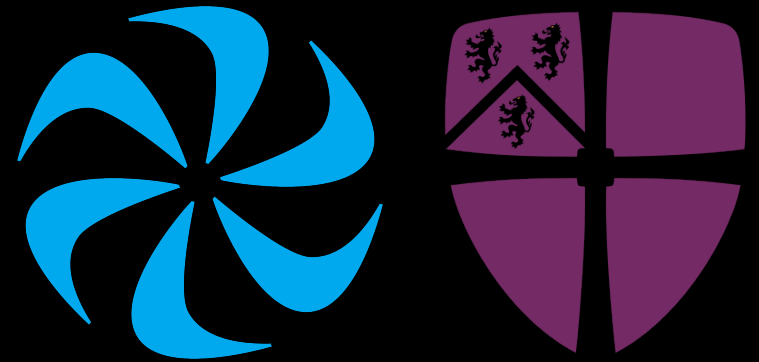


Black hole archaeology with gravitational waves

Djuna Lize Croon (TRIUMF → IPPP Durham)

MPI Heidelberg, June 2021

dcroon@triumf.ca | djunacroon.com



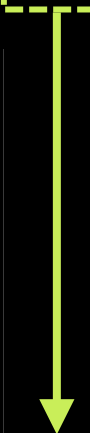
The era of gravitational waves

Many opportunities for particle and nuclear astrophysics

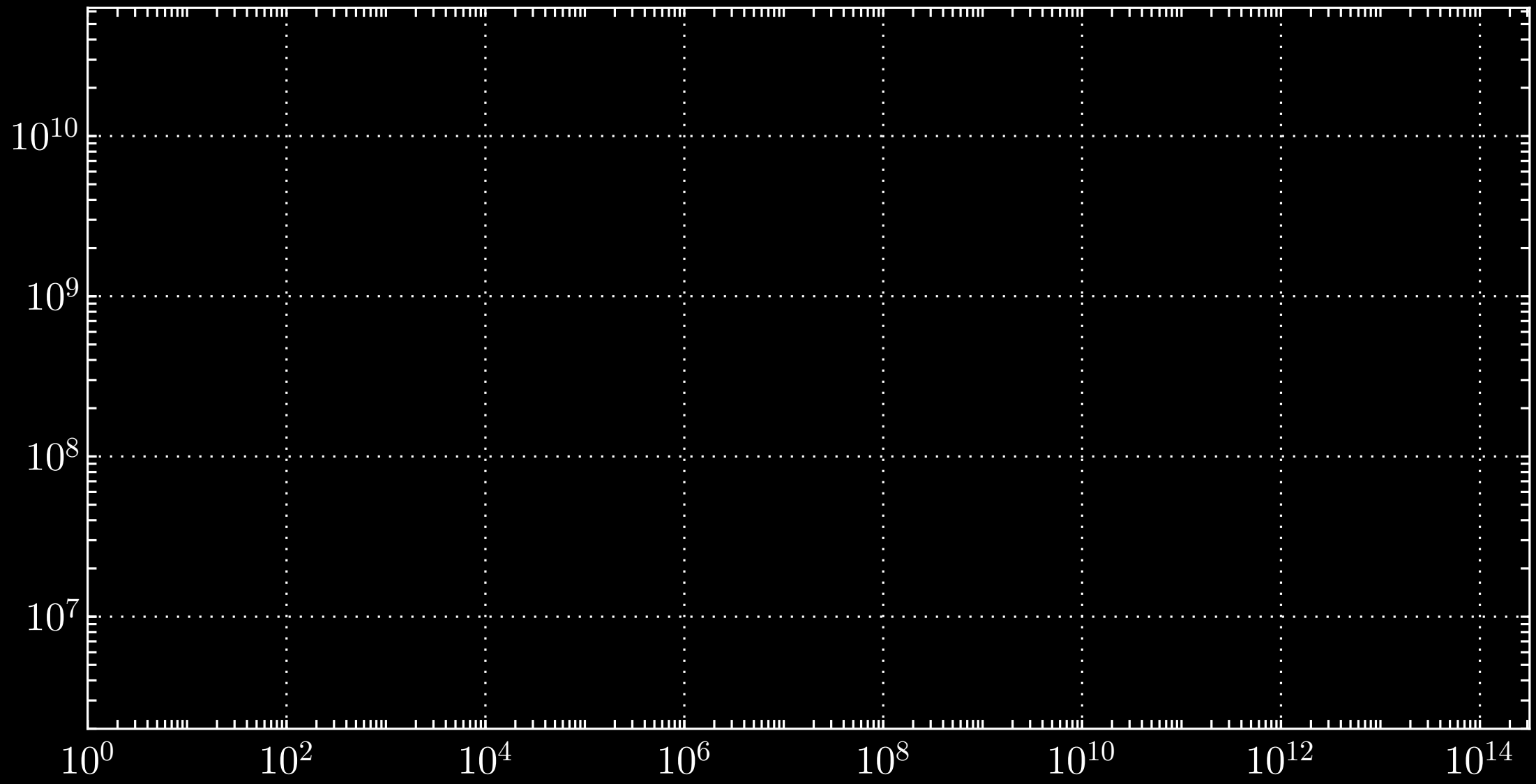
- + A unique observational window: a **probe of the dark**, a **memory of the past**
- + A growing dataset: ~ 50 merger events (and counting!),
- + Many decades in frequency: experiments planned / under construction

Gravitational waves may help shed light on many open questions

Temperature



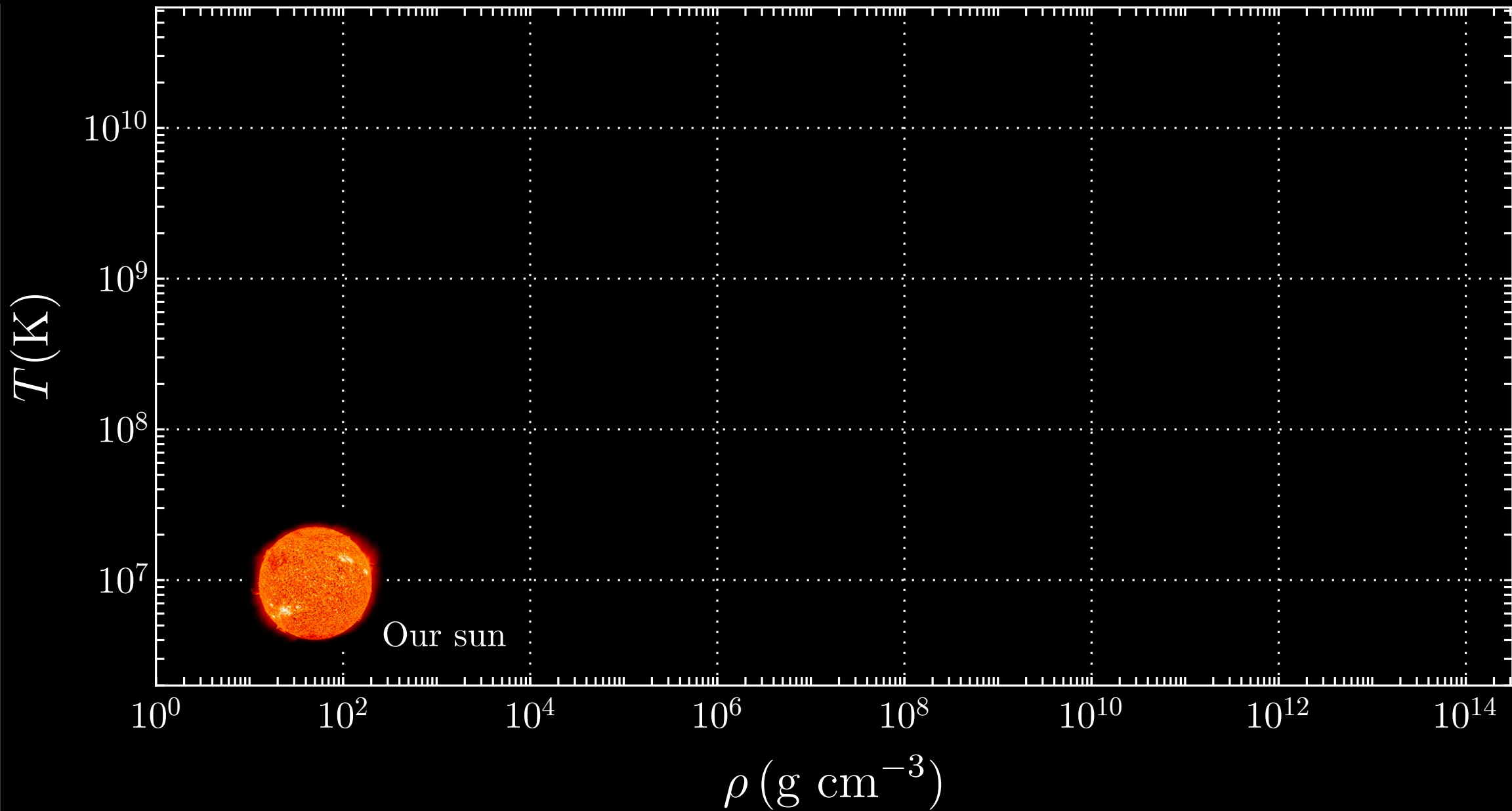
$T(\text{K})$

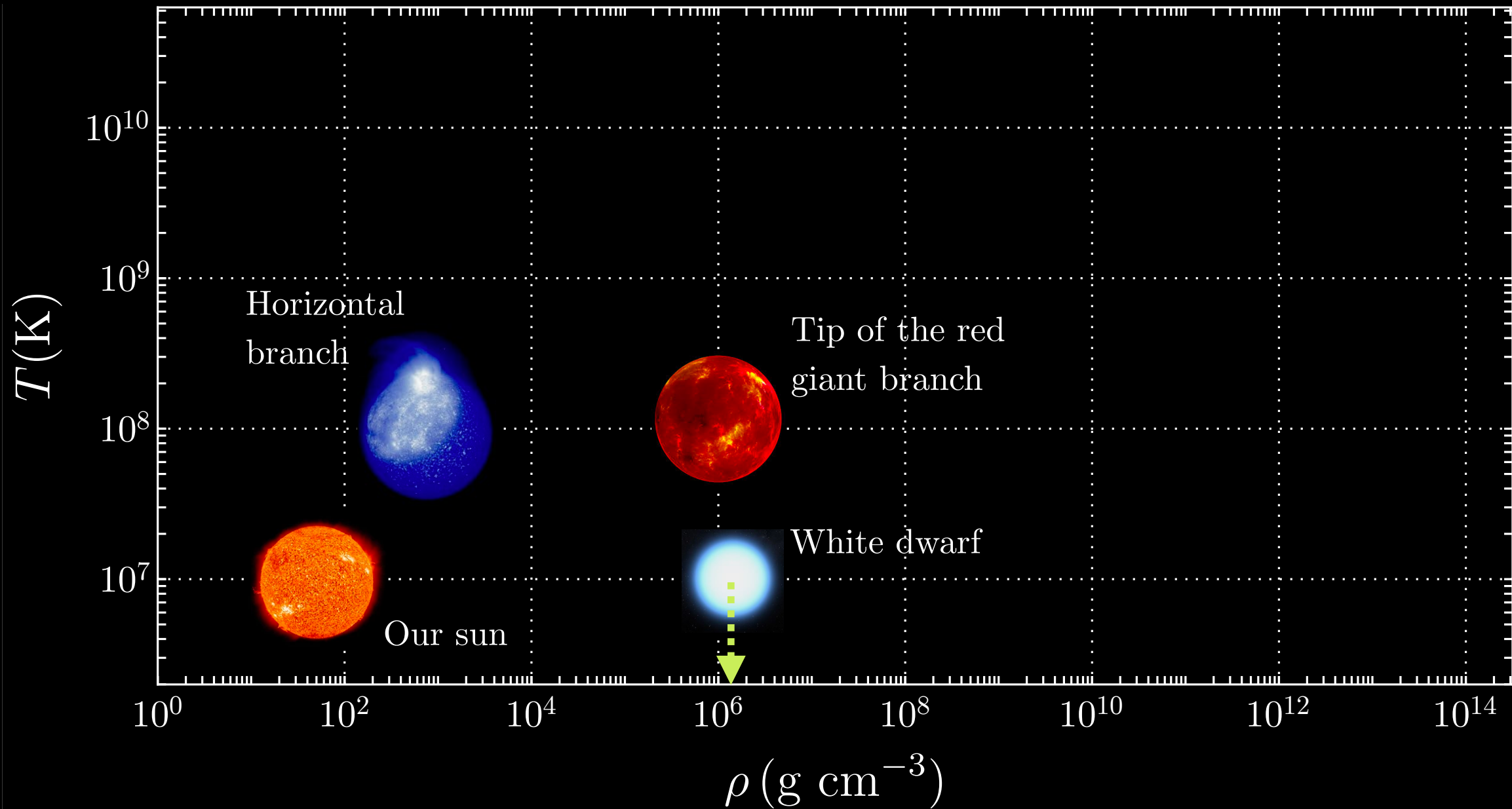


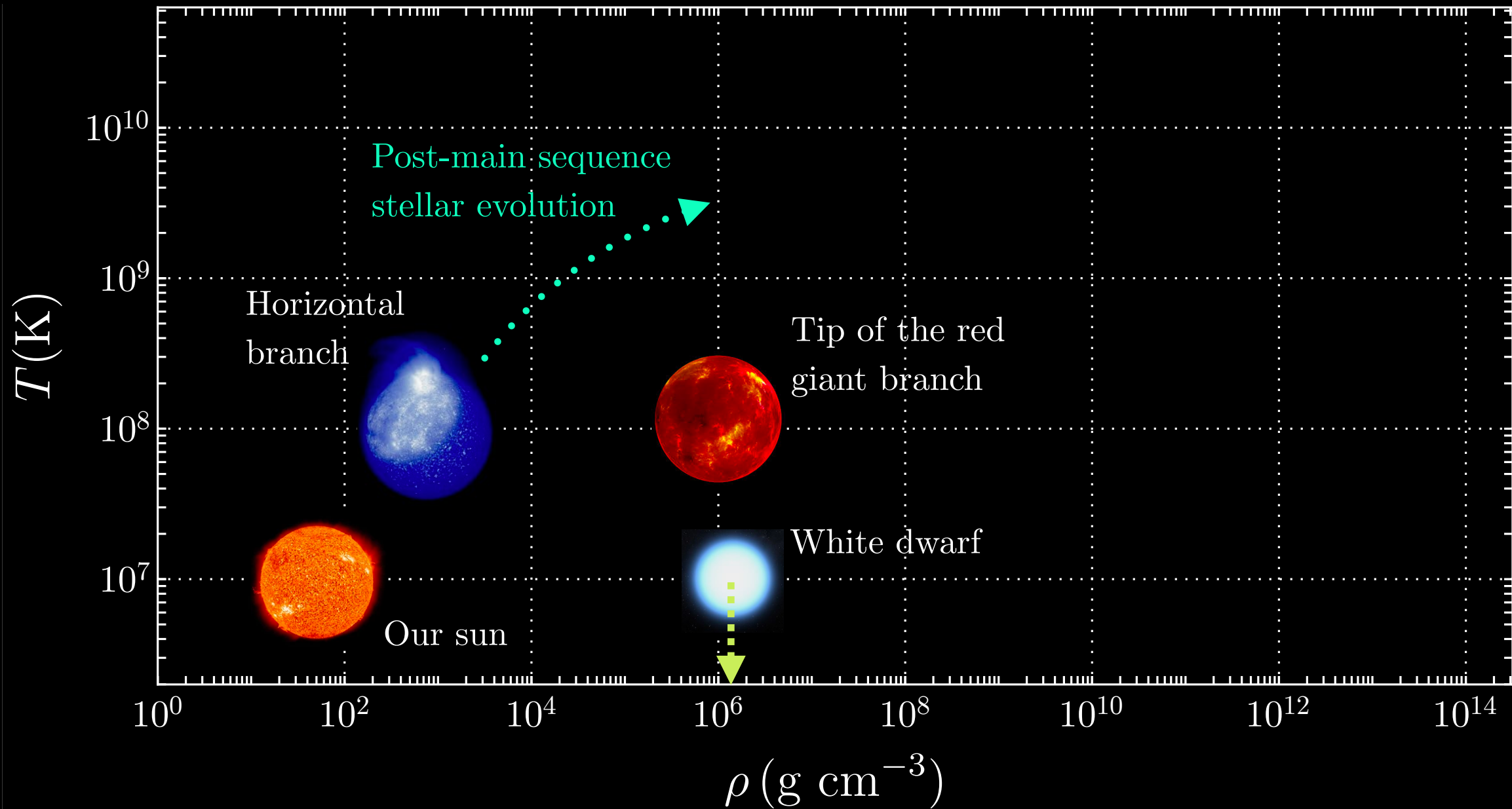
$\rho (\text{g cm}^{-3})$

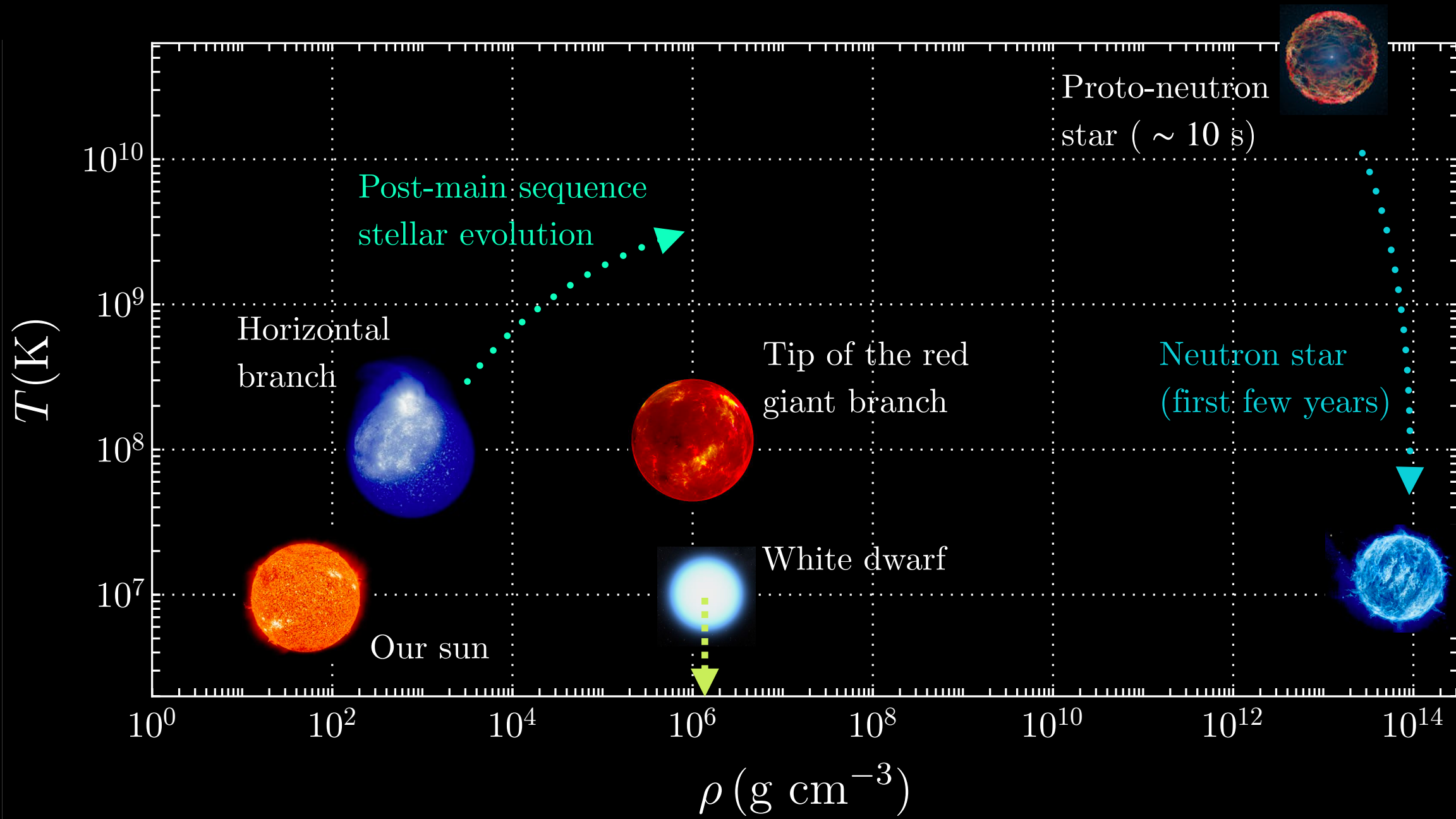
Density

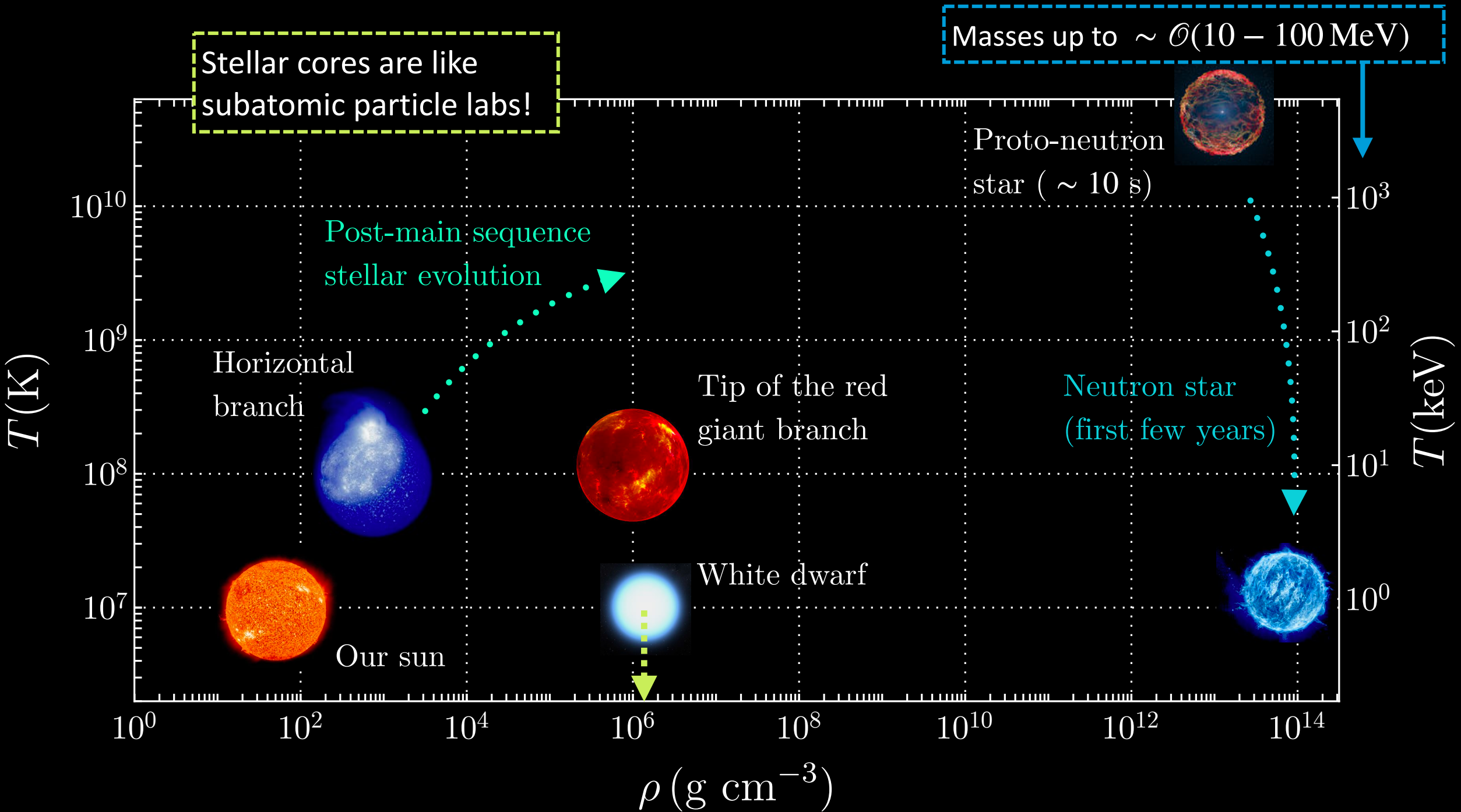




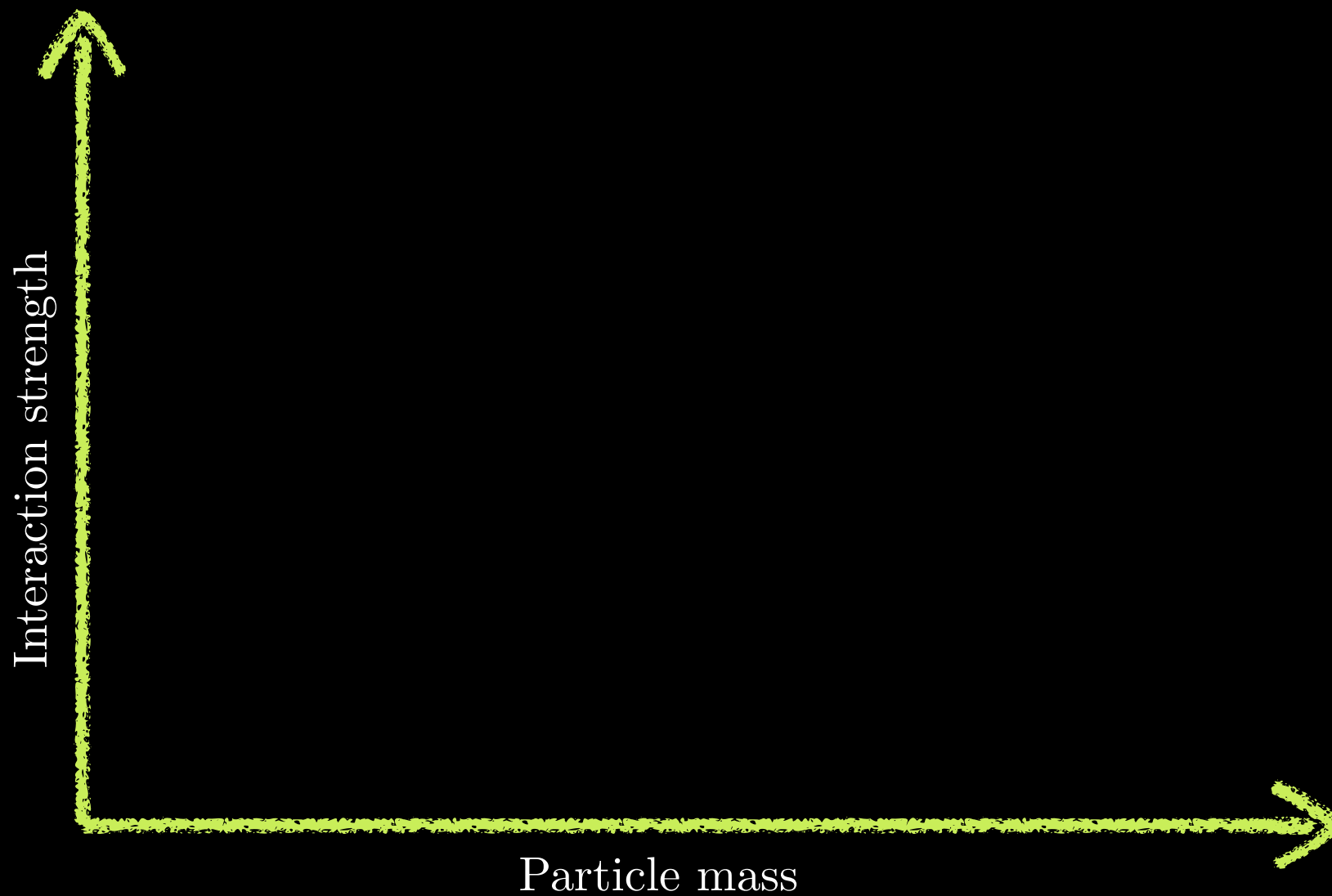




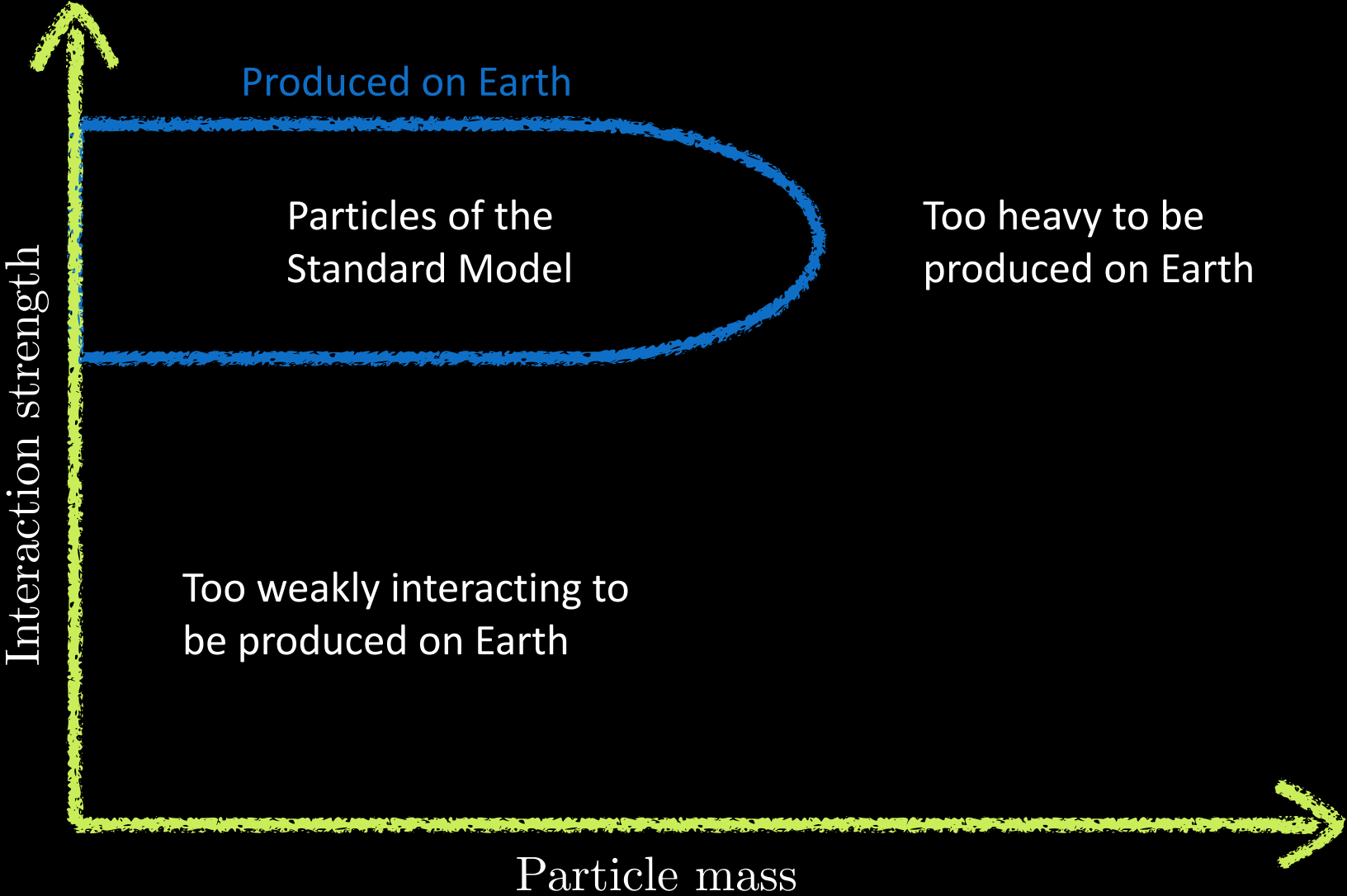




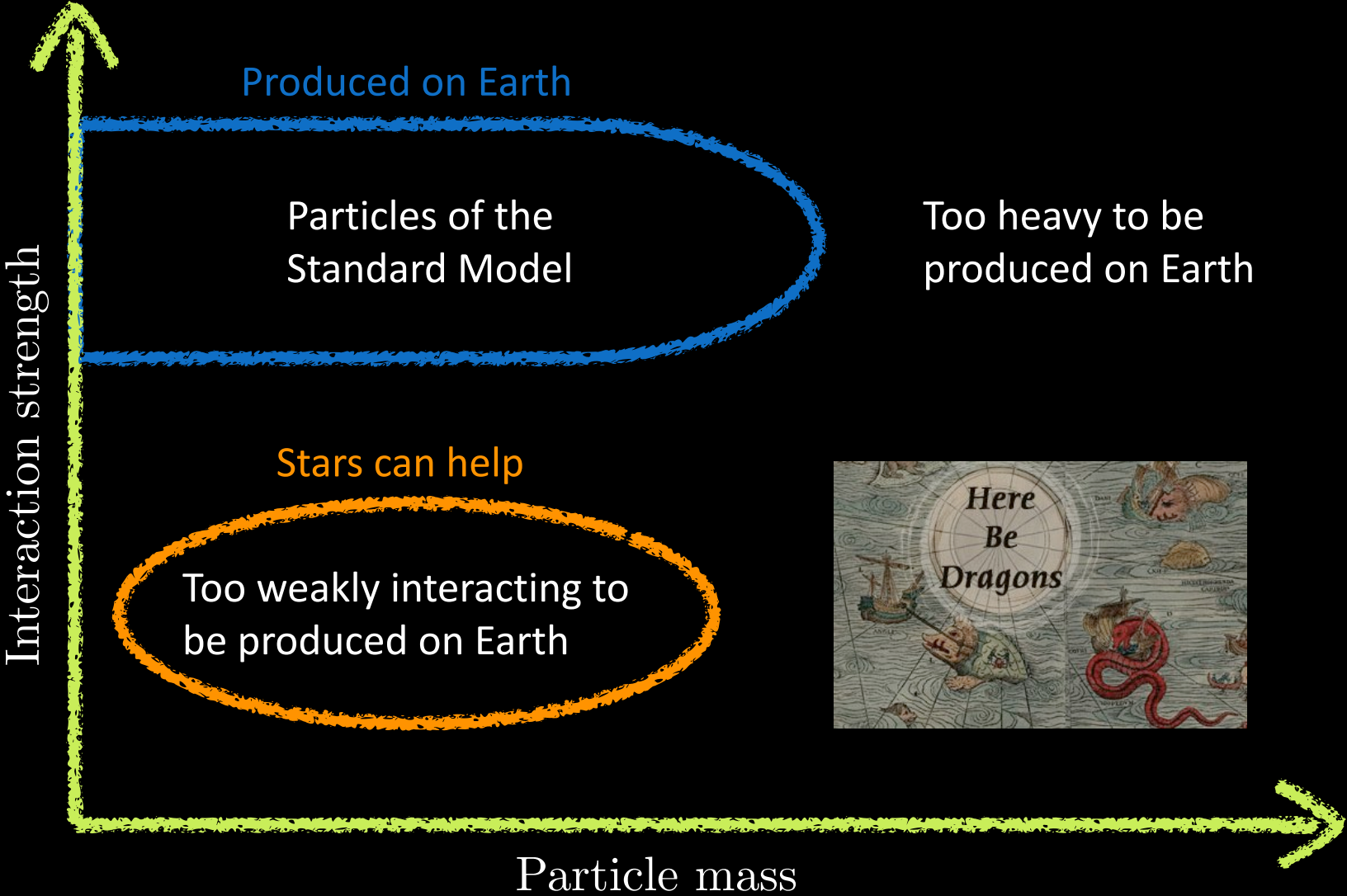
Particle physics in stars



Particle physics in stars

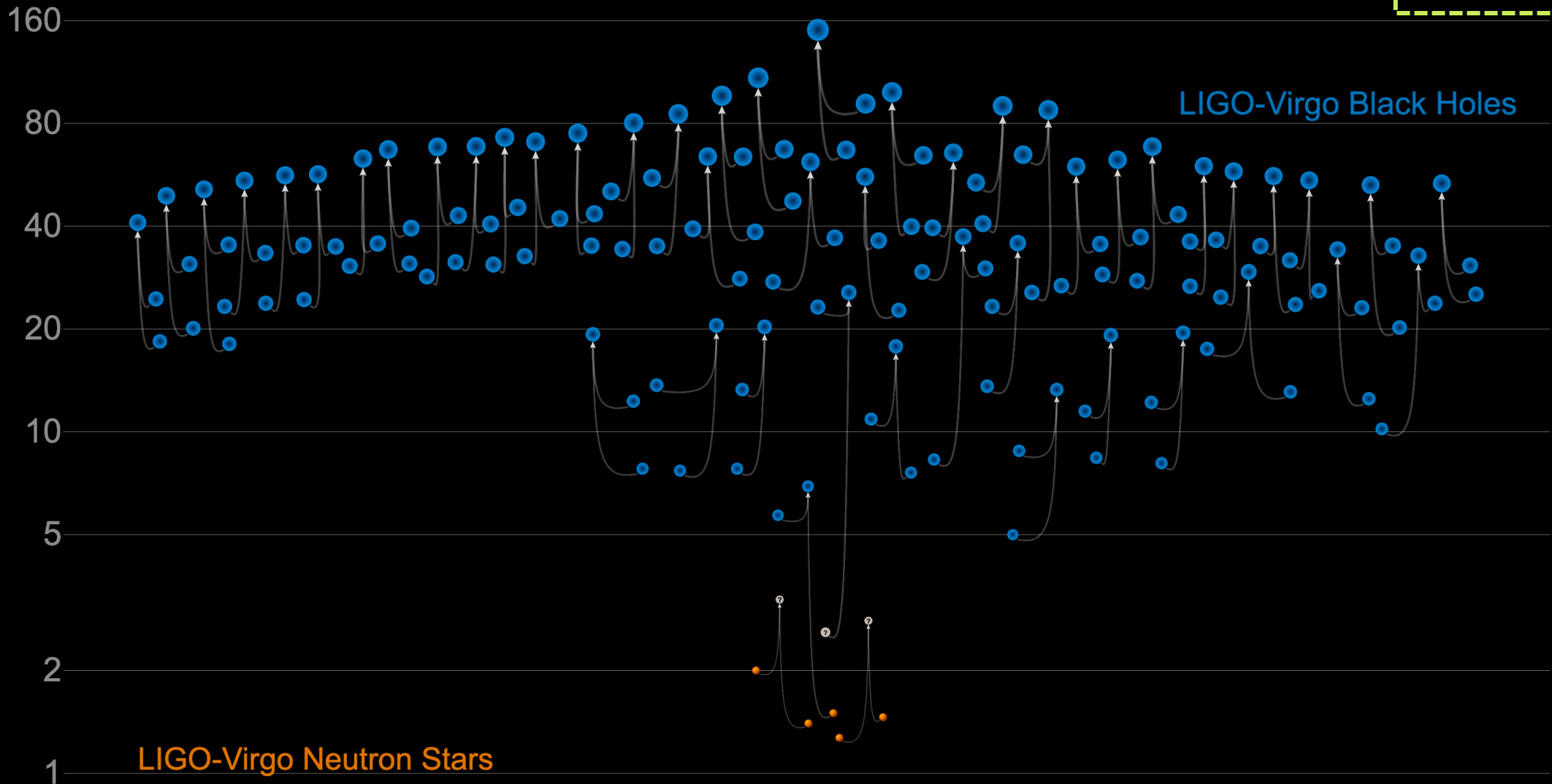


Particle physics in stars



Binary mergers in LIGO/Virgo O1-3a

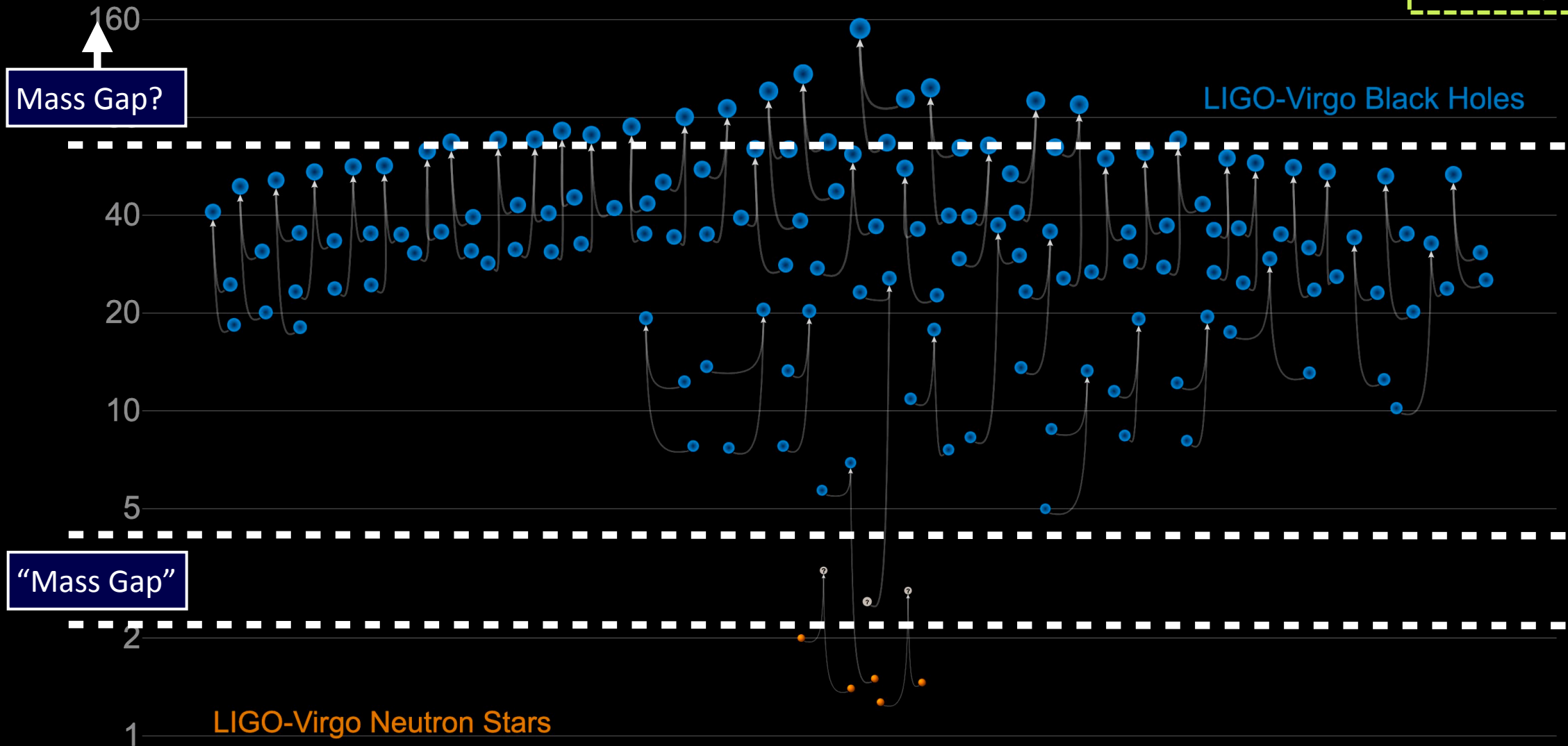
“The Stellar Graveyard”



