferometry





Low-Energy Precision Physics

Physikalisches Institut Universität Heidelberg

Case 2: nuclear spin gyroscopes



Typical(?) experiment/theory interaction



TC: "How many EDM experiments do we need?"

MJRM: "Only the one that discovers an EDM"

TC: "Oh – so now I know which one to work on…?"



Our motivation, and a sense of scale





State of the art: catch/pour ...with 0.1% success



New approach: catch them all, directly in many bottles



1 lost

(2) Challenge the second: observation time

"Never measure anything but frequency"

–Arthur Schawlow (1981 Physics Nobel Prize)





But... how to store or cool ensembles?

Wave optics, with massive particles!

"Cold" beams: O(500 m/s)

particles fly through most experiments in milliseconds





"Ultracold" traps: O(5 m/s)

particles stored for minutes (>10⁵ ms)



Well, suppose the scale of new physics is far above the SM...

...or imagine we couldn't access the heavy gauge bosons we already know



If the scale of new physics is >> TeV, it looks the same whether we probe it at TeV or neV!

EXP

Orders of Magnitude

Some recent experimental EDM limits:

$$\begin{array}{l} n: \ |d| < 1.8 \times 10^{-26} \ e \ \mathrm{cm} \ (90\% \ \mathrm{C.L.}) \\ ^{129}\mathrm{Xe:} \ |d| < 1.4 \times 10^{-27} \ e \ \mathrm{cm} \ (95\% \ \mathrm{C.L.}) \\ \mathrm{ThO:} \ |d| < 1.1 \times 10^{-29} \ e \ \mathrm{cm} \ (90\% \ \mathrm{C.L.}) \end{array}$$

$$10^{-26}e \text{ cm} \times \frac{1 \text{ MV}}{m} \times \frac{1}{2\pi\hbar} = 24 \text{ nHz}$$

$$10^{-22} \text{ eV sensitivity}$$
(cf. 10¹³ eV for new physics scale)

Another example of jumping across several orders:

neutron capture $\rightarrow 10^{6-7}$ eV products nEDM systematics $\rightarrow 10^{-7}$ eV shifts

... so how to get 10⁻¹⁴ in energy (or equivalent in momentum)?

Some handy conversion factors:

$$1 \text{ neV} = 1 \frac{\text{GeV}}{c^2} \times 1 \text{ cm} \times g$$
$$1 e \text{ cm} = 10^{13} e \text{ fm}$$



"Permanent Electric Dipole Moment" = ?

Quantum eigenfrequencies:

$$\hbar\omega_E \propto -dm{S}\cdotm{E}$$

Classical moments:

$$\mathbf{d} = \int \mathbf{r} \rho(\mathbf{r}) d\mathbf{r}$$
$$\boldsymbol{\mu} = \frac{1}{2} \int \mathbf{r} \times \mathbf{J}(\mathbf{r}) d\mathbf{r}$$

$$\hbar\omega_B \propto -\mu oldsymbol{S} \cdot oldsymbol{B}$$







Is it different from a molecular dipole?

...or, "a warm-up for non-relativistic quantum methods"



What happens in an electric field?



Find the eigenstates



The *energy* eigenstates are:

$$\frac{1}{\sqrt{2}}\left(\left|1\right\rangle\pm\left|2\right\rangle\right)$$

Check the limiting cases



$$E_{\pm} = E_0 + \sqrt{A^2 + d^2 E^2}$$

The *energy* eigenstates are:

$$\frac{1}{\sqrt{2}}\left(\left|1\right\rangle\pm\left|2\right\rangle\right)$$

No surprises, actually





The *energy* eigenstates are:

$$\frac{1}{\sqrt{2}}\left(\left|1\right\rangle\pm\left|2\right\rangle\right)$$

So... is it different from a molecular dipole?

dE





New Physics, in Familiar Terms

- Non-conservation of *P* and *T* already apparent in EDM term
- Consistency with zero vs. consistency with SM





A Taxonomy of Form Factors*

*which are not just for composite particles!



