$\mu \to e\gamma$: *from* m_μ *to the*

Scale of New Physics

1601.07166, 1601.01949

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1. $\mu \to e\gamma$: contact interaction parametrisation and bound at m_μ

2. Effective Field Theory: a recipe to approximate evolution of $\mathcal{L}$ with scale
   correctly include mass thresholds in Dim Reg

3. top-down: $\mu \to e\gamma$ in the 2HDM
   - the simplest EFT analysis
   - the "exact" result: did I get the right answer?

4. bottom-up: what does the $\mu \to e\gamma$ bound constrain at $\Lambda_{NP} \gg m_W$?
   - constraints at the New Physics scale

5. Summary useful, not so simple...lots to do!

Looking for flavour change with the muon

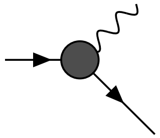
LFV $\equiv$ FCNC of charged leptons @ a point (ν osc not count)

New Physics that exists! Just not know rate...

1 *assume* New Particles giving LFV are heavy: $\Lambda_{NP} \gg m_W$

2 focus on $\mu \leftrightarrow e$ (for simplicity, only impose $\mu \to e\gamma$)

To parametrise effect of this NP in $\mu \to e\gamma$:



$$\begin{aligned} \delta\mathcal{L}_{meg} &= -\frac{4G_F}{\sqrt{2}} m_\mu \left(C_R^D \overline{\mu_R} \sigma^{\alpha\beta} e_L F_{\alpha\beta} + C_L^D \overline{\mu_L} \sigma^{\alpha\beta} e_R F_{\alpha\beta} \right) \\ BR(\mu \to e\gamma) &= 384\pi^2 (|C_R^D|^2 + |C_L^D|^2) < 5.7 \times 10^{-13} \\ &\Rightarrow |C_X^D| \lesssim 10^{-8} \end{aligned}$$

MEG expt, PSI

How big does one expect C to be? Suppose operator coefficient

			$n=0$	$n=1$	$n=2$
$C\dfrac{m_\mu}{v^2} \sim \dfrac{ev}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 30\ 000$ TeV	3000 TeV	300 TeV
$C\dfrac{m_\mu}{v^2} \sim \dfrac{em_\mu}{(16\pi^2)^n \Lambda^2}$	$\Rightarrow$	probes	$\Lambda \lesssim 1000$ TeV	100 TeV	10 TeV

What to do as a theorist?

Want to identify new particles + interactions responsable for LFV

$\Rightarrow$ build beautiful models (symmetries, particle content,parameter ranges)
+ calculate their predictions

? but what can a (stupid-about-"beauty") phenomenologist do?
assume New Physics is heavy: $\Lambda_{NP} \gg m_W$
$\Rightarrow$ parametrise at accessible scales with contact interactions

dream = reconstruct the fundamental Lagrangian at Λ_{NP},
from low-E observations/the effective Lagrangian at m_W?
"simple" first step: translate exptal bounds from low E to New Physics scale.
NB: this is a *Standard Model* problem

Between m_μ and the NP scale using EFT

Georgi, EFT, ARNPP 43(93) 209
(one of my all-time favourite papers)

$$\boxed{\mathcal{L}_{SM} + \mathcal{L}_{NP}}$$

match ——————————————————— Λ_{NP}

$$\mathcal{L}_{SM} + \mathcal{L}(\text{SM invar. Contact Interactions})$$

$$\uparrow$$

$\{Z, W, \gamma, g, h, t, f\}$ $SU(3) \times SU(2) \times U(1)$ RGEs

match ——————————————————— m_W, m_h, m_t

$$\mathcal{L}_{QCD*QED} + \mathcal{L}'(\text{QCD}, \text{QED invar. Contact Interactions})$$

$$\uparrow$$

$\{\gamma, g, f\}$ RGEs of QCD $\times$ QED

(above here τ, c, b)

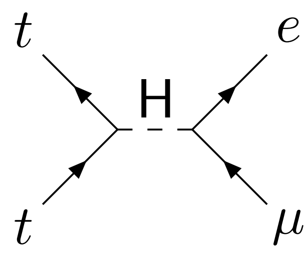
$$\boxed{\mu \to e\gamma}$$

Matching

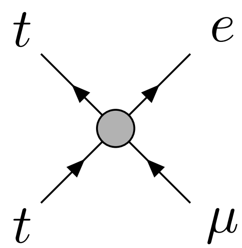
Wilsonian renormalisation: as "integrate out" short-distance modes, the couplings constants (also of non-renorm ops) run.

But I want to use Dim Reg! How to remove heavy particles?
$\Rightarrow$ change theories at heavy particle scale, $match$ Greens fns of the two theories!

Ex: 2HiggsDoubletModel in the decoupling limit $M_H \gg m_W$



$\Lambda_{NP} \simeq M_H$



At M_H, match Greens fns of SM + Contact Interactions onto Greens fns of 2HDM

match to desired order in cplings, loops. If at loop order, NB, integrate $\int_{-\infty}^{+\infty} d^4k/(2\pi)^4$ above and below matching scale

Match onto...what basis of contact interactions?

Simple approximation: $assume$ that lowest order in Λ is sufficient.
For LFV, usually means **dimension six** operators $\propto 1/\Lambda^2$.

(is that a good approx? Depends on how precise an answer you want...)

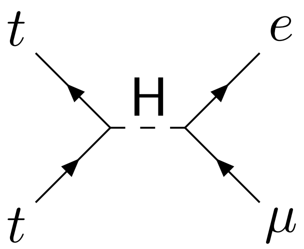
Contact Interactions should respect relevant **symmetries**:

- above m_W $\exists$ "Buchmuller-Wyler" basis of $SU(3) \times SU(2) \times U(1)$ invariant, dimension six operators

Buchmuller-Wyler
Gradkowski etal

- below m_W, use QCD$\times$QED invariant dimension six (+7) operators

Kuno-Okada

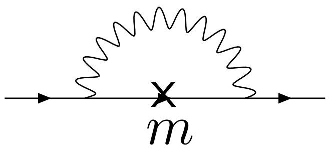


$$= (\bar{t}P_R t)(\bar{e}P_R \mu)\frac{i\lambda_t\lambda_{\mu e}}{p^2-\Lambda^2_{NP}}$$

$$= -i\lambda_t\lambda_{\mu e}(\bar{t}P_R t)(\bar{e}P_R \mu)\left(\frac{1}{\Lambda^2_{NP}} + \frac{p^2}{\Lambda^4_{NP}} + ...\right)$$

Running

Recall running mass of $\overline{MS}$:



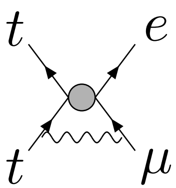
$$\supset i4m\frac{g^2}{16\pi^2}\left(\frac{1}{\epsilon}+...\right) \quad \Rightarrow \quad \mathcal{L} \supset Z_m Z_2 m\overline{\psi}\psi$$

Then, 2pt fn should be μ indep $\quad \Rightarrow -\frac{\mu}{m}\frac{\partial m}{\partial\mu} = \frac{\mu}{Z_m}\frac{\partial Z_m}{\partial\mu} \equiv \gamma_m(\alpha)$