Adapted from LIGO-Virgo, Frank Elavsky, Aaron Geller

Binary mergers in LIGO/Virgo O1-3a

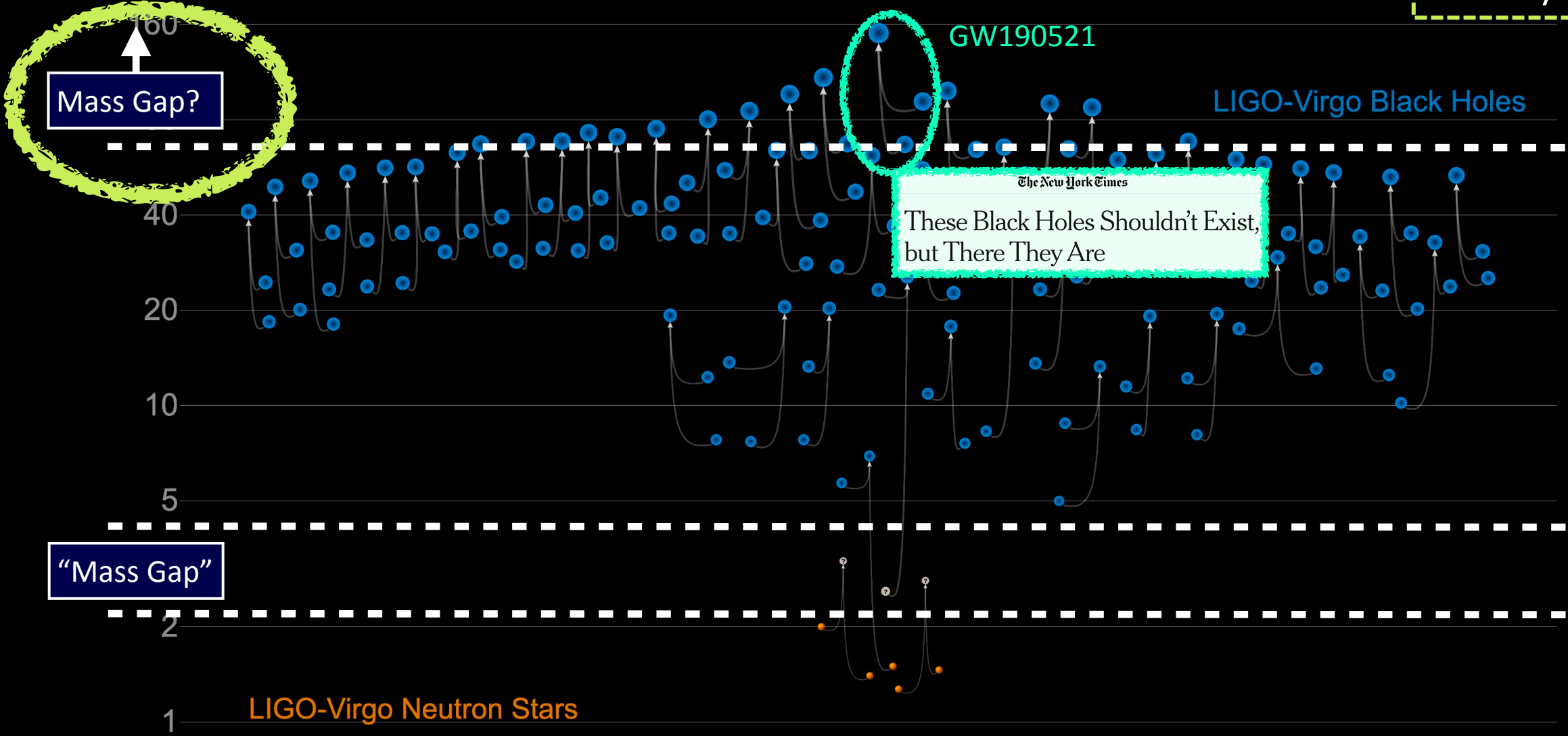
“The Stellar Graveyard”



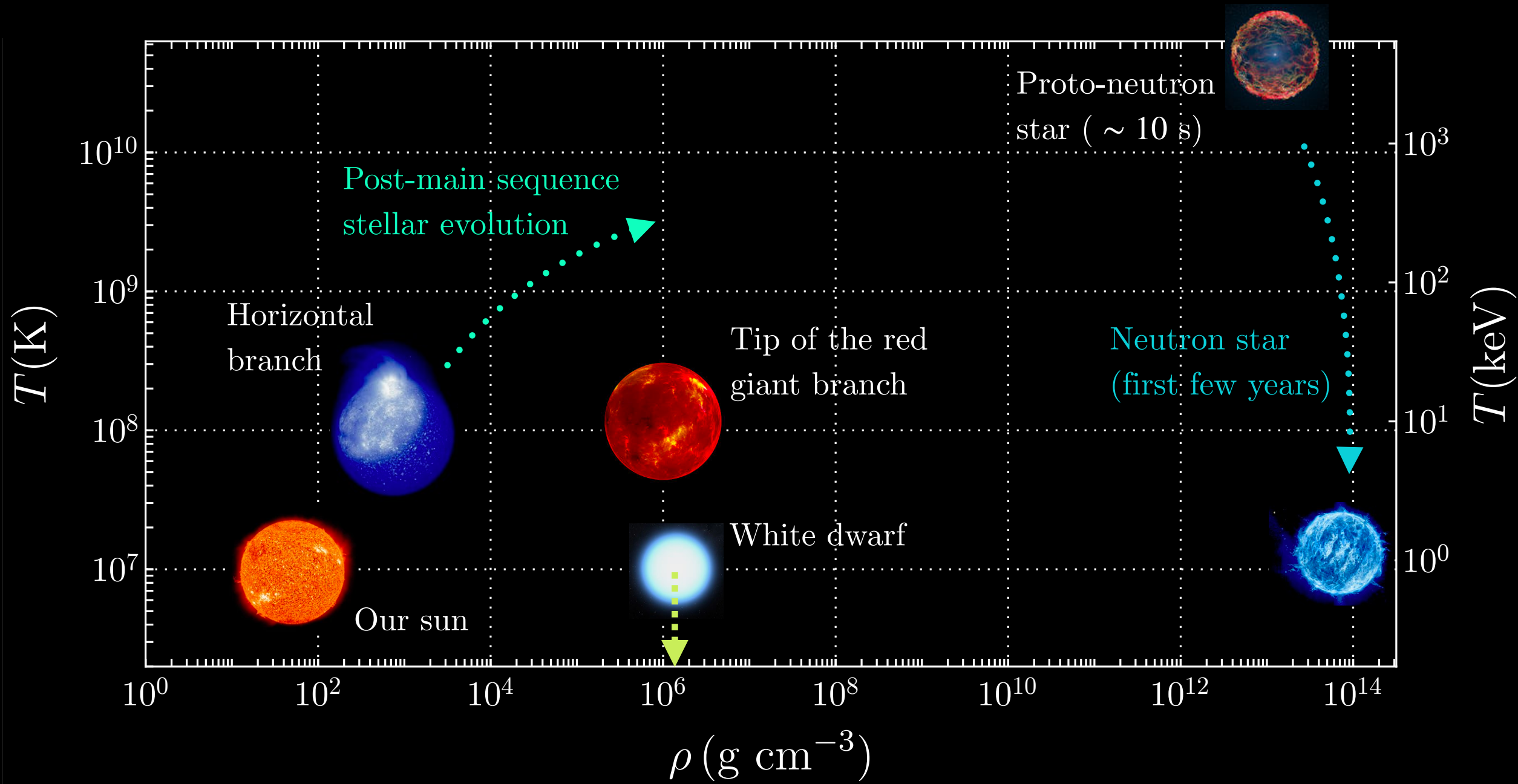
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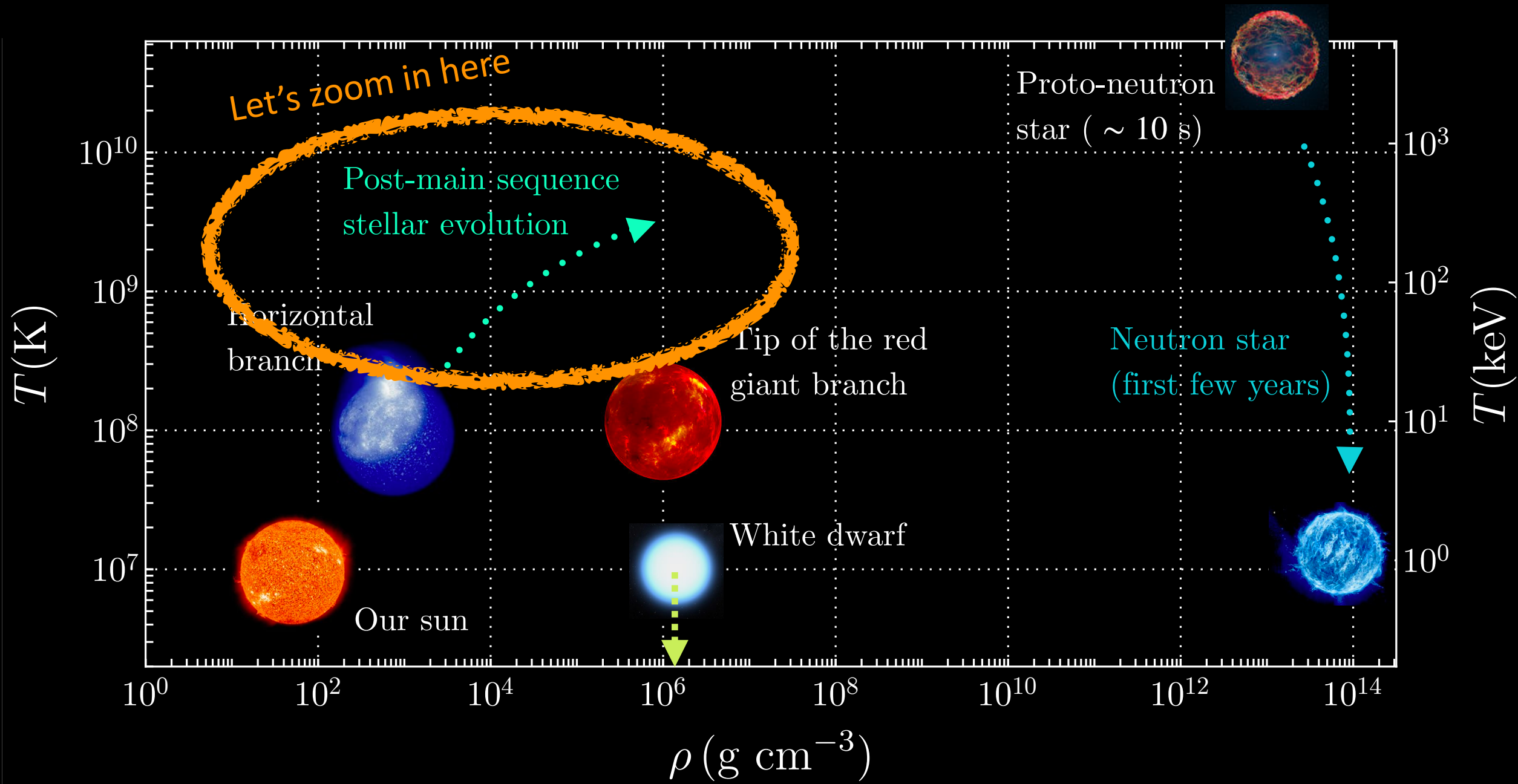
Binary mergers in LIGO/Virgo O1-3a

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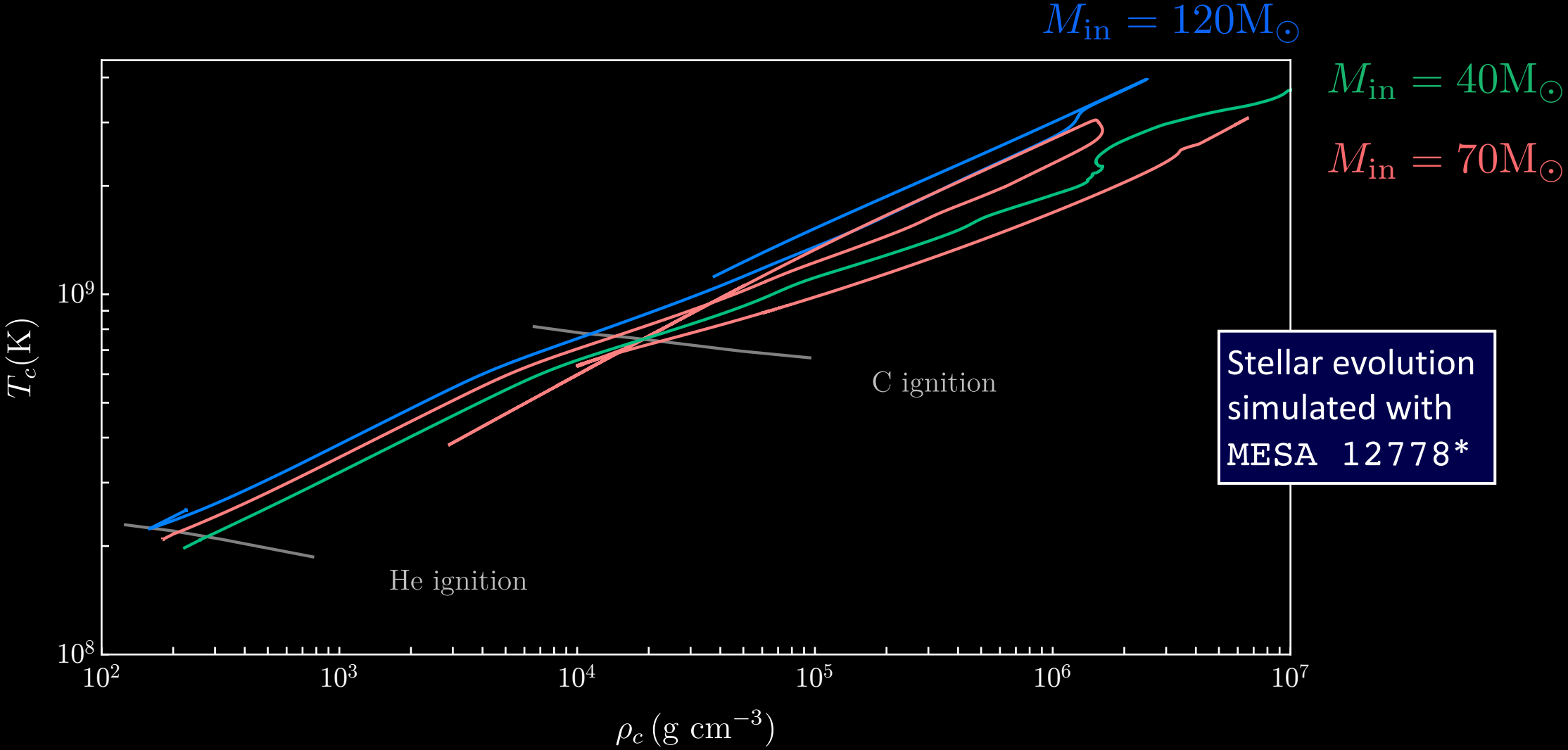


Adapted from LIGO-Virgo, Frank Elavsky, Aaron Geller



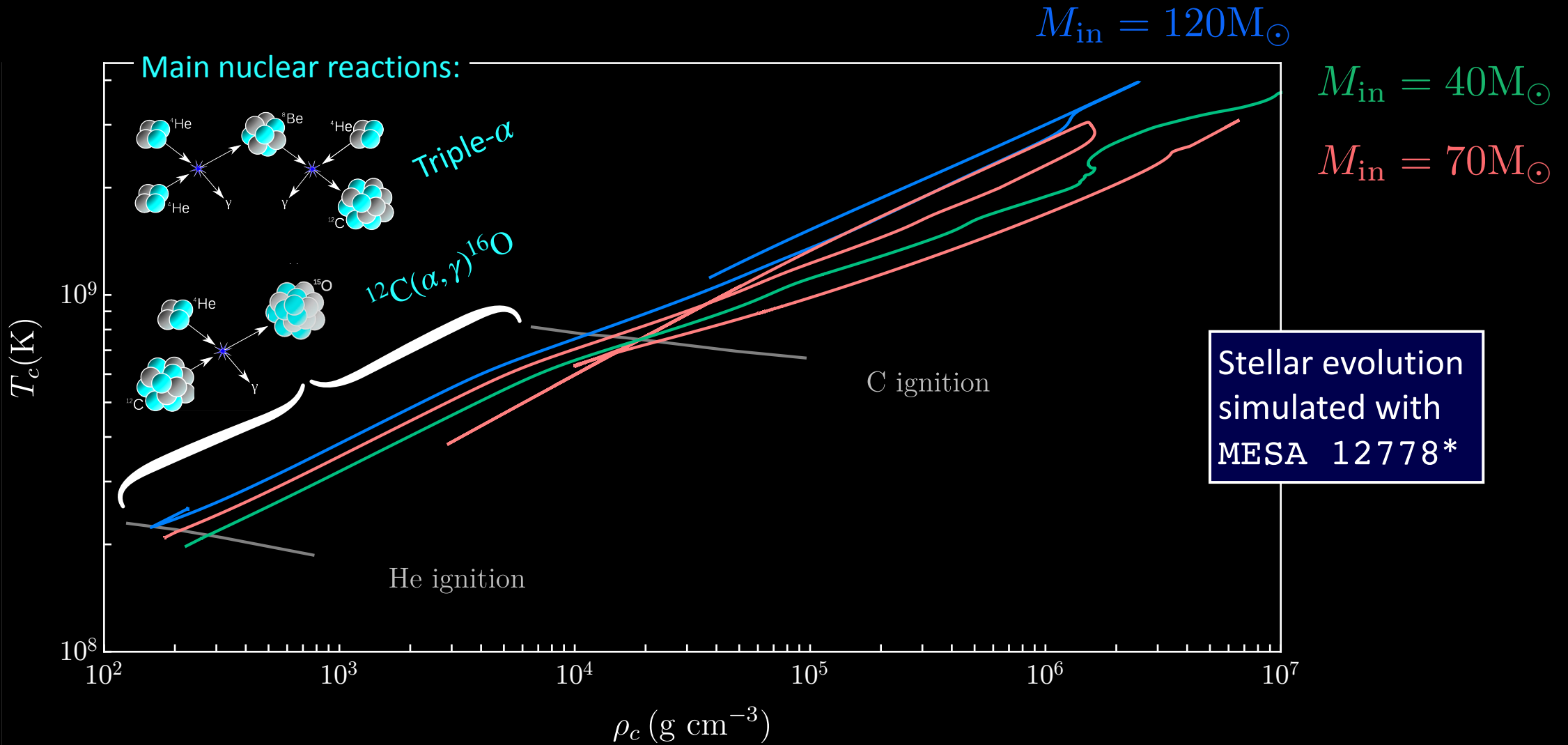


Late evolution of heavy stars



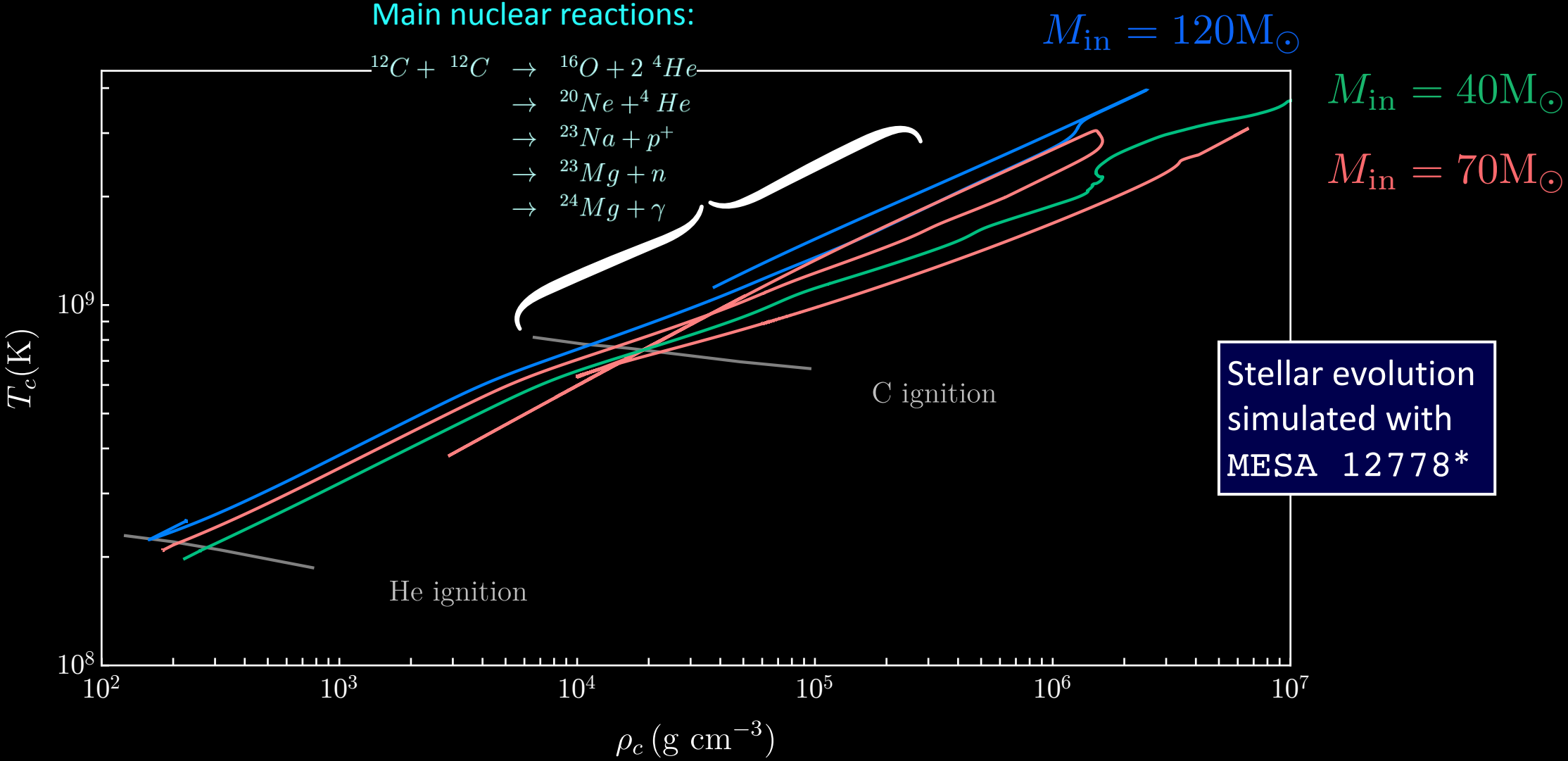
*Paxton et al, arXiv:1710.08424 [astro-ph.SR]

Late evolution of heavy stars



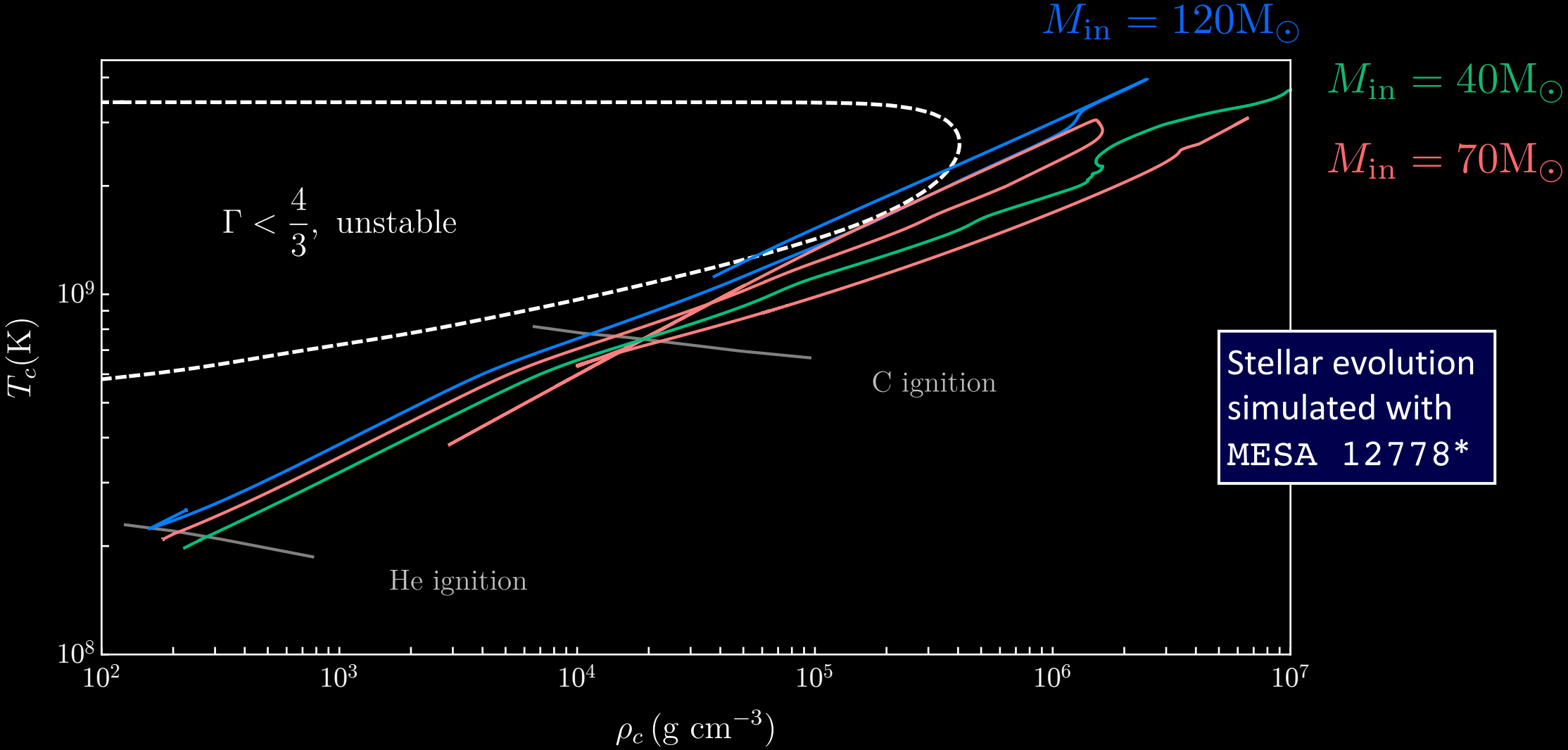
*Paxton et al, arXiv:1710.08424 [astro-ph.SR]

Late evolution of heavy stars



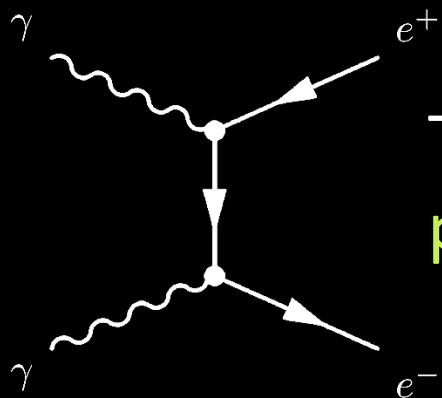
*Paxton et al, arXiv:1710.08424 [astro-ph.SR]

Late evolution of heavy stars



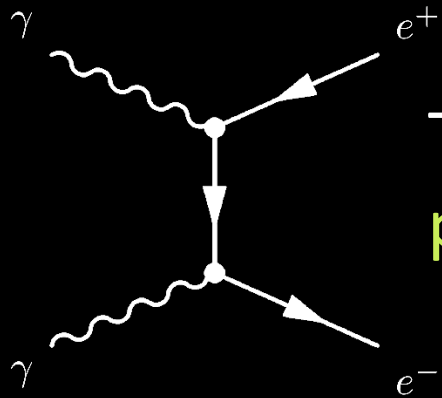
*Paxton et al, arXiv:1710.08424 [astro-ph.SR]

The danger zone: pair-instability



The high temperatures of stellar cores mean **electron-positron pairs** can be created from photons: $\gamma\gamma \rightarrow e^+e^-$

The danger zone: pair-instability

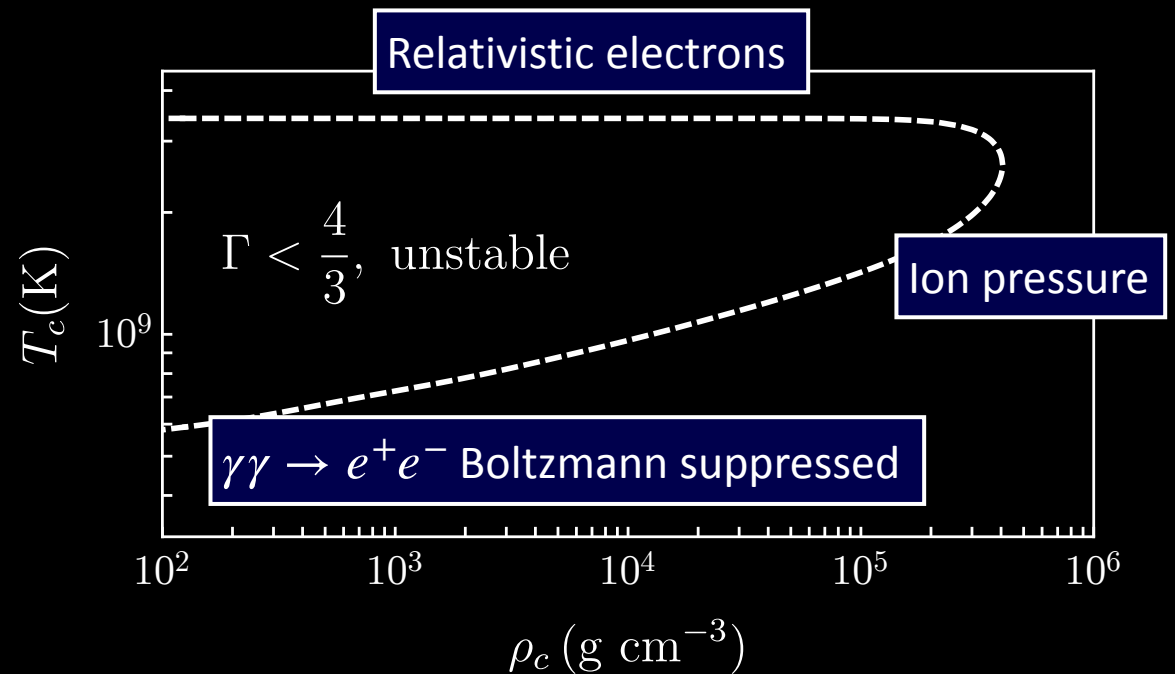


The high temperatures of stellar cores mean **electron-positron pairs** can be created from photons: $\gamma\gamma \rightarrow e^+e^-$

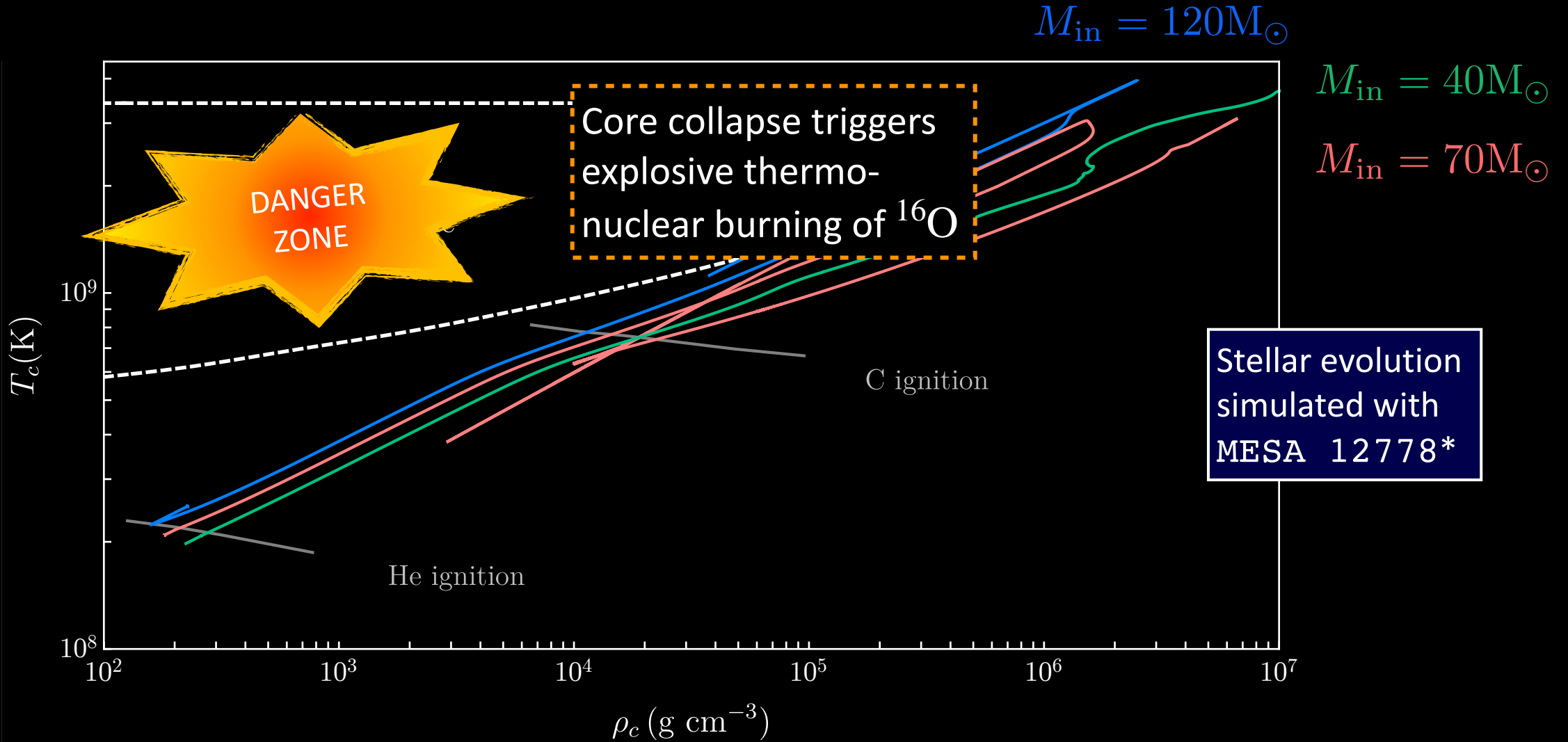
Unstable, because:

