Gravity matters:
from gravitational waves to dark matter

Xavier Calmet

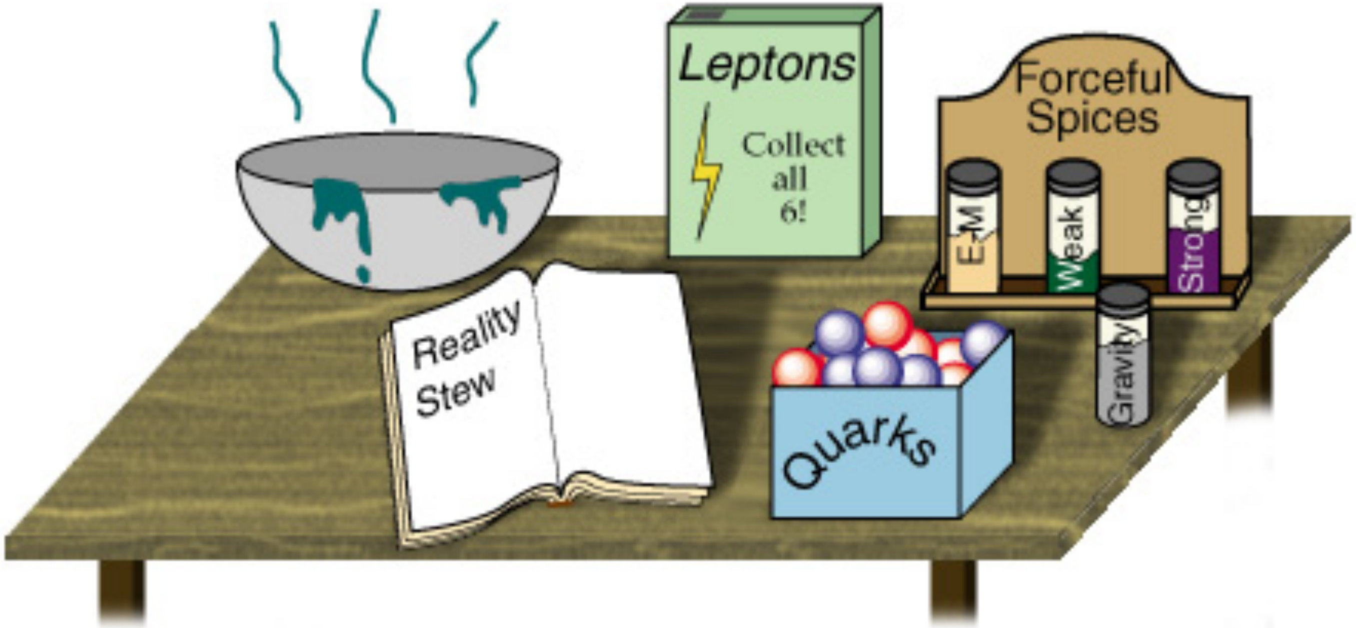
Physics & Astronomy
University of Sussex

University of Sussex

Outline

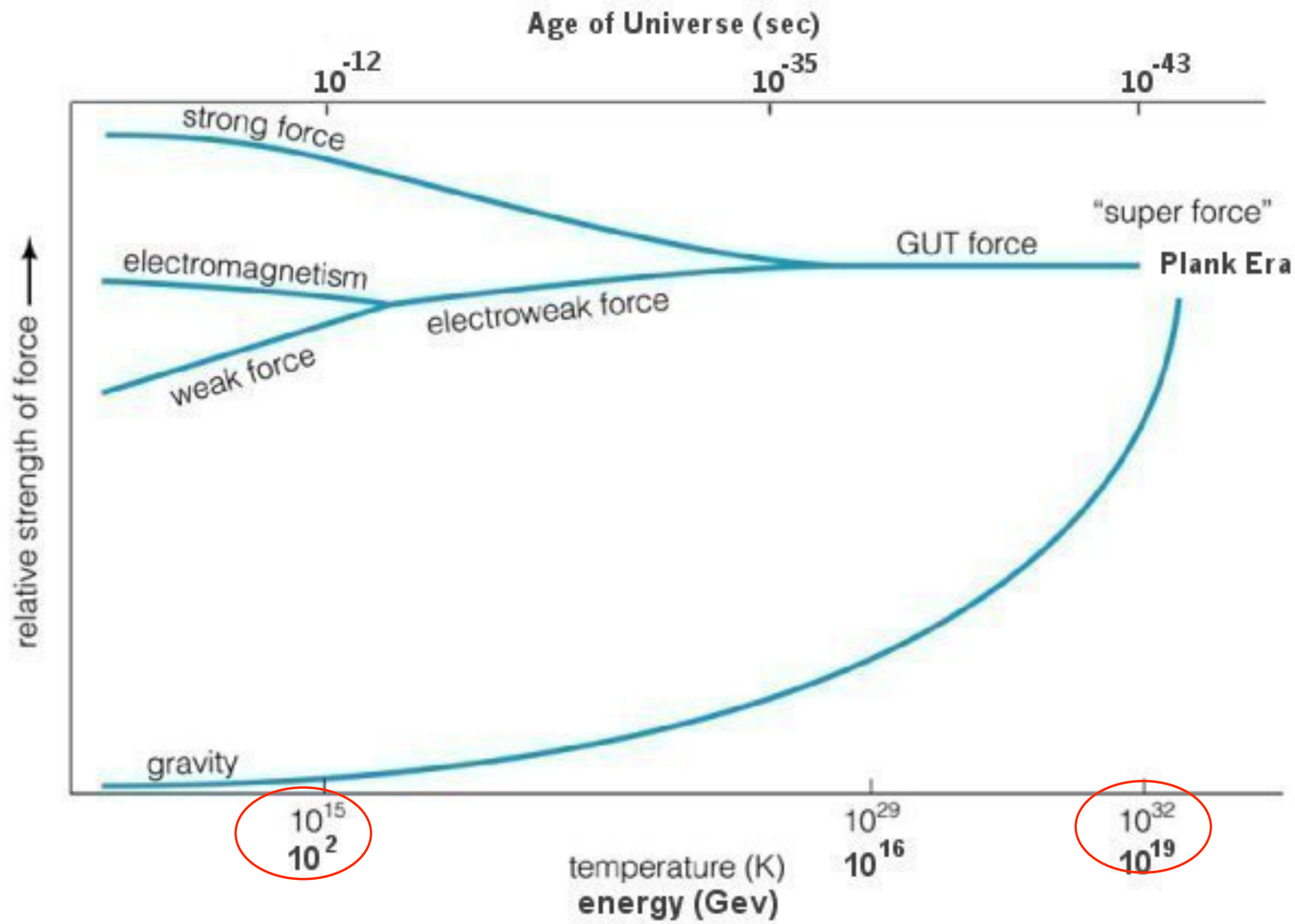
- Introduction
- Effective field theory methods applied to quantum gravity
- Applications to dark matter, black holes, gravitational waves, inflation and grand unification.
- Conclusions

Particle Physics and Gravity



- Gravity is at odd with other forces of nature.
- It has a dimensionful coupling constant: Newton’s constant.
- It is a very weak force in comparison to the standard model interactions.