A Taxonomy of Form Factors

Summary of Motivation

Problem: 1 extra baryon in every 10^9

- Asymmetry: $\eta = \frac{n_B n_{\bar{B}}}{n_{\gamma}}$
- n_{γ} comes from CMB decoupling
- Actually normalize to entropy density *s*, since universe expands

Requirements: Sakharov's criteria

- Baryon-number (B) violation
- ullet C and CP-violation
- Departure from thermal equilibrium

Solution: ???

- No complex antimatter nuclei
- No annihilation fronts
- No adequate symmetry-breaking

Prediction:

- New *CP*-violating physics
- Coupling to Standard Model baryons
- Polarization of bound states

Dimensional Analysis

Naïve estimate for generic new physics:

$$d_n \propto \frac{m_q}{\Lambda^2} \cdot e \cdot \phi_{\rm CPV}$$

Current experiments: 10⁻²⁶ e cm $\longrightarrow \Lambda \sim 10-100 {\rm ~TeV}$

Analysis: neutron in Global context

Naïve estimate for generic new physics:

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Current experiments: 10⁻²⁶ e cm $\longrightarrow \Lambda \sim 10-100~{\rm TeV}$

Standard Model CKM: $10^{-32} e \text{ cm}$ Standard Model QCD: ??? $\longrightarrow d_n \approx (10^{-16} e \text{ cm})\bar{\theta}$

Neutron EDM from CP-violating pion couplings:



Pospelov & Ritz, Annals of Physics 318 (2005): 119-169



Analysis: neutron in Global context

Define a matrix ∂_{ij} according to $d_i = \sum_i \alpha_{ij} C_j$, e.g.,

$$d_n \approx \bar{d}_n^{\rm sr} + 1.6 \times 10^{-14} \bar{g}_{\pi}^{(0)} - 8.6 \times 10^{-16} \bar{g}_{\pi}^{(1)} + 1.5 \times 10^{-18} C_T^{(0)}$$

for global analysis at the atomic/nuclear level.

Lattice calculations would* also give us some control at the hadronic level:

$$\begin{split} d_n^{\rm sr} &= g_T^{(n,u)} d_u + g_T^{(n,d)} d_d + g_T^{(n,s)} d_s & \text{FLAG 2021, 5-10\% for } u, d \\ &- (0.55 \pm 0.28) e \tilde{d}_u - (1.1 \pm 0.55) e \tilde{d}_d & \text{*QCD sum rules} \\ &+ \text{Weinberg} + 4\text{-fermion} & \text{*Naïve dim. analysis} \end{split}$$

"Global analysis" (hadronic/nuclear)

Define a matrix ∂_{ii} according to $d_i = \sum \alpha_{ij} C_j$,

values: Rev. Mod. Phys. 91, 015001 (2019)

$$\alpha_{ij} = \begin{pmatrix} 1.0 & 1.6 \times 10^{-14} & -8.6 \times 10^{-16} & 1.5 \times 10^{-18} \\ 1.9 \times 10^{-5} & -8.6 \times 10^{-21} & -2.1 \times 10^{-19} & -6.1 \times 10^{-21} \\ -5.7 \times 10^{-4} & -1.3 \times 10^{-17} & 1.9 \times 10^{-17} & 3.1 \times 10^{-20} \\ 1.2 \times 10^{-2} & 2.2 \times 10^{-15} & -8.1 \times 10^{-15} & -8.4 \times 10^{-19} \end{pmatrix} \begin{bmatrix} \vec{d}_n^{(\text{sr})} \\ \vec{g}_n^{(0)} \\ \vec{g}_n^{(1)} \\ C_T^{(0)} \end{bmatrix}$$