The **photons** give the star outward pressure

The **electron-positron** pairs imply extra gravity but no pressure
 → *the core starts to collapse*



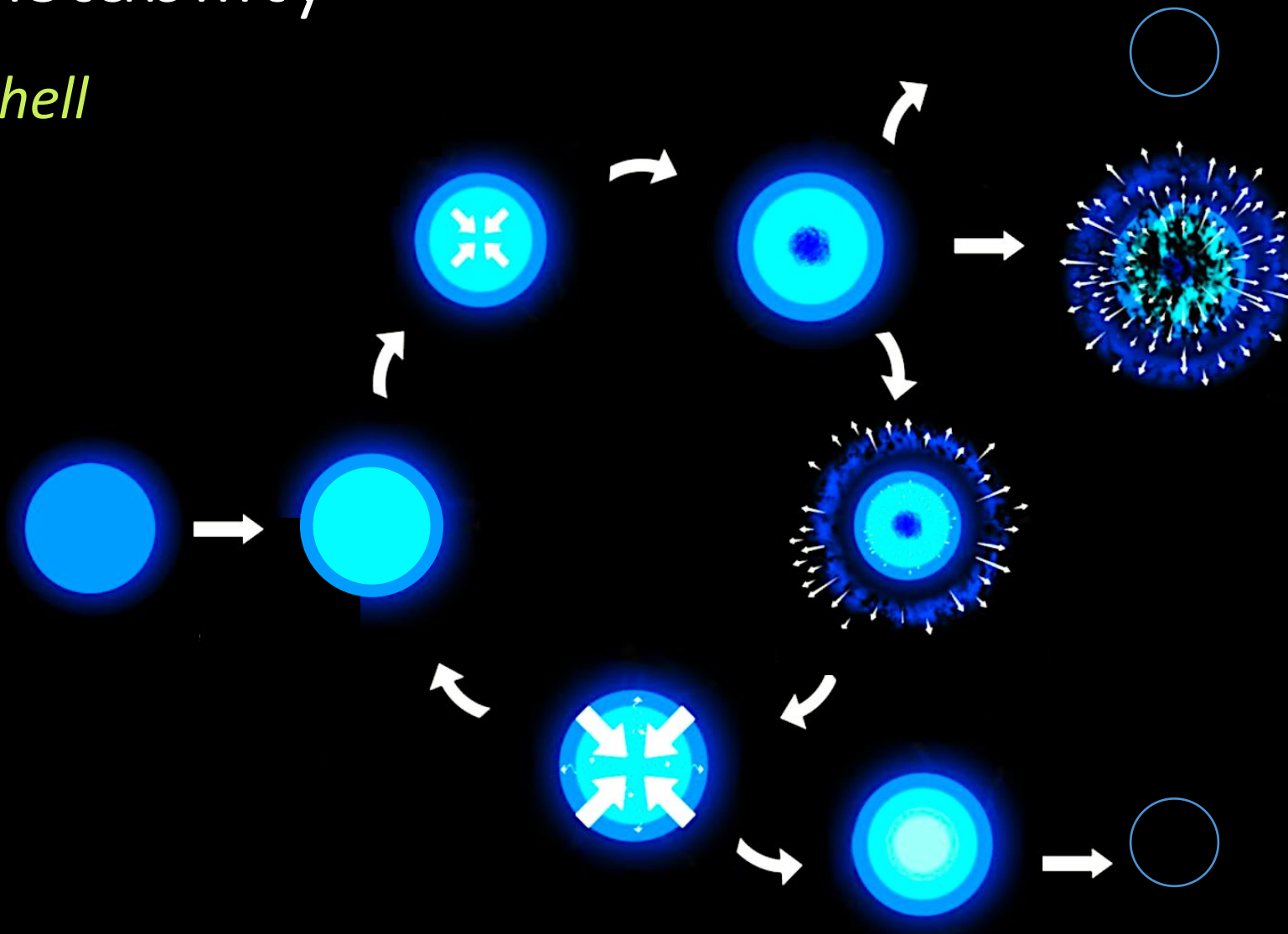
Evolution of old population-III stars



*Paxton et al, arXiv:1710.08424 [astro-ph.SR]

Pair instability

in a nutshell

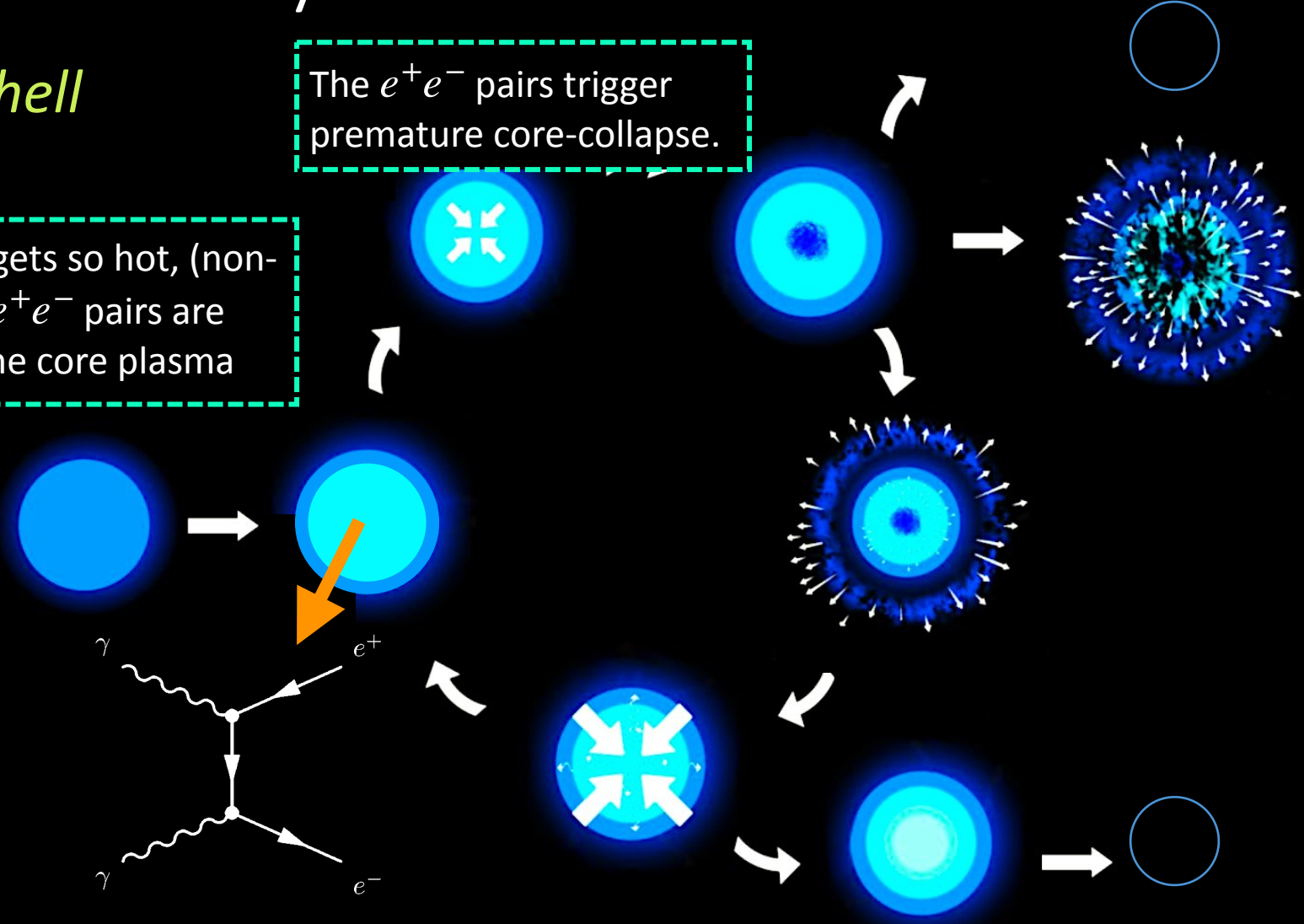


Pair instability

in a nutshell

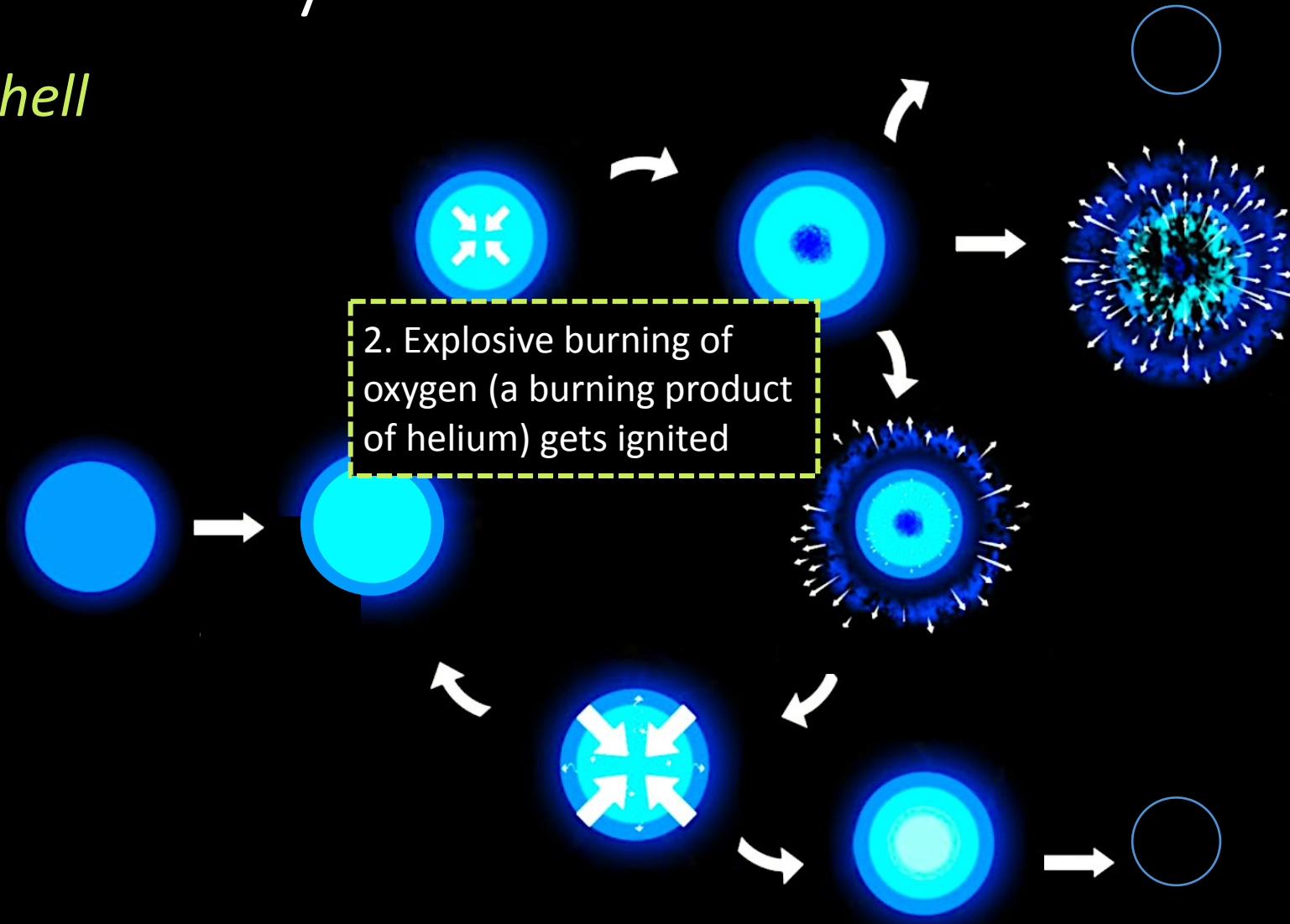
1. The core gets so hot, (non-relativistic) e^+e^- pairs are created in the core plasma

The e^+e^- pairs trigger premature core-collapse.



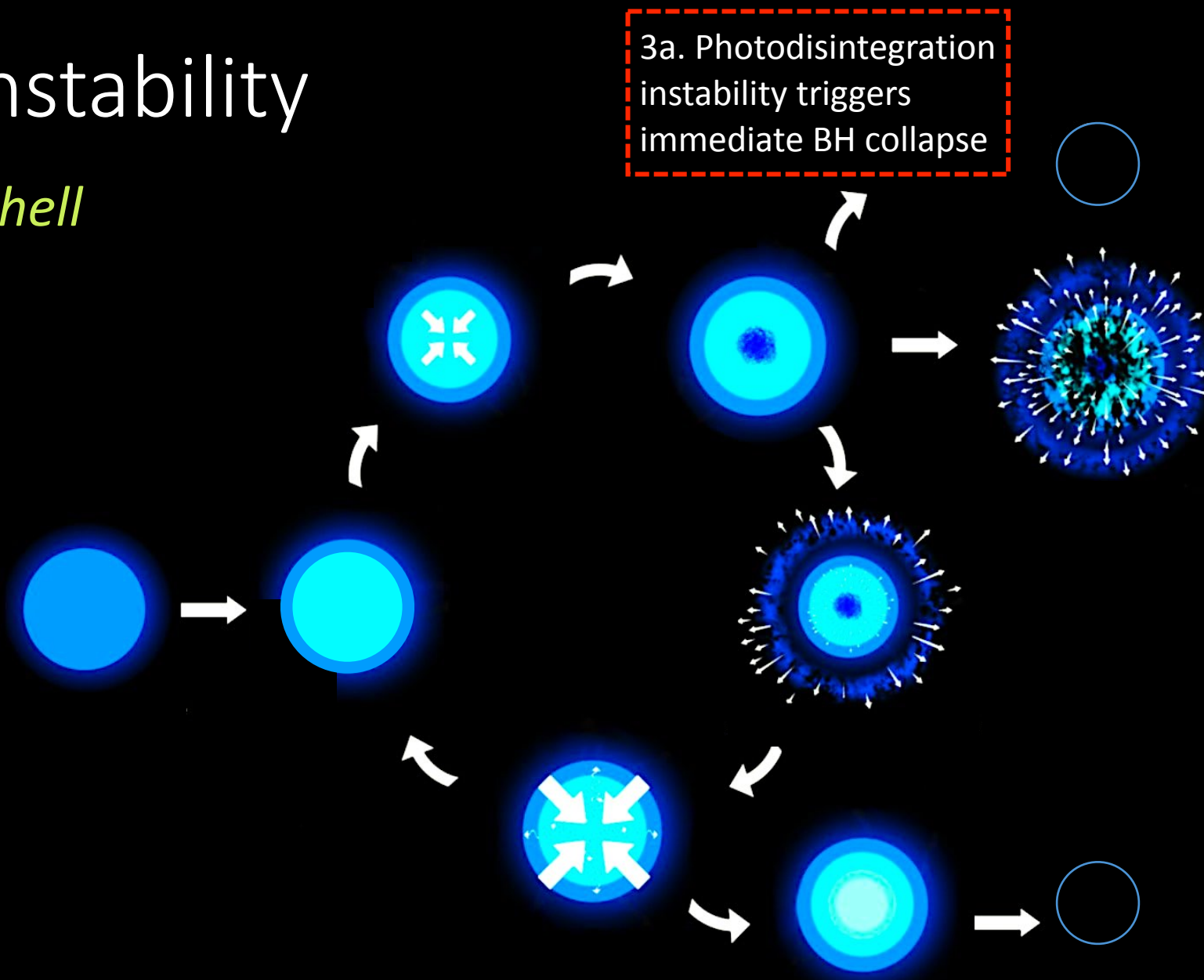
Pair instability

in a nutshell



Pair instability

in a nutshell

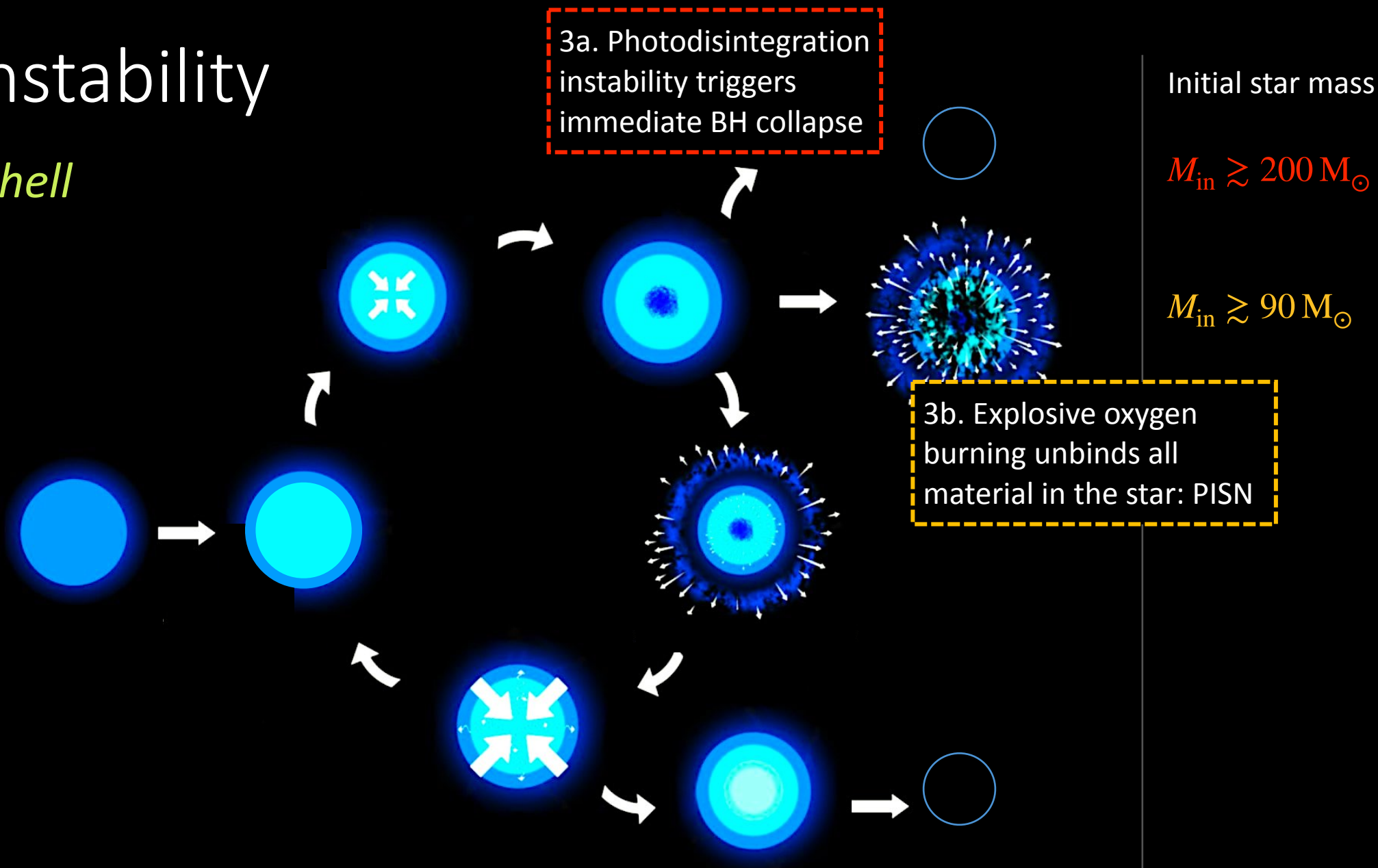


Initial star mass

$$M_{\text{in}} \gtrsim 200 M_{\odot}$$

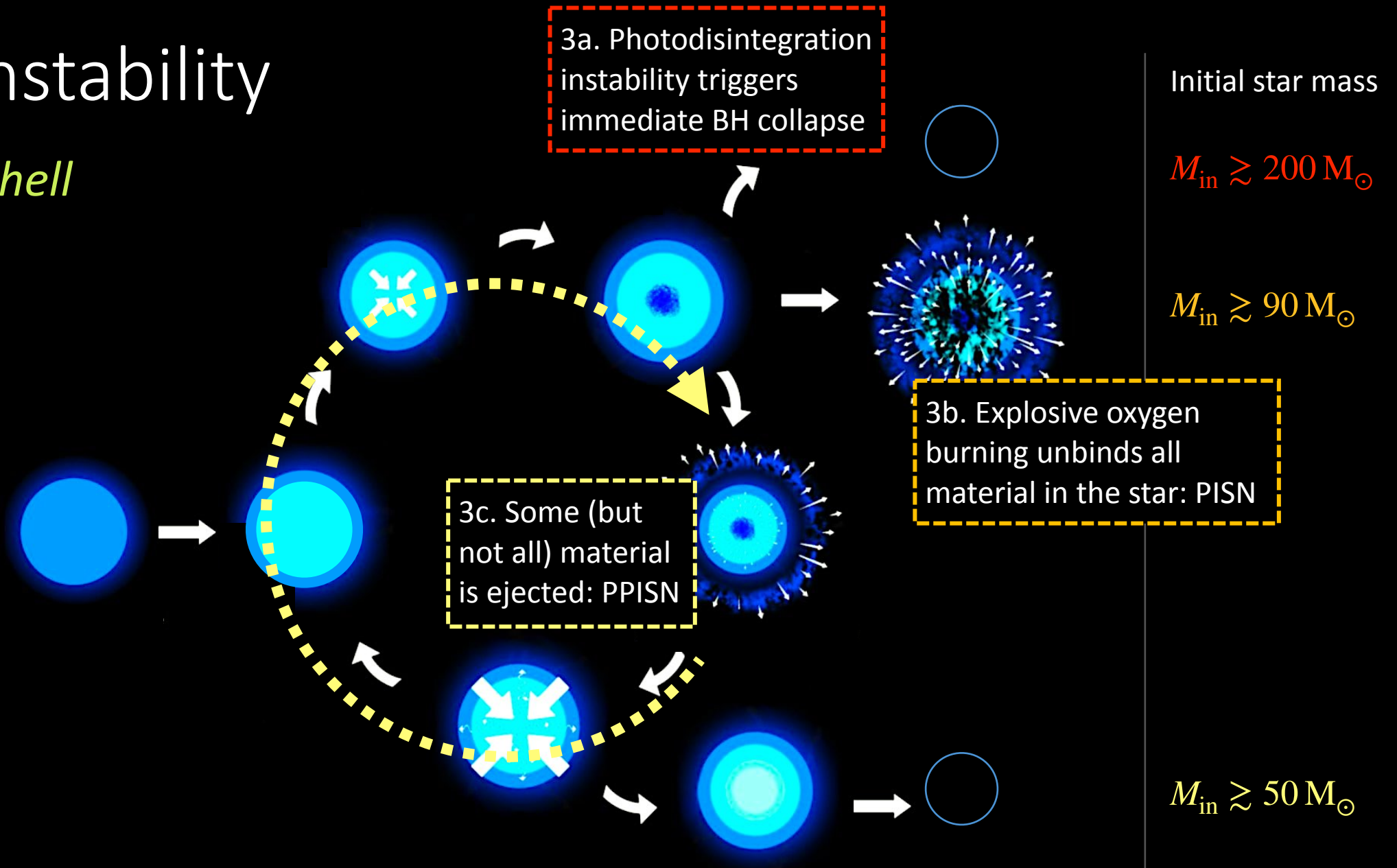
Pair instability

in a nutshell

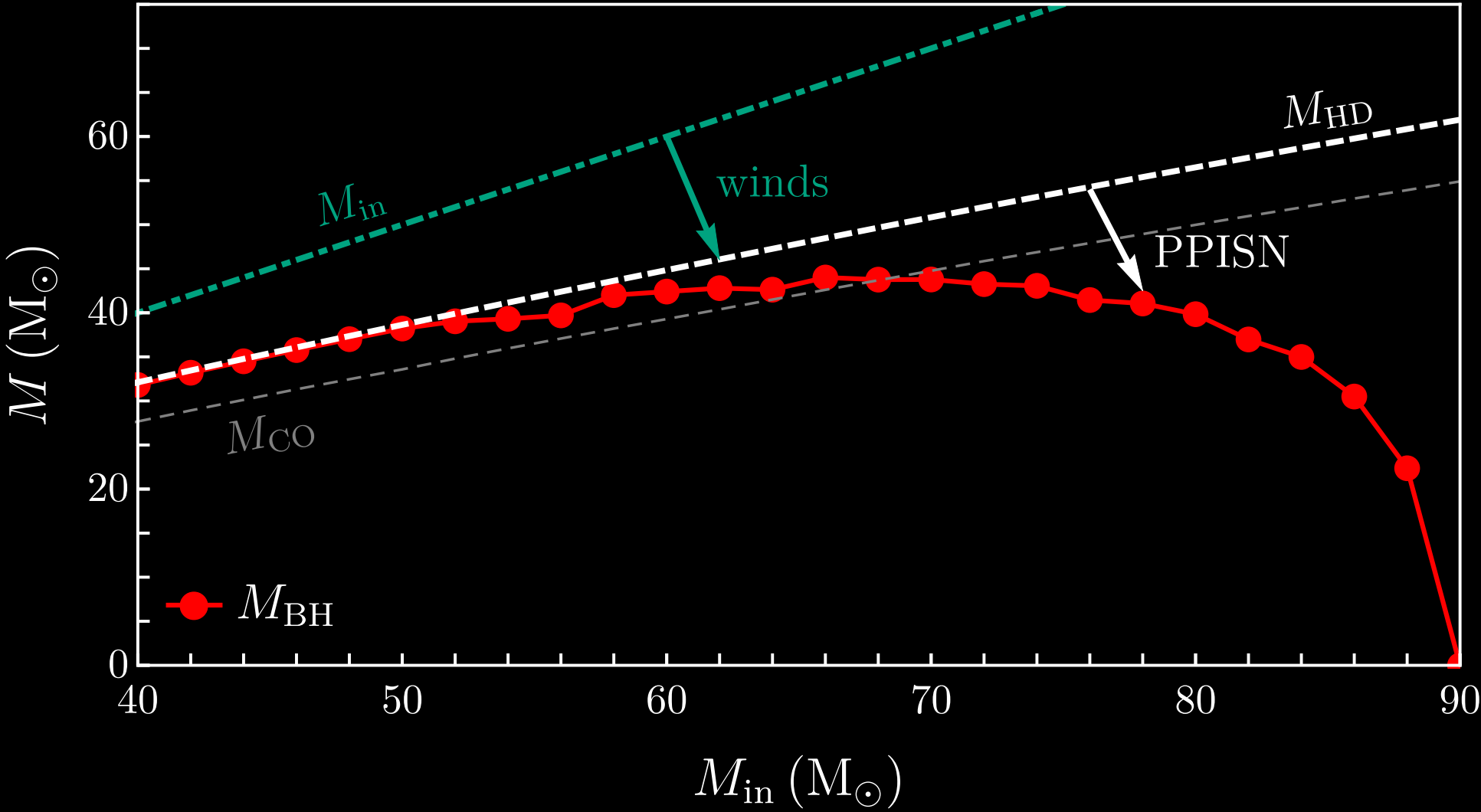


Pair instability

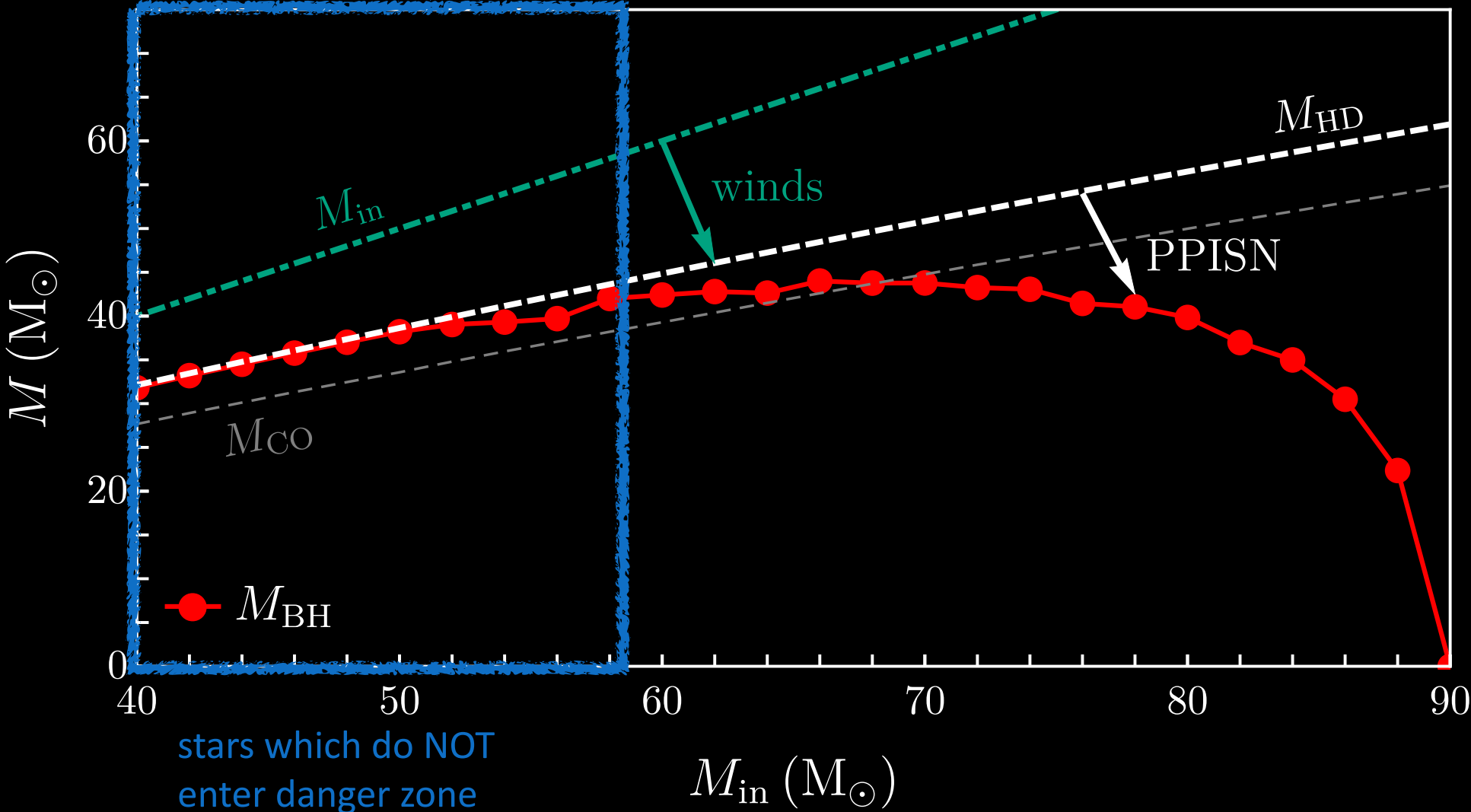
in a nutshell



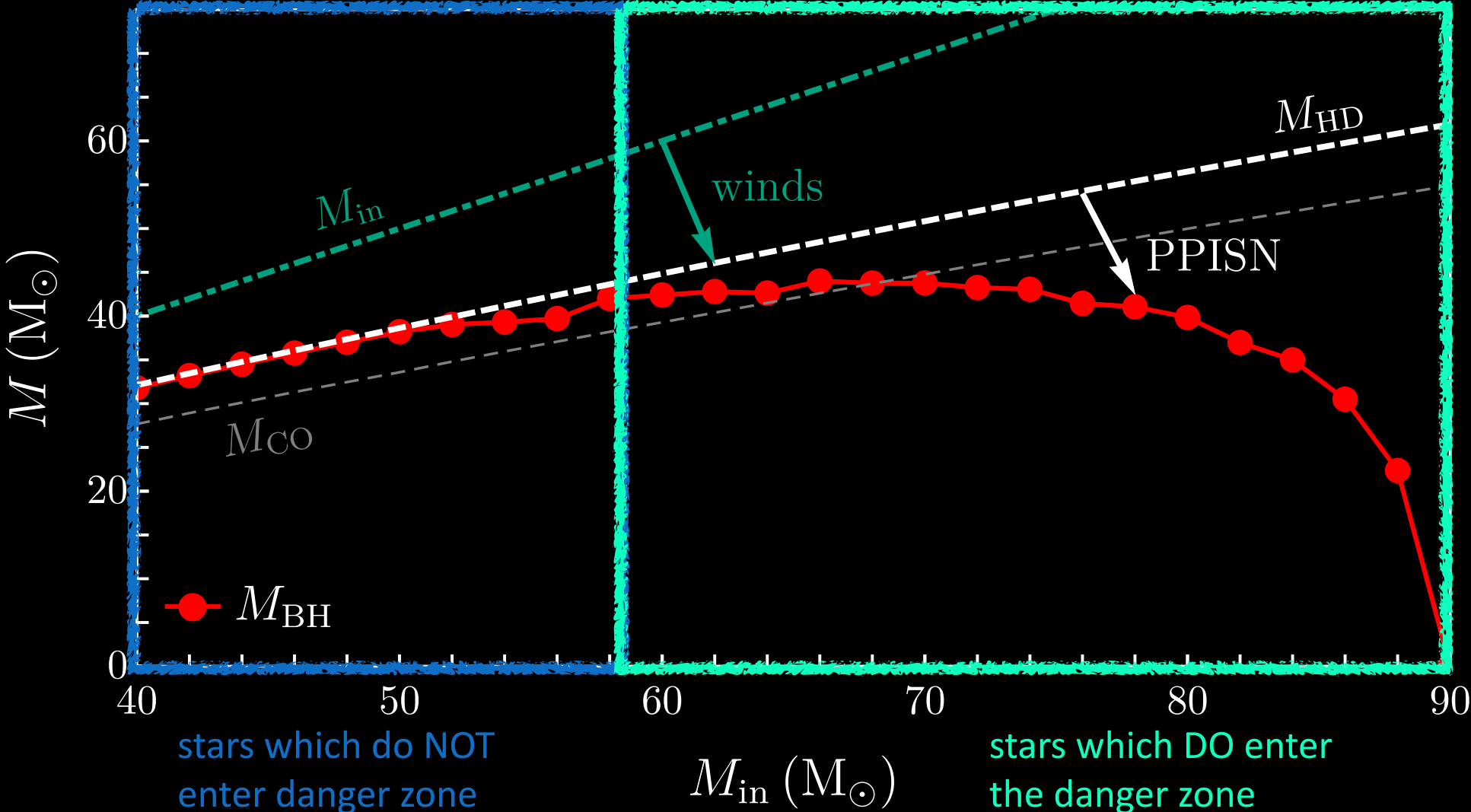
Resulting black hole masses



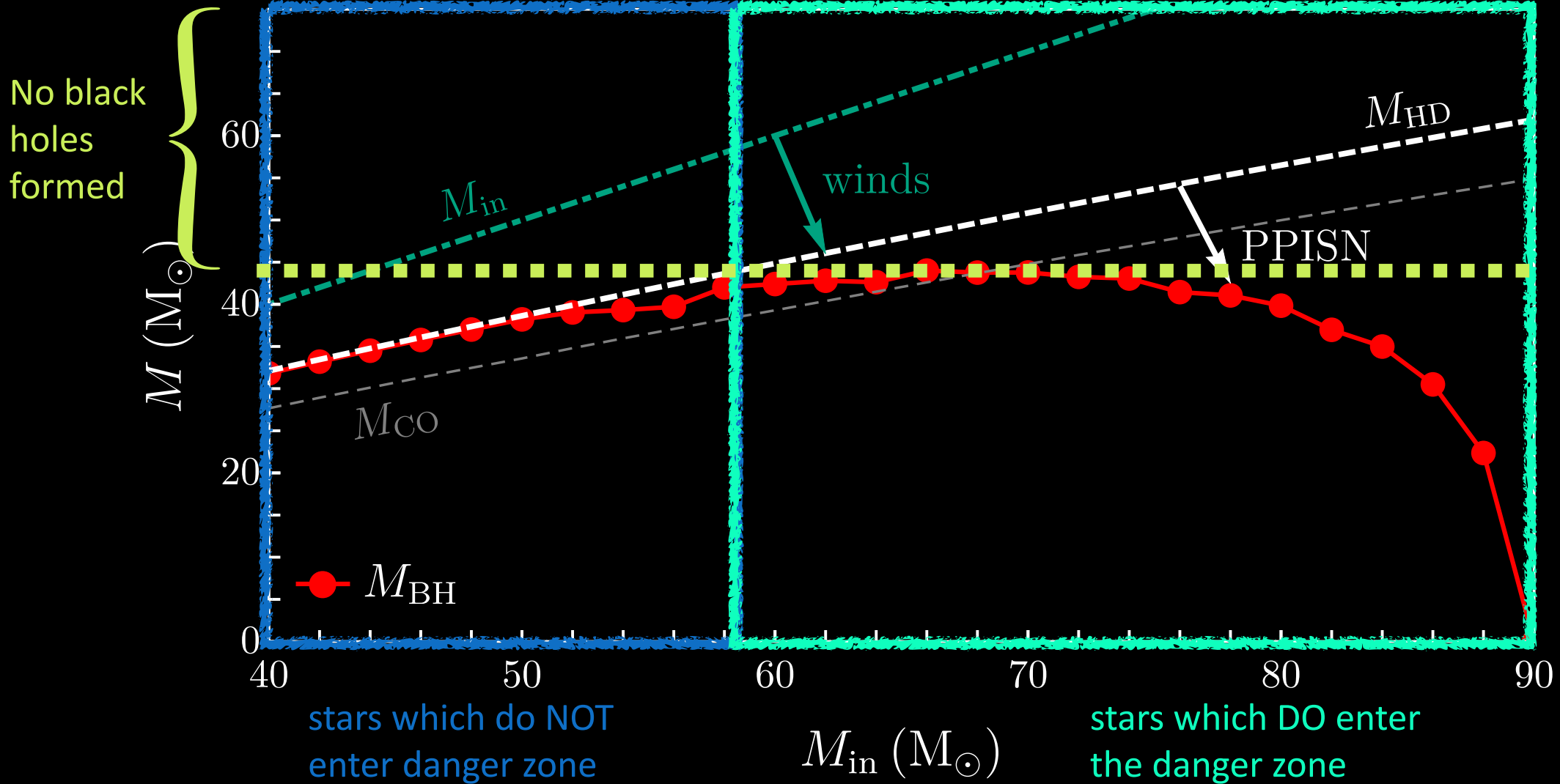
Resulting black hole masses



Resulting black hole masses



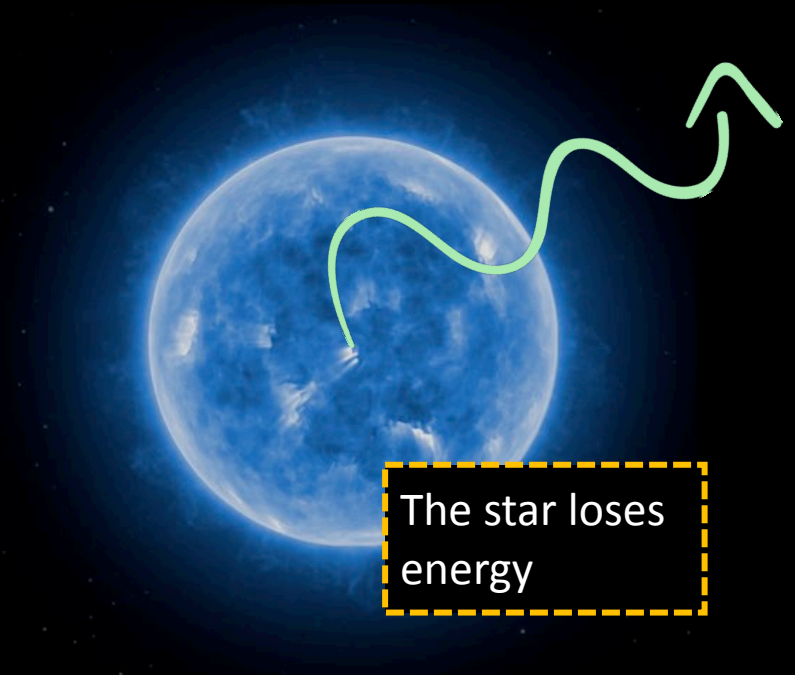
Resulting black hole masses



What about new particles?