When does gravity become comparable in strength to other forces?

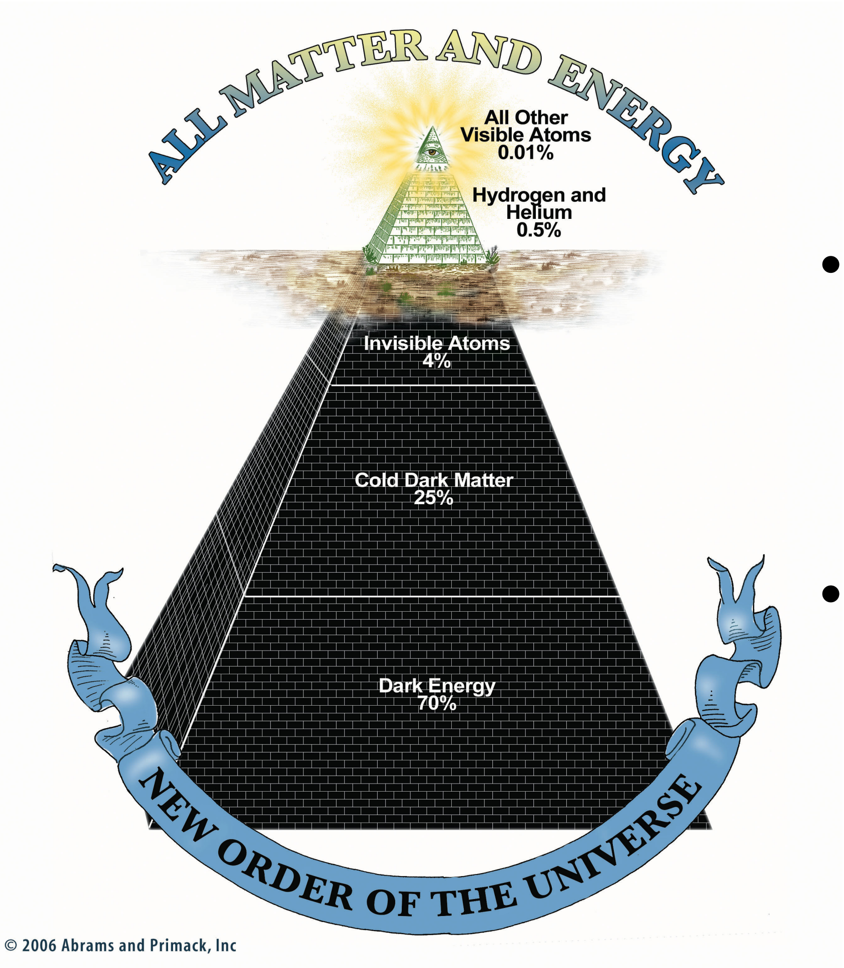


Traditionally the Planck mass is taken to be of the order of 10^{19} GeV.

$$M_P = \sqrt{\frac{\hbar c}{G_N}}$$

- For this reason, it is usually assumed by most particle physicists that the issue of quantum gravity can safely be ignored at low energy.
- However, I will show you that gravity matters!

Missing ingredient in the standard model: dark matter



- Can gravity/quantum gravity account for the shortcomings of the standard model?

- For example, can it say anything about dark matter, inflation or the unification of forces?

- We have known for about 20 years now that the Planck scale is a dynamical quantity and it could be well below 10^{19} GeV,

- It is an exciting field as gravity provides a new way to study our universe with gravitational waves.

Effective action for GR

- How can we describe general relativity quantum mechanically?
- Well known issues with linearized GR: it is not renormalizable.
- This is the reason d'être of string theory, loop quantum gravity etc…
- How much can we understand using QFT techniques?
- We have good reasons to think that length scales smaller than the Planck scale are not observables due to the formation of small black holes.
- Effective field theories might be all we need to discuss physics at least up to the Planck scale.

- The goal is to try to make the link with observables.

- Or at least with with thought experiments.

- It is very conservative.

- What can we learn using techniques we actually understand well, and which are compatible with nature as we know it: standard model and GR?

- I am going to assume general covariance (diffeomorphism invariance)

- Quantum gravity has only 2 dofs namely the massless graviton (which has 2 helicity states).

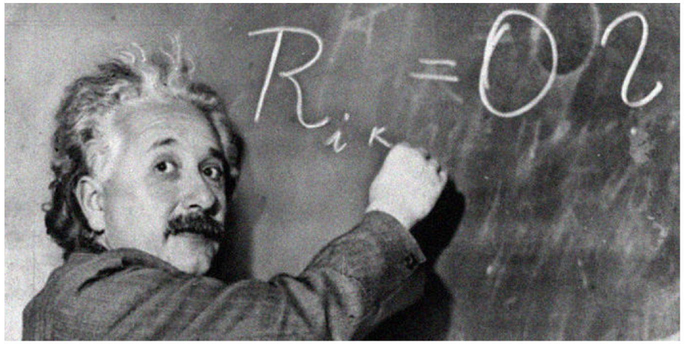
- We know the particle content of the “matter theory” (SM, GUT, inflation etc).

- We can write down an effective action for quantum gravity.

- This program was started by Feynman in the 60’s using linearized GR.

- Try to find/calculate observables

- Try to find consistency conditions which could guide us on our path towards a quantization of GR.