...and invert it:

$$\begin{pmatrix} \bar{d}_{n}^{(\mathrm{sr})} \\ \bar{g}_{\pi}^{(0)} \\ \bar{g}_{\pi}^{(1)} \\ \bar{g}_{\pi}^{(1)} \\ C_{T}^{(0)} \end{pmatrix} = \begin{pmatrix} 1.0 & 1.6 \times 10^{-14} & -8.6 \times 10^{-16} & 1.5 \times 10^{-18} \\ 1.9 \times 10^{-5} & -8.6 \times 10^{-21} & -2.1 \times 10^{-19} & -6.1 \times 10^{-21} \\ -5.7 \times 10^{-4} & -1.3 \times 10^{-17} & 1.9 \times 10^{-17} & 3.1 \times 10^{-20} \\ 1.2 \times 10^{-2} & 2.2 \times 10^{-15} & -8.1 \times 10^{-15} & -8.4 \times 10^{-19} \end{pmatrix} \begin{pmatrix} d_n \\ d_{\mathrm{Xe}} \\ d_{\mathrm{Hg}} \\ d_{\mathrm{Ra}} \end{pmatrix}$$

Status from 2019: (hadronic/nuclear)



Updates now in progress...



Many Parameters / Many Experiments

Sensitivity: System:	Paramagnetic	Diamagnetic	"Particle"
Тгар	Tl, Cs, PbO, HfF⁺, Fr, BaF,	¹⁹⁹ Hg, ¹²⁹ Xe, ²²⁵ Ra, Rn, Pa, RaO,	n (ultra-cold)
Beam	YbF, ThO, WC	TIF	n (cold)
Storage ring	TaO⁺	?	p, d, ³ He ⁺⁺ , μ,

Other: solid state (Gd₃Ga₅O₁₂, Eu_{0.5}Ba_{0.5}TiO₃), colliders (τ , Λ , ν , ...), crystal (n scattering on quartz), ...

How could you measure an EDM?



$$\hbar(\omega_+ - \omega_-) = 4dE$$

... up to drift, gradients, etc.

Time-Domain Interferometry

Ramsey's method to measure frequencies*:



*we'll come back to *frequency* vs. *phase*

How could you measure an EDM?



How could you measure an EDM?

What if we could measure continuously?



The HeXe Experiment: ¹²⁹Xe

Use the best magnetic shields available (at least to start with...)



The HeXe Experiment

We do not expect a large Schiff enhancement in ¹²⁹Xe

$$S = s_N d_N + \frac{m_N g_A}{F_{\pi}} [a_0 \bar{g}_{\pi}^{(0)} + a_1 \bar{g}_{\pi}^{(1)} + a_2 \bar{g}_{\pi}^{(2)}]$$

$$\longrightarrow d_A(\text{dia}) = \kappa_S S - [k_{C_T}^{(0)} C_T^{(0)} + k_{C_T}^{(1)} C_T^{(1)}]$$

Octupole-deformed nuclei can have enhanced EDMs:

04

Laser box

$$S \propto \frac{\eta \beta_1 \beta_3^2 A^{\frac{2}{3}} r_0^3}{E_+ - E_-}$$

...but new challenges for those.



HeXe EDM

The HeXe Experiment

HeXe EDM



The HeXe Experiment



A rapidly-moving field!



Our result from HeXe:

 $d_A(^{129}\text{Xe}) = (1.4 \pm 6.6_{\text{stat}} \pm 2.0_{\text{syst}}) \times 10^{-28} \ e \text{ cm}$

Phys. Rev. Lett. 123, 143003 (2019)



Near-simultaneous from MiXed:

 $d_{\rm Xe} = (-4.7 \pm 6.4) \cdot 10^{-28} \ e {\rm cm}$

Phys. Rev. A 100, 022505 (2019)

A rapidly-moving field!



150 d_{Xe} (10⁻²⁸ ecm) 100 50 50 - 100 - 150 - 200 2 8 3 7 9 5 6 run#

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A United Future at the PI



By contrast: neutrons disappear faster!