Neglecting running α (!), solve as $m(\mu_H) = m(\mu_L)\{1 - \gamma_m \ln\frac{\mu_H}{\mu_L} + ...$

$\Rightarrow$1 loop RGEs give logs

Loop corrections to Contact Interactions/Operators cause running *and mixing* of coefficients:



Write a row vector $\vec{C}$ of operator coefficients

obtain $\mu\partial\vec{C}/\partial\mu = \vec{C}[\gamma]$

solve as $\vec{C}(\mu_H) = \vec{C}(\mu_L)\{1 - [\gamma]\ln\frac{\mu_H}{\mu_L} + [\gamma\gamma]\ln^2 + ...$

$\Rightarrow$ RGEs cause operators to mix into other operators, one-loop diagrams give mixing $\propto (\alpha\log)^n$

...to what order in loops and couplings?

common first-attempt in EFT:

match @tree, run at one-loop

its easy and $independent$ of the renorm-scheme for contact interactions! :)

...but not come close to the correct results in $\mu \to e\gamma$:(

The Two Higgs Doublet Model(2HDM) with LFV

Add a second doublet to the SM (suppose CP-conserving potential)
8 real fields - 3 goldstones = 5 Higgses = $\{h, H, A, H^\pm\}$

1. a more complicated Higgs potential

$$
\begin{aligned}
\mathcal{V} \quad = \quad & M_{11}^2 H_1^\dagger H_1 + M^2 H_2^\dagger H_2 - [M_{12}^2 H_1^\dagger H_2 + \mathrm{h.c.}] \\
& + \frac{1}{2}\lambda_1 (H_1^\dagger H_1)^2 + \frac{1}{2}\lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\
& + \left\{ \frac{1}{2}\lambda_5 (H_1^\dagger H_2)^2 + \left[\lambda_6 (H_1^\dagger H_1) + \lambda_7 (H_2^\dagger H_2)\right] H_1^\dagger H_2 + \mathrm{h.c.} \right\} ,
\end{aligned}
$$

2. independent Yukawa matrices $[Y^F]$, $[\rho^F]$ for each Higgs ($\Leftrightarrow$ FCNC = LFV)

$$
\begin{aligned}
-\mathcal{L}_{\mathrm{Y}} \quad = \quad & \left(\overline{Q}_j \widetilde{H}_1 K_{ij}^* Y_i^U U_i + \overline{Q}_i H_1 Y_i^D D_i + \overline{L}_i H_1 Y_i^E E_i \right) \\
& + \overline{Q}_i \widetilde{H}_2 [K^\dagger \rho^U]_{ij} U_j + \overline{Q}_i H_2 [\rho^D]_{ij} D_j + \overline{L}_i H_2 [\rho^E]_{ij} E_j + \mathrm{h.c.} ,
\end{aligned}
$$

(Q, L SU(2) doublets, E, U, D singlets, K = CKM, $\widetilde{H} = i\sigma_2 H^*$).
LFV $\Rightarrow$ no discrete symmetry $\Rightarrow$ no $\tan\beta$.
(I suppose discrete sym, except for μ-e).

The Two Higgs Doublet Model(2HDM) with LFV

Take decoupling limit $\Leftrightarrow \{H, A, H^{\pm}\}$ of mass $\Lambda_{NP} > 10m_W$

$\Rightarrow$ lowest order in the EFT expansion ($=$ dim 6 operators) should work check later

$\Rightarrow$ h almost in the doublet containing vev and goldstones, misaligned by $c_{\beta-\alpha}$.

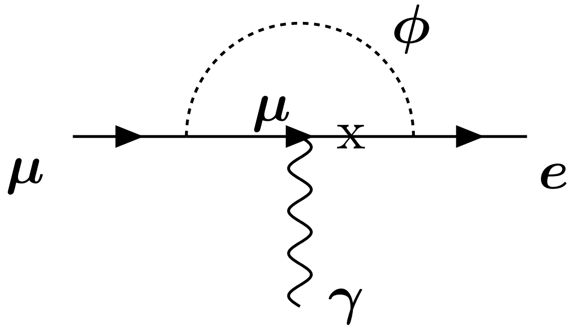
NB, for counting powers of $1/\Lambda^2$: $c_{\beta-\alpha} \simeq \lambda_6 v^2/\Lambda^2$ Gunion-Haber

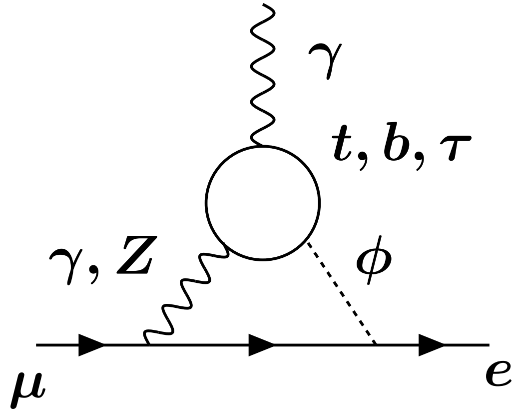
$c_{\beta-\alpha}$ suppresses h couplings to $\bar{e}\mu$ and to $\tan\beta$-enhanced Yukawas, and HWW.

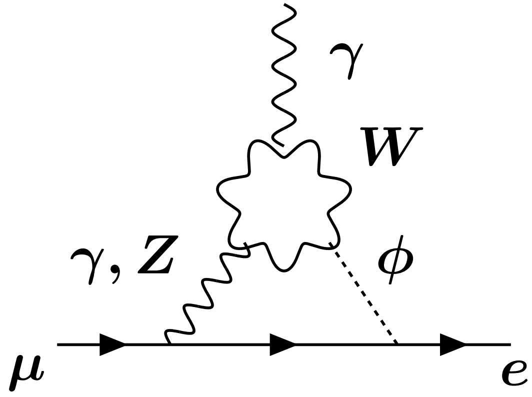
LFV couplings of the neutral Higgses:

$$\frac{[\rho^{E\dagger}]_{e\mu}}{\sqrt{2}} c_{\beta-\alpha}\, h(\overline{e}P_L\mu) \qquad -\frac{[\rho^{E\dagger}]_{e\mu}}{\sqrt{2}}\, H(\overline{e}P_L\mu) \qquad -i\frac{[\rho^{E\dagger}]_{e\mu}}{\sqrt{2}}\, A(\overline{e}P_L\mu)$$

$\mu \to e\gamma$ in the Two Higgs Doublet Model(2HDM) with LFV







Bjorken-Weinberg

$\phi \in \{h, H, A\}$ $\qquad$ $1 \text{ loop} \propto \frac{m_\mu^2 \rho_{e\mu}}{16\pi^2} \ll 2 \text{ loop} \propto \frac{m_t^2 \rho_{e\mu}}{(16\pi^2)^2}$

ChangHouKeung
HisanoEtal
...

Amplitudes in literature... use neutral Higgs and QED diagrams of CHK

compare to EFT of dim6 operators, matching at tree and running with QED at one-loop

Is it ok to neglect dimension eight operators?