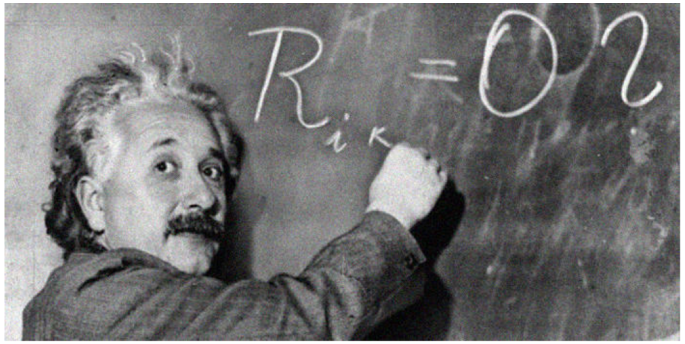


Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- The Ricci scalar R and tensor $R_{\mu\nu}$ contain two derivatives of the metric.
- They thus have mass dimension 2, this is important to organize the effective field theory.
- G is Newton's constant, it is related to the Planck mass.
- $T_{\mu\nu}$ is the energy-momentum tensor: this is your particle physics model.
- It can be derived from the Hilbert-Einstein action:

$$S = \int d^4x \sqrt{-g} \frac{2}{\kappa^2} R \qquad \kappa^2 = 32\pi G \qquad g = \det(g_{\mu\nu})$$



Effective action for quantum gravity

The Hilbert-Einstein action

$$S = \int d^4x \sqrt{-g}\frac{2}{\kappa^2}R$$

receives corrections when integrating out fluctuations of the graviton (and other matter fields depending on the energy under consideration), one obtains:

$$S = \int d^4x \sqrt{-g}\left[\left(\frac{1}{2}M^2 + \xi H^\dagger H\right)\mathcal{R} - \Lambda_C^4 + c_1\mathcal{R}^2 + c_2\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + c_4\Box\mathcal{R}\right.$$
$$\left. -b_1\mathcal{R}\log\frac{\Box}{\mu_1^2}\mathcal{R} - b_2\mathcal{R}_{\mu\nu}\log\frac{\Box}{\mu_2^2}\mathcal{R}^{\mu\nu} - b_3\mathcal{R}_{\mu\nu\rho\sigma}\log\frac{\Box}{\mu_3^2}\mathcal{R}^{\mu\nu\rho\sigma} + \mathcal{L}_{SM} + \mathcal{O}(M_\star^{-2})\right]$$

The non-local part of the EFT

- The Wilson coefficients of the non-local operators are universal predictions of quantum gravity:

$$-b_1 R \log \frac{\Box}{\mu_1^2} R - b_2 R_{\mu\nu} \log \frac{\Box}{\mu_2^2} R^{\mu\nu} - b_3 R_{\mu\nu\rho\sigma} \log \frac{\Box}{\mu_3^2} R^{\mu\nu\rho\sigma}$$

	b_1	b_2	b_3
real scalar	$5(6\xi-1)^2/(11520\pi^2)$	$-2/(11520\pi^2)$	$2/(11520\pi^2)$
Dirac spinor	$-5/(11520\pi^2)$	$8/(11520\pi^2)$	$7/(11520\pi^2)$
vector	$-50/(11520\pi^2)$	$176/(11520\pi^2)$	$-26/(11520\pi^2)$
graviton	$430/(11520\pi^2)$	$-1444/(11520\pi^2)$	$424/(11520\pi^2)$

NB: they are calculated using dim-reg.

(see e.g. Birrell and Davies, Quantum Fields in Curved Space-Time, more recently Donoghue et al.)

- The Wilson coefficients of the local operators on the other hand are not calculable: this is the price to pay.

Non-locality propagates into the matter sector

XC, Croon & Fritz (2015)

- Gravity leads to non-local effects in Matter
- Let's consider the graviton propagator

$$iD^{\alpha\beta,\mu\nu}(q^2) = \frac{i\left(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu}\right)}{2q^2\left(1 - \frac{NG_N q^2}{120\pi}\log\left(-\frac{q^2}{\mu^2}\right)\right)}$$

- Using this propagator we can now calculate the dressed amplitude for the gravitational scattering of 2 scalar fields.

- We see that the non-locality feeds back into matter, e.g.:

$$\mathcal{O}_8 = \frac{2}{15}G^2 N\left(\partial_\mu\phi(x)\partial^\mu\phi(x) - m^2\phi^2\right)\log\left(\frac{\Box}{\mu^2}\right)\left(\partial_\nu\phi(x)\partial^\nu\phi(x) - m^2\phi^2\right)$$

Field content of the EFT

- By linearizing the EFT (or mapping it to the Einstein frame), one can easily identify the field content.
- Calculating

$$T^{(1)\mu\nu}(k)D_{\mu\nu\alpha\beta}(k)T^{(2)\alpha\beta}(k)$$

- we find

$$\frac{\kappa^2}{4}\left[\frac{T^{(1)}_{\mu\nu}T^{(2)\mu\nu}-\frac{1}{2}T^{(1)\mu}_{\ \ \mu}T^{(2)\nu}_{\ \ \nu}}{k^2}\right.$$

$$\left.-\frac{T^{(1)}_{\mu\nu}T^{(2)\mu\nu}-\frac{1}{3}T^{(1)\mu}_{\ \ \mu}T^{(2)\nu}_{\ \ \nu}}{k^2-\frac{2}{\kappa^2\left(-c_2+(b_2+4b_3)\log\left(\frac{-k^2}{\mu^2}\right)\right)}}+\frac{T^{(1)\mu}_{\ \ \mu}T^{(2)\nu}_{\ \ \nu}}{k^2-\frac{1}{\kappa^2\left((3c_1+c_2)-(3b_1+b_2+b_3)\log\left(\frac{-k^2}{\mu^2}\right)\right)}}\right]$$

Masses of the new states

- The masses are given by the poles of the Green's function.
- We find:
  - Massless spin-2 field (classical graviton)
  - Massive spin-2 field with a complex mass

$$m_2^2 = \frac{2}{(b_2+4b_3)\kappa^2 W\left(-\frac{2\exp\frac{-c_2}{(b_2+4b_3)}}{(b_2+4b_3)\kappa^2\mu^2}\right)}$$

  - Massive spin-0 field with a complex mass

$$m_0^2 = \frac{-1}{(3b_1+b_2+b_3)\kappa^2 W\left(\frac{\exp\frac{-3c_1-c_2}{(3b_1+b_2+b_3)}}{(3b_1+b_2+b_3)\kappa^2\mu^2}\right)}$$

  - Note that the poles are complex ones, we can identify a mass and width

$$m_i^2 = (M_i - i\Gamma_i/2)^2$$

  - the badly behaving ones can be eliminated by a proper choice of the contour integrals.