New particles...

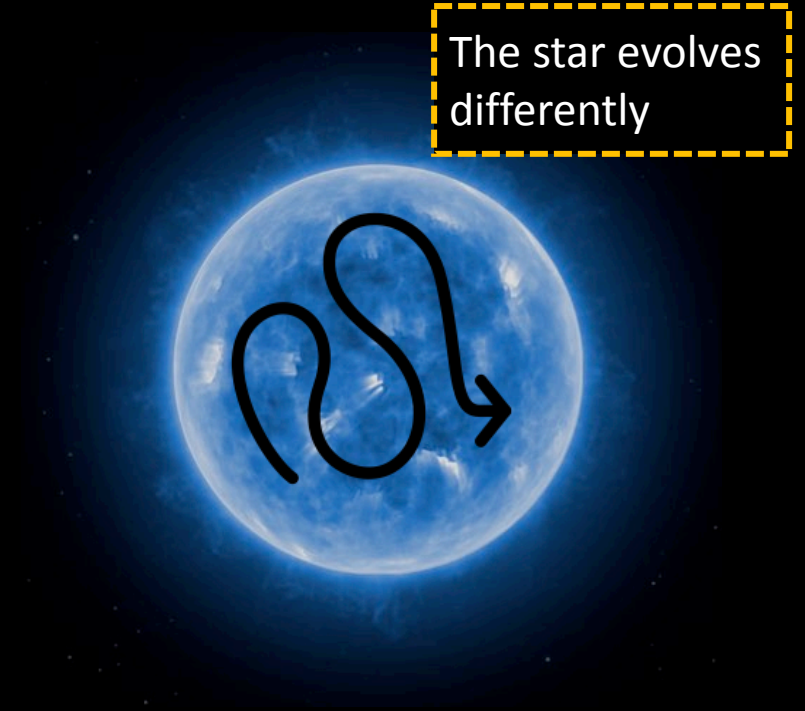
- May be produced in the star and *free stream out*



What about new particles?

New particles...

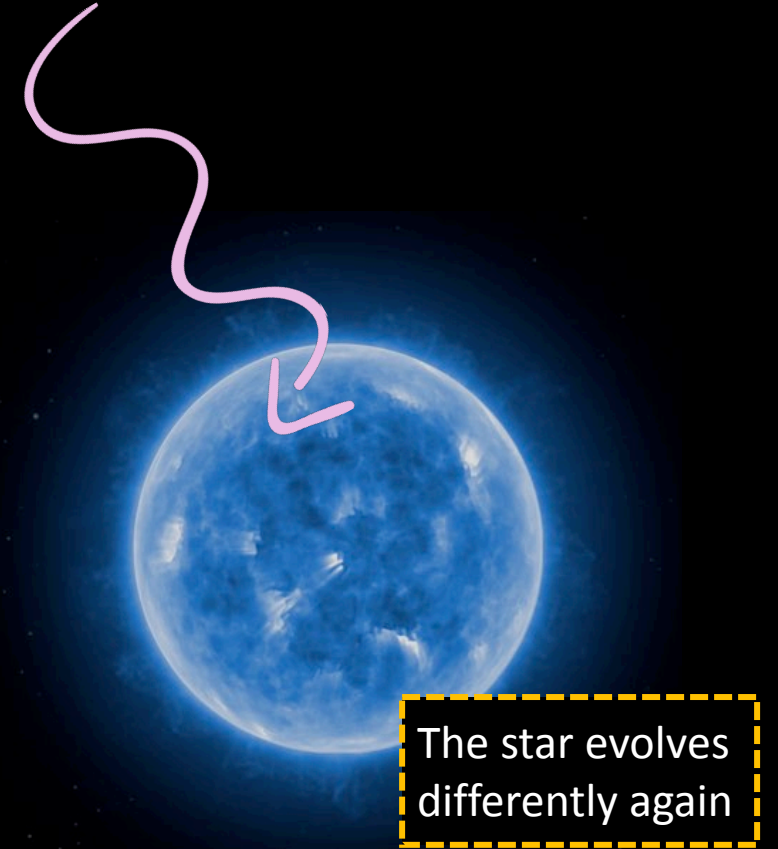
- May be produced in the star and *free stream out*
- May be produced in the star and *get trapped*



What about new particles?

New particles...

- May be produced in the star and *free stream out*
- May be produced in the star and *get trapped*
- May collect in the star and annihilate in the core

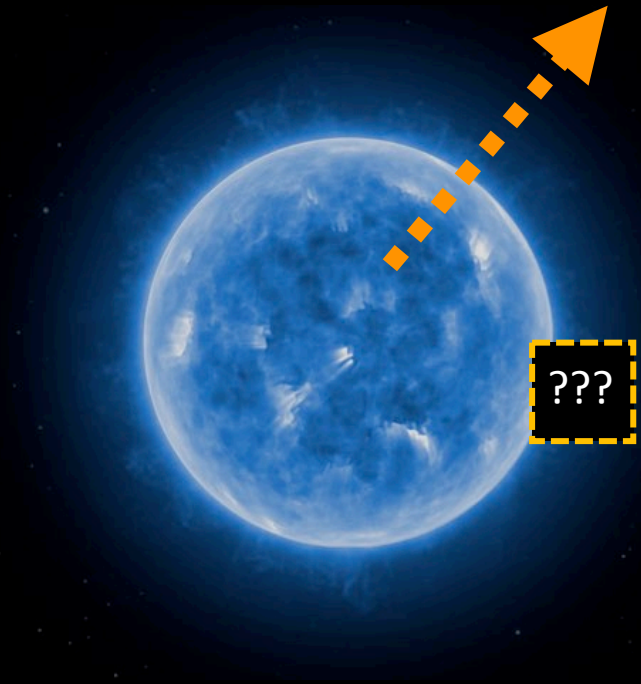


The star evolves differently again

What about new particles?

New particles...

- May be produced in the star and *free stream out*
- May be produced in the star and *get trapped*
- May collect in the star and annihilate in the core
- May modify other rates in the star



What about new particles?

New particles...

- May be produced in the star and *free stream out*
- May be produced in the star and *get trapped*
- May collect in the star and annihilate in the core
- May modify other rates in the star

Nuclear astrophysics: pair-instability is a sensitive probe of $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$

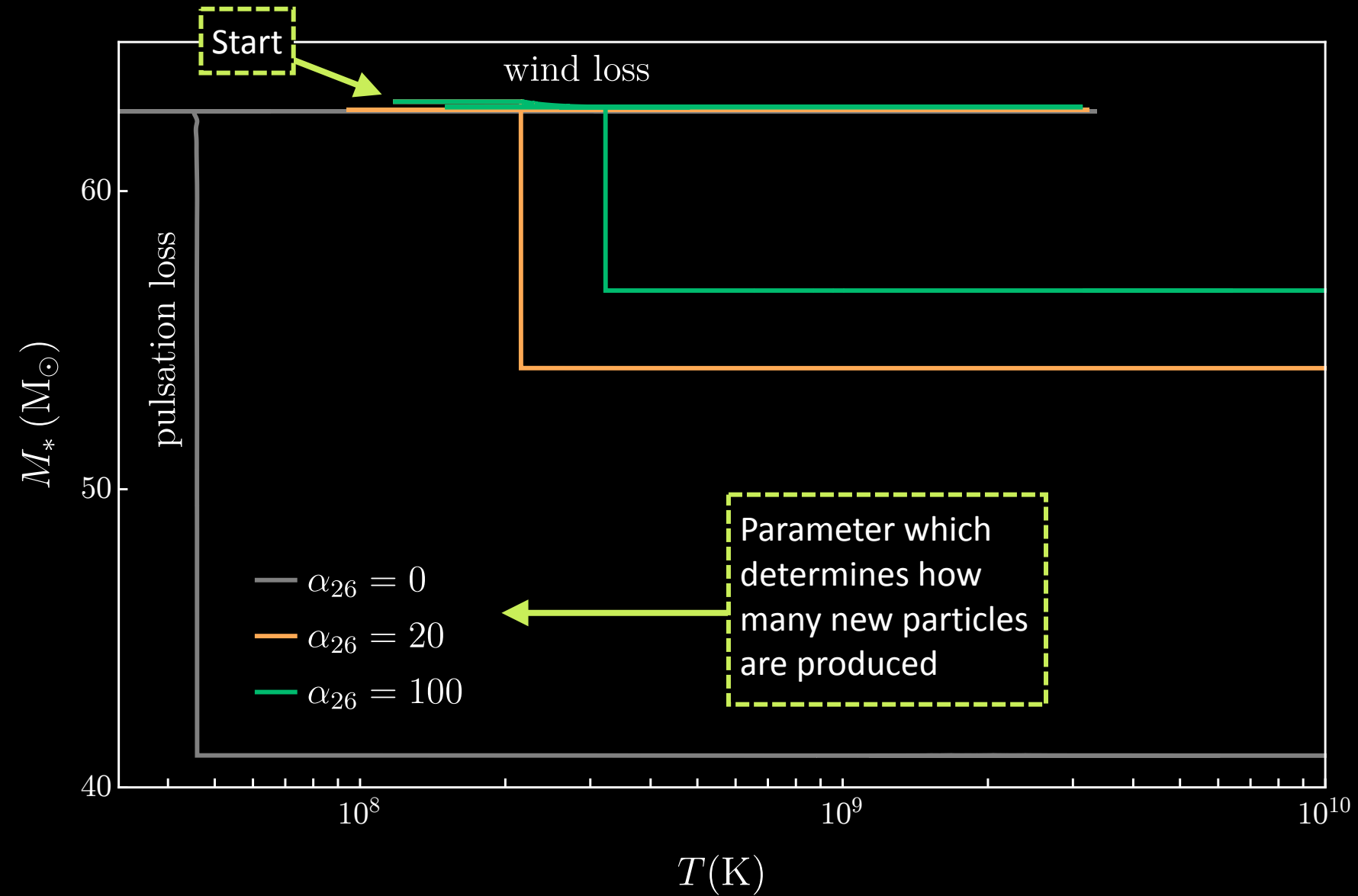
*Farmer, Renzo, de Mink,
Fishbach, Justham
arXiv:2006.06678*

Gravity: the BHMG is a test of G_N in stellar cores

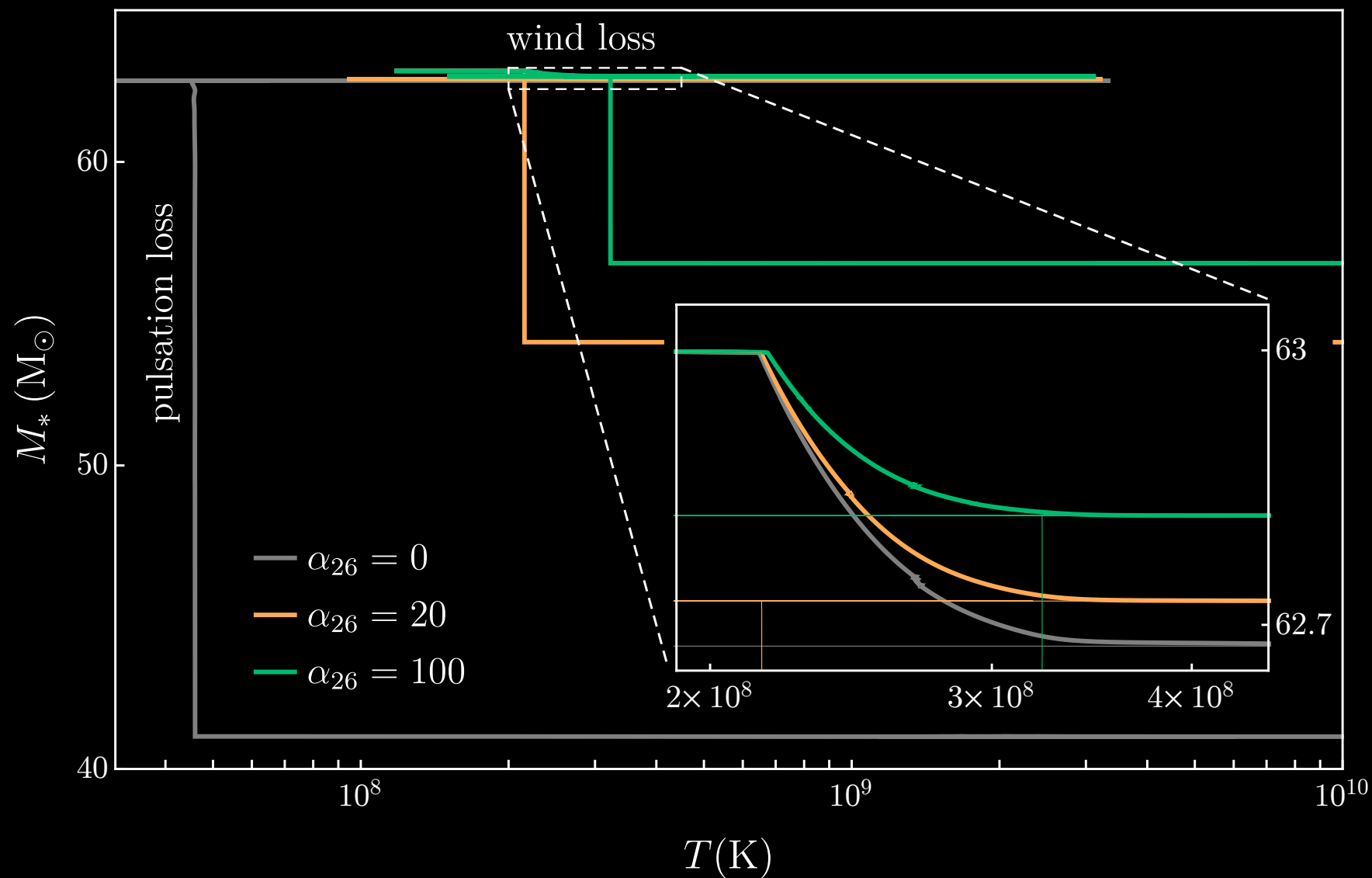
*Straight, Sakstein, Baxter,
arXiv: 2009.10716*

→ Testing the BHMG hypothesis with GW data

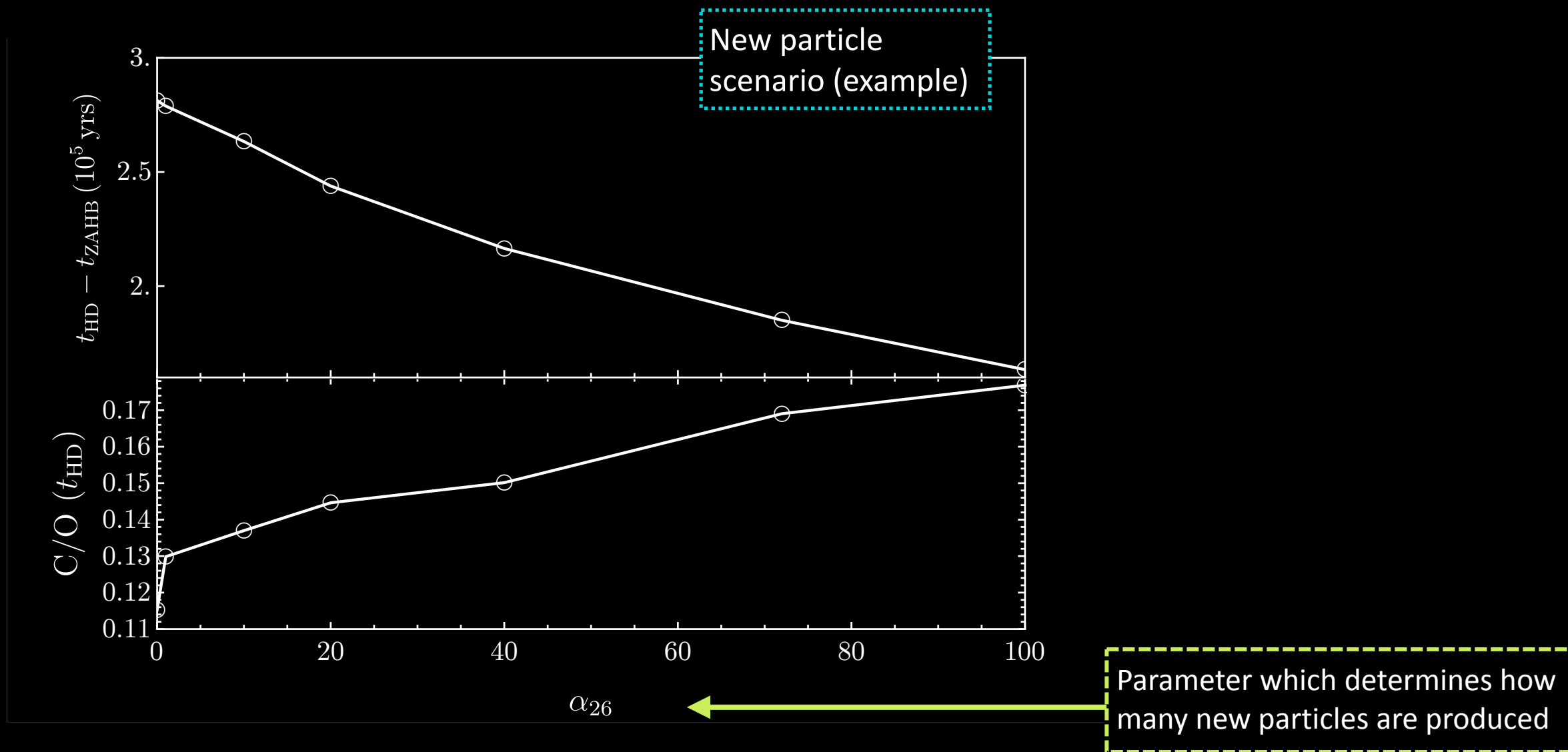
Implications of particles free streaming out



Implications of particles free streaming out



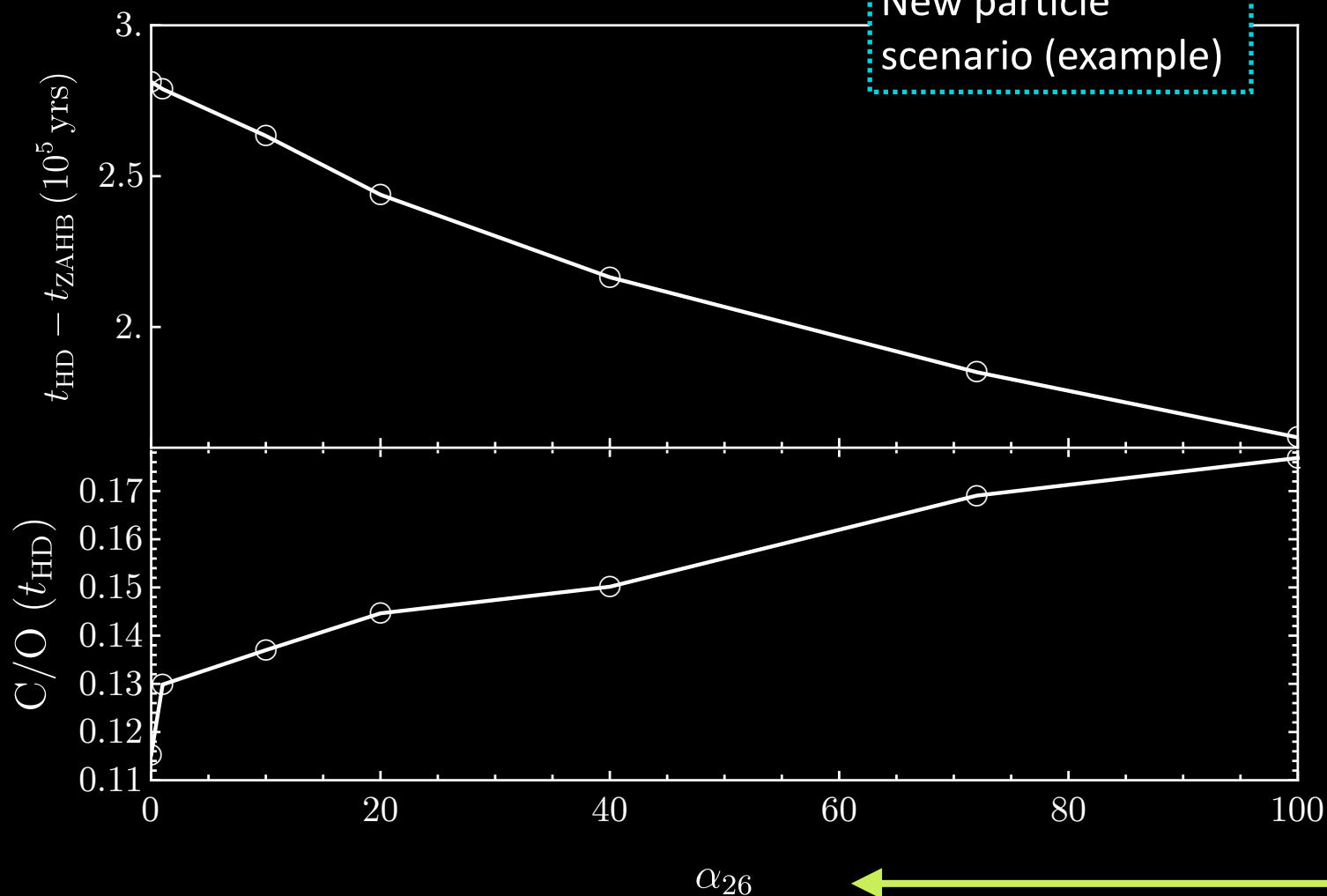
What does the extra energy loss do?



What does the extra energy loss do?

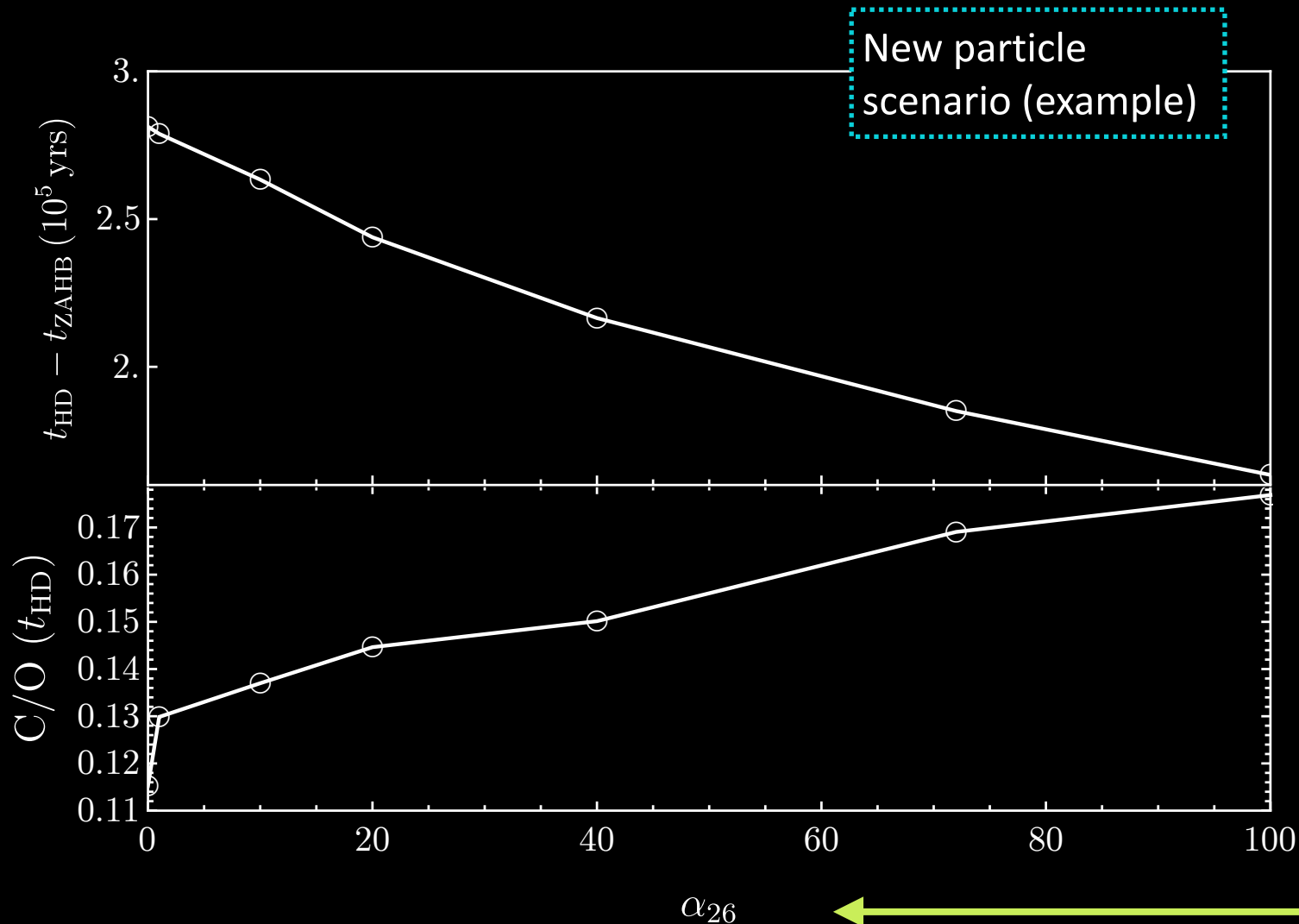
New particle scenario (example)

Greater energy losses lead to shorter *He-burning* phases



Parameter which determines how many new particles are produced

What does the extra energy loss do?



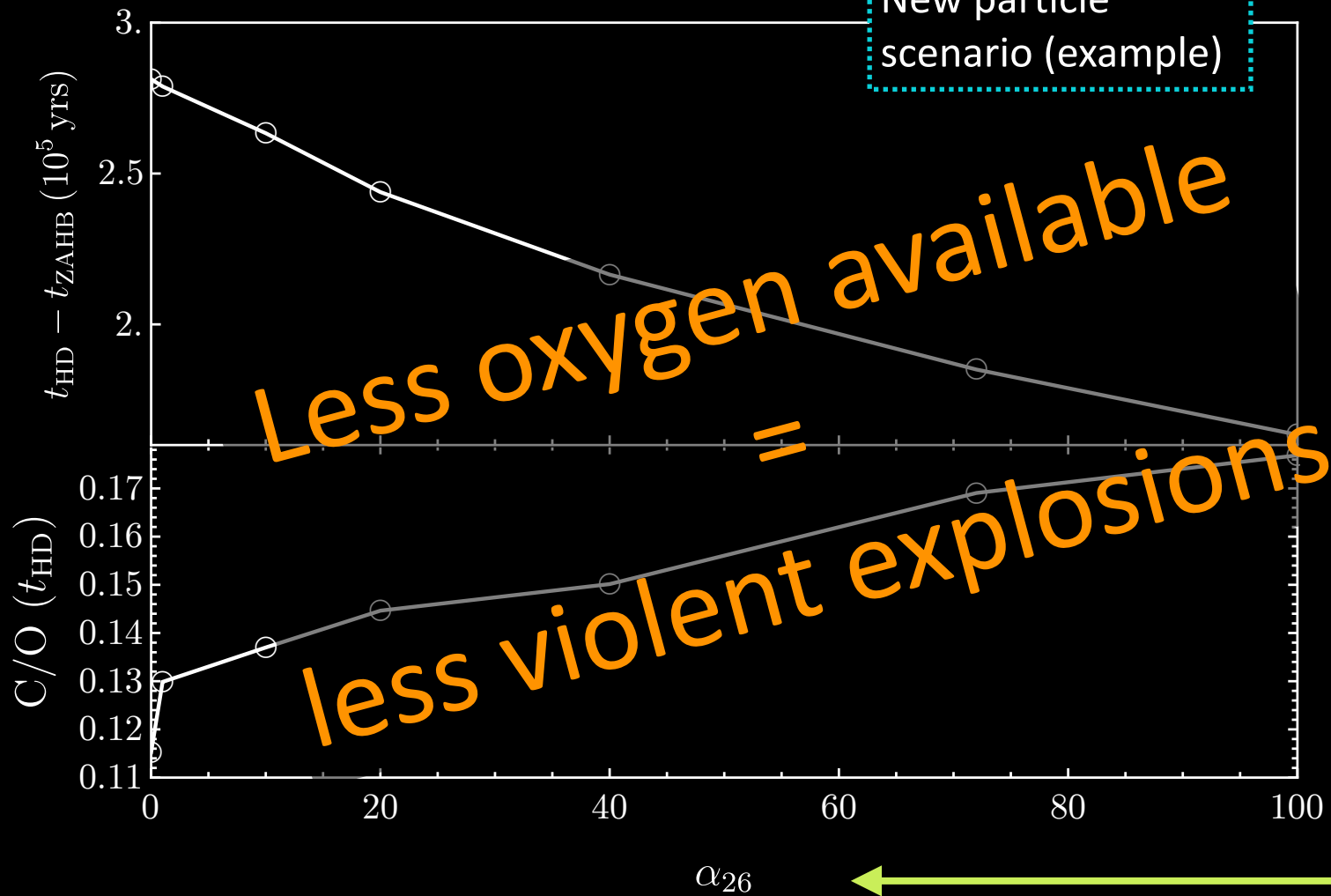
Greater energy losses lead to *shorter He-burning phases*

That means *more carbon and less oxygen* at the end!

Parameter which determines how many new particles are produced

What does the extra energy loss do?

New particle scenario (example)

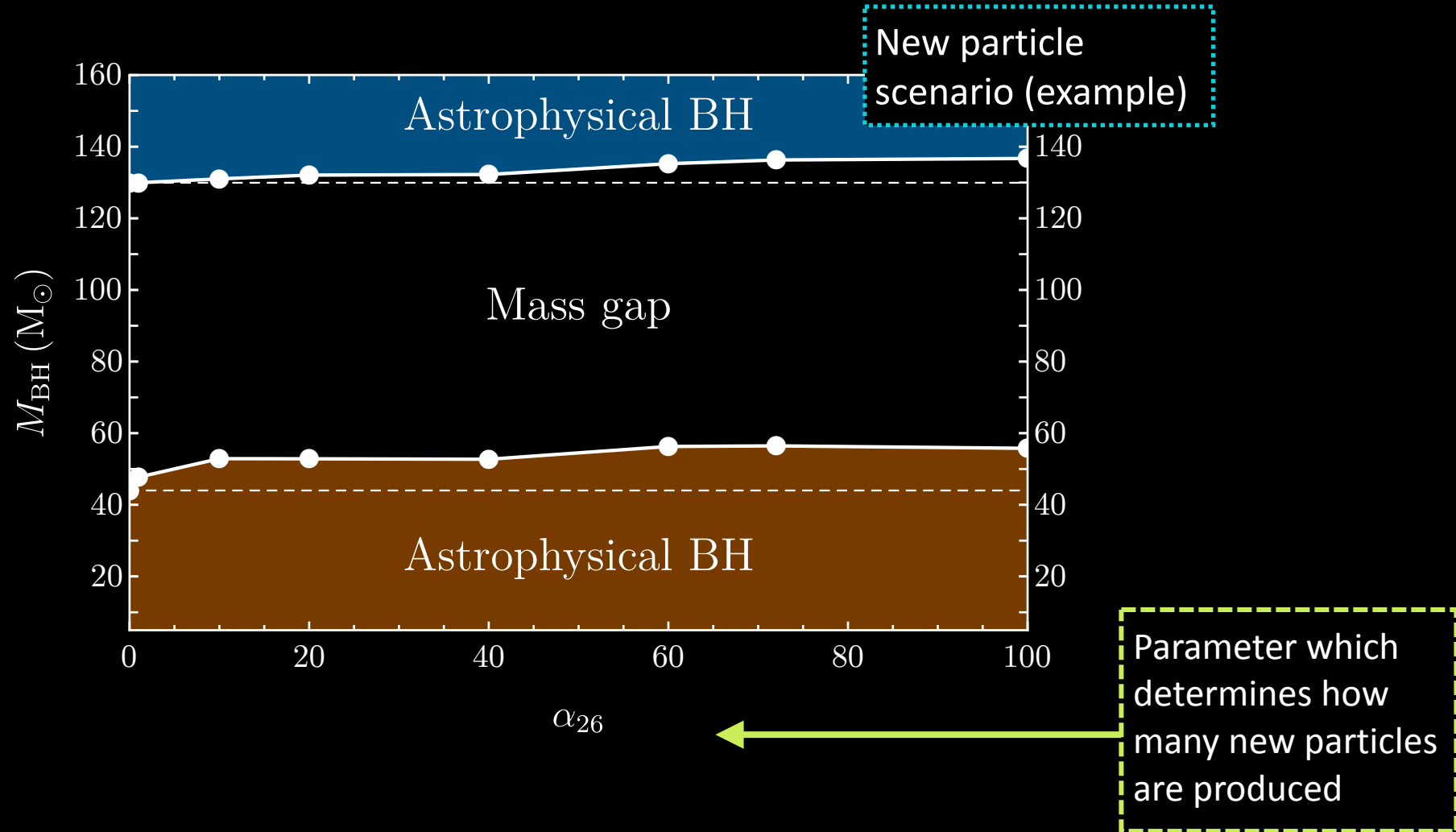


Greater energy losses lead to shorter He-burning phases

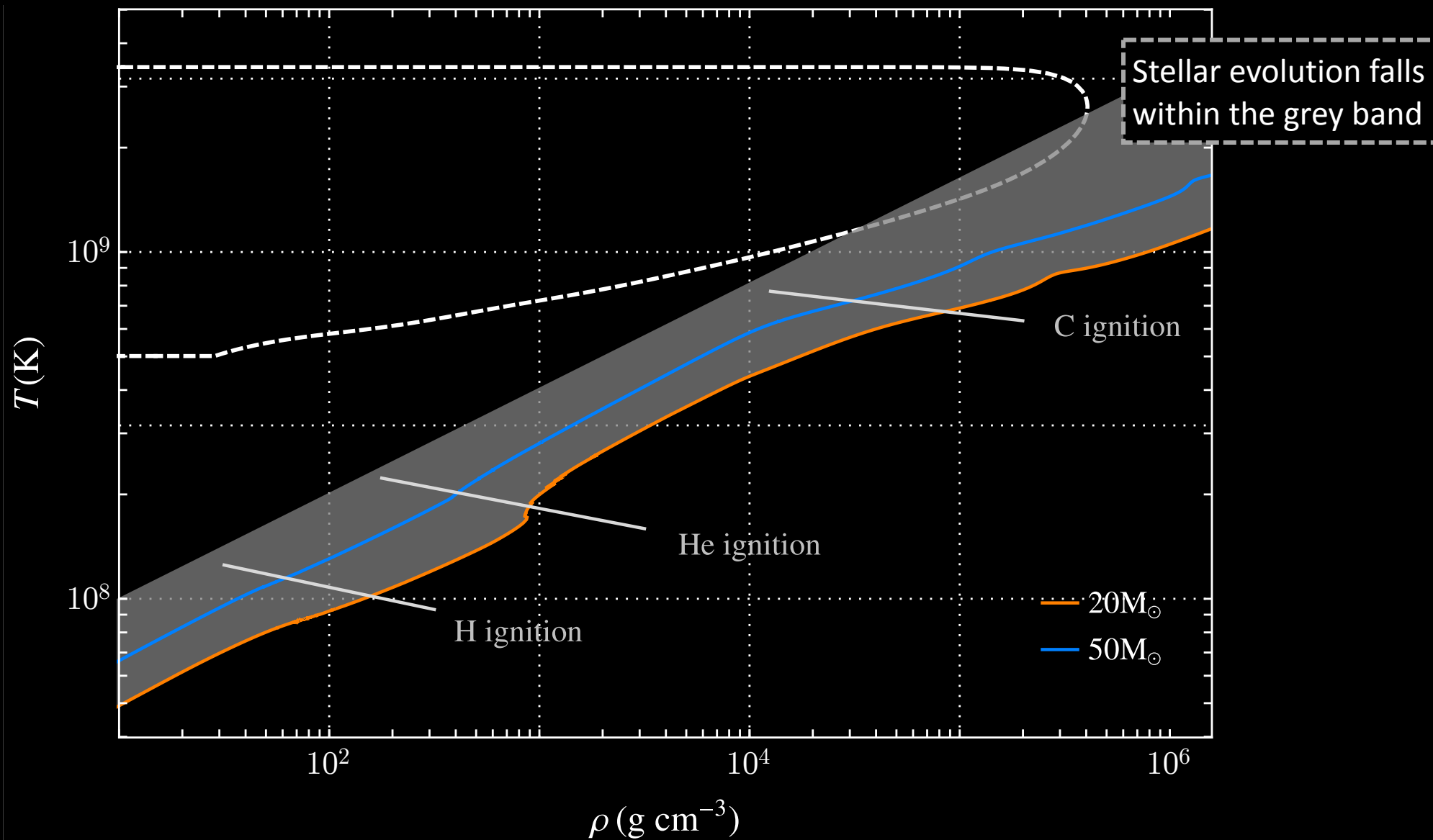
That means more carbon and less oxygen at the end!