Consider 2HDM in decoupling limit, $\Lambda_{NP} \gtrsim 10v$.

compare $1/\Lambda_{NP}^2$ (= dim6) and $1/\Lambda_{NP}^4$ (= dim8) parts of $\mu \to e\gamma$ amplitude of CHK:

$$\frac{\mathrm{dim8}}{\mathrm{dim6}} \sim \lambda_i \tan\beta \frac{v^2}{\Lambda_{NP}^2} \quad , \quad \frac{m_W^2}{\Lambda_{NP}^2} \ln^2 \frac{m_W^2}{\Lambda_{NP}^2}$$

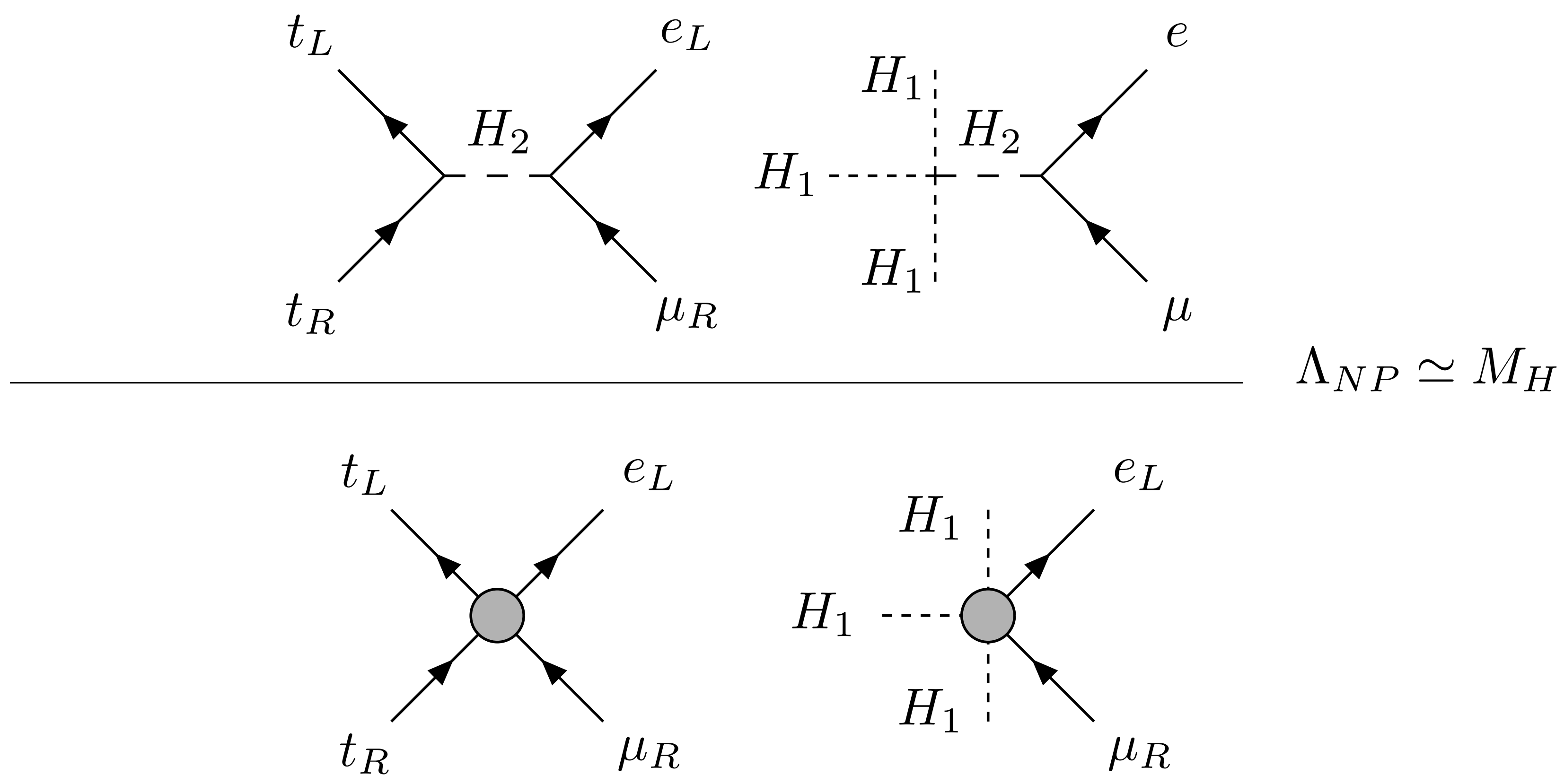
$\Rightarrow$ For reasonable Higgs potential parameters $\{\lambda_i\}$, and $\cot\beta$, $\tan\beta \lesssim 50$, the $1/\Lambda_{NP}^2$ parts are larger than the $1/\Lambda_{NP}^4$ terms.

But: would need dimension 8 to get numerically reliable result? ($z \ln^2 z \sim 0.2$ for $z \sim 0.01$!)

* At dim6, one should find: Barr-Zee for H, A and t-loop,
Barr-Zee for h and W, t, b, τ-loop, and one-loop with h

matching at $\Lambda_{NP} \simeq M_{A,H}$

match tree- level Greens fns mediated by heavy doublet:



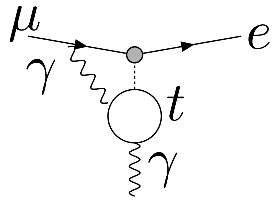
$$-\frac{[\rho^E]_{e\mu}[\rho^{U\dagger}]_{tt}}{\Lambda_{NP}^2}(\overline{L}_e^A E_\mu)\epsilon_{AB}(\overline{Q}_t^B U_t) \quad , \quad -\frac{[\rho^E]_{e\mu}\lambda_6}{\Lambda_{NP}^2} H_1^\dagger H_1 \overline{L}_e H_1 E_\mu$$

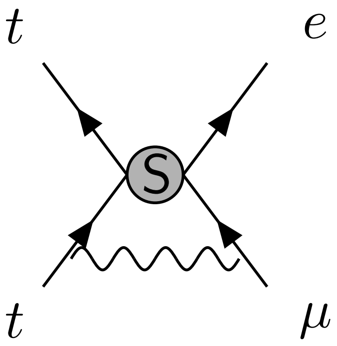
Run with QED (!) down to m_W

$$\mu \frac{\partial}{\partial \mu} \vec{C} = \frac{\alpha_{em}}{4\pi} \vec{C} \mathbf{\Gamma}$$

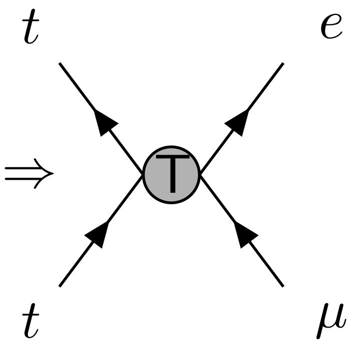
$\mathbf{\Gamma}$ = anomalous dimension matrix
$\mathbf{\Gamma}_{AB} \sim$ coefficient of $\frac{e^2}{16\pi^2 \epsilon}$ div, when dress operator A with a photon to obtain Greens-fn of op B