Much better at low temperature: Neulinger *et al.*, EPJA 58, 141 (2022)

The PanEDM Experiment

- Double chamber Ramsey interferometer at room temperature (but $T_{UCN} \sim 5$ mK)
- ¹⁹⁹Hg magnetometers with few-fT resolution
- Cs magnetometers (also at high voltage)
- Magnetic shielding factor: 6×10⁶ at 1 mHz
- Simultaneous spin detection for up/down
- SuperSUN UCN source at ILL in 2 phases: Phase I: unpolarized UCN with 80 neV peak Phase II: polarized UCN, magnetic storage
- Ongoing installation of parts, commissioning with UCN production in 2023-2024

EPJ Web of Conferences 219, 02006 (2019)

Much lower statistics!

SuperSUN	Phase I			
Saturated source				
density [cm ⁻³]	330			
Diluted density [cm ⁻³]	63			
Density in cells [cm ⁻³]	3.9			
PanEDM Sensitivity [1	$\sigma, e \text{ cm}]$		-	
Per run	5.5×10^{-25}	 ΔE /	$\Delta t \geq$	$\hbar/2$
Per day	3.8×10^{-26}	L		
Per 100 days	3.8×10^{-27}			

EPJ Web of Conferences 219, 02006 (2019)

D. Wurm, PhD 2021

The PanEDM Experiment

EPJ Web of Conferences	219,	02006	(2019)
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- 1: EDM cells2:3: HV feed4:5: Inner shield6:7: Outer shield door
- 2: Vac. Chamber
 4: B₀ & B₁ coil
 6: Outer shield

PanEDM @ ILL, 2021

The PanEDM Experiment

Rev. Sci. Inst. 85(7), 075106 (2014) J. Appl. Phys. 117(18), 183903 (2015)

EPJ Web of Conferences 219, 02006 (2019)

 Vac. pumps
 SP
 UCN optics
 MSR

D. Wurm, PhD 2021

Reality always looks messier!

UCN and Production in He-II

SuperSUN Neutron Source: Cutaway

UCN out

Proof-of-principle: quantum sensing

SEMINAR!

"Quantum Sensing" for Neutrons

Spin-dependent shifts of the wall potential (magnetic field increased for visibility)

SEMINAR!

The next generation... scaling up!

Thematic Recap

Questions?

what-if.xkcd.com

Special thanks to:

V. Cirigliano, J. de Vries, U. Schmidt

PI mechanical workshop PI technical design office

Institut Laue-Langevin, NPP & SANE BNC GINA team and user support

PanEDM collaboration HeXe collaboration

xkcd.com

EDMs in the SM do not vanish

• CP violation from three sources (ignoring neutrinos):

$$\mathcal{L}_{\text{CPV}} = \mathcal{L}_{\text{CKM}} + \mathcal{L}_{\bar{ heta}} + \mathcal{L}_{\text{BSM}}$$

• CKM CP-violation (Standard Model):

$$\mathcal{L}_{\text{CKM}} = -\frac{ig_2}{\sqrt{2}} \sum_{p,q} V^{pq} \bar{U}_L^p \mathcal{W}^+ D_L^q + \text{H.c.}$$

• Strong CP-violation (Standard Model):

$$\mathcal{L}_{\bar{\theta}} = -\frac{\alpha_S}{16\pi^2} \bar{\theta} \mathrm{Tr}(G^{\mu\nu} \tilde{G}_{\mu\nu})$$

details:

Rev. Mod. Phys. **91**, 015001 (2019) Phys. Rev. C **91**, 035502 (2015) Prog. Part. Nucl. Phys. **71**, 21 (2013)

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*recently called into question: arXiv:2205.15093, 2001.07152, 1912.03941, 2106.11369

Effective Field Theory for EDMs

General Effective Lagrangian:

$$\mathscr{L}_{\text{eff}} = \mathscr{L}_{\text{SM}} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_{i} \frac{C_{i}^{(6)}}{\Lambda^{2}} O_{i}^{(6)} + \dots$$