Parameter which determines how many new particles are produced

Less violent explosions = heavier black holes



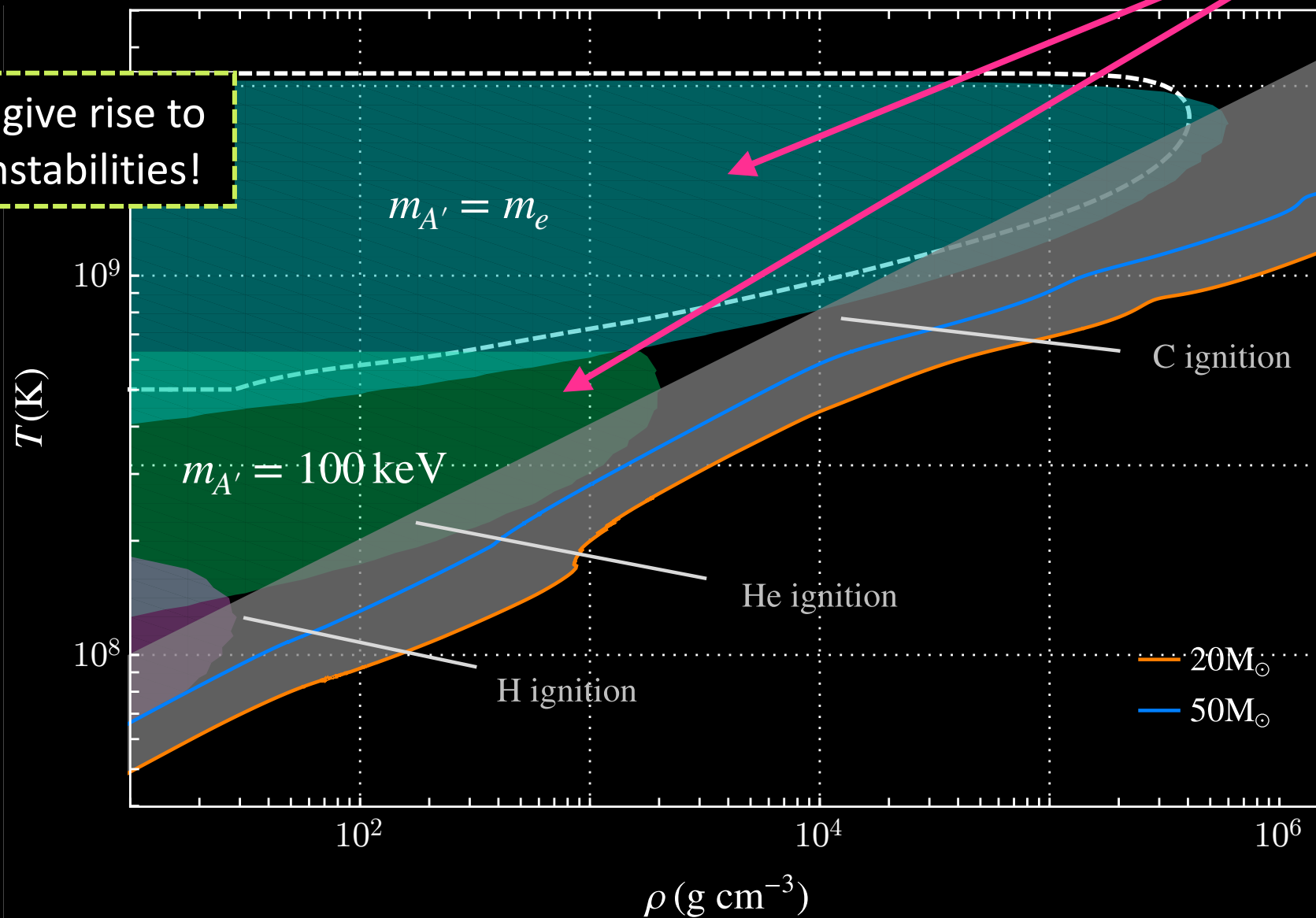
What about new trapped particles?



What about new trapped particles?

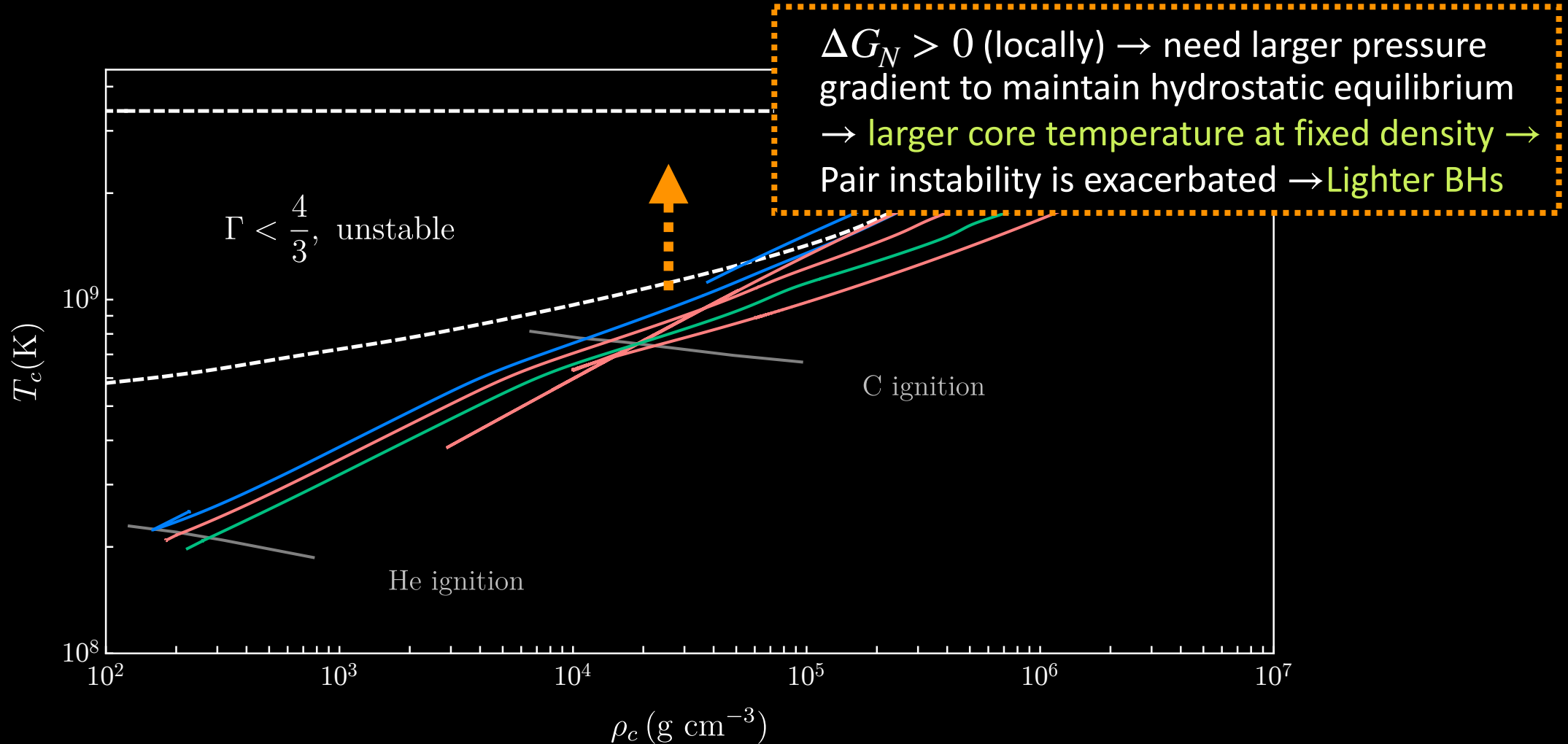
New particle A'

... can give rise to new instabilities!



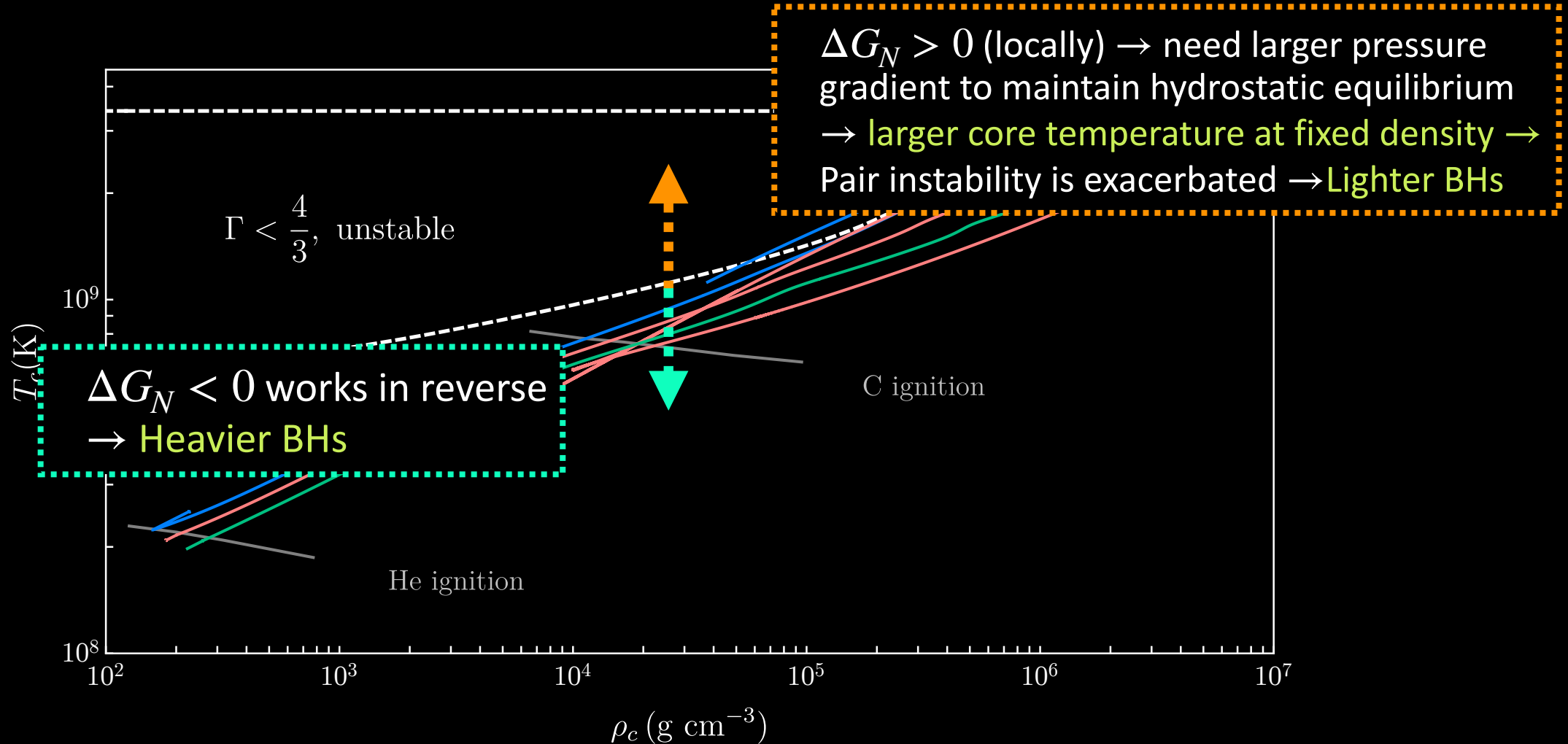
New forces

Screened modified gravity: new force as effective ΔG_N



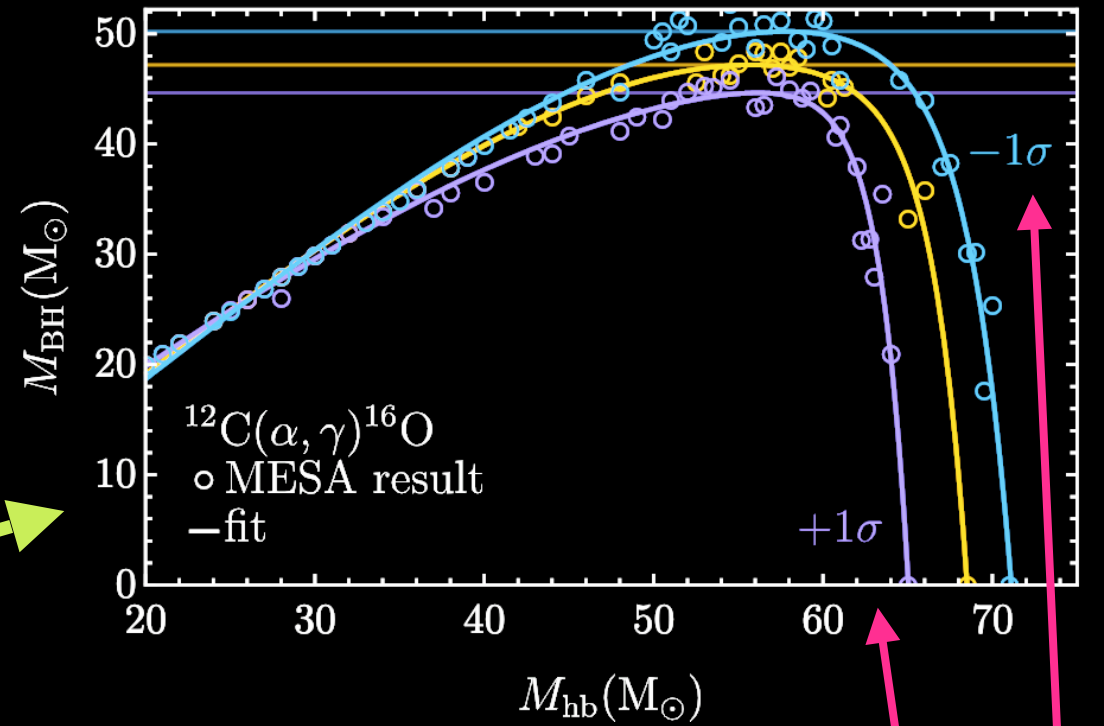
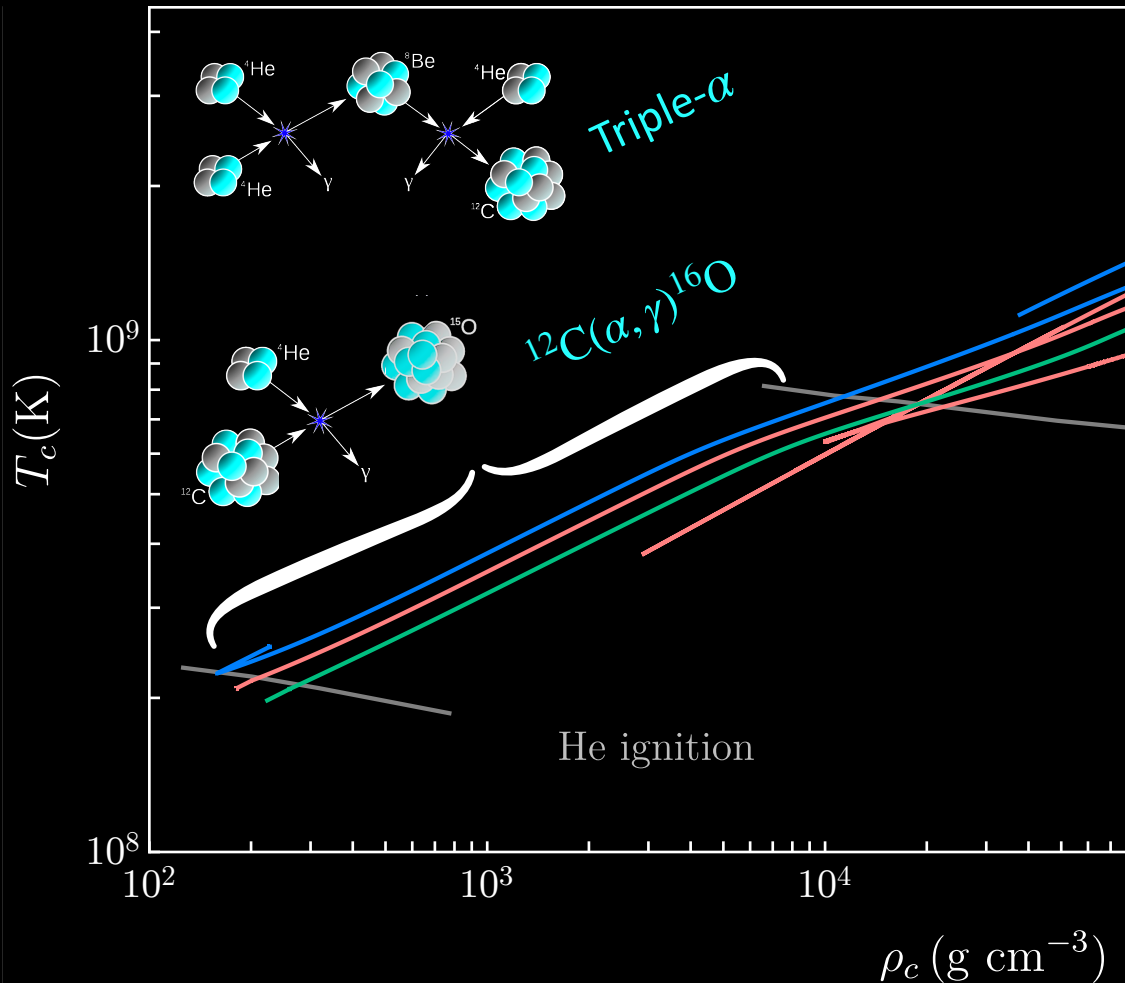
New forces

Screened modified gravity: new force as effective ΔG_N



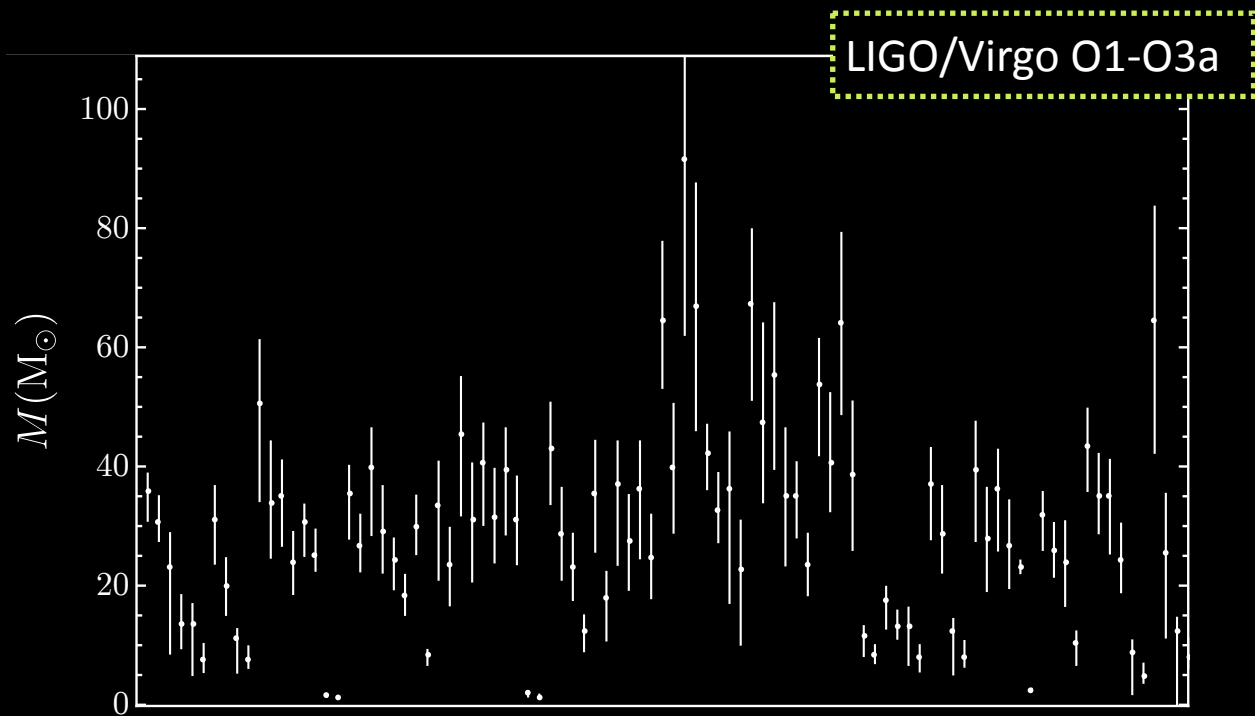
Nuclear physics

Particularly sensitive to $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$

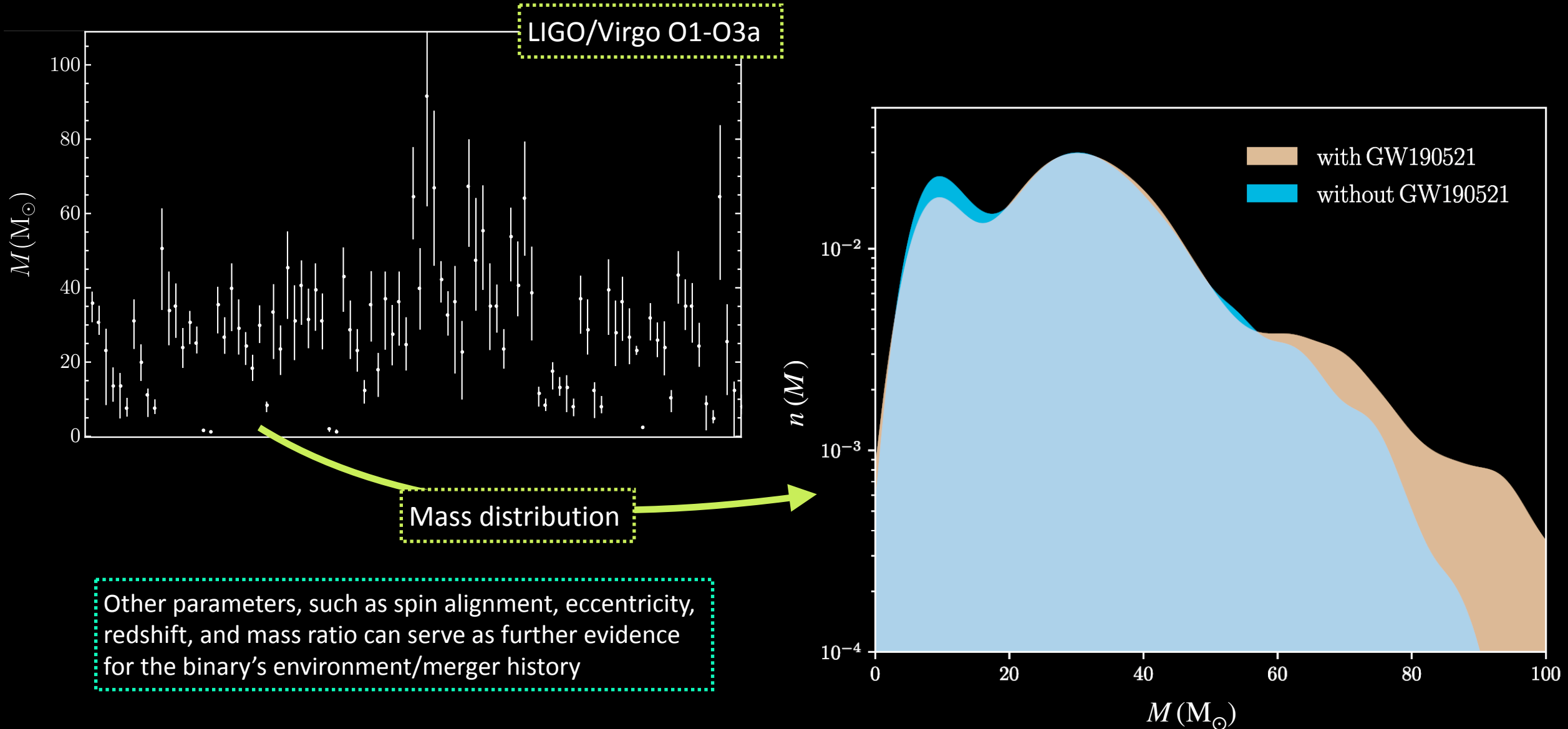


deBoer et al, RMP, arXiv:1709.03144 [hep-ex]

Black hole archeology: what about the data?

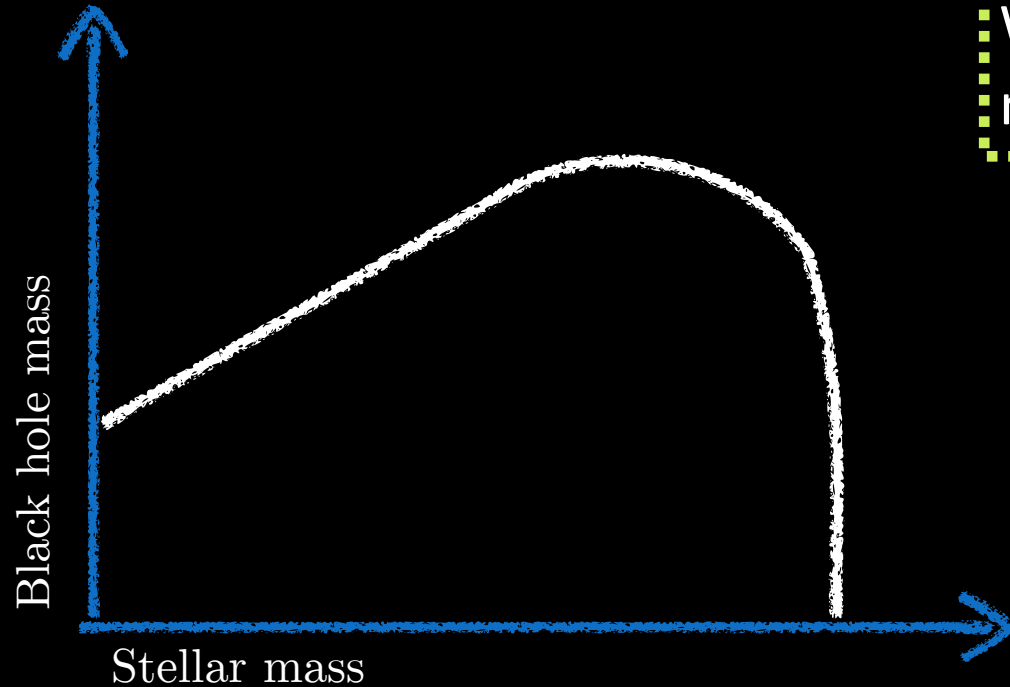


Black hole archeology: what about the data?

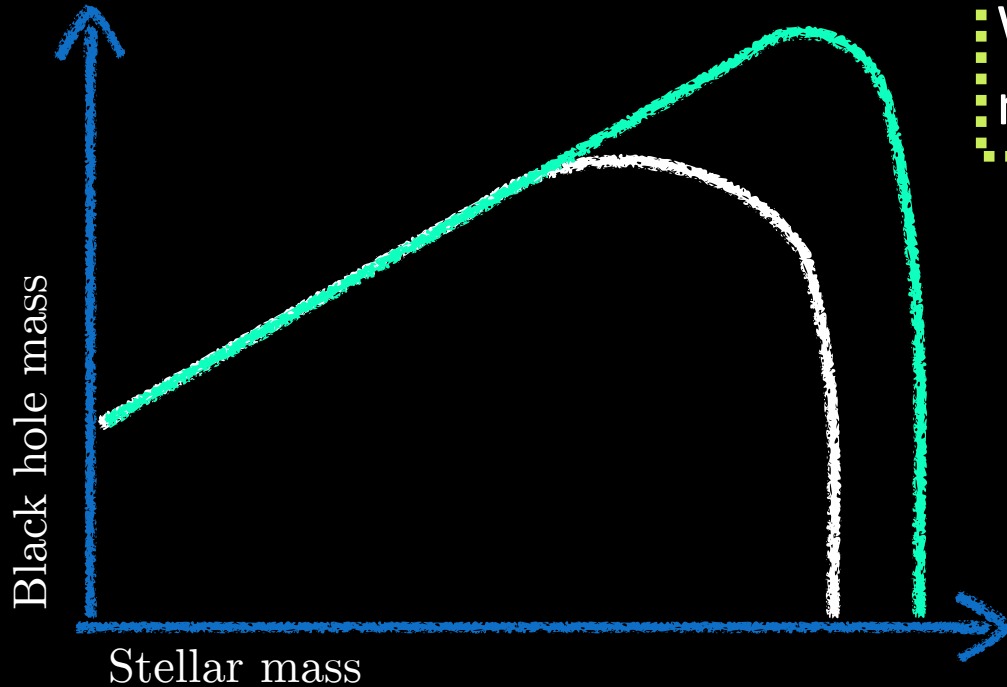


Pair-instability and black hole populations

We can predict black hole masses from stellar masses through stellar evolution simulations



Pair-instability and black hole populations



We can predict black hole masses from stellar masses through stellar evolution simulations

New particles or different nuclear physics may **change** this prediction

DC, McDermott, Sakstein arXiv:2007.00650 [hep-ph]

DC, McDermott, Sakstein, PRD (editor's suggestion), arXiv:2007.07889 [gr-qc]

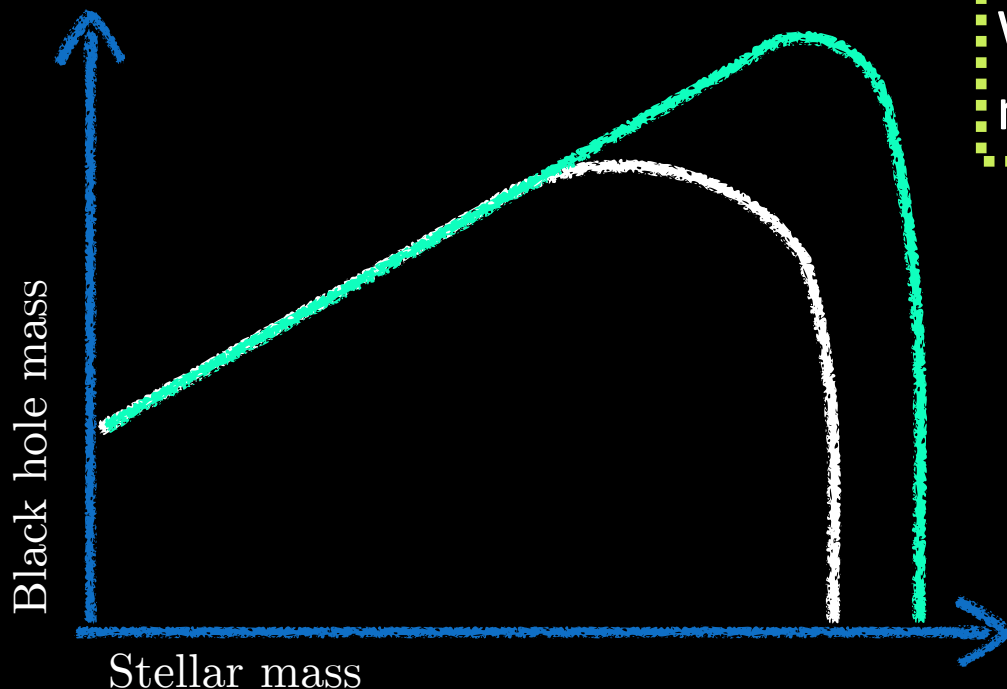
Straight, Sakstein, Baxter, PRD, arXiv:2009.10716 [gr-qc]

Sakstein, DC, McDermott, Straight, Baxter, PRL, arXiv:2009.01213 [gr-qc]

Ziegler, Freese arXiv:2010.00254 [astro-ph]

...More work in progress

Pair-instability and black hole populations



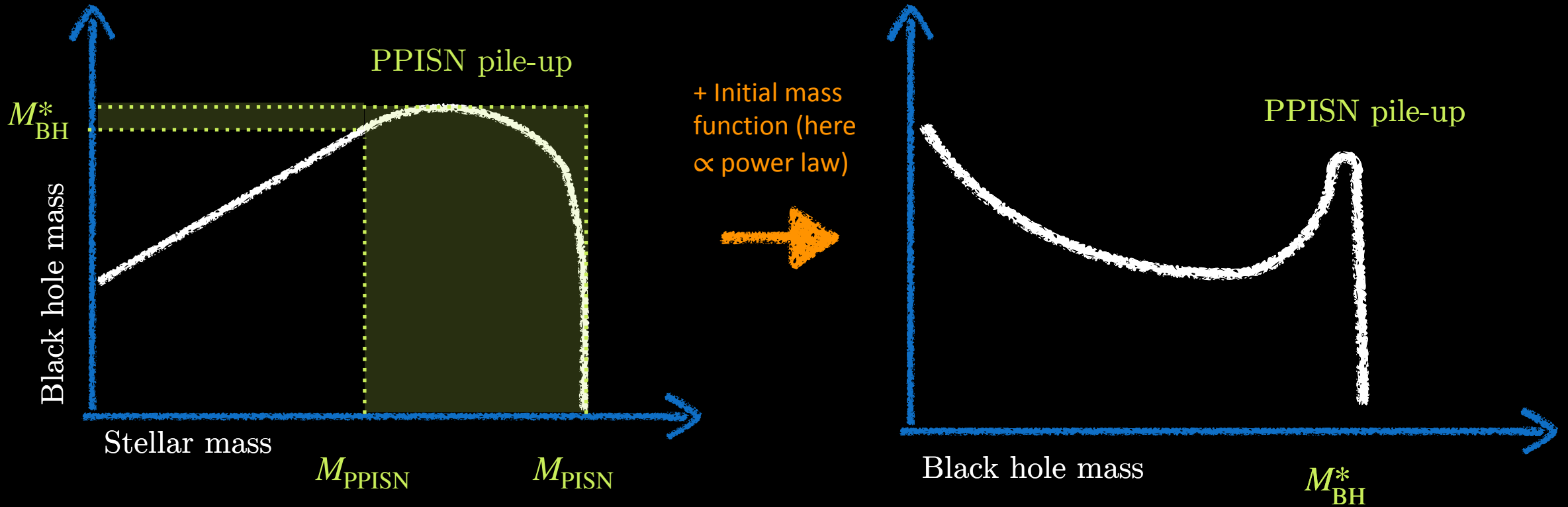
We can predict black hole masses from stellar masses through stellar evolution simulations

New particles or different nuclear physics may **change** this prediction

How can we use *data* to test that?