Run with QED (!) down to m_W



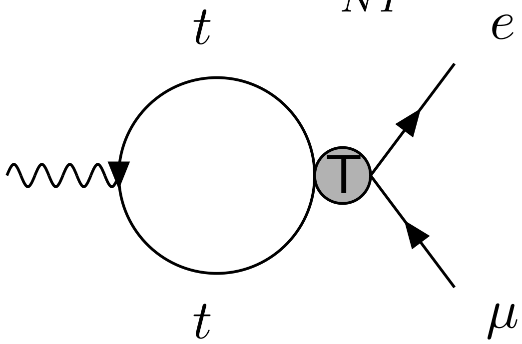


$+ \ldots \quad \Rightarrow$

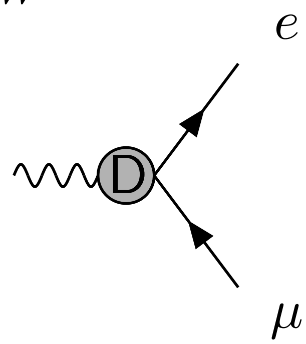


$$(\bar{t}\sigma t)(\bar{e}\sigma\mu)$$

$$\to \frac{C_S}{\Lambda_{NP}^2}\frac{\alpha}{4\pi}(2Q_t)\ln\frac{\Lambda}{m_W}$$



$\Rightarrow$

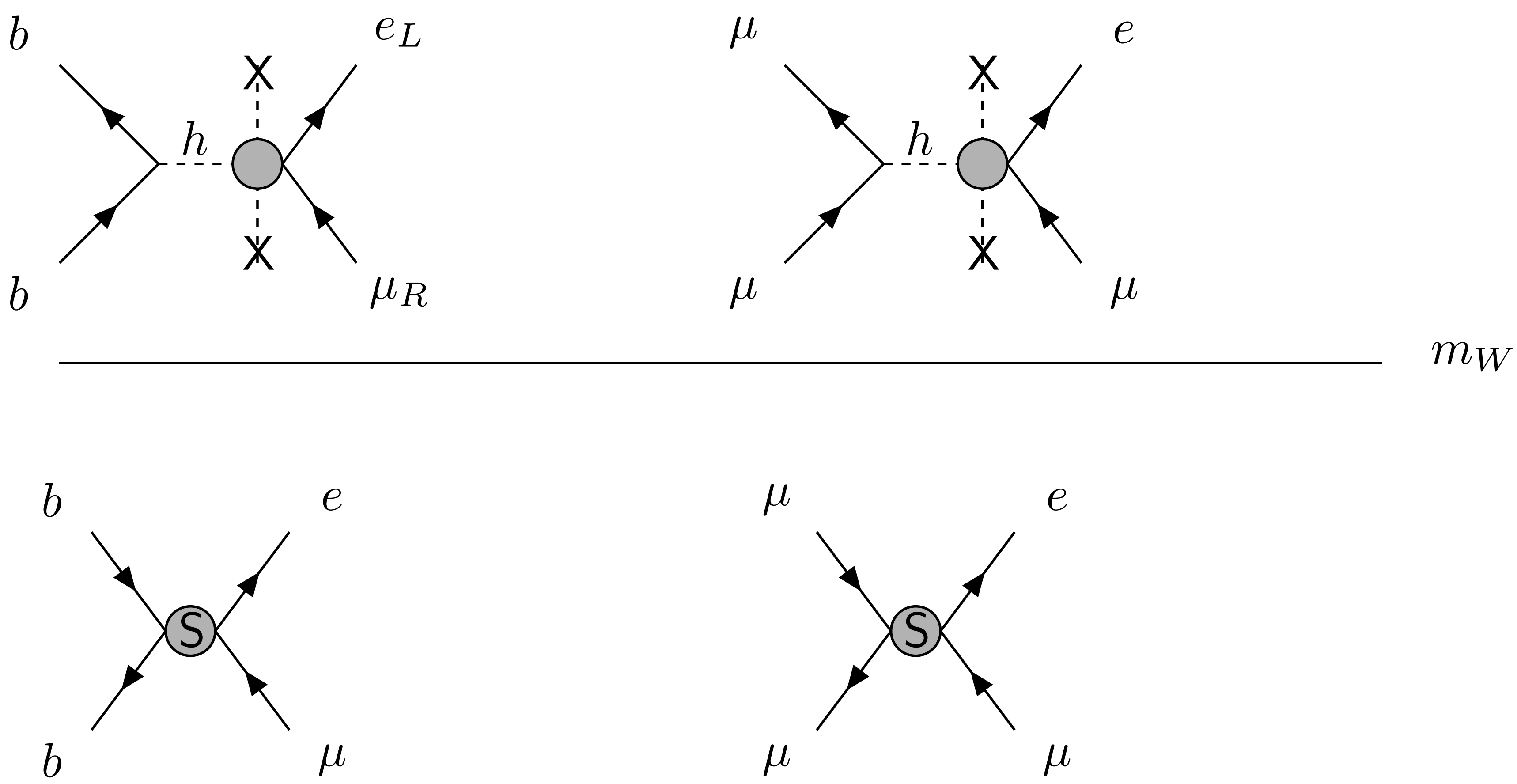


$$\to \quad \frac{C_T}{\Lambda_{NP}^2}\frac{\alpha}{4\pi}\left(\frac{8N_c Q_t m_t}{e}\right)\ln\frac{\Lambda}{m_W}$$

$$= \quad \frac{e\alpha}{32\pi^3\Lambda^2}3Q_t^2 m_t[\rho^E]_{e\mu}[\rho^{U\dagger}]_{tt}\log^2\frac{m_t^2}{\Lambda^2}$$

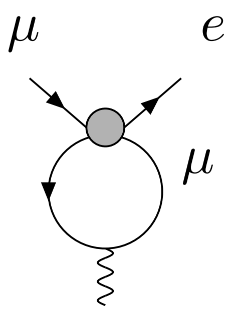
!get $\mathcal{O}([\alpha\ln]^2/\Lambda^2)$ part of Barr-Zee diagram (for t, heavy Higgs) via 1-loop RGES!

Matching at $m_W \approx m_h \approx m_t$

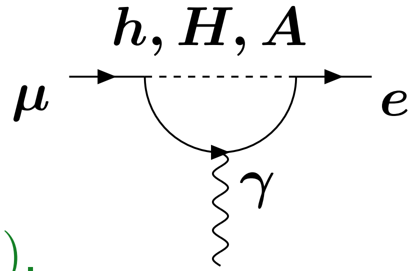


$$\frac{[Y^D]_{bb}[\rho^E]_{e\mu}\lambda_6 v^2/2}{m_h^2\Lambda_{NP}^2}(\overline{\mu}P_R e)(\overline{b}P_R b) \quad , \quad \frac{[Y^E]_{\mu\mu}[\rho^E]_{e\mu}\lambda_6 v^2/2}{m_h^2\Lambda_{NP}^2}(\overline{e}P_R \mu)(\overline{\mu}P_R \mu)$$

Run with QED from m_W to m_μ



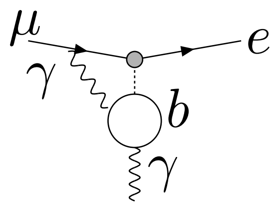
correctly reproduce the $\mathcal{O}(\alpha \ln /\Lambda^2)$ part of one-loop diagram:

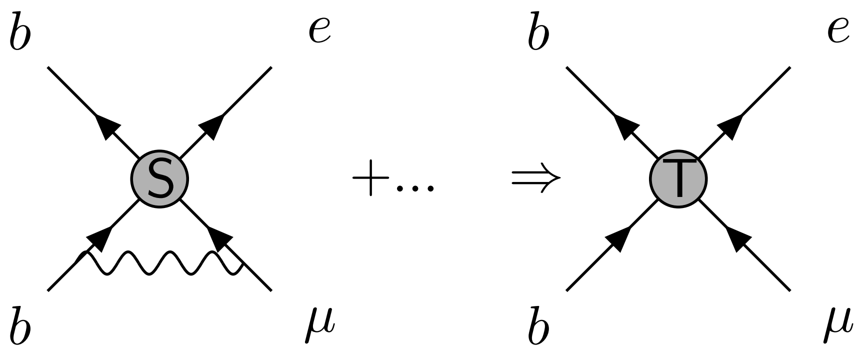


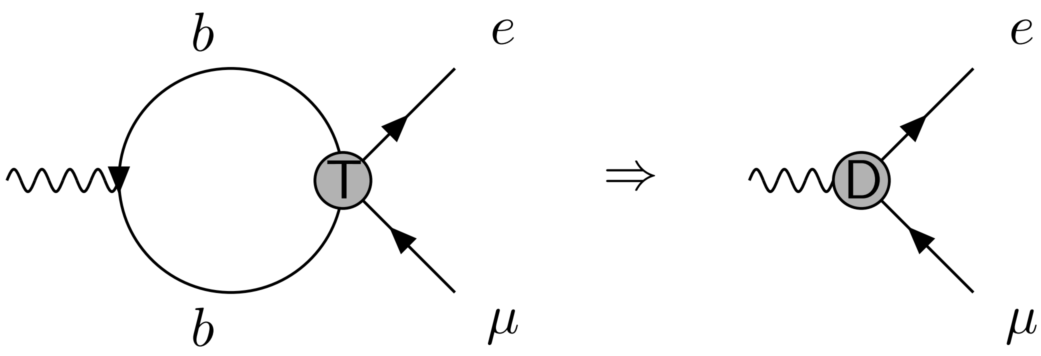
only light Higgs h contributes at $\mathcal{O}(1/\Lambda^2)$; heavy Higgses are $\mathcal{O}(1/\Lambda^4)$.

Comment: important to change basis to QCD*QED invar ops below m_W:
no SU(2)-invar $(\overline{e}P_R\mu)(\overline{\mu}P_R\mu)$ — hypercharge only allows $(\overline{L}_e P_R E_\mu)(\overline{E}P_L L)$.
$\Leftrightarrow$ EFT can reproduce the one-loop diagram only if use the QCD*QED basis below m_W.

2nd order QED running from m_W to μ_μ







$$= \quad -\frac{e\alpha}{64\pi^3\Lambda_{NP}^2}3Q_b^2 m_b[\rho^E]_{e\mu}[Y^D]_{bb}\log^2\frac{m_\mu^2}{m_h^2}$$

get $\mathcal{O}([\alpha\ln]^2/\Lambda^2)$ part of Barr-Zee diagram (for b, τ, light Higgs) via 1-loop RGES
(again, only if use the QCD*QED basis below m_W)

What did I learn computing $\mu \to e\gamma$ in the 2HDM with EFT?

dimension six operators are probably an ok approximation
(but worry about logs and coupling hierachies)

its important to change operator basis at m_W, from SU(2)-invariant operators above, to QCD*QED invariant operators below.

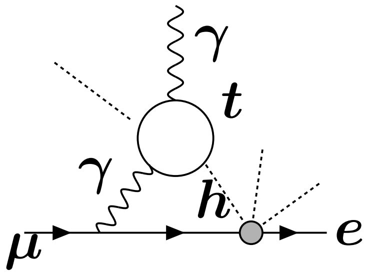
EFT works! Obtained all $[\alpha \ln]^n/\Lambda^2$ terms, via trivial calculations. In particular, obtained dominant part of Barr-Zee (2-loop!) diagrams of t-loop with heavy Higgses, and b or τ-loop with light Higgs h.

my simplest-EFT, with tree-matching and one-loop running, missed the numerically most important diagrams = Barr-Zee of h with t and W loop!... they arise in 2-loop matching (finite) at m_W.

$\Rightarrow$ solution = go cherry-picking!
include n-loop matching if its finite and numerically relevant

Why are there finite and relevant loop matching contributions at m_W ?

1. operator dimensions change at m_W (Higgs field becomes vev)
   rule of thumb: if run with 1-loop RGEs, match at tree — reasonable if same diagram gives matching and running. But... Dim 6 LFV Higgs vertices: $H^\dagger H \overline{L}_\mu H E_e$ contribute in loops to *dim 8* dipole $H^\dagger H(\overline{L}_e H\sigma \cdot F E_\mu)$, so not mix in RG running above m_W to the dim6 dipole, but do contribute in matching at m_W.



2. above m_W, perturb in g_i, loops and yukawas (many expansion parameters!)...two-loop$\times y_t^2, g^2 \gg$ one-loop$\times y_\mu^2$

Finally, back to translating the $\mu \to e\gamma$ bound to Λ

$\boxed{\mathcal{L}_{SM} + \mathcal{L}_{NP}}$

match Λ_{NP}

$\mathcal{L}_{SM} + \mathcal{L}(\text{SM invar. Contact Interactions})$

$\{Z, W, \gamma, g, h, t, f\}$ $SU(3) \times SU(2) \times U(1)$ RGEs

match m_W, m_h, m_t

$\mathcal{L}_{QCD*QED} + \mathcal{L}'(\text{QCD, QED invar. Contact Interactions})$

$\{\gamma, g, f\}$ RGEs of QCD $\times$ QED

(above here τ, c, b)

$\boxed{\mu \to e\gamma}$

Finally, back to translating the $\mu \to e\gamma$ bound to Λ

1. make a list of QCD$\times$QED-invar operators, representing all 3,4 point interactions of μ with e and γ, g, u, d, s, and τ, c, b.

Kuno+Okada... (see backup)

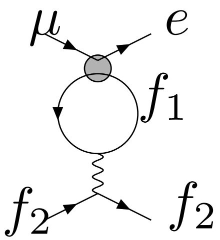
Finally, back to translating the $\mu \to e\gamma$ bound to Λ

1. make a list of QCD$\times$QED-invar operators, representing all 3,4 point interactions of μ with e and γ, g, u, d, s, and τ, c, b.