Dimension-Six terms for the neutron:

$$\begin{aligned} \mathscr{L}_{\text{eff}}^{(6)} &= -\frac{i}{2} \sum_{l,q} d_q \bar{q} \sigma_{\mu\nu} \gamma^5 F^{\mu\nu} q \\ &- \frac{i}{2} \sum_q \tilde{d}_q g_s \bar{q} \sigma_{\mu\nu} \gamma^5 G^{\mu\nu} q \\ &+ d_W \frac{g_s}{6} G \tilde{G} G + \sum_i C_i^{(4f)} O_i^{(4f)} \end{aligned}$$

Global Analysis: T. Chupp, M. Ramsey-Musolf *Rev. Mod. Phys.* **91**, 015001 (2019) *Phys. Rev. C* **91**, 035502 (2015)

Prog. Part. Nucl. Phys. 71, 21 (2013)

Wilson coefficient	Operator (dimension)	Number
$\bar{ heta}$	Theta term (4)	1
δ_e	Electron EDM (6)	1
Im $C^{(1,3)}_{\ell equ}$, Im $C_{\ell eqd}$	Semi-leptonic (6)	3
δ_q	Quark EDM (6)	2
$\tilde{\delta}_{q}$	Quark chromo EDM (6)	2
$C_{\tilde{G}}$	Three-gluon (6)	1
$\operatorname{Im} C_{auad}^{(1,8)}$	Four-quark (6)	2
Im $C_{\varphi ud}$	Induced four-quark (6)	1
Total		13

Interpreting EDM bounds

neutron: diamagnetic: paramagnetic:

$$\bar{d}_n^{sr}, \ \bar{g}_\pi^{(0)}, (\ \bar{g}_\pi^{(1)})$$

$$\bar{g}_\pi^{(0,1)}, \ C_T^{0,1}$$

$$d_e, \ C_S^{(0)}$$

$$\mathcal{L}_{\pi NN} = \bar{N} \left[\bar{g}_{\pi NN}^{(0)} \vec{\tau} \cdot \vec{\pi} + \bar{g}_{\pi NN}^{(1)} \pi^0 + \bar{g}_{\pi NN}^{(2)} (3\tau_3 \pi^0 - \vec{\tau} \cdot \vec{\pi}) \right] N$$
$$\mathcal{L}_T = \frac{8G_F}{\sqrt{2}} \bar{e} \sigma^{\mu\nu} e v_{\nu} \bar{N} \left[C_T^{(0)} + C_T^{(1)} \tau_3 \right] S_{\mu} N$$
$$\mathcal{L}_S = -\frac{G_F}{\sqrt{2}} \bar{e} i \gamma_5 e \ \bar{N} \left[C_S^{(0)} + C_S^{(1)} \tau_3 \right] N$$

 Schiff's theorem: the field due to an EDM induces a displacement of the bound charges, which exactly cancels it*

$$H_0 = \sum \frac{p^2}{2m} + U(\mathbf{r})$$

Hamiltonian of the charge-system (no EDM)

*Schiff: Phys. Rev. **132**, 2194 (1963) J. Engel: elegant formulation used here

 Schiff's theorem: the field due to an EDM induces a displacement of the bound charges, which exactly cancels it

$$H_0 = \sum \frac{p^2}{2m} + U(\mathbf{r})$$

Add constituent EDMs As a perturbation...

$$\mathbf{d}_{ ext{tot}} = \sum_i \mathbf{d}_i$$

(sum over constituents)

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$$egin{aligned} H &= H_0 - \sum \mathbf{d} \cdot \mathbf{E} \ &= H_0 + \sum \mathbf{d} \cdot rac{
abla U(\mathbf{r})}{q} \ &= H_0 + \sum rac{i}{q} \left[\mathbf{d} \cdot \mathbf{p}, H_0
ight] \end{aligned}$$

Now see what effect this has...