The poles in two different limits: c_i=0

- Let's look at the limit where c_i=0, i.e, we consider only the nonlocal operators

- In the standard model, $N_S = 4$, $N_f = 45$ and $N_V = 12$.

- For the spin-2 pole, we thus find: $(7- i\ 3) \times 10^{18}$ GeV

  using $p_0^2 = (m - i\Gamma/2)^2$

- It thus corresponds to a state with mass 7×10^{18} GeV

and width 6×10^{18} GeV

- Clearly in that limit these poles only matter at very high energy!

The poles in two different limits: $c_i \gg b_i$

- Since the c_i are free parameters, they could be much larger than the Wilson coefficients of the non-local operators.

- In that case, we obtain

$$m_2^2 = -\frac{2}{\kappa^2 c_2} - i\pi \frac{2}{\kappa^2 c_2^2}(b_2 + 4b_3)$$

$$M_2 = \sqrt{\frac{2}{c_2}\frac{M_P}{2}}$$

$$\Gamma_2 = \frac{73 M_2^3}{360\pi\sqrt{2} M_P^2}$$

- So we need to pick $c_2 < 0$ and

$$m_0^2 = \frac{1}{\kappa^2(3c_1 + c_2)} - i\pi \frac{1}{\kappa^2(3c_1 + c_2)^2}(3b_1 + b_2 + b_3)$$

$$M_0 = \sqrt{\frac{1}{(3c_1 + c_2)}\frac{M_P}{2}} \qquad \Gamma_0 = \frac{7 M_0^3}{72\pi M_P^2}$$

Consistency condition

- The c_i must be such that the masses are not tachyonic!

$$\frac{\kappa^2}{4}\left[\frac{T^{(1)}_{\mu\nu}T^{(2)\mu\nu}-\frac{1}{2}T^{(1)\mu}_{\ \ \mu}T^{(2)\nu}_{\ \ \nu}}{k^2} - \frac{T^{(1)}_{\mu\nu}T^{(2)\mu\nu}-\frac{1}{3}T^{(1)\mu}_{\ \ \mu}T^{(2)\nu}_{\ \ \nu}}{k^2-\frac{2}{\kappa^2\left(-c_2+(b_2+4b_3)\log\left(\frac{-k^2}{\mu^2}\right)\right)}} + \frac{T^{(1)\mu}_{\ \ \mu}T^{(2)\nu}_{\ \ \nu}}{k^2-\frac{1}{\kappa^2\left((3c_1+c_2)-(3b_1+b_2+b_3)\log\left(\frac{-k^2}{\mu^2}\right)\right)}}\right]$$

- Consistency check for any UV completion of the EFT:

$$(3c_1+c_2)-(3b_1+b_2+b_3)\log\left(\frac{-k^2}{\mu^2}\right) > 0$$

$$-c_2+(b_2+4b_3)\log\left(\frac{-k^2}{\mu^2}\right) > 0$$

Lagrangian

- We recover standard general relativity with two new massive modes

$$S = \int d^4x \Bigg[\left(-\frac{1}{2} h_{\mu\nu} \Box h^{\mu\nu} + \frac{1}{2} h_\mu{}^\mu \Box h_\nu{}^\nu - h^{\mu\nu} \partial_\mu \partial_\nu h_\alpha{}^\alpha + h^{\mu\nu} \partial_\rho \partial_\nu h^\rho{}_\mu \right) + \Bigg(-\frac{1}{2} k_{\mu\nu} \Box k^{\mu\nu} + \frac{1}{2} k_\mu{}^\mu \Box k_\nu{}^\nu - k^{\mu\nu} \partial_\mu \partial_\nu k_\alpha{}^\alpha + k^{\mu\nu} \partial_\rho \partial_\nu k^\rho{}_\mu - \frac{M_2^2}{2} \left(k_{\mu\nu} k^{\mu\nu} - k_\alpha{}^\alpha k_\beta{}^\beta \right) \Bigg) + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{M_0^2}{2} \sigma^2 - \sqrt{8\pi G_N} (h_{\mu\nu} - k_{\mu\nu} + \frac{1}{\sqrt{3}} \sigma \eta_{\mu\nu}) T^{\mu\nu} \Bigg] .$$

- Note that this extends the classical result of Stelle.
- It is crucial to realize that these are classical fields.

- We see that the “classical” graviton plays the role of the metric and determines the geometry.

- It couples in a universal manner to matter as usual (this is just GR).

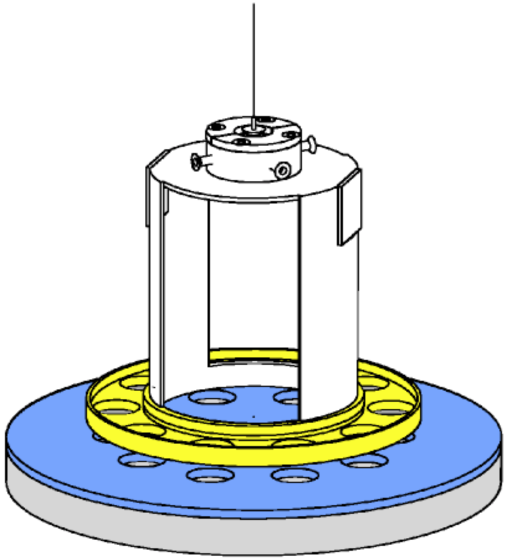
- The massive classical fields are not gravitational fields in the sense that they do not affect the invariant length or geometry.