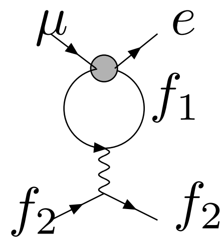
2. translate bound on $\Gamma(\mu \to e\gamma)$ to bounds on operator coefficients *at tree*

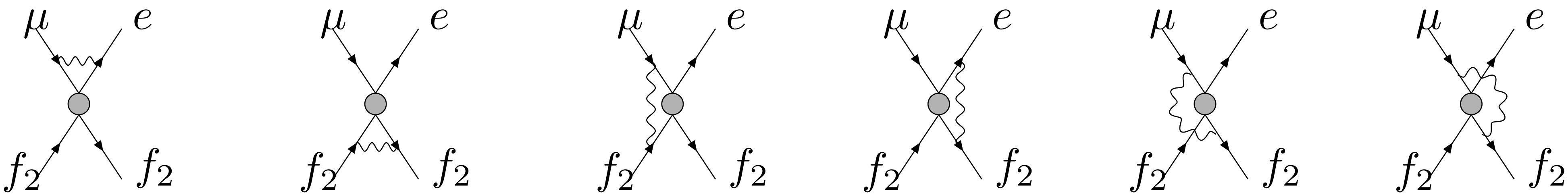
   ($\Gamma(\mu - e \text{ conv.})$ and $\Gamma(\mu \to e\bar{e}e)$ in progress with Crivellin,Pruna,Signer...quark FC laaater...)

3. run up to m_W with *one-loop* RGEs of QED



3: Run from m_W to m_μ with *one-loop* RGEs of QED





$$\mu \frac{\partial}{\partial \mu} \vec{C} = \frac{\alpha_{em}}{4\pi} \vec{C} \mathbf{\Gamma}$$

QED: mixes ops, $\alpha_{em} \ll \Rightarrow$ solve in pert theory:

$$C_A(m_W) \left(\delta_{AB} - \frac{\alpha_{em}}{4\pi} [\Gamma]_{AB} \log \frac{m_W}{m_\tau} + \frac{\alpha_{em}^2}{32\pi^2} [\Gamma\Gamma]_{AB} \log^2 \frac{m_W}{m_\tau} + .. \right) = C_B(m_\tau)$$

DegrassiGiudice

NB: at one loop: $\mathbf{\Gamma} = \left[\begin{array}{cc} \mathbf{\Gamma}_V & 0 \\ 0 & \mathbf{\Gamma}_{STD} \end{array} \right]$ ... V $\rightarrow$dipole mixing arises at 2-loop

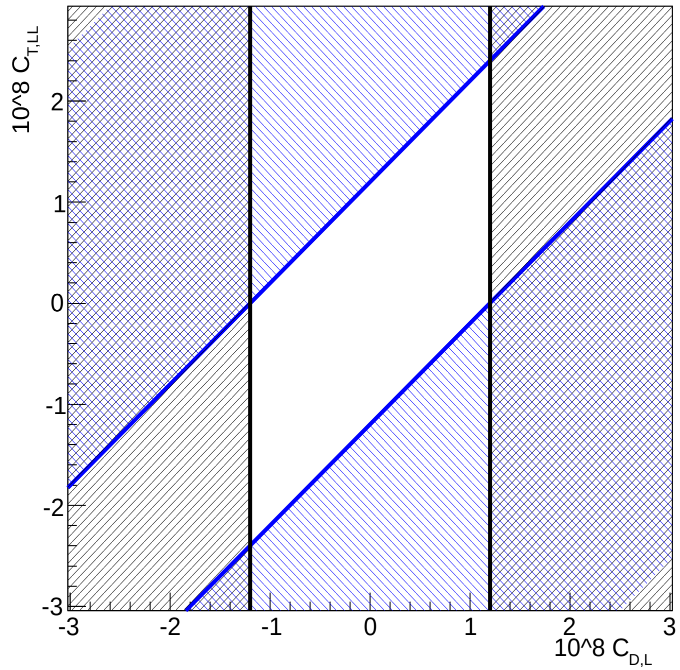
(include vectors later...)

Bounds at m_W

Translated to m_W, the $\mu \to e\gamma$ bound constrains two linear combinations of operators:

$$C^{e\mu}_{D,L}(m_\tau) \simeq C^{e\mu}_{D,L}(m_W) - 1.0C^{e\mu cc}_{T,LL}(m_W) + 1.0C^{e\mu\tau\tau}_{T,LL}(m_W) + 1.8C^{e\mu bb}_{T,LL}(m_W)$$
$$+10^{-3}\left\{7.6C^{e\mu\mu\mu}_{S,LL}(m_W) + 4.6C^{e\mu\tau\tau}_{S,LL}(m_W) + 1.4C^{e\mu bb}_{S,LL}(m_W) + 1.5C^{e\mu cc}_{S,LL}(m_W)\right\}$$

Suppose charm tensor and the dipole:



black (blue) lines is bound at m_μ (m_W)

Finally, back to translating the $\mu \to e\gamma$ bound to Λ

1. make a list of QCD×QED-invar operators, representing all 3,4 point interactions of μ with e and γ, g, u, d, s, and τ, c, b.

2. translate bound on $\Gamma(\mu \to e\gamma)$ to bounds on operator coefficients *at tree*

   ($\Gamma(\mu - e \text{ conv.})$ and $\Gamma(\mu \to e\bar{e}e)$ in progress with Crivellin,Pruna,Signer...quark FC laaater...)

3. run up to m_W with *one-loop* RGEs of QED

4. at m_W, perform *finite* matching to SU(2)-invar "BWP" operators — *exercise SM art of finding include largest finite contributions, be they tree, one- or two-loop*

5. run up with SM RGEs *at one loop*

Results (1-loop RGEs, finite matching)

$\mu \to e\gamma$ constrains 2 linear combos of coefficients. Bounds "one-at-a-time":

γ dipole	$C < 1.2 \times 10^{-8}$
Z dipole $\times \ln \frac{\Lambda}{m_W}$	$C < 3.0 \times 10^{-6}$
LFV h cpling	$C < 7.5 \times 10^{-7}$
$LFVZ$ penguin	$C < 1.2 \times 10^{-5}$
t tensor$\times \ln \frac{\Lambda}{m_W}$	$C < 2.0 \times 10^{-10}$
c tensor$\times \ln \frac{\Lambda}{m_W}$	$C < 2.8 \times 10^{-8}$
t scalar $\times \ln^2 \frac{\Lambda}{m_W}$	$C < 3.1 \times 10^{-7}$
c scalar $\times \ln^2 \frac{\Lambda}{m_W}$	$C < 6.0 \times 10^{-6}$

Operator coefficients $-2\sqrt{2}G_F C$, except dipole $-2\sqrt{2}G_F C y_\mu \Rightarrow C \sim v^2/\Lambda^2$

(neglected op renorm, only include bds $C < 10^{-4}$...maybe means ok to neglect dim 8?)

Summary : LFV in EFT

Did two things:
1 in decoupling limit of 2HDM, reproduce in EFT the numerically largest parts of the (diagrammatic) $\mu \to e\gamma$ amplitude
2 translate the $\mu \to e\gamma$ bound from m_μ to a scale Λ_{NP}, where it constrains a linear combo of coefficients of contact interactions

This *could be* useful and interesting because
$\star$ for constraining models, its much easier to calculate finite matching coefficients at Λ_{NP}, than rates for $\mu \to e\gamma$, $\mu \to e\bar{e}e$, and $\mu - e$ conv..
$\star$ EFT should simplfy caln (does! got 2-loop Barr-Zee from one-loop divs). Would be easy to include leading QCD effects, etc.