FIND THE GAP

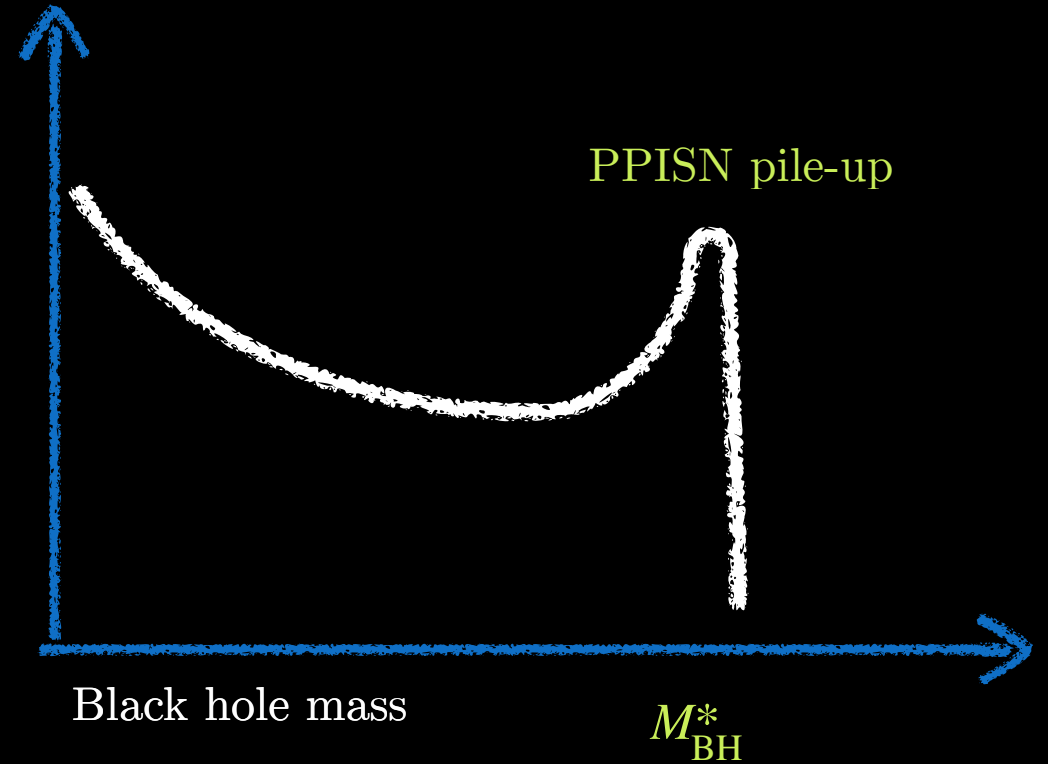
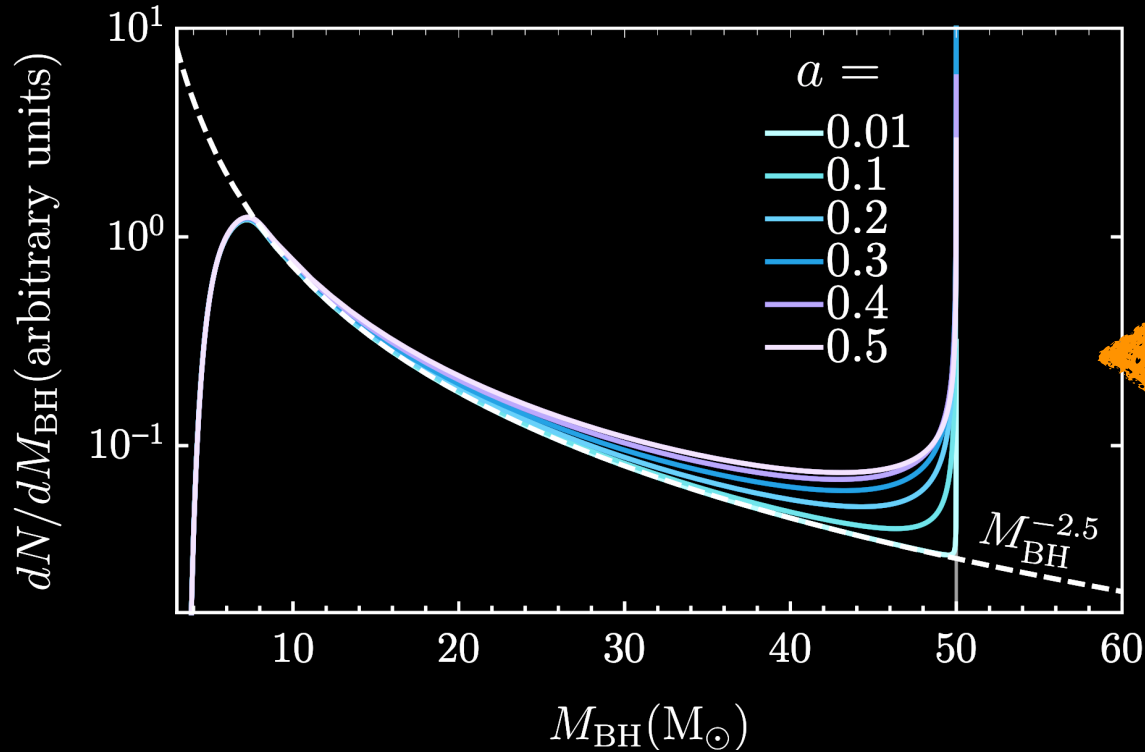
Pair-instability and black hole populations



+ Initial mass function (here \propto power law)

+ Initial mass function (here assumed to be a power law)

Pair-instability and black hole populations



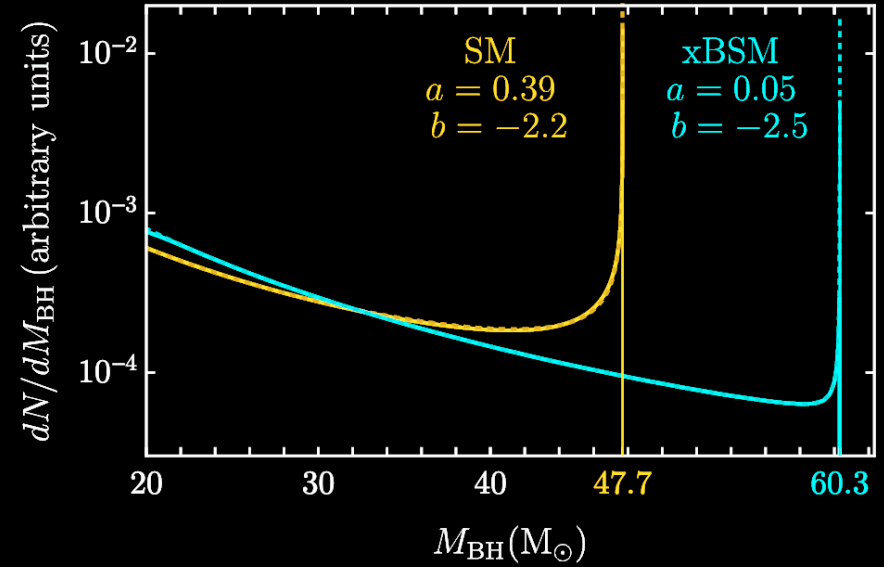
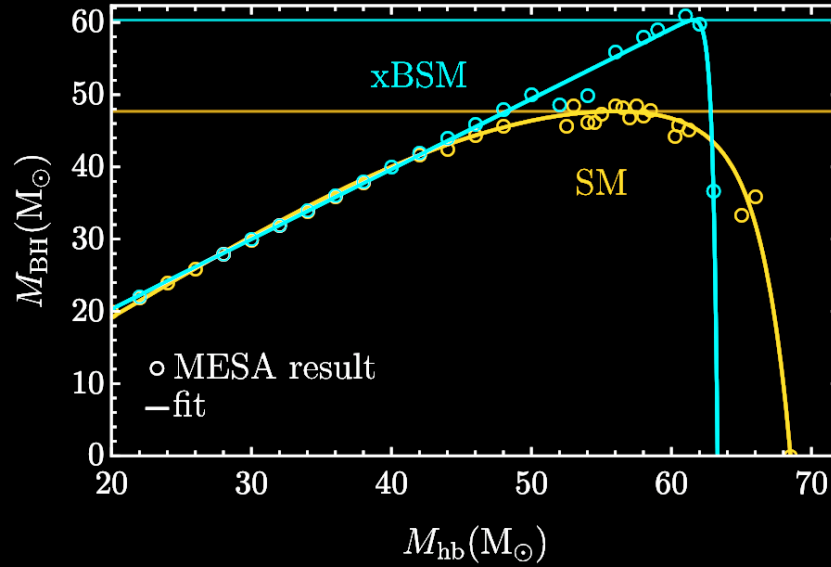
$$\frac{dN_{\text{BH}}^{(1g)}}{dM_{\text{BH}}} \propto M_{\text{BH}}^b \left[1 + \frac{2a^2 M_{\text{BH}}^{1/2} (M_{\text{BHMG}} - M_{\text{BH}})^{a-1}}{M_{\text{BHMG}}^{a-1/2}} \right]$$

Baxter, DC, McDermott, Sakstein, arXiv:2104.02685

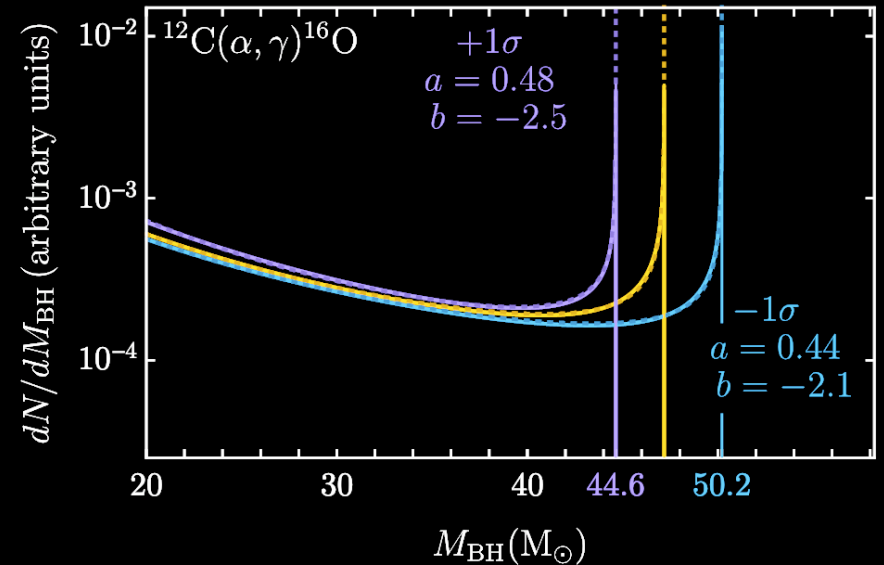
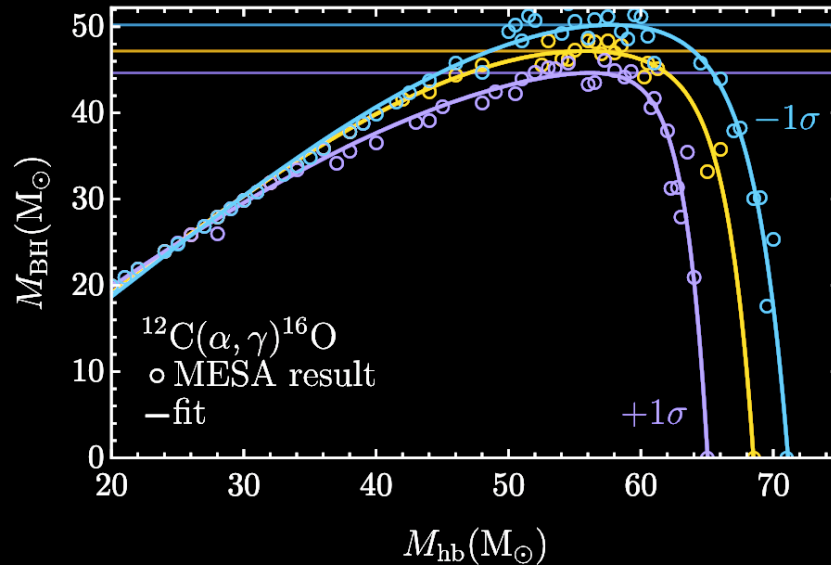
Pair-instability and black hole populations

FIND THE GAP

Testing BSM
particle physics



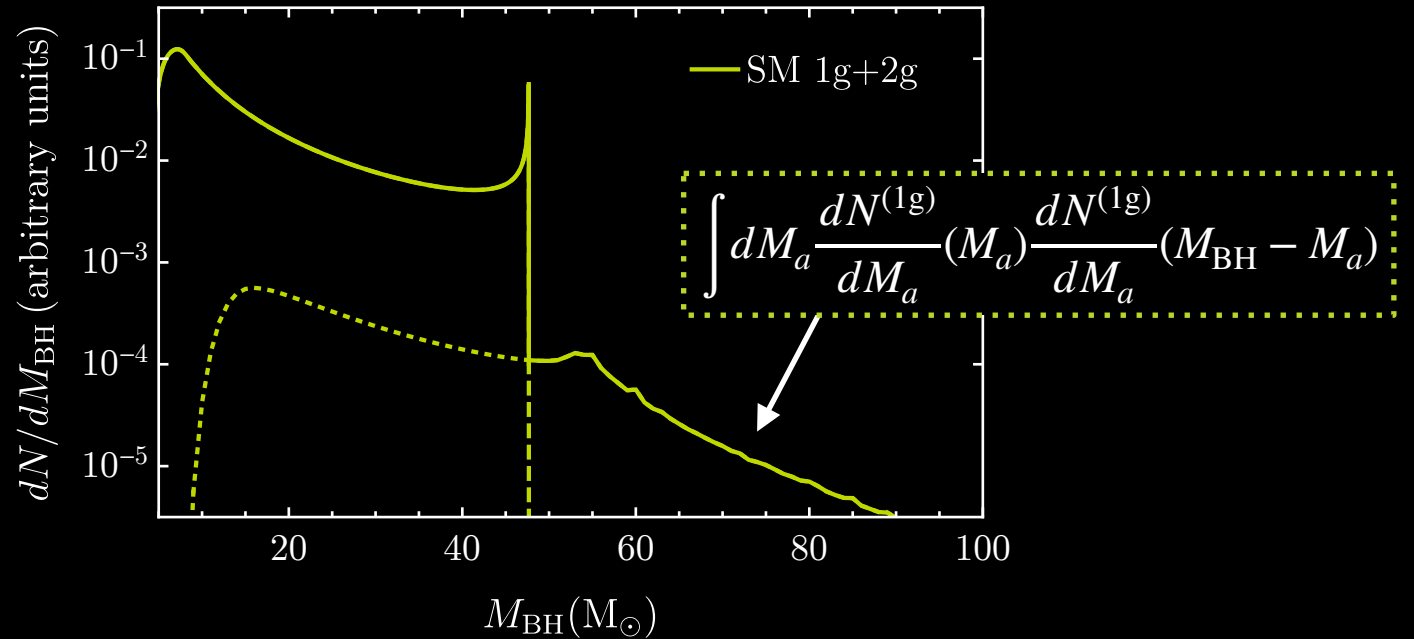
Testing nuclear
(astro) physics



Dynamical mergers and black hole genealogy

Black holes formed in **prior mergers** may in principle populate the mass gap.

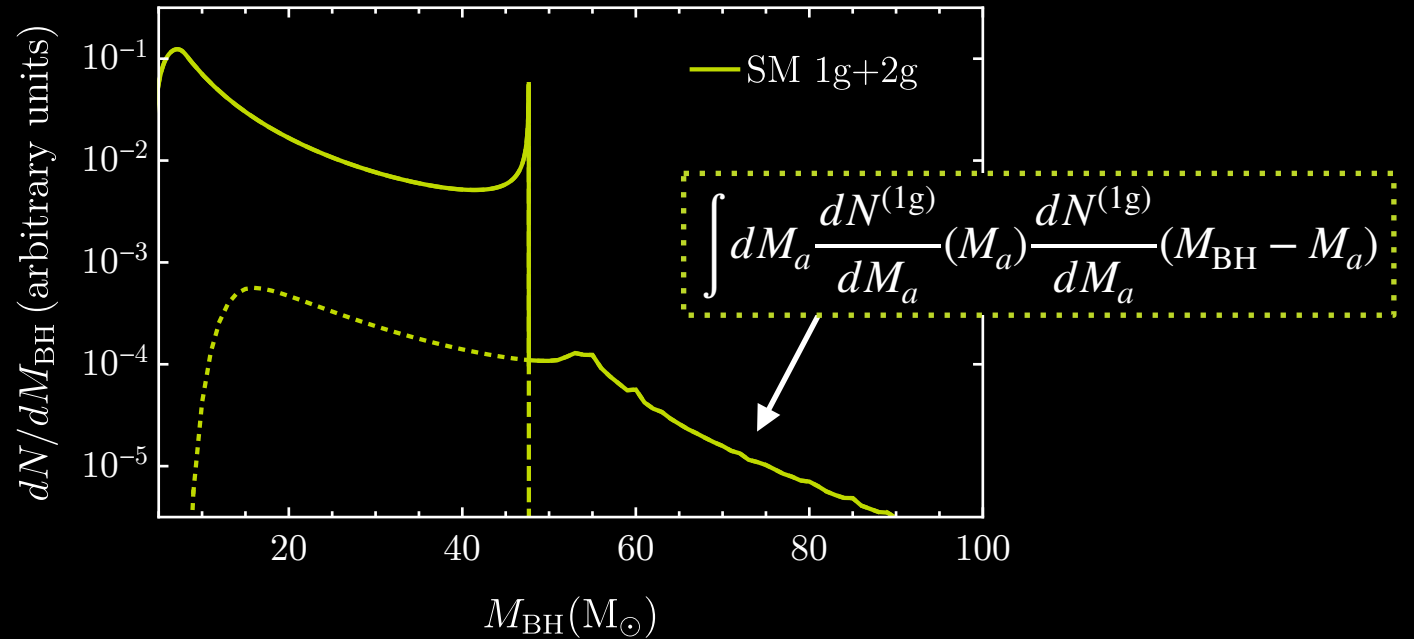
Their mass distribution inherits from the **1g** mass distribution.



Dynamical mergers and black hole genealogy

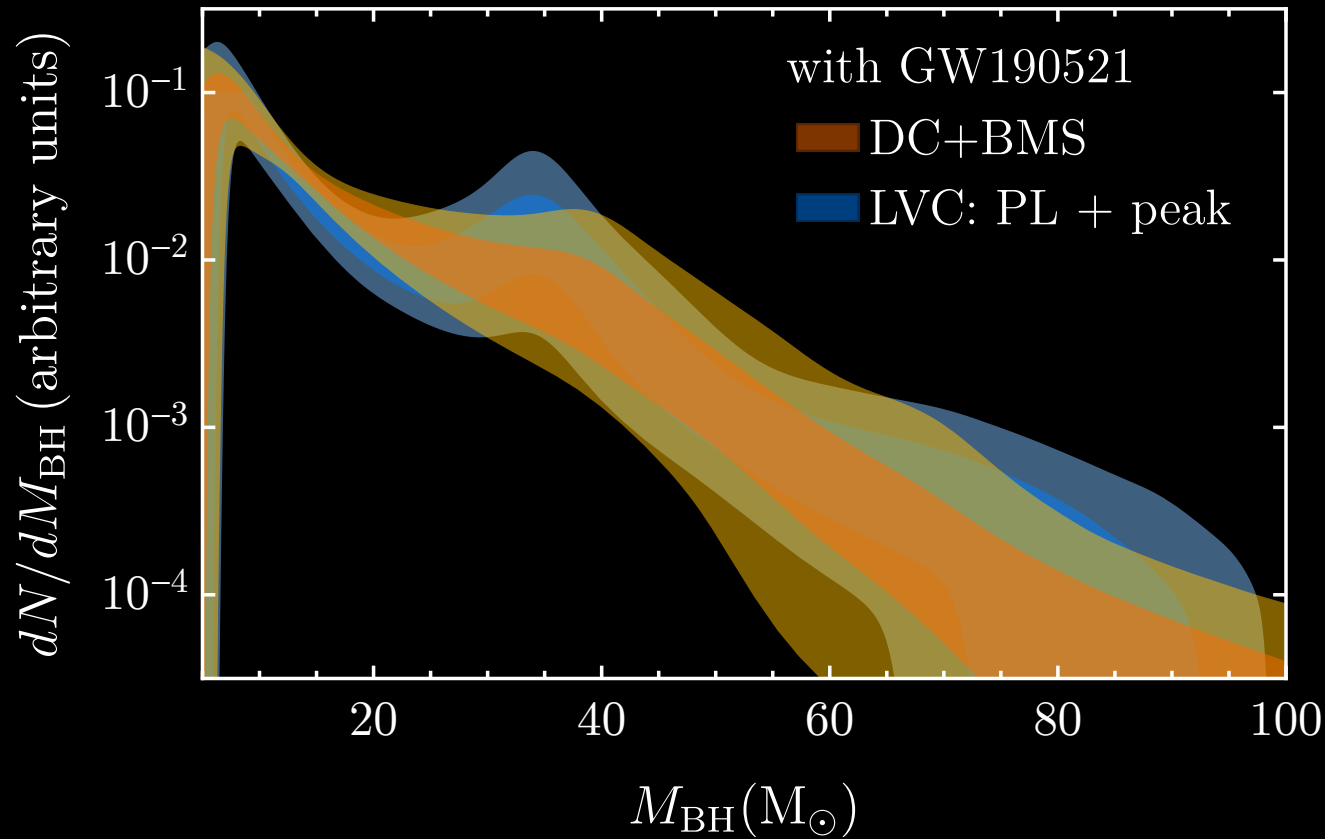
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Their mass distribution inherits from the 1g mass distribution.



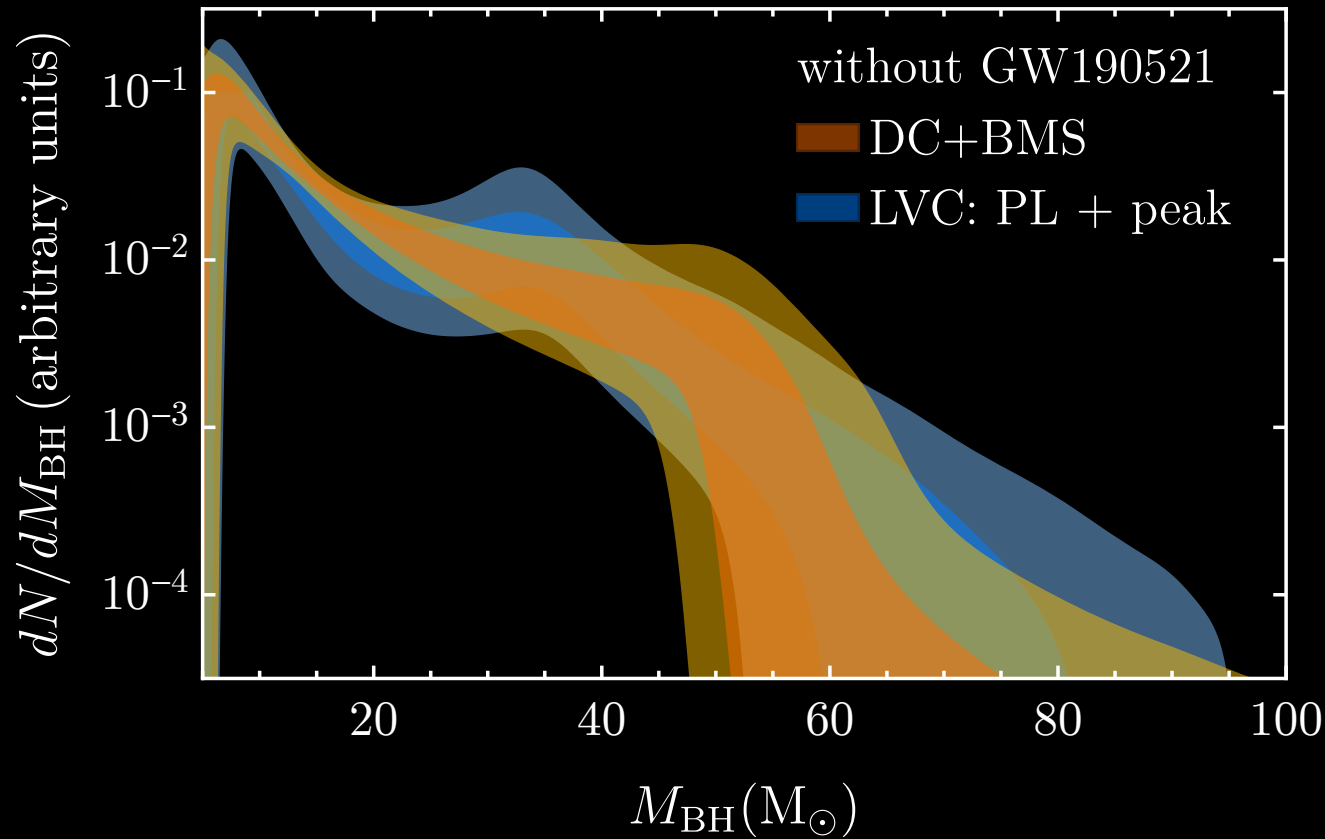
$$\frac{dN}{dM_{\text{BH}}} = \frac{dN_{\text{BH}}^{(1g)}}{dM_{\text{BH}}} + \frac{dN_{\text{BH}}^{(2+g)}}{dM_{\text{BH}}} \left\{ \begin{array}{l} \frac{dN_{\text{BH}}^{(1g)}}{dM_{\text{BH}}} \propto M_{\text{BH}}^b \left[1 + \frac{2a^2 M_{\text{BH}}^{1/2} (M_{\text{BHMG}} - M_{\text{BH}})^{a-1}}{M_{\text{BHMG}}^{a-1/2}} \right] : \text{first generation black holes } (a, b, M_{\text{BHMG}}) \\ \frac{dN_{\text{BH}}^{(2+g)}}{dM_{\text{BH}}} \propto \lambda \min \left[1, \left(\frac{M_{\text{BH}}}{M_{\text{BHMG}} + M_{\text{min}} + \delta_m/2} \right)^d \right] : \text{“Pollutant” population (2g+) } (d, \lambda) \end{array} \right.$$

Binary mergers in LIGO/Virgo O3a



This work, Eq. (7)	with GW190521
$\log_{10} \lambda$	$-0.88^{+0.41}_{-1.46}$
$M_{\text{BHMG}} [M_{\odot}]$	$46.23^{+16.83}_{-6.15}$
a	$0.23^{+0.17}_{-0.16}$
b	$-1.95^{+0.51}_{-0.54}$
d	$-5.95^{+1.75}_{-2.07}$
$M_{\text{min}} [M_{\odot}]$	$3.38^{+1.50}_{-1.56}$
$\delta_m [M_{\odot}]$	$5.12^{+2.97}_{-3.19}$
LVC: PL+peak	with GW190521
α	$2.72^{+0.38}_{-0.48}$
$M_{\text{max}} [M_{\odot}]$	85^{+10}_{-8}
λ_{peak}	$0.113^{+0.032}_{-0.094}$
$\mu_m [M_{\odot}]$	$34.0^{+2.2}_{-1.7}$
$\sigma_m [M_{\odot}]$	$4.7^{+1.8}_{-3.5}$
$M_{\text{min}} [M_{\odot}]$	$4.40^{+1.3}_{-0.89}$
$\delta_m [M_{\odot}]$	< 4.75

Binary mergers in LIGO/Virgo O3a

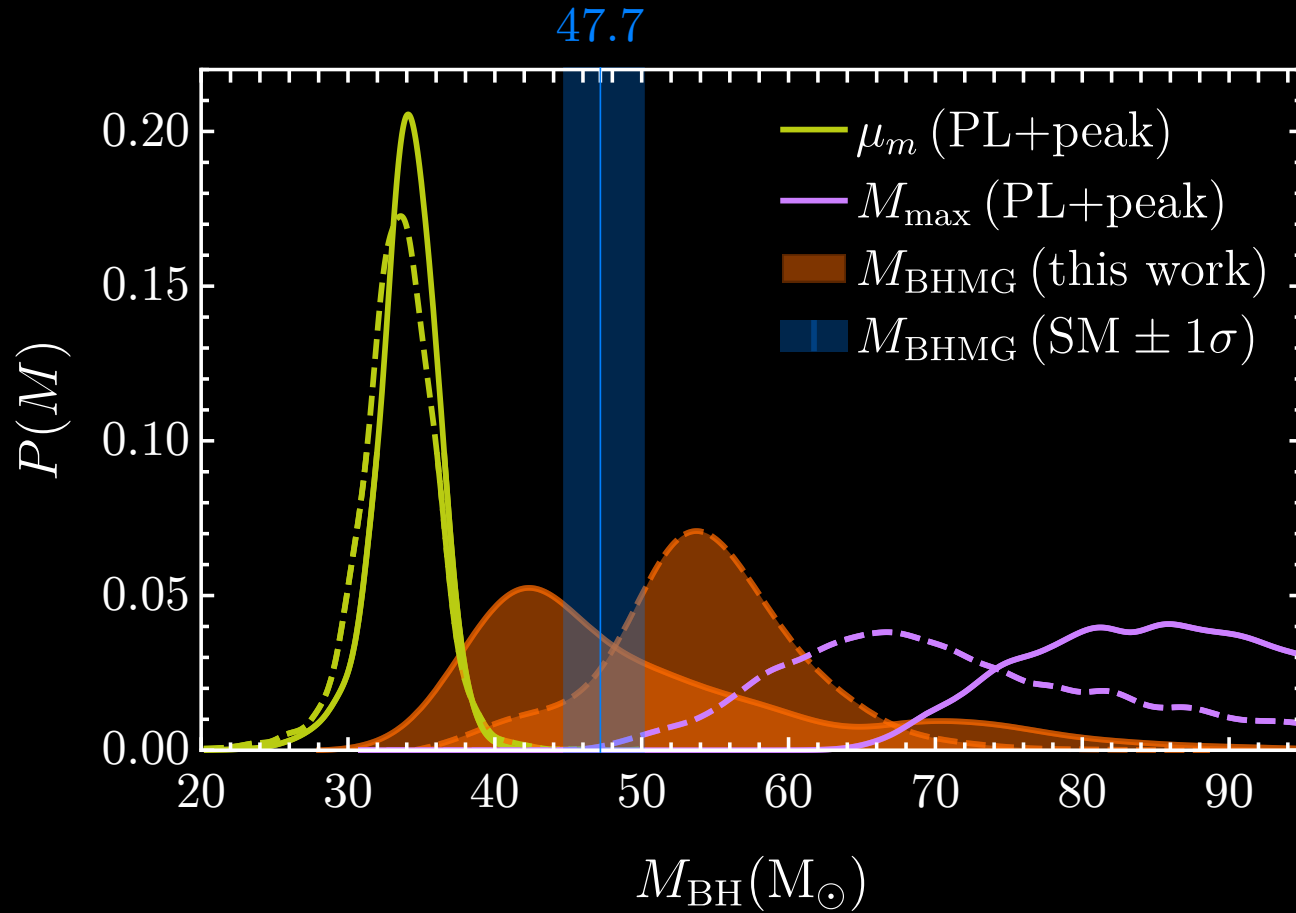


This work, Eq. (7)	no GW190521
$\log_{10} \lambda$	$-3.92^{+2.40}_{-2.02}$
$M_{\text{BHMG}} [\text{M}_{\odot}]$	$54.11^{+5.85}_{-4.96}$
a	$0.23^{+0.18}_{-0.16}$
b	$-1.98^{+0.45}_{-0.42}$
d	$-5.79^{+3.54}_{-2.81}$
$M_{\text{min}} [\text{M}_{\odot}]$	$3.33^{+1.47}_{-1.66}$
$\delta_m [\text{M}_{\odot}]$	$5.15^{+2.97}_{-3.15}$
LVC: PL+peak	no GW190521
α	$3.08^{+0.51}_{-1.2}$
$M_{\text{max}} [\text{M}_{\odot}]$	72^{+9}_{-10}
λ_{peak}	$0.107^{+0.029}_{-0.092}$
$\mu_m [\text{M}_{\odot}]$	$33.4^{+2.5}_{-2.1}$
$\sigma_m [\text{M}_{\odot}]$	> 4.49
$M_{\text{min}} [\text{M}_{\odot}]$	$4.56^{+1.3}_{-0.77}$
$\delta_m [\text{M}_{\odot}]$	< 4.04

GW190521 and the mass gap

The New York Times

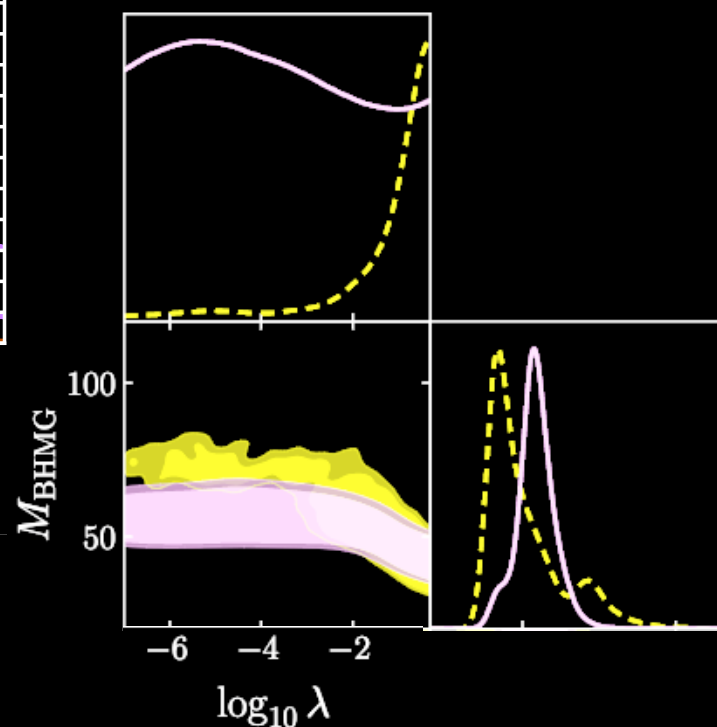
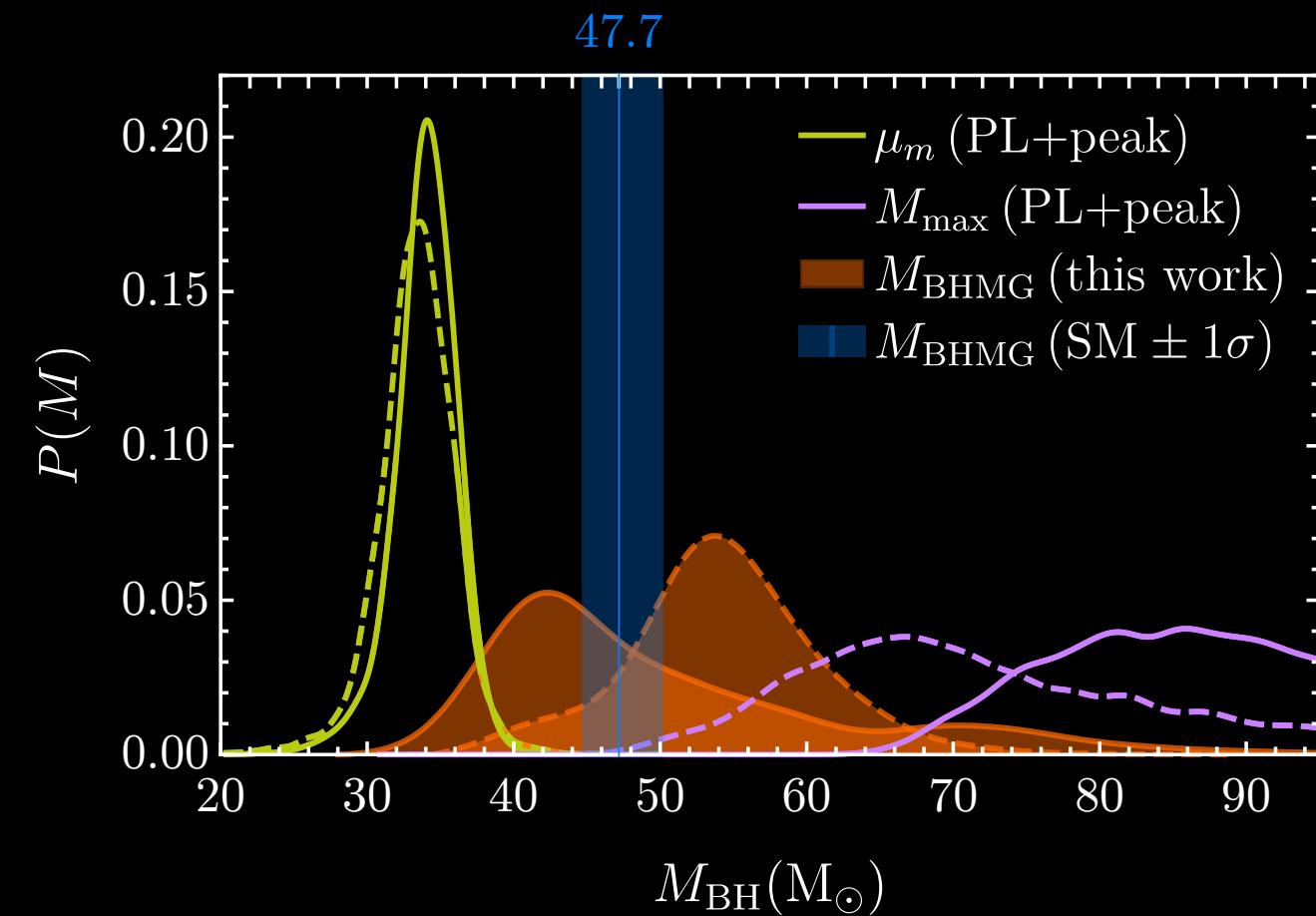
These Black Holes Shouldn't Exist,
but There They Are



GW190521 and the mass gap

The New York Times

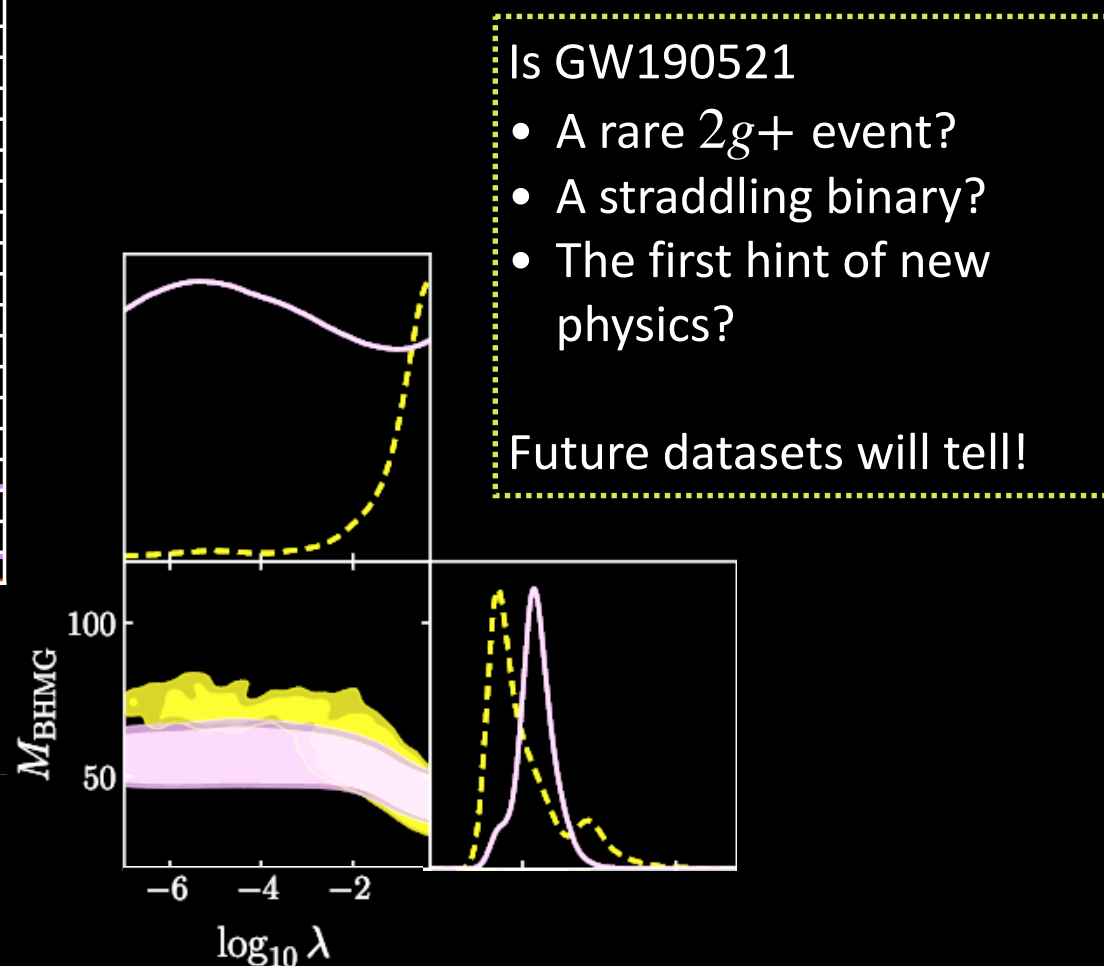
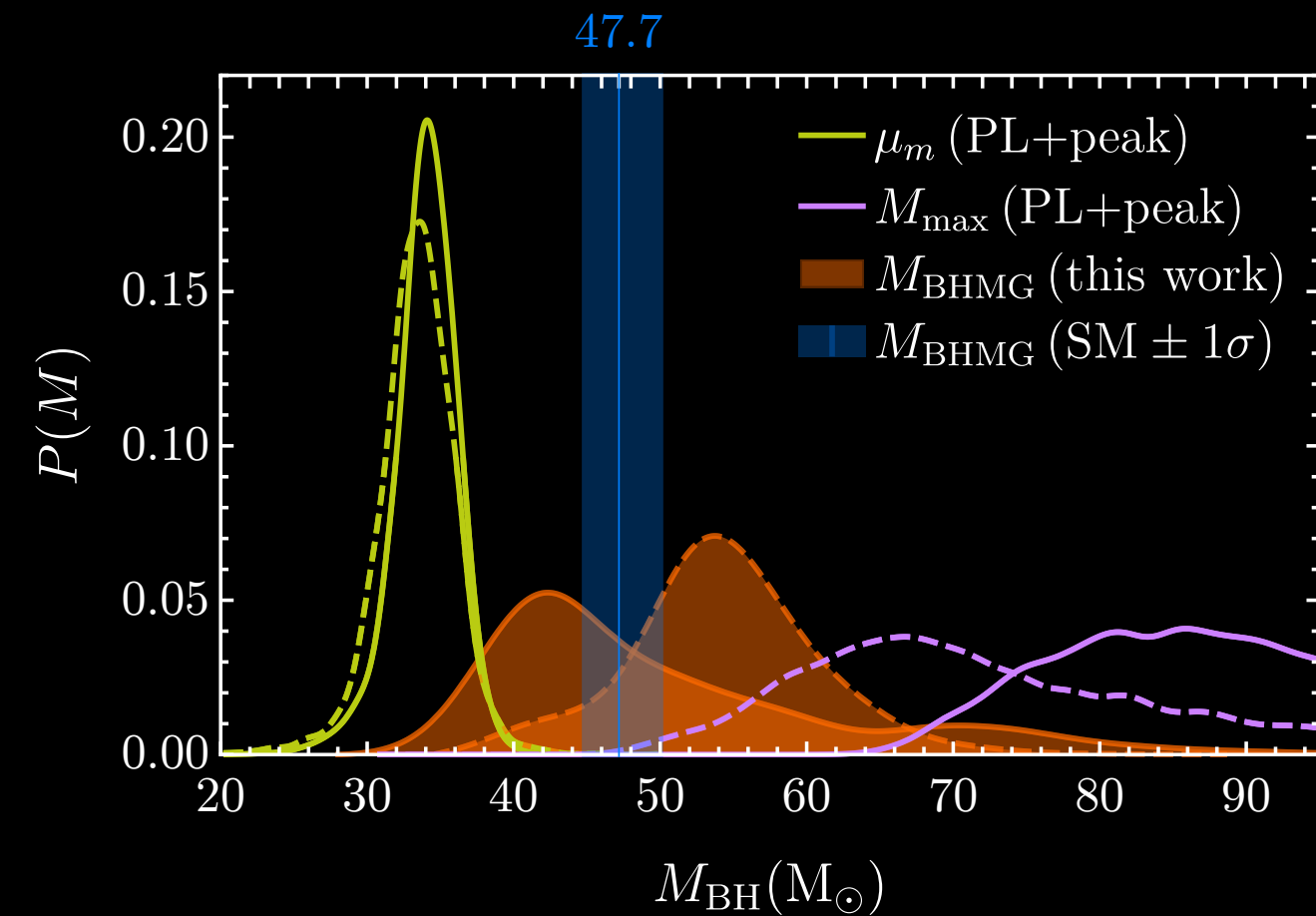
These Black Holes Shouldn't Exist,
but There They Are



GW190521 and the mass gap

The New York Times

These Black Holes Shouldn't Exist,
but There They Are



To conclude,

- Gravitational waves offer an **exciting new opportunity** to study open questions in stellar astrophysics and particle physics
- **Pair-instability supernovae** lead to unpopulated space in the stellar graveyard → the **black hole mass gap** is an entirely new probe of particle & nuclear physics
- Black hole population studies will allow us to study stellar evolution → **black hole archeology**

Thank you!