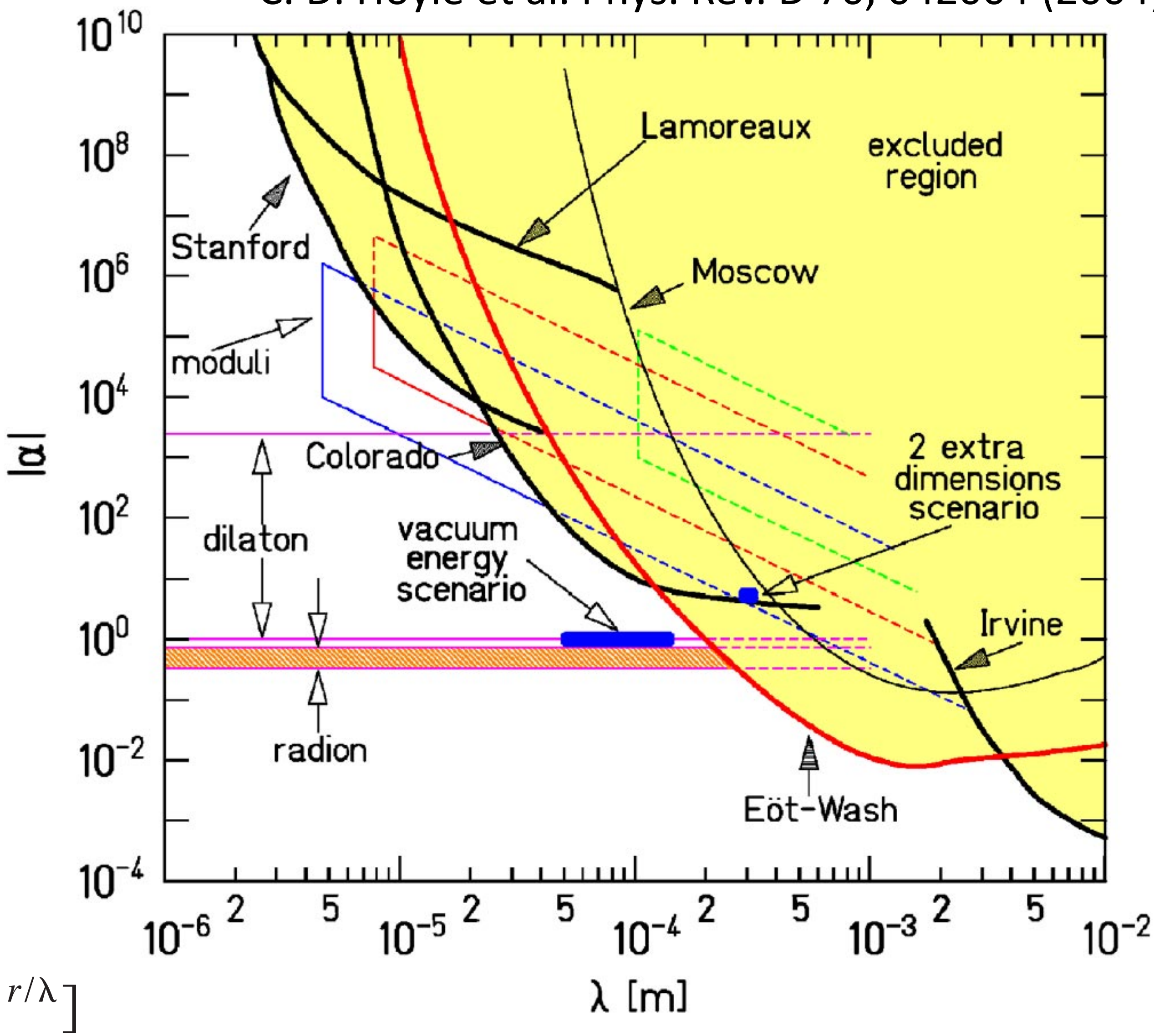
- The coupling of the massive spin-2 object to matter is universal while that of the massive spin-0 is not: it does not couple to massless vector fields as it couples to the trace of the energy-momentum tensor.

- As we are dealing with a classical field, the fact that the massive spin-2 is a ghost is not an obvious problem, it simply means that it couples to matter with a negative Planck mass.

Eöt-Wash pendulum experiment

C. D. Hoyle et al. Phys. Rev. D 70, 042004 (2004)





$$V(r) = -G\frac{m_1 m_2}{r}[1 + \alpha e^{-r/\lambda}]$$

Quantum gravity correction to Newton's Law

- We can easily deduce the quantum gravitational corrections to the Newtonian potential of a point mass

$$\Phi(r) = -\frac{Gm}{r}\left(1 + \frac{1}{3}e^{-Re(m_0)r} - \frac{4}{3}e^{-Re(m_2)r}\right)$$

- Note that the imaginary parts of the masses cancel out.

- In the absence of accidental fine cancellations between both Yukawa terms, the current bounds imply m_0 , $m_2 > (0.03\ \text{cm})^{-1} = 6.6 \times 10^{-13}$ GeV.

- Note that the experiment performed by Hoyle et al. is probing separations between 10.77 mm and 137 μm, a cancelation between the two Yukawa terms on this range of scales seems impossible without modifying general relativity with new physics to implement a screening mechanism.

Summary of bounds on the EFT parameters

- We can describe any theory of quantum gravity below the Planck scale using effective field theory techniques:

$$S=\int d^4x\,\sqrt{-g}\left[\left(\frac{1}{2}M^2+\xi H^\dagger H\right)\mathcal{R}-\Lambda_C^4+c_1\mathcal{R}^2+c_2\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}+\mathcal{L}_{SM}+\mathcal{O}(M_\star^{-2})\right]$$

- Planck scale $(M^2+\xi v^2)=M_P^2$ $\quad M_P=2.4335\times 10^{18}$ GeV
- $\Lambda_C \sim 10^{-12}$ GeV; cosmological constant.
- $M_\star >$ few TeVs from QBH searches at LHC and cosmic rays.
- Dimensionless coupling constants ξ, c_1, c_2
  - c_1 and $c_2 < 10^{61}$ [xc, Hsu and Reeb (2008)]

    R^2 inflation requires $c_1=9.7\times 10^8$ (Faulkner et al. astro-ph/0612569]).
  - $\xi < 2.6\times 10^{15}$ [xc & Atkins, 2013]

    Higgs inflation requires $\xi \sim 10^4$.

Can quantum gravity account for dark matter?

- If the massive spin-0 and spin-2 fields are components of the dark matter content of the universe nowadays, their masses have to be such that none of these partial decay widths should enable these fields to decay faster than the current age of the universe.

- From the requirement that the lifetime of the spin-0 σ is longer than current age of the universe, we can thus get a bound on c_2 using the gravitational decay width.