But EFT expansion in $1/\Lambda^2$ does not absolve from dealing with the complexity of the SM (SSB, hierarchies in couplings)...*e.g.* in matching@m_W, needed 1- or 2-loop diagrams, finite and not contributing in running...

...lots to do! (best bd of every observable on every op at 2-loop?)

Backup

The operator basis for $\mu \to e$ flavour change below m_p

$$m_\mu(\overline{e}\sigma^{\alpha\beta}P_Y\mu)F_{\alpha\beta} \qquad\qquad dim\ 5$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_Y e) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{e}\gamma_\alpha P_X e)$$

$$(\overline{e}P_Y\mu)(\overline{e}P_Y e) \qquad\qquad dim\ 6$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{u}\gamma_\alpha P_Y u) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{u}\gamma_\alpha P_X u)$$

$$(\overline{e}\gamma^\alpha P_Y\mu)(\overline{d}\gamma_\alpha P_Y d) \qquad (\overline{e}\gamma^\alpha P_Y\mu)(\overline{d}\gamma_\alpha P_X d)$$

$$(\overline{e}P_Y\mu)(\overline{u}P_Y u) \qquad (\overline{e}P_Y\mu)(\overline{u}P_X u)$$

$$(\overline{e}P_Y\mu)(\overline{d}P_Y d) \qquad (\overline{e}P_Y\mu)(\overline{d}P_X d)$$

$$(\overline{e}\sigma P_Y\mu)(\overline{u}\sigma P_Y u) \qquad (\overline{e}\sigma P_Y\mu)(\overline{d}\sigma P_Y d)$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}G^{\alpha\beta} \qquad\qquad dim\ 7$$

$$\frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}F^{\alpha\beta} \quad , \quad \frac{1}{m_t}(\overline{e}P_Y\mu)G_{\alpha\beta}\widetilde{G}^{\alpha\beta} \qquad \frac{1}{m_t}(\overline{e}P_Y\mu)F_{\alpha\beta}\widetilde{F}^{\alpha\beta}$$

...zzz...

(plus operators with $d \leftrightarrow s$). $\mu \to e\gamma$, $\mu \to e\bar{e}e$, and $\mu - e$ conv. are sensitive to all but a few of new 3 or 4-point μ-e interactions

$(P_X, P_Y = (1 \pm \gamma_5)/2)$

Some more operators above m_τ, m_c, m_b

At a slightly higher scale, operators containing c, b μ and τ bilinears should be included:

$$
\begin{array}{lll}
\text{4 lepton} & \mathcal{O}^{e\mu ll}_{YY} = \frac{1}{2}(\overline{e}\gamma^\alpha P_Y \mu)(\bar{l}\gamma^\alpha P_Y l) \quad , & \mathcal{O}^{e\mu ll}_{YX} = \frac{1}{2}(\overline{e}\gamma^\alpha P_Y \mu)(\bar{l}\gamma^\alpha P_X l) \\
 & \mathcal{O}^{e\mu ll}_{S,YY} = (\overline{e} P_Y \mu)(\bar{l} P_Y l) & \mathcal{O}^{e\mu\tau\tau}_{S,YX} = (\overline{e} P_Y \mu)(\overline{\tau} P_X \tau) \\
 & \mathcal{O}^{e\mu\tau\tau}_{T,YY} = (\overline{e}\sigma P_Y \mu)(\overline{\tau}\sigma P_Y \tau) & \\
 & & \\
\text{2 lepton 2 quark} & \mathcal{O}^{e\mu qq}_{YY} = \frac{1}{2}(\overline{e}\gamma^\alpha P_Y \mu)(\overline{q}\gamma^\alpha P_Y q) \quad , & \mathcal{O}^{e\mu qq}_{YX} = \frac{1}{2}(\overline{e}\gamma^\alpha P_Y \mu)(\overline{q}\gamma^\alpha P_X q) \\
 & \mathcal{O}^{e\mu qq}_{S,YY} = (\overline{e} P_Y \mu)(\overline{q} P_Y q) \quad , & \mathcal{O}^{e\mu qq}_{S,YX} = (\overline{e} P_Y \mu)(\overline{q} P_X q) \\
 & \mathcal{O}^{e\mu qq}_{T,YY} = (\overline{e}\sigma P_Y \mu)(\overline{q}\sigma P_Y q) &
\end{array}
$$

where $l \in \{\mu, \tau\}$, $q \in \{c, b\}$, $X, Y \in \{L, R\}$, and $X \neq Y$.

Constraining the operator-zoo with 3 processes?

some processes	current sensitivities	future sensitivities?
$BR(\mu \to e\gamma)$	$< 5.7 \times 10^{-13}$	$\sim 10^{-14}$ (2016, MEG)
$BR(\mu \to e\bar{e}e)$	$< 1.0 \times 10^{-12}$	$\sim 10^{-14} \to 10^{-16}$ (2018, PSI)
$\frac{\sigma(\mu + Au \to e + Au)}{\sigma(\mu \text{ capture})}$	$< 7 \times 10^{-13}$	$\sim \text{few } 10^{-17}$ (Mu2e,COMET)...

Trivia about Fiertz and tensors for chiral fermions

Tensor operators are always:

$$(\overline{e}\sigma^{\alpha\beta}P_X\mu)(\overline{\psi}\sigma_{\alpha\beta}P_X\chi)$$

because $\sigma_{\mu\nu} = \frac{i}{2}\varepsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}\gamma_5$, which implies

$$(\overline{e}\sigma^{\alpha\beta}P_L\mu)(\overline{\psi}\sigma_{\alpha\beta}P_R\chi) \equiv 0$$

So the only SU(2)-invariant, dimension-six tensors that one can construct, are

$$(\overline{L}_e^A\sigma^{\mu\nu}E_\mu)\epsilon_{AB}(\overline{Q}_n^B\sigma_{\mu\nu}U_m) \qquad (\overline{L}_\mu^A\sigma^{\mu\nu}E_e)\epsilon_{AB}(\overline{Q}_n^B\sigma_{\mu\nu}U_m)$$

that is, tensors with d-type quarks or leptons only exist at dim 6 $below\ m_W$.

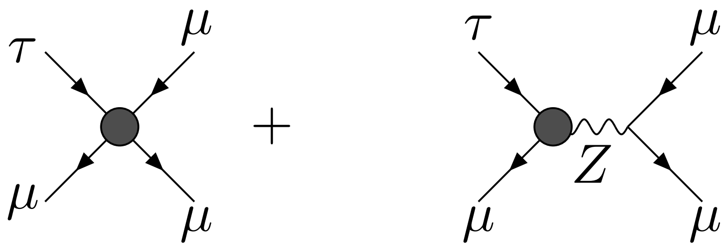
(This is relevant for $\mu \to e\gamma$, because tensors mix to the dipole in QED running.)