...ask me anything you like!

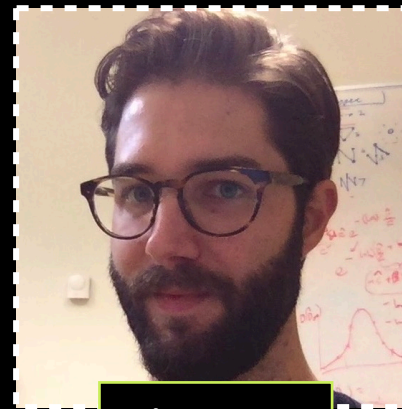
dcroon@triumf.ca | djunacroon.com



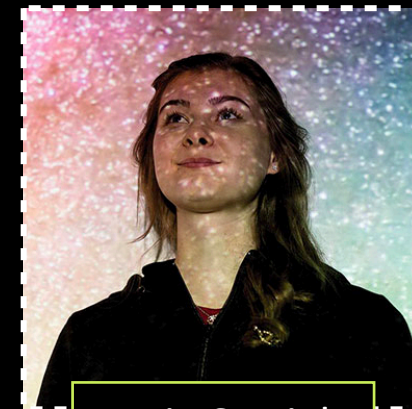
Sam McDermott



Jeremy Sakstein



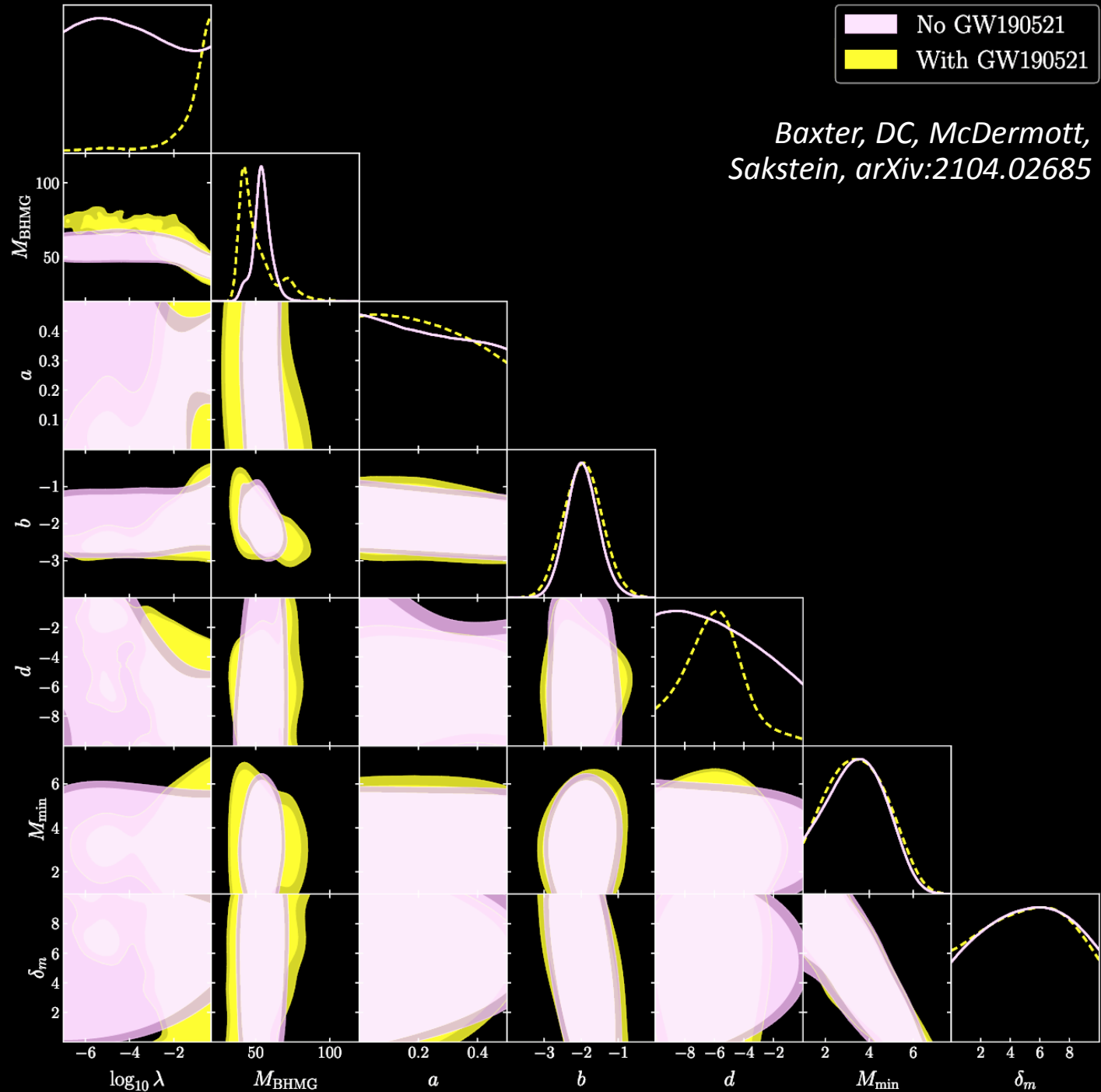
Eric Baxter



Maria Straight



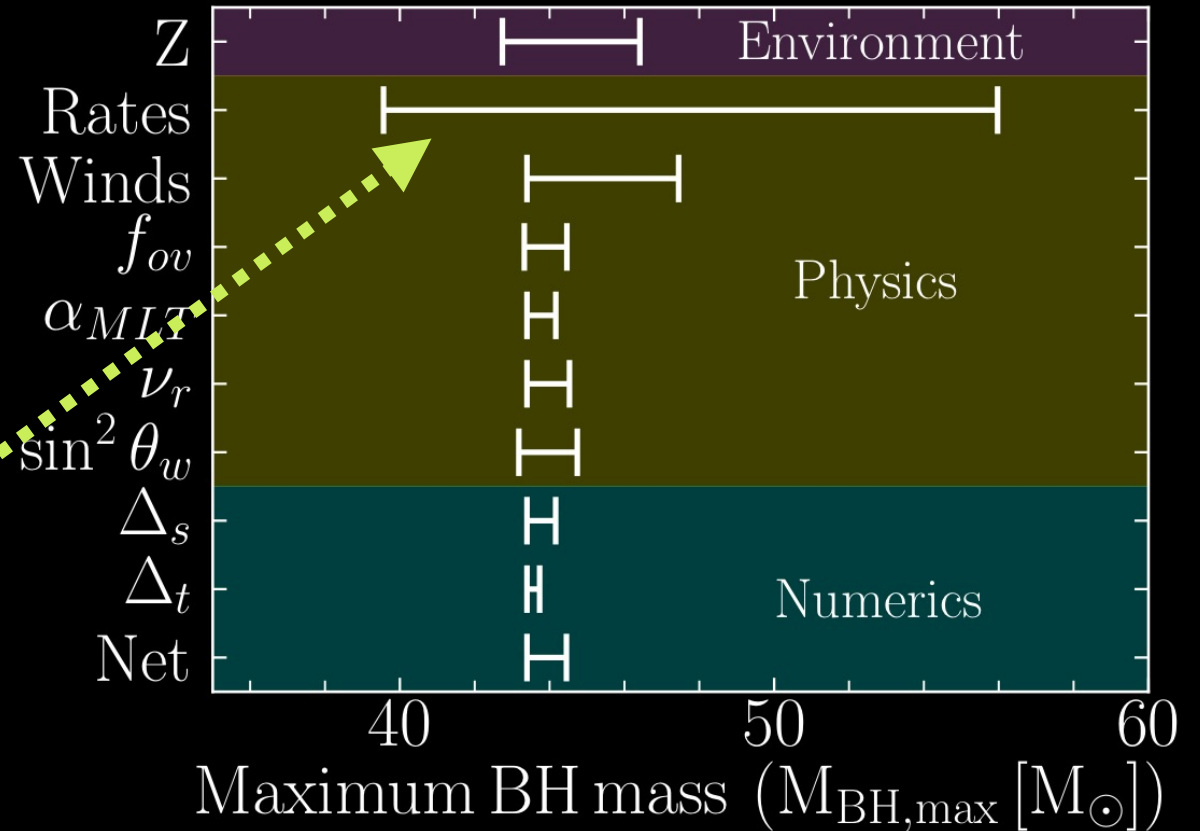
Baxter, DC, McDermott,
Sakstein, arXiv:2104.02685



Physics dependence of the BHMG

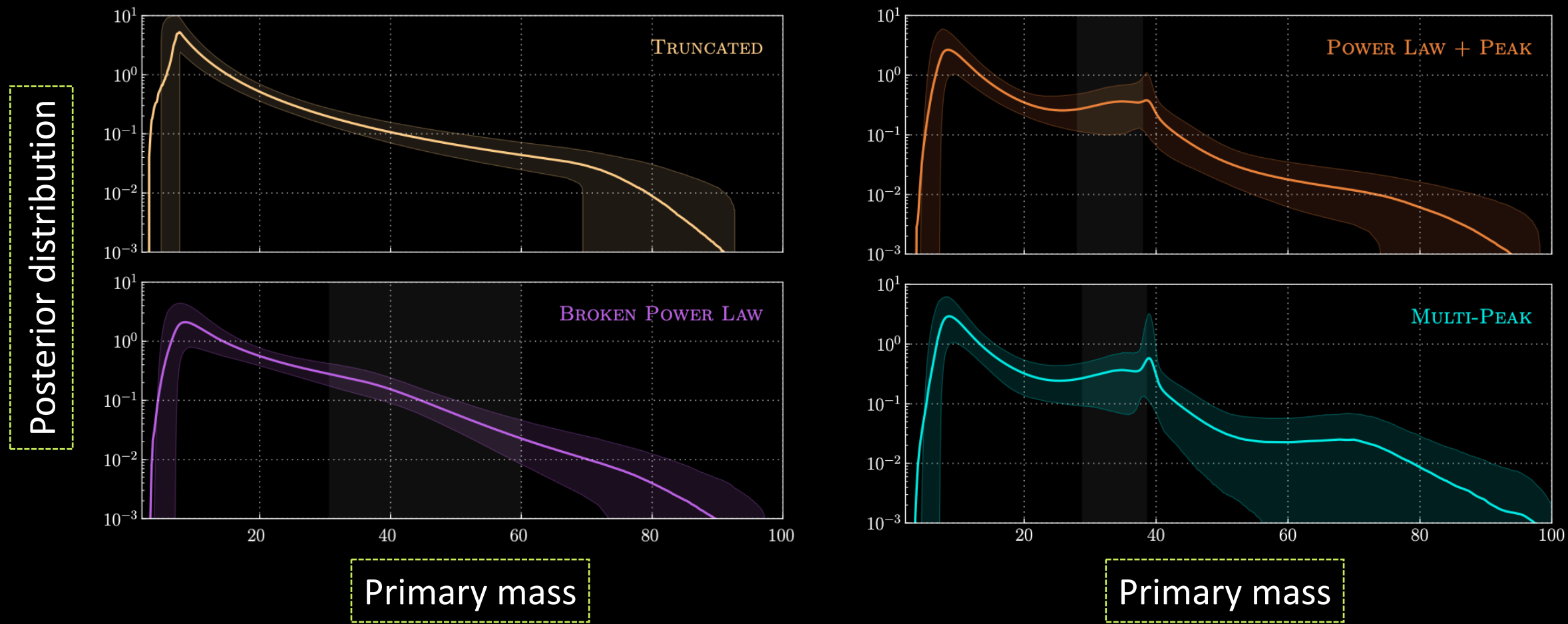
- Astrophysical + nuclear + numerical dependence
- Most important dependence: $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate
- Using updated deBoer et al rate, BHMG found at $48^{+2}_{-3} M_{\odot}$

deBoer et al arXiv:1709.03144 [hep-ex]
 Farmer, Renzo, de Mink, Fishbach, Justham
 arXiv:2006.06678 [astro-ph.SR]



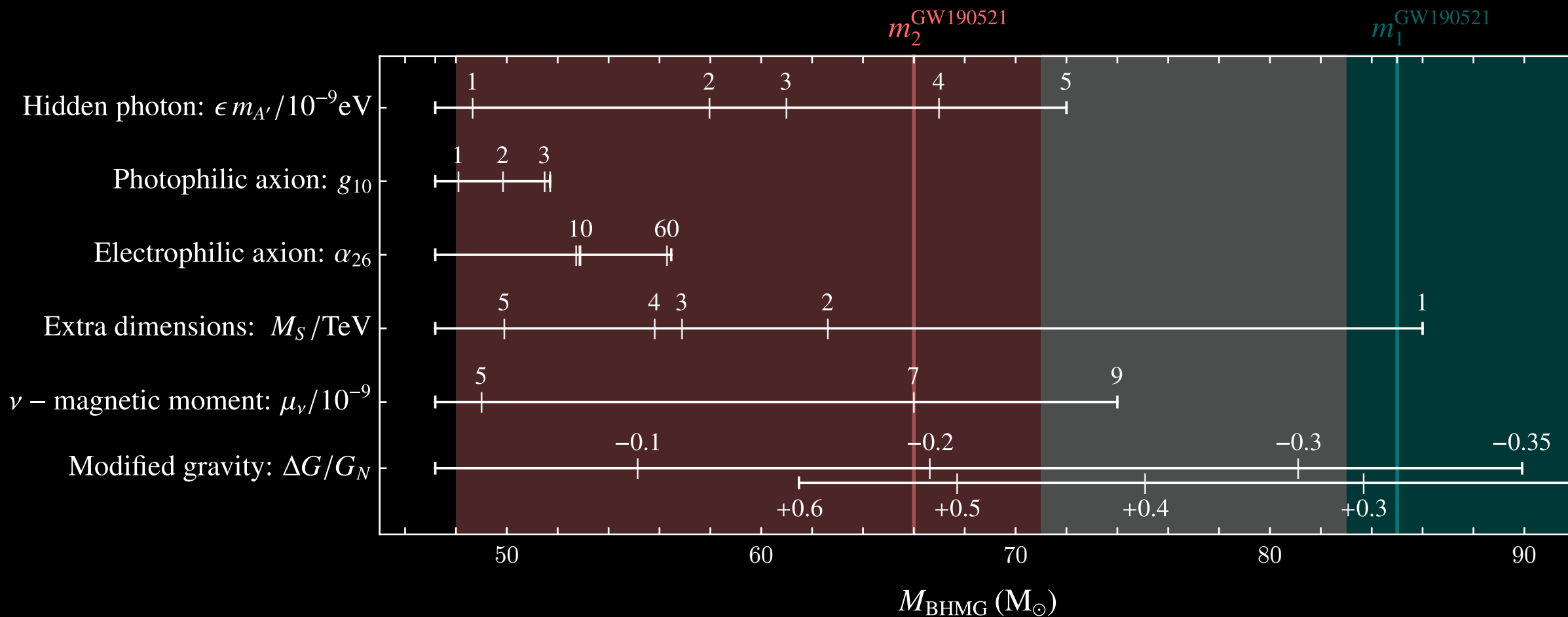
Farmer, Renzo, de Mink, Marchant, Justham
 arXiv:1910.12874 [astro-ph.SR]

LIGO/Virgo O3a population analysis

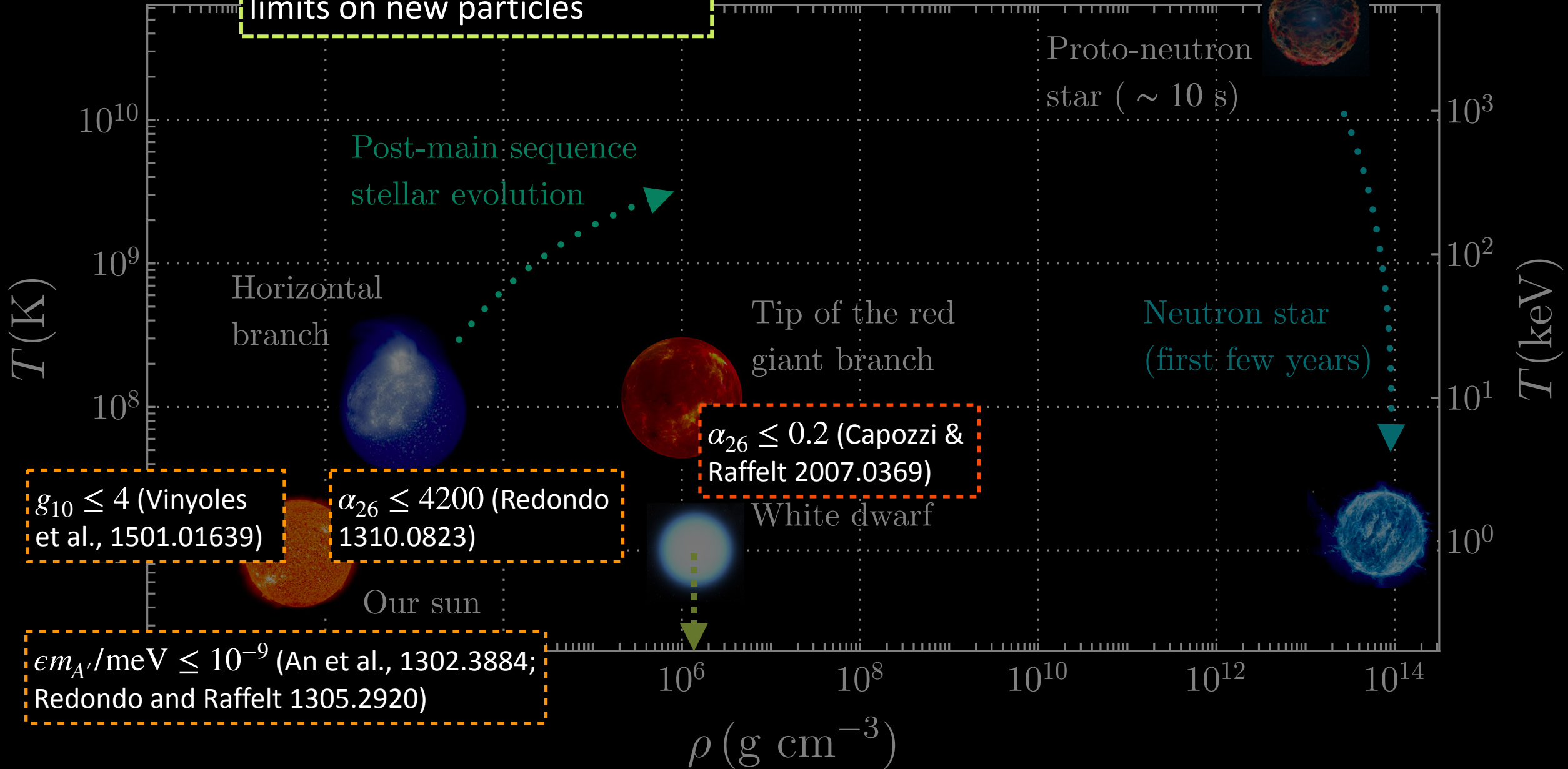


GW190521, the impossible black holes

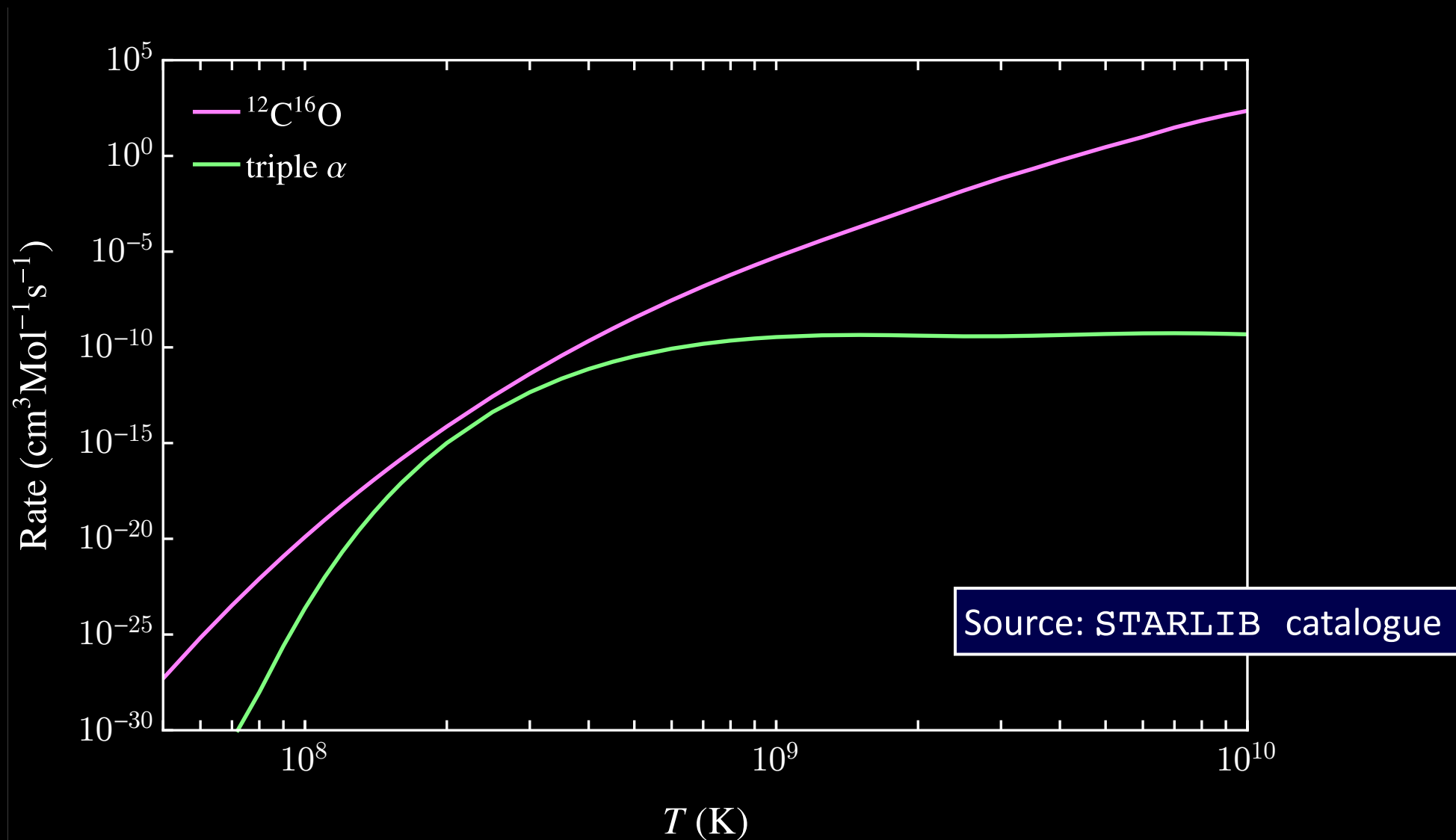
... and Beyond the Standard Model physics

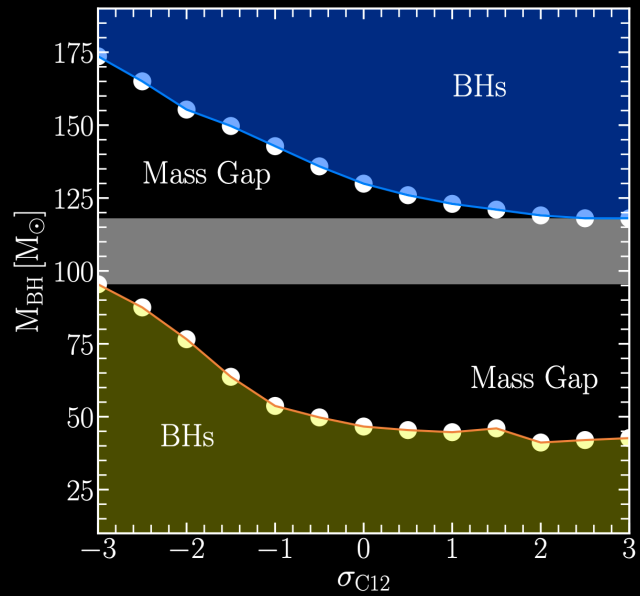


An illustration of astrophysical limits on new particles



Helium burning rates as a function of T

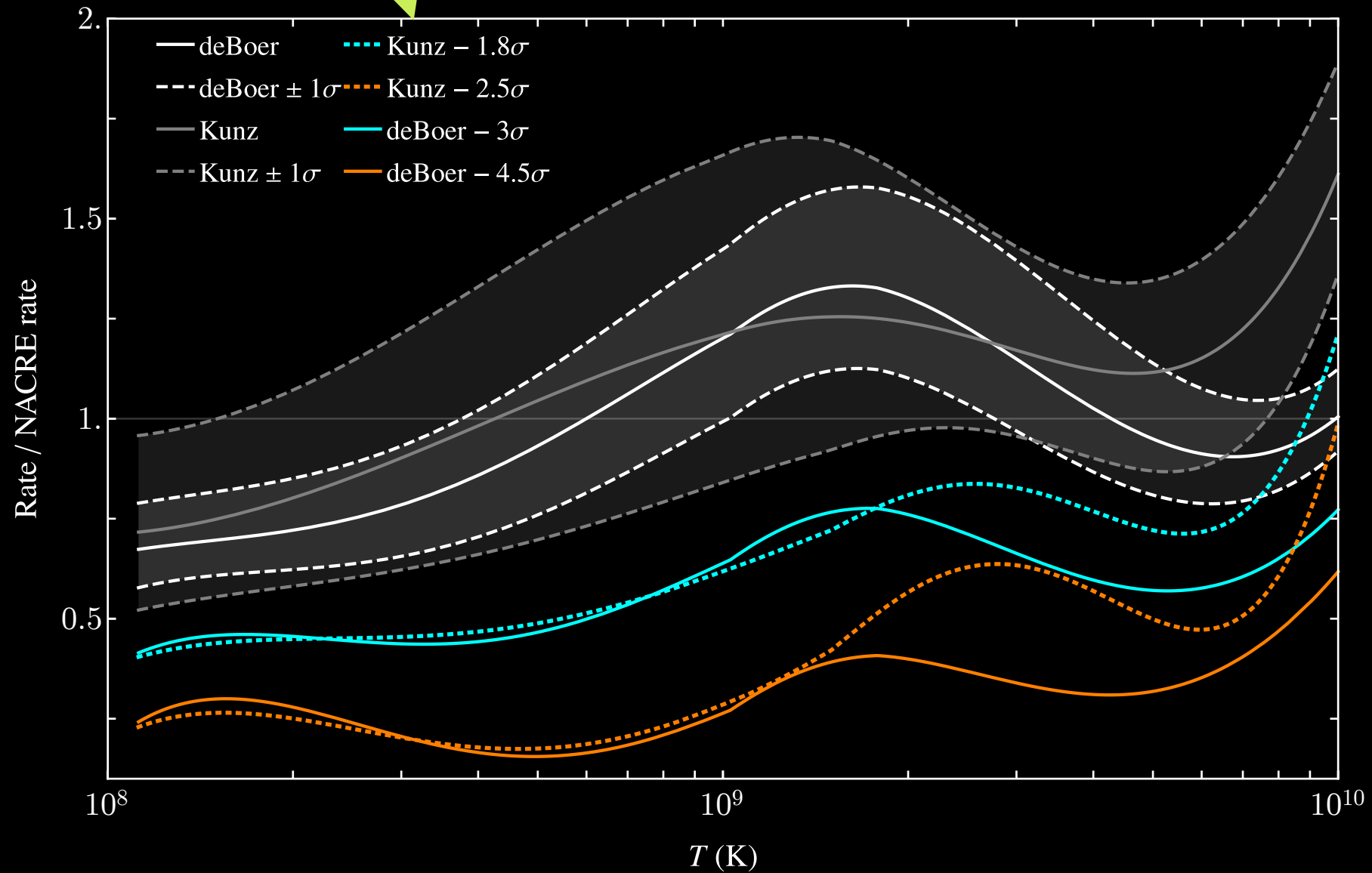


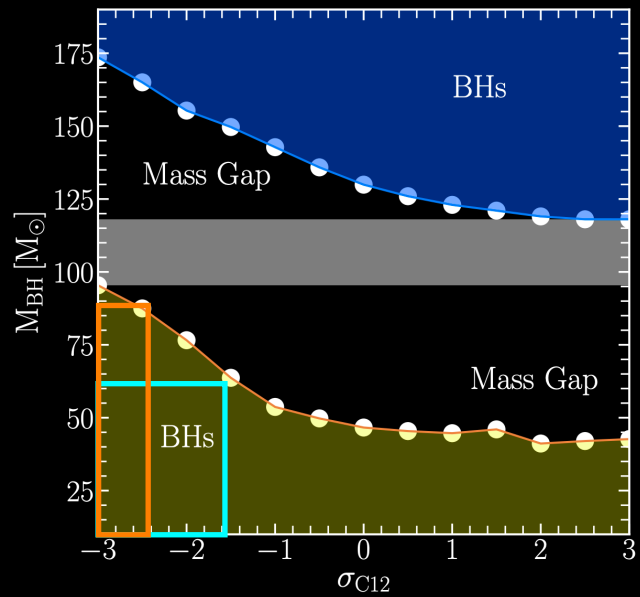


(Kunz is currently used in STARLIB)



Fitting GW190521 masses m_1 and m_2

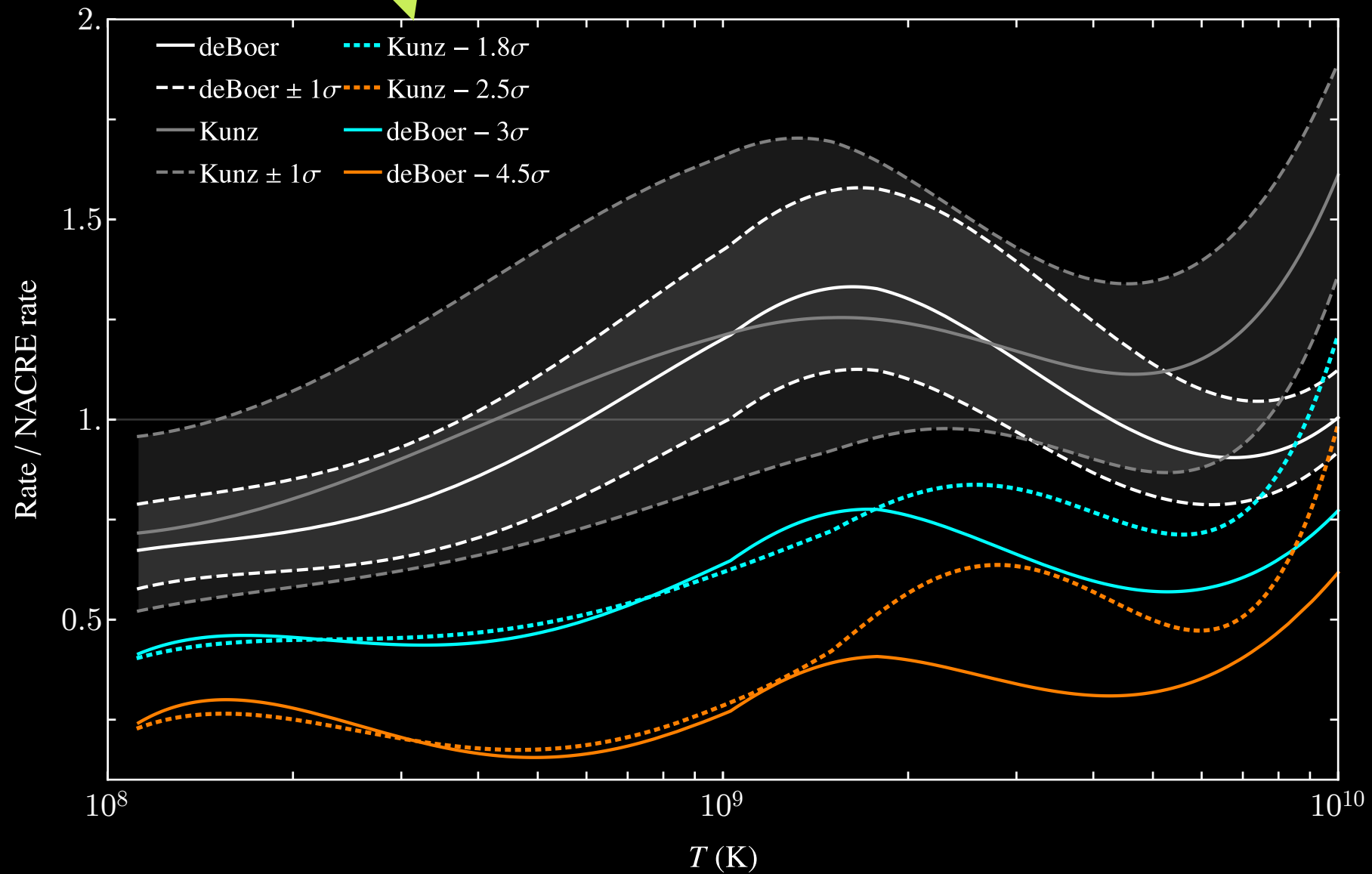




(Kunz is currently used in STARLIB)



Fitting GW190521 masses m_1 and m_2



Large black hole in LB-1?