- We find $\tau = 1/\Gamma = 7.2 \times 10^{-17}\sqrt{c_2^3}\ \mathrm{GeV}^{-1} > 13.77 \times 10^9 \mathrm{y}$

and thus $c_2 > 4.4 \times 10^{38}$ and a similar bound on $3c_1 + c_2$

Quantum Gravity as Dark Matter

- Note that the Eöt-Wash experiment implies $c_2 < 10^{61}$.

- We thus find a bound:

$$4.4\times10^{38} < c_2 < 10^{61} \quad \text{or} \quad 1\times10^{-12}\,\text{GeV} < M_0 < 0.16\ \text{GeV}.$$

- Again a similar bound applies to the combination $3c_1 + c_2$ and thus to M_2.
- Clearly such light dark matter candidates could not decay to the massive gauge bosons of the standard model, its charged leptons such as the electron or the quarks.

- They could however decay to gluons, photons and potentially neutrinos.

- The decay to photons might be of astrophysical relevance and could be observable by gamma-ray experiments.

Production of Dark Matter

- The fact that our dark matter candidates are light points towards the vacuum misalignment mechanism.

- Indeed, in an expanding universe both σ and k have an effective potential in which they oscillate.

- The amount of dark matter produced by this mechanism becomes simply a randomly chosen initial condition for the value of the field in our patch of the universe.

- Quantum gravity could thus easily account for dark matter, maybe in conjunction with primordial black holes.

Gravitational Waves

- The new modes could affect the dynamics between two astrophysical bodies via the modified Newtonian potential.

- Furthermore, the new classical fields could be produced in high energetic astrophysical or cosmological events.

- In binary system, only the massive spin-2 can be produced, it has 5 polarizations. As the trace of the energy-momentum tensor is conserved the spin-0 cannot be produced.

- In phase transitions, both the massive spin-0 and spin-2 modes could be produced.

- There are thus three kinds of waves in quantum gravity: the massless gravitational waves that have just been observed and massive waves.

- A typical merger has enough energy to produce the new waves:
  $36\,M_\odot + 29\,M_\odot \rightarrow 62M_\odot + 3M_\odot$(gravitational wave)

- But, the bound on the quantum gravitational corrections to Newton's potential imply that quantum gravity could only impact the final moments of the inspiraling of binary of two neutron stars or of two black holes.

- Their effect will only become relevant at distances shorter than 0.03 cm.

- This distance is well within the Schwarzschild radius of any astrophysical black hole and clearly tools from numerical relativity need to be employed to obtain a reliable computation.

- Maybe the situation is not so bad for black holes as the mass is centered around the "singularity".

- A lengthy calculation leads to a remarkable result: the energy E carried away by the massive spin-2 mode from a binary system per frequency is identical to that of massless spin-2 mode:

$$\frac{dE_{massive}}{d\omega} = \frac{G_N}{45}\omega^6 \langle Q_{ij}Q^{ij}\rangle \theta(\omega - m_2)$$

- The total wave emission by a binary system is thus given by

$$\frac{dE}{d\omega} = \frac{dE_{massless}}{d\omega} + \frac{dE_{massive}}{d\omega}$$

- This is the confirmation of what I had told you already, the massive spin-2 is not a ghost, it is simply a field that couples to matter with the opposite sign in comparison to the massless spin-2 classical graviton.

- Note that the massive waves will be damped, e.g.:

$$m_0^2 = \frac{1}{\kappa^2(3c_1 + c_2)} - i\pi \frac{1}{\kappa^2(3c_1 + c_2)^2}(3b_1 + b_2 + b_3)$$

Quantum Corrections to Black Holes

- Quantum effects could also impact black holes themselves and thus matter for gravitational wave production.

- We have thus revisited the issue of quantum corrections to the Schwarzschild black hole solution which have been studied in the past by Duff and Donoghue et al. .

- We identify a complication which has not been realized previously, namely that of how to define a black hole.

- A mathematically consistent way to define a black hole is to define it as a static vacuum solution, i.e., an eternal black hole.

- If this definition is adopted, we obtain a result that differs from previous investigations by Duff or Donoghue et al.

- In particular, we have shown that there are no correction at 2^{nd} order in curvature to the Schwarzschild solution.

- Leading order quantum corrections to vacuum solutions appear at 3rd order in curvature.

- On the other hand, we found that if we look at quantum corrections to a collapsing star modeled by a ring of matter, we do find corrections at 2^{nd} order in curvature.

Absence of perturbative correction to Schwarzschild black hole

- We write the metric as follows

$$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{Sch.}} + g_{\mu\nu}^{\mathrm{q}}$$

where g^q represents the quantum correction to Schwarzschild solution.

- Linearizing the field equations around g^{Sch}, one finds

$$G_{\mu\nu}^{\mathrm{L}}\left[g^{\mathrm{q}}\right] + H_{\mu\nu}\left[g^{\mathrm{Sch.}}\right] + H_{\mu\nu}^{\mathrm{q}}\left[g^{\mathrm{Sch.}}\right] = 0$$

- It is well known that $H_{\mu\nu}[g^{Sch}] = 0$

- A lengthy calculation shows that $H_{\mu\nu}{}^{q}[g^{Sch}] = 0$.

- There are no correction to Schwarzschild's metric at 2nd order in curvature

- While there are no corrections at quartic order in curvature, which is in sharp contrast with previous results, there will be corrections at higher order for example higher dimensional operators such as

$$c_6 R^{\mu\nu}{}_{\alpha\sigma} R^{\alpha\sigma}{}_{\delta\gamma} R^{\delta\gamma}{}_{\mu\nu}$$

- will lead to quantum corrections of the Schwarzschild solution.

- We are doing perturbation around the standard Schwarzschild solution

$$A(r) = 1 - \frac{2MG}{r} + h(r)$$

- Far away from the hole, we find

$$h(r) = c_6 \frac{576\pi G_N^3 M^2}{r^6}$$

- This simply demonstrates that the Schwarzschild solution is not a solution of the field equations when higher dimensional operators of dimensions d≥6 are included.

Corrections to the gravitational field of a static source

- It is interesting to understand the effect of the quantum induced non-locality on the field of a static spherically symmetric object such as a star.