And,btw

$$(\overline{e}\sigma^{\alpha\beta}P_Y\mu)(\overline{\psi}\sigma_{\alpha\beta}P_Y\chi) = \frac{1}{2}(\overline{e}\sigma^{\alpha\beta}\mu)(\overline{\psi}\sigma_{\alpha\beta}\chi)$$

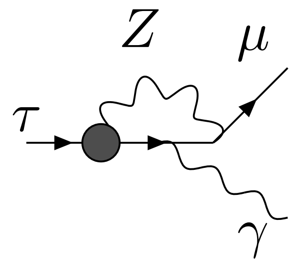
Why not just the best bound on each operator?
$\Leftrightarrow$ why want contribution of each operator to each observable?

1. A Z penguin gives $\bar{\tau} \not{Z} \mu$, which contributes at tree to $\tau \to \mu \bar{l} l$, in combination with $(\bar{\mu}\Gamma\tau)(\bar{l}\Gamma l)$:



2. Can ask "is is interesting for the LHC to search for $Z \to \tau^{\pm}\mu^{\mp}$?"
For LHC8 to see, need penguin coefficient $\gtrsim$ "naive" bound from $\tau \to \mu\bar{l}l$
("naive" = neglect possible cancellation with 4-f operator).

$\Rightarrow$ cancellations possible; but what about the bound on the penguin from $\tau \to \mu\gamma$?



$\tau \to \mu\gamma$ bound negligeable, so interesting for LHC to look for $\tau \to \mu\gamma$.
Same argument suggests they should not see $Z \to \mu^{\pm}e^{\mp}$.

The BWP basis: 2q2l and 4l

$$\mathcal{O}_{LQ}^{(1)e\mu nm} = \frac{1}{2}(\overline{L}_e\gamma^\alpha L_\mu)(\overline{Q}_n\gamma^\alpha Q_m) \qquad \mathcal{O}_{LQ}^{(3)e\mu nm} = \frac{1}{2}(\overline{L}_e\gamma^\alpha \tau^a L_\mu)(\overline{Q}_n\gamma^\alpha \tau^a Q_m)$$

$$\mathcal{O}_{EQ}^{e\mu nm} = \frac{1}{2}(\overline{E}_e\gamma^\alpha E_\mu)(\overline{Q}_n\gamma^\alpha Q_m)$$

$$\mathcal{O}_{LU}^{e\mu nm} = \frac{1}{2}(\overline{L}_e\gamma^\alpha L_\mu)(\overline{U}_n\gamma^\alpha U_m) \qquad \mathcal{O}_{LD}^{e\mu nm} = \frac{1}{2}(\overline{L}_e\gamma^\alpha L_\mu)(\overline{D}_n\gamma^\alpha D_m)$$

$$\mathcal{O}_{EU}^{e\mu nm} = \frac{1}{2}(\overline{E}_e\gamma^\alpha E_\mu)(\overline{U}_n\gamma^\alpha U_m) \qquad \mathcal{O}_{ED}^{e\mu nm} = \frac{1}{2}(\overline{E}_e\gamma^\alpha E_\mu)(\overline{D}_n\gamma^\alpha D_m)$$

$$\mathcal{O}_{LEQU}^{e\mu nm} = (\overline{L}_e^A E_\mu)\epsilon_{AB}(\overline{Q}_n^B U_m) \qquad \mathcal{O}_{LEQU}^{\mu enm} = (\overline{L}_\mu^A E_e)\epsilon_{AB}(\overline{Q}_n^B U_m)$$

$$\mathcal{O}_{LEDQ}^{e\mu nm} = (\overline{L}_e E_\mu)(\overline{D}_n Q_m) \qquad \mathcal{O}_{LEDQ}^{\mu enm} = (\overline{L}_\mu E_e)(\overline{D}_n Q_m)$$

$$\mathcal{O}_{T,LEQU}^{e\mu nm} = (\overline{L}_e^A \sigma^{\mu\nu} E_\mu)\epsilon_{AB}(\overline{Q}_n^B \sigma_{\mu\nu} U_m) \qquad \mathcal{O}_{T,LEQU}^{\mu enm} = (\overline{L}_\mu^A \sigma^{\mu\nu} E_e)\epsilon_{AB}(\overline{Q}_n^B \sigma_{\mu\nu} U_m)$$

$$\mathcal{O}_{LL}^{e\mu ii} = \frac{1}{2}(\overline{L}_e\gamma^\alpha L_\mu)(\overline{L}_i\gamma^\alpha L_i)$$

$$\mathcal{O}_{LE}^{e\mu ii} = \frac{1}{2}(\overline{L}_e\gamma^\alpha L_\mu)(\overline{E}_i\gamma^\alpha E_i) \qquad \mathcal{O}_{LE}^{iie\mu} = \frac{1}{2}(\overline{L}_i\gamma^\alpha L_i)(\overline{E}_e\gamma^\alpha E_\mu)$$

$$\mathcal{O}_{EE}^{e\mu ii} = \frac{1}{2}(\overline{E}_e\gamma^\alpha E_\mu)(\overline{E}_i\gamma^\alpha E_i)$$

$$-\frac{1}{2}\mathcal{O}_{LE}^{e\tau\tau\mu} = (\overline{L}_e E_\mu)(\overline{E}_\tau L_\tau) \qquad -\frac{1}{2}\mathcal{O}_{LE}^{\mu\tau\tau e} = (\overline{L}_\mu E_e)(\overline{E}_\tau L_\tau)$$

The BWP basis: 2l

$$\mathcal{O}_{EH}^{e\mu} = H^\dagger H \overline{L}_e H E_\mu \qquad \mathcal{O}_{EH}^{\mu e} = H^\dagger H \overline{L}_\mu H E_e$$

$$\mathcal{O}_{eW}^{e\mu} = y_\mu (\overline{L}_e \vec{\tau}^a H \sigma^{\alpha\beta} E_\mu) W_{\alpha\beta}^a \qquad \mathcal{O}_{eW}^{\mu e} = y_\mu (\overline{L}_\mu \vec{\tau}^a H \sigma^{\alpha\beta} E_e) W_{\alpha\beta}^a$$

$$\mathcal{O}_{eB}^{e\mu} = y_\mu (\overline{L}_e H \sigma^{\alpha\beta} E_\mu) B_{\alpha\beta} \qquad \mathcal{O}_{eB}^{\mu e} = y_\mu (\overline{L}_\mu H \sigma^{\alpha\beta} E_e) B_{\alpha\beta}$$

$$\mathcal{O}_{HL}^{(1)e\mu} = i(\overline{L}_e \gamma^\alpha L_\mu)(H^\dagger \overleftrightarrow{D}_\alpha H)$$

$$\mathcal{O}_{HL}^{(3)e\mu} = i(\overline{L}_e \gamma^\alpha \vec{\tau} L_\mu)(H^\dagger \overleftrightarrow{D}_\alpha \vec{\tau} H)$$

$$\mathcal{O}_{HE}^{e\mu} = i(\overline{E}_e \gamma^\alpha E_\mu)(H^\dagger \overleftrightarrow{D}_\alpha H)$$

where $i(H^\dagger \overleftrightarrow{D}_\alpha H) \equiv i(H^\dagger D_\alpha H) - i(D_\alpha H)^\dagger H$, and $D_\alpha = \partial_\alpha + i\frac{g}{2} W_\alpha^a \tau^a + i\frac{g'}{2} B_\alpha$. (The sign in the covariant derivative fixes the sign of the penguin operator and the SM Z vertex.)