 Schiff's theorem: the field due to an EDM induces a displacement of the bound charges, which exactly cancels it

$$H_0 = \sum \frac{p^2}{2m} + U(\mathbf{r})$$
$$H = H_0 - \sum \mathbf{d} \cdot \mathbf{E}$$
$$= H_0 + \sum \mathbf{d} \cdot \frac{\nabla U(\mathbf{r})}{q}$$
$$= H_0 + \sum \frac{i}{q} [\mathbf{d} \cdot \mathbf{p}, H_0]$$

Eigenstates receive an energy shift due to the perturbation:

$$|0\rangle \rightarrow |\tilde{0}\rangle = |0\rangle + \sum_{n} \frac{|n\rangle \langle n| \sum \frac{i}{q} [\mathbf{d} \cdot \mathbf{p}, H_{0}] |0\rangle}{E_{0} - E_{n}}$$
$$= \left(1 + \sum \frac{i}{q} \mathbf{d} \cdot \mathbf{p}\right) |0\rangle$$

• What is the total, observable, dipole moment after this shift?

$$\begin{split} \tilde{\mathbf{d}} &= \sum \mathbf{d} + \langle \tilde{0} | \sum q \mathbf{r} | \tilde{0} \rangle \\ &= \sum \mathbf{d} + \langle \tilde{0} | \left(1 - \sum \frac{i}{q} \mathbf{d} \cdot \mathbf{p} \right) \sum q \mathbf{r} \left(1 + \sum \frac{i}{q} \mathbf{d} \cdot \mathbf{p} \right) | \tilde{0} \rangle \\ &= \sum \mathbf{d} + i \langle 0 | \left[\sum q \mathbf{r}, \sum \frac{1}{q} \mathbf{d} \cdot \mathbf{p} \right] | 0 \rangle \\ &= \sum \mathbf{d} - \sum \mathbf{d} \\ &= 0 \end{split}$$

But some details can save us!

- Schiff's theorem assumes:
 - pointlike particles \rightarrow *incorrect for nuclei*

$$oldsymbol{S} = rac{1}{10} \left\langle r^2 oldsymbol{d}
ight
angle - rac{1}{6Z} \left\langle r^2
ight
angle \left\langle oldsymbol{d}
ight
angle$$

...see Prog. Part. Nucl. Phys. **71**, 21 (2013)

• non-relativistic treatment → *incorrect for atomic electrons*

$$U_{ ext{lab}} = -d_{ ext{lab}} \cdot E = -d_{ ext{rest}} \cdot E + rac{\gamma}{1+\gamma} (oldsymbol{eta} \cdot d) (oldsymbol{eta} \cdot E)$$

...see American Journal of Physics 75, 532 (2007)

A Key Systematic Effect

Motional magnetic field:
$$B = \frac{1}{c^2} E \times v$$

Magnetic field gradients:

$$oldsymbol{B}_{\perp} = -rac{oldsymbol{R}}{2}rac{\partial B_z}{\partial z}$$

Leading false EDM effect at 2nd order:

$$d_{\rm false}^{\rm (slow)} = -\frac{\hbar}{4c^2} \left(\frac{v_\perp}{B_z}\right)^2 \frac{\partial B_z}{\partial z} \quad \mu$$

Adiabatic (UCN)

 $d_{\rm false}^{\rm (fast)} = -\frac{\hbar}{8c^2}\gamma^2 R^2 \frac{\partial B_z}{\partial z}$

Diabatic (atomic magnetometer)

Baryon asymmetry

Consider

$$p + \bar{p} \leftrightarrow \gamma + \gamma$$

$$\eta_p = \frac{n_p - n_{\bar{p}}}{n_{\gamma}}$$

Threshold energy/temperature:

$$T \gg \frac{2~{\rm GeV}}{k_{\rm B}} \sim 2 \times 10^{13}~{\rm K}$$

After freeze-out:

$$n_{\gamma} = \int_0^\infty \frac{d\omega}{c^3 \pi^2} \frac{\omega^2}{e^{\frac{\hbar\omega}{k_{\rm B}T}} - 1} = 16\pi \left(\frac{k_{\rm B}T}{hc}\right)^3 \zeta(3)$$

Statistics of the photon gas

$$\langle E \rangle = \int_0^\infty \hbar \omega \frac{g(\omega)}{e^{\frac{\hbar \omega}{k_{\rm B}T}} - 1}$$

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$$n_{-} - n_{-}$$

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