- We aim for a simplified treatment and thus we only consider $\alpha R \ln\left(\frac{\Box}{\mu^2}\right) R$

- We use a perturbative approach again compatible with our EFT approach.

$$g = -B(r)\mathrm{d}t^2 + A(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

- The solution to Einstein equation for a constant density star is known in closed form.

$$A(r) = \left[1 - \frac{2G\mathcal{M}(r)}{r}\right]^{-1}, \quad \mathcal{M}(r) = \int^r \rho\,\mathrm{d}V = \int_0^r 4\pi r'^2 \rho(r')\,\mathrm{d}r.$$

$$B(r) = \exp\left\{-\int_r^\infty \frac{2G}{r'^2}[\mathcal{M}(r') + 4\pi r'^3 P(r')]A(r')\,\mathrm{d}r'\right\}.$$

In these equations ρ is the density of the star and P its pressure.

- Outside the star, the non-locality introduces a non-trivial contribution

$$G^{\mathrm{L}}_{\mu\nu} = \alpha(16\pi G_N)^2(\nabla_\mu\nabla_\nu - g_{\mu\nu}\Box)\int_{\mathrm{S}} d^4x'\sqrt{g}\,L(x,x';\mu)\,T$$

where the integral extends only over the source region,
$T = \rho_0 - 3P$ is the trace of the energy-momentum tensor, ρ_0 is the mass density and P is the pressure.

- Both the pressure and metric functions are known in the interior of the star

$$P(r) = \rho_0 \frac{(1-2GMr^2/R_S^3)^{1/2} - (1-2GM/R_S)^{1/2}}{(1-2GM/R_S)^{1/2} - 3(1-2GMr^2/R_S^3)^{1/2}}$$

where R_S is the radius of the star.

- Far away from the source, we find $g^{\mathrm{q}}_{\mathrm{tt}} = \dfrac{18\alpha l_{\mathrm{P}}^2}{R_S^2}\dfrac{2G_N M}{r}, \quad g^{\mathrm{q}}_{\mathrm{rr}} = \dfrac{12G_N M\alpha l_{\mathrm{P}}^2}{r^3}$
- The limit R_S to 0 is ill-defined, we can't recover the eternal black hole result in that limit.

- While eternal black holes are mathematically well defined, they may not capture the full physical picture.

- A real, astrophysical, black hole is the final state of the evolution of a matter distribution, for example of a heavy star, after it has undergone gravitational collapse.

- This process is certainly not happening in vacuum.

- This raises the question of how to define a real astrophysical black hole and of how to calculate quantum corrections to its metric.

Quantum gravity and minimalistic inflation models

- Given the lack of new physics at the LHC, it is crucial to investigate whether the standard model Higgs and/or general relativity can describe inflation.

- The EFT approach to quantum gravity is the right framework for this.

- We need scalar degrees of freedom: Higgs boson or the scalar hidden in higher gravitational operators such as R^2 (Starobinsky inflation).

- Quantum gravity offers diverse model building options for inflation: Higgs inflation, Starobinsky or Higgs-Starobinsky inflation.

Higgs inflation

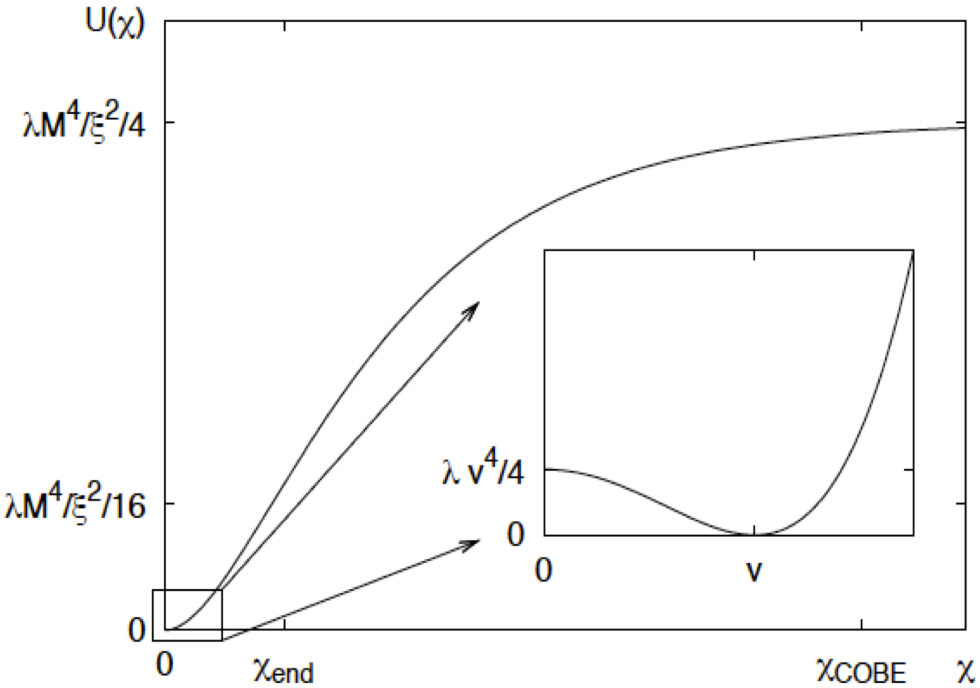


Fig. 1. Effective potential in the Einstein frame.

$$S = -\int d^4x\,\sqrt{-g}\left(\frac{1}{2}M^2 + \xi H^\dagger H\right) R$$

$$\xi \sim 10^4$$

A large non-minimal coupling
does not introduce a new energy scale
below the Planck scale

Our work demonstrates that
the potential is stable and the model
viable!

R^2 inflation

- The model is defined by the action in the Jordan frame

$$S^{J}_{Starobinsky} = \int d^4x \sqrt{g} \frac{1}{2} \left(M_P^2 R + c_S R^2 \right)$$

- which corresponds to an Einstein frame action given by

$$S^{E}_{Starobinsky} = \int d^4x \sqrt{g} \left(\frac{M_P^2}{2} R - \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{M_P^4}{c_S} \left(1 - \exp\left(-\sqrt{\frac{2}{3}} \frac{\sigma}{M_P} \right) \right)^2 \right)$$