- Last year, a $70 M_{\odot}$ black hole was reported in a binary with a high-metallicity smaller star (from the radial velocity variability of the $H\alpha$ emission line, suggesting an accretion disk)
- It was suggested (1911.12357) that it was formed due to the core-collapse of a high metallicity progenitor with reduced stellar winds
- However, those simulations did not include pulsations (they were stopped at carbon burning)
- The observation has since also been disputed (1912.04185 and 1912.03599) - apparent shifts instead originate from shifts in the luminous star's $H\alpha$ absorption line

Binary merger events ($M_1 \approx M_2$)

- >50 LIGO/Virgo observations
 - 2017 Nobel Prize in Physics
- *Can be used to learn about new physics in various ways*
- Most GW radiation from the **inspiral phase**, ending in f_{ISCO}
- Solvable in a (v/c) expansion
 - Weak gravity, small velocity

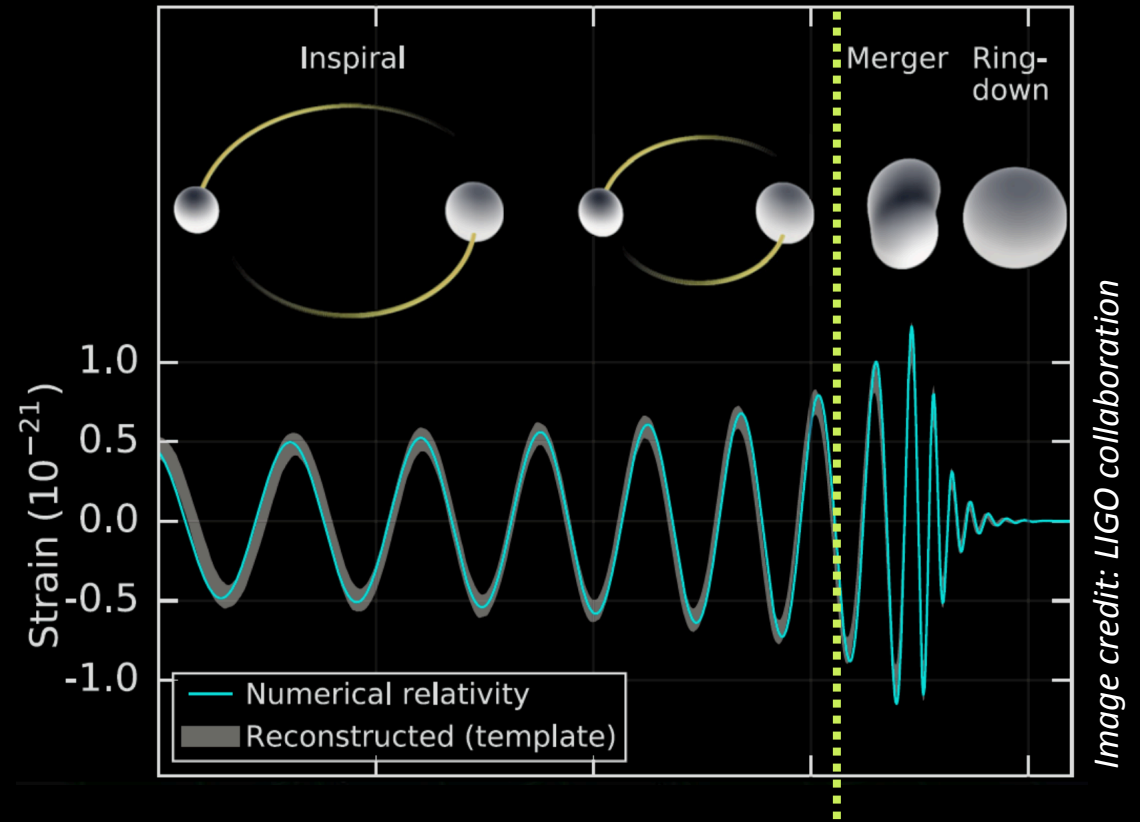


Image credit: LIGO collaboration

$$f_{\text{ISCO}} = \frac{C_*^{3/2}}{3^{3/2} \pi G_N (M_1 + M_2)}$$

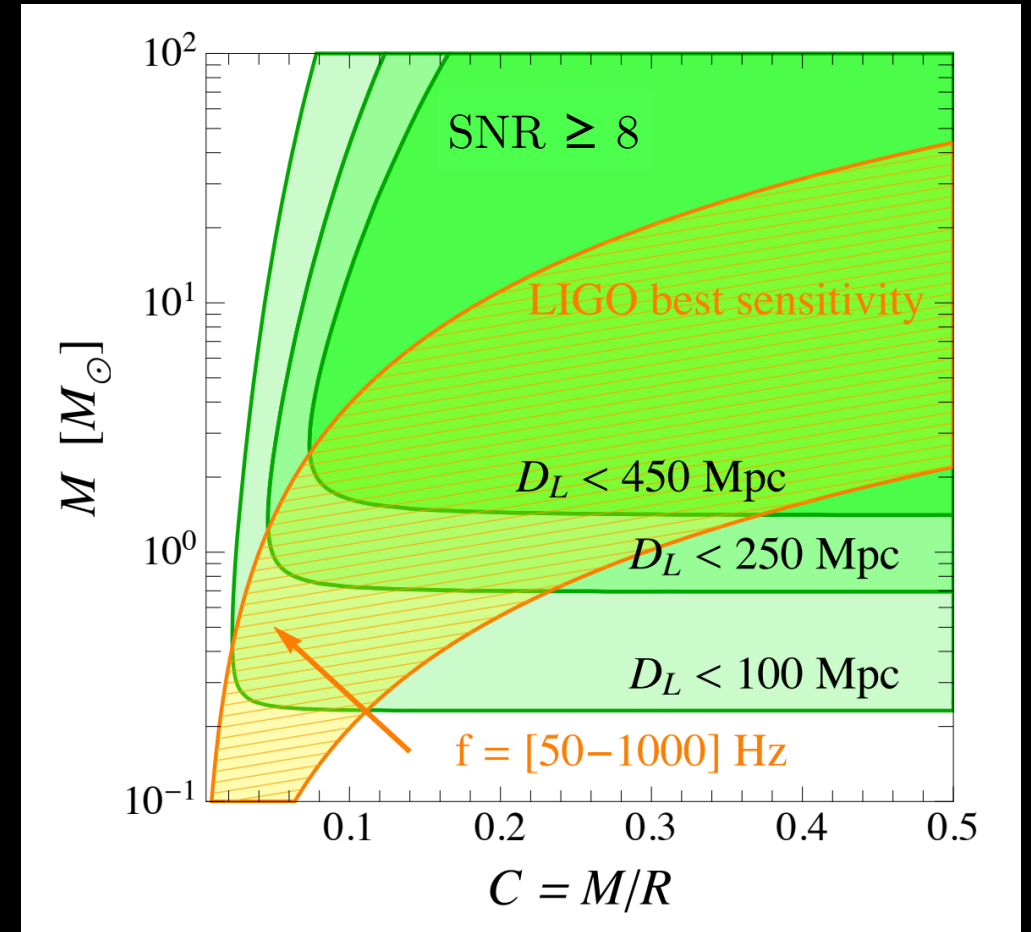
Compact object merger sensitivity

- Best detection prospects for $f_{\min} < f_{\text{peak}} \sim f_{\text{ISCO}} < f_{\max}$
- Defines an **CO sensitivity band**

$$f_{\text{ISCO}} = \frac{C_*^{3/2}}{3^{3/2} \pi G_N (M_1 + M_2)} \quad C_* = \frac{G_N M_*}{R_*}$$

$C_{\odot} = 2 \times 10^{-6}$	$C_{\text{BH}} = 0.5$
$C_{\oplus} = 7 \times 10^{-10}$	$C_{\text{NS}} \sim 0.1$

- Sensitivity determined by masses, **compactness** and luminosity distance



Giudice, McCullough, Urbano [JCAP, 1605.01209]

What can we learn from the *inspiral* waveform?*

A lot, for example,

1. Component masses
2. Tidal effects → equation of state
3. Dynamical friction → environmental effects
4. Long-range (dark) forces → BSM effects
5. Extra dissipation channels → BSM effects
6. Redshift distribution of events → age of objects
7. “Hair”: multipolar metric deviations (EMRIs) → tests of GR

So what about new physics? May show up in various ways, I will give a (unabashedly biased) selection of examples

Hints of mass-gap mergers:

- GW190814 → downgraded mass gap probability <1% → publication June '20
- GW190924 (24 September '19)
- GW190930 (30 September '19)

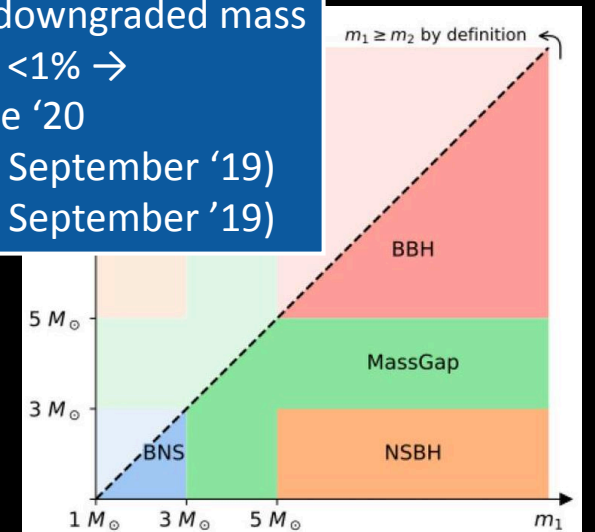


Image credit: LIGO collaboration

*Further information could come (for example) from multi-messenger signals (or absence thereof), or post-merger quasi-normal modes or “echoes”

The BHMG and BSM cooling

- Scenario: new, light particles coupled to material in the star introduce **new loss channels**

Extra scenarios: large extra dimensions ($d = 4 + 2$) and neutrino magnetic moment work through *essentially the same mechanism*

- Case studies: $\mathcal{L}_{\text{SM}} + \dots$

- the electrophilic axion $\mathcal{L}_{ae} = -ig_{ae}\bar{\psi}_e\gamma_5\psi_e a$ (will also work with $\alpha_{26} \equiv 10^{26}g_{ae}^2/4\pi$ for convenience)*

- the photophilic axion $\mathcal{L}_{a\gamma} = -\frac{1}{4}g_{a\gamma}aF_{\mu\nu}\widetilde{F}^{\mu\nu}$ (will also define $g_{10} \equiv 10^{10}g_{a\gamma}$ GeV)

- the hidden photon $\mathcal{L}_{A'\gamma} = -\frac{\epsilon}{2}F'_{\mu\nu}F^{\mu\nu} + \frac{m_{A'}^2}{2}A'_\mu A'^\mu$ (and define nothing)

*Interesting in light of the XENON1T excess, arXiv:2006.09721 [hep-ex]

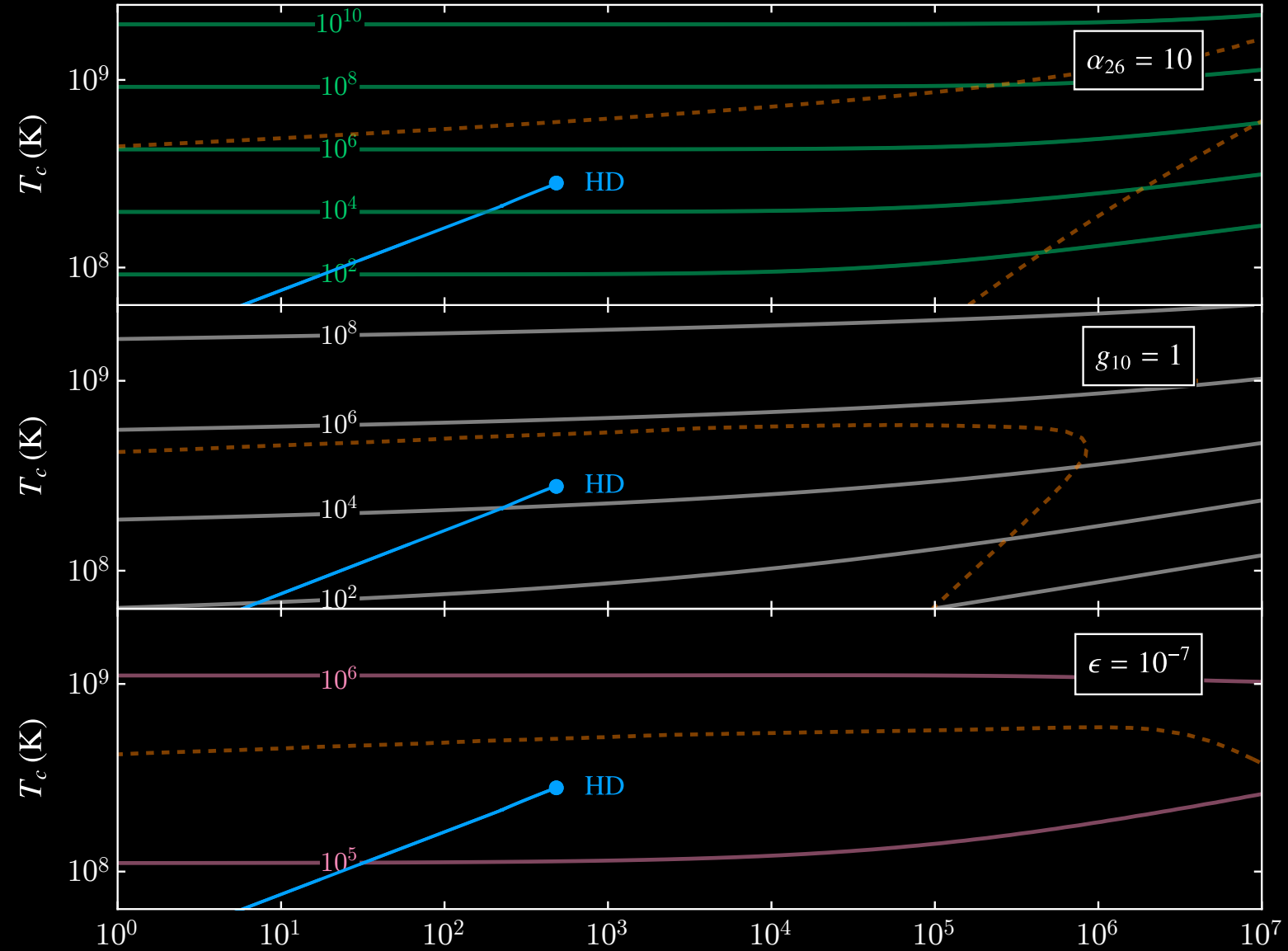
Loss rates

Electrophilic axion: $\mathcal{Q}_{ae} \propto T^6$
 $(e + \gamma \rightarrow e + a)$

Photophilic axion: $\mathcal{Q}_{a\gamma} \propto T^4$
 $((Z, A) + \gamma \rightarrow (Z, A) + a)$

Hidden photon: $\mathcal{Q}_{A'} \propto T$
 (resonant emission)

Central losses: $Q_{ae}, Q_{a\gamma}, Q_{A'}$ (erg g⁻¹ s⁻¹)



Example track of $M_{in} = 55 M_{\odot}$ progenitor ρ_c (g cm⁻³)

Energy loss due to electrophilic axions

- Semi-Compton scattering, $e + \gamma \rightarrow e + a$:

$$Q_{\text{sc}} = \frac{40 \zeta_6 \alpha_{\text{EM}}^2 g_{ae}^2}{\pi^2} \frac{Y_e T^6}{m_N m_e^4} F_{\text{deg}} \simeq 33 \alpha_{26} Y_e T_8^6 F_{\text{deg}} \frac{\text{erg}}{\text{g} \cdot \text{s}} \quad \left(T_8 \equiv \frac{T}{10^8 \text{K}} \right)$$

$$F_{\text{deg}} = \frac{2}{n_e} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} f_{e^-} (1 - f_{e^-}), \text{ where } f_{e^-} \text{ is the Fermi-Dirac distribution}$$

- Bremsstrahlung, $e + (Z, A) \rightarrow e + (Z, A) + a$:

$$Q_{b,\text{ND}} = \frac{32 \alpha_{\text{EM}}^2 g_{ae}^2 \rho T^{5/2}}{45 \sqrt{\frac{\pi^3}{2}} m_N^2 m_e^{7/2}} F_{b,\text{ND}} \simeq 582 \alpha_{26} \rho_6 T_8^{5/2} F_{b,\text{ND}} \frac{\text{erg}}{\text{g} \cdot \text{s}} \quad \left(\rho_6 \equiv \frac{\rho}{10^6 \text{g cm}^{-3}} \right)$$

$$Q_{b,\text{D}} = \frac{\pi}{60} \frac{Z^2}{A} \frac{\alpha_{\text{EM}}^2 g_{ae}^2 T^4}{m_N m_e^2} F_{b,\text{D}} \simeq 10.8 \alpha_{26} T_8^4 F_{b,\text{D}} \frac{\text{erg}}{\text{g} \cdot \text{s}}$$

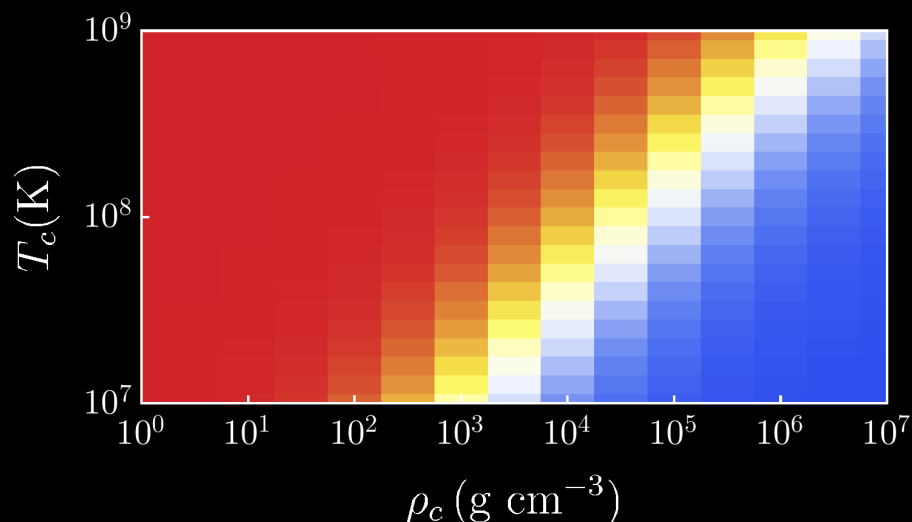
Energy loss due to electrophilic axions

- Semi-Compton scattering, $e + \gamma \rightarrow e + a$:

$$Q_{\text{sc}} = \frac{40 \zeta_6 \alpha_{\text{EM}} g_{ae}^2}{\pi^2} \frac{Y_e T^6}{m_N m_e^4} F_{\text{deg}} \simeq 33 \alpha_{26} Y_e T_8^6 F_{\text{deg}} \frac{\text{erg}}{\text{g} \cdot \text{s}} \quad \left(T_8 \equiv \frac{T}{10^8 \text{K}} \right)$$

$$F_{\text{deg}} = \frac{2}{n_e} \int \frac{d^3 \mathbf{p}}{(2\pi)^3} f_{e^-} (1 - f_{e^-}), \text{ where } f_{e^-} \text{ is the Fermi-Dirac distribution}$$

$$0 < F_{\text{deg}} < 1$$

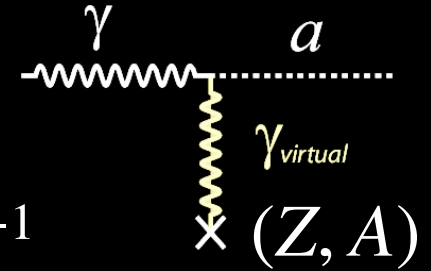


Semi-Compton emission dominates throughout the Helium burning phase

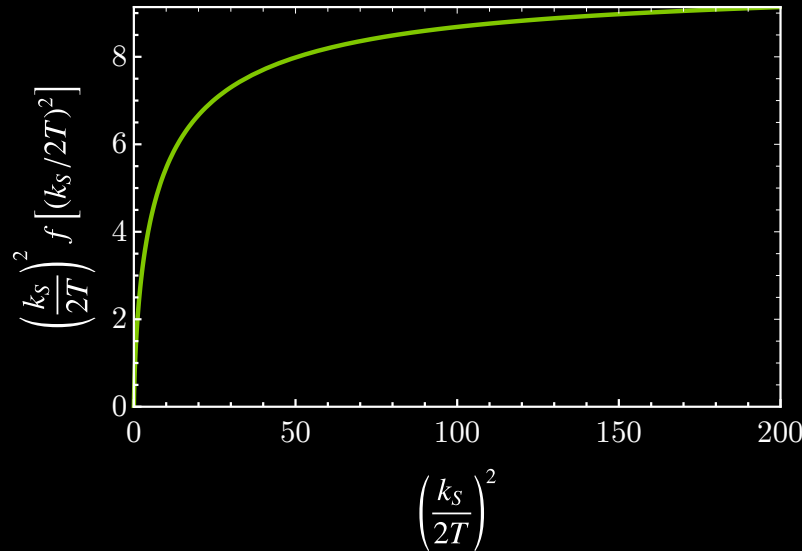
Energy loss due to photophilic axions

- Primakov effect $(Z, A) + \gamma \rightarrow (Z, A) + a$

$$Q_{a\gamma} = \frac{g_{a\gamma}^2 T^7}{4\pi^2 \rho} \left(\frac{k_S}{2T}\right)^2 f[(k_S/2T)^2] \simeq 283.16 \frac{\text{erg}}{\text{g} \cdot \text{s}} g_{10}^2 T_8^7 \rho_3^{-1} \times \left(\frac{k_S}{2T}\right)^2 f[(k_S/2T)^2], \text{ where } \left(\frac{k_S}{2T}\right)^2 = 0.166 \frac{\rho_3}{T_8^3} \sum_j Y_j Z_j^2$$



Screened at high T and low ρ



Energy loss due to hidden photons

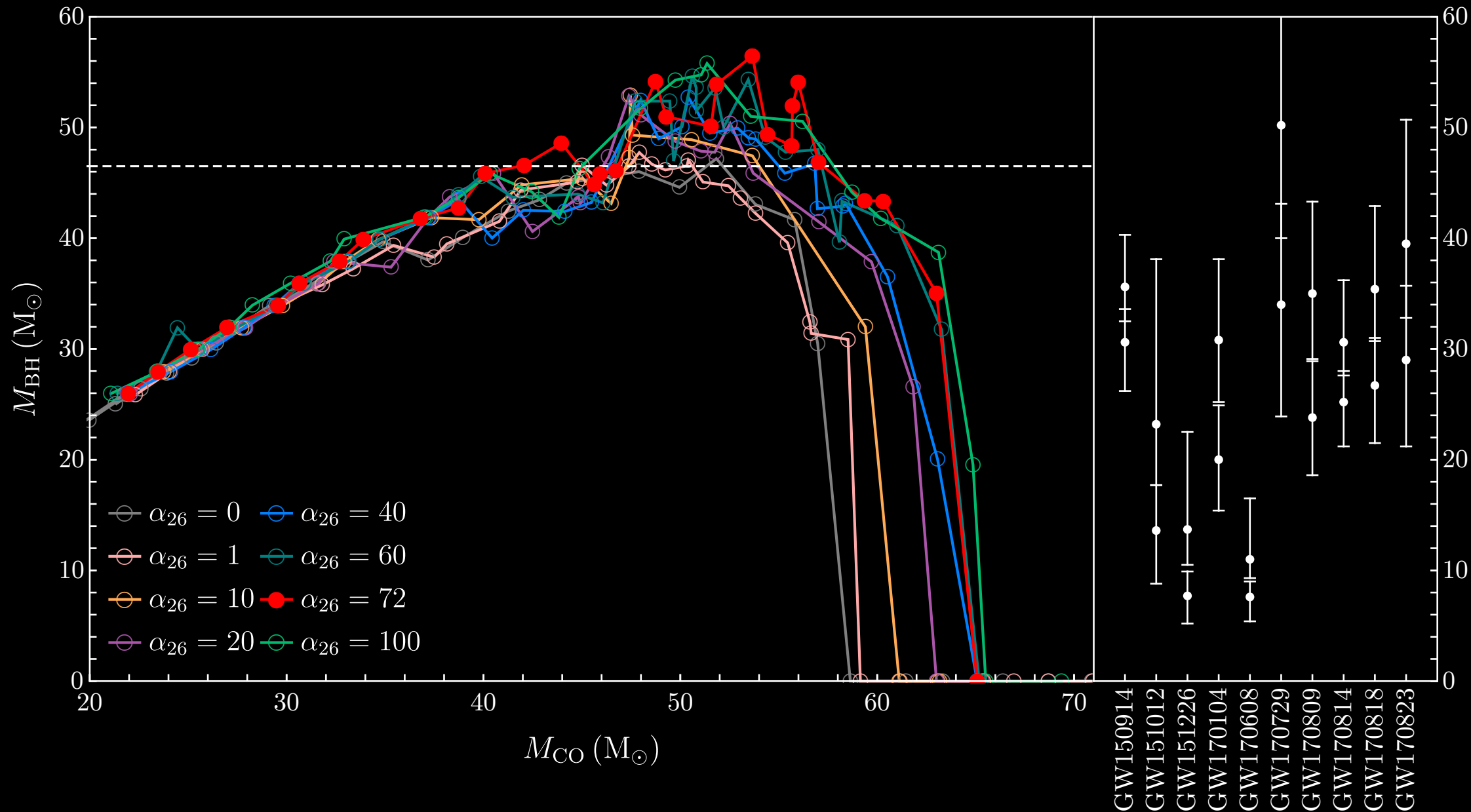
- Plasma production, dominated by longitudinal modes (in a non-relativistic plasma)

$$Q_{A'} = \frac{\epsilon^2 m_{A'}^2 \omega_p^3}{4\pi \rho e^{\omega_p/T} - 1} \simeq \frac{\epsilon^2 m_{A'}^2 \omega_p^2 T}{4\pi \rho} \simeq 1.8 \times 10^3 \frac{\text{erg}}{\text{g} \cdot \text{s}} \frac{Z}{A} T_8 \left(\frac{\epsilon}{10^{-7}} \frac{m_{A'}}{\text{meV}} \right)^2$$

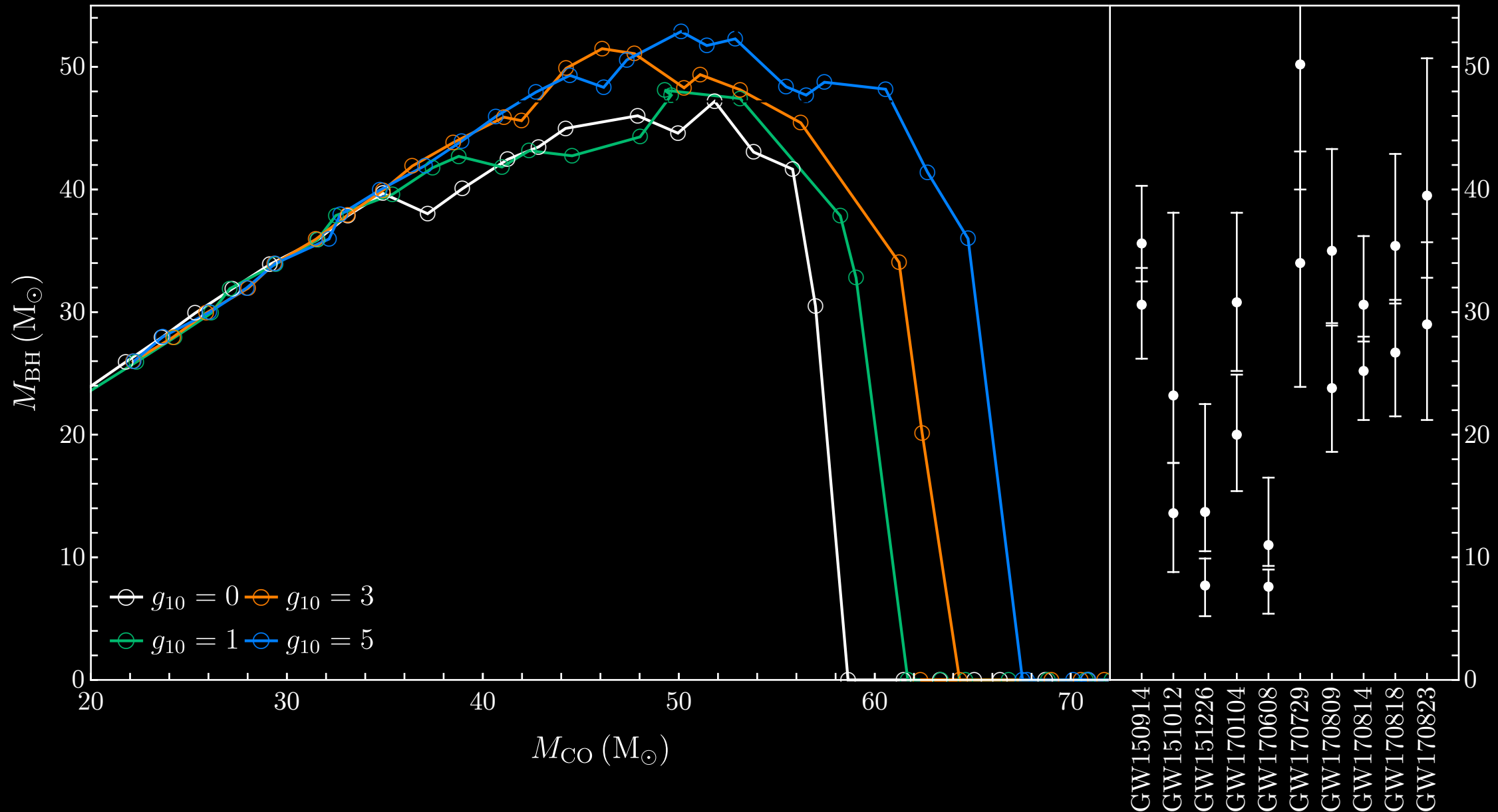
In the limit $\omega_p \ll T$

- Where photons have plasma mass $\omega_p \simeq \sqrt{\frac{4\pi\alpha_{\text{EM}}n_e}{m_e}} \simeq 654\text{eV} \sqrt{\frac{Z}{A}\rho_3}$

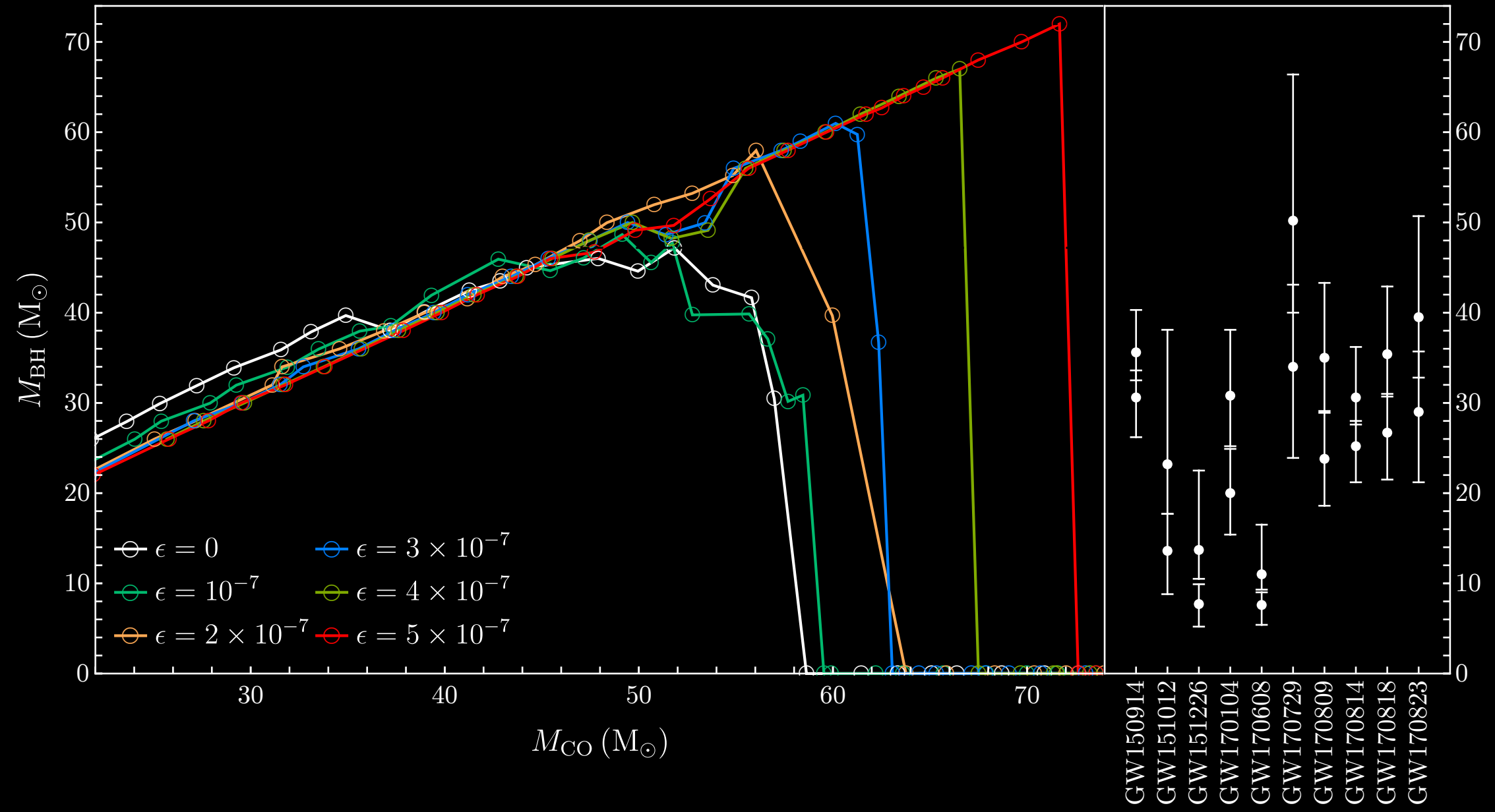
Electrophilic axion: $m_a \ll \text{keV}$, $Z = 10^{-5}$



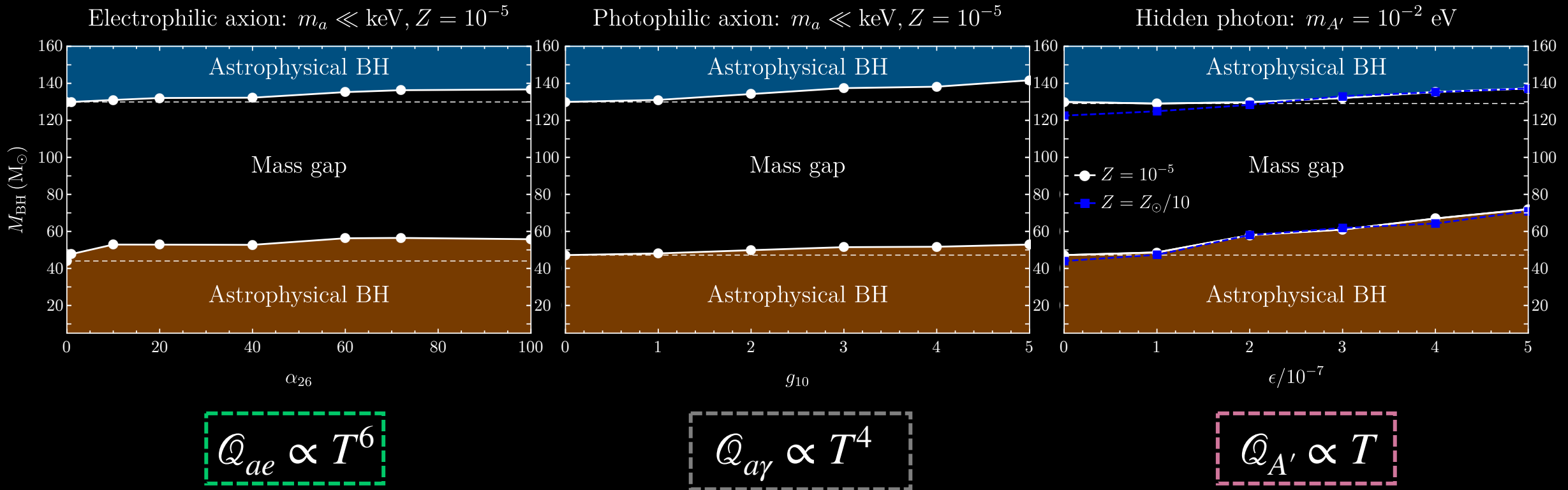
Photophilic axion: $m_a \ll \text{keV}$, $Z = 10^{-5}$



Hidden photon: $m_{A'} = 10^{-2} \text{eV}, Z = 10^{-5}$



BSM cooling and the black hole mass gap



Important extra cooling = large shifts of the mass gap!

What about trapped new physics?

- Heavier and more strongly coupled degrees of freedom may instead *remain in the star*
- Then, they affect the stellar structure equations

From *Stellar Structure and Evolution* (2nd edition), Kippenhahn, Weigert, Weiss

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho},$$

Mass function

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4},$$

Hydrostatic equilibrium

$$\frac{\partial l}{\partial m} = \varepsilon_n - \varepsilon_v - c_P \frac{\partial T}{\partial t} + \frac{\delta}{\rho} \frac{\partial P}{\partial t},$$

Energy flux

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P} \nabla,$$

Convection

$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho} \left(\sum_j r_{ji} - \sum_k r_{ik} \right), \quad i = 1, \dots, I.$$

Reactions

+ BSM physics contributions?

Heavier degrees of freedom?

DC, McDermott, Sakstein arXiv:2007.07889 [gr-qc]

+ work in progress