- Assuming that the scalar field σ hidden in R^2 takes large values in the early universe, a successful prediction of the density perturbation δϱ/ϱ requires

$$c_S = 0.97 \times 10^9$$

Strength of gravity and Grand Unification

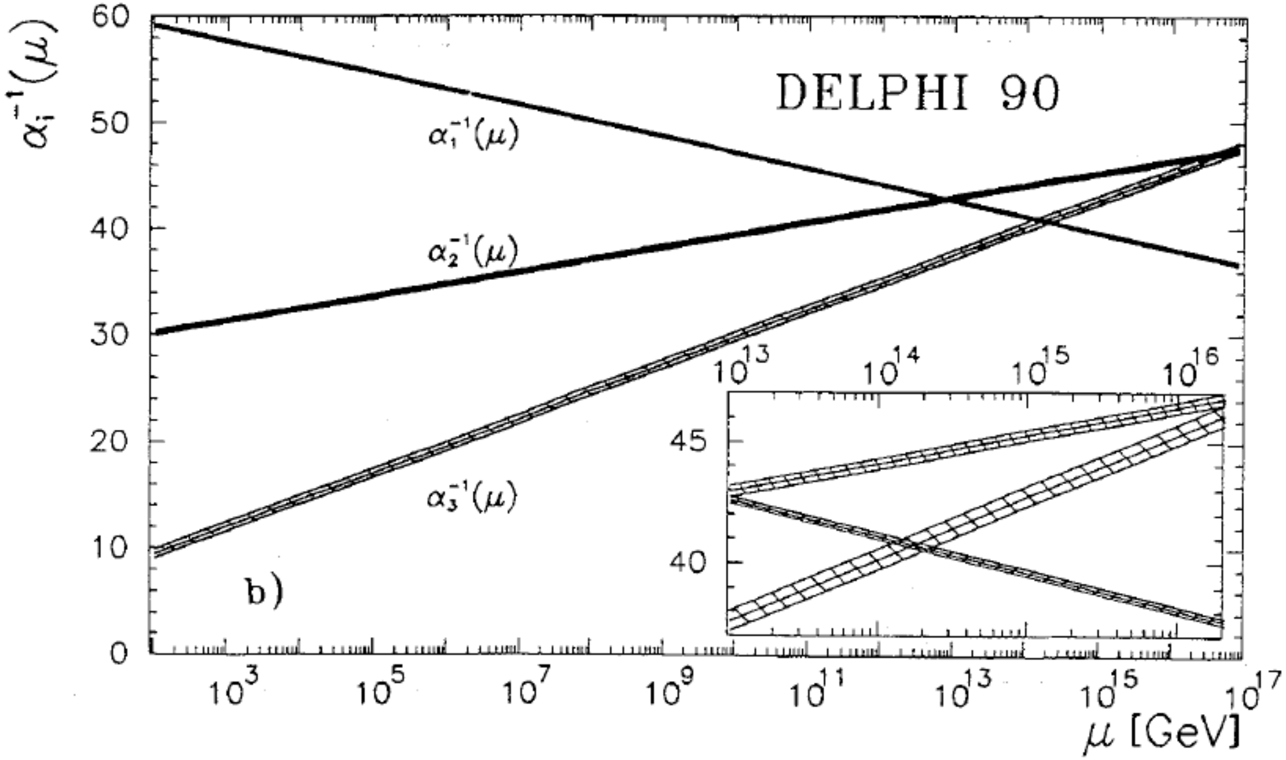
- EFT techniques can be applied in models of grand unified theory.

- Already 1000 new particles can affect M_P and reduce its value by one order of magnitude!

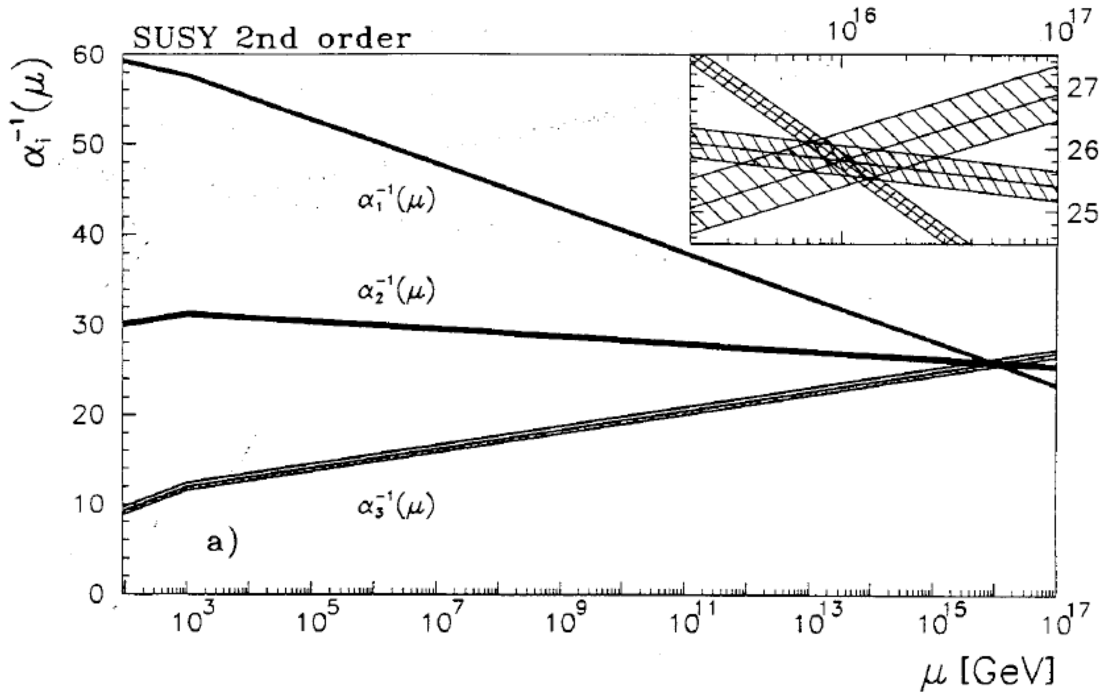
- 1000 new particles: this is quite common in GUTs!

- That's important for grand unified theories:

$$\mathcal{O} = \frac{c}{M_P} \, \mathrm{Tr}(G_{\mu\nu} G^{\mu\nu} H)$$



Typical argument for supersymmetry



We expect strong gravitational effects at an energy scale of (linked to perturbative unitarity bound and resummation/self-healing).

$$\sim \sqrt{\frac{120\pi}{NG_N}}$$

So M_P in our dim 5 operator should really be that scale!

$$\mathcal{O} = \frac{c}{M_P}\,\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}H)$$

The new operators change the unification condition

$$\begin{aligned}\alpha_G &= (1+\epsilon_1)\,\alpha_1(M_X) = (1+\epsilon_2)\,\alpha_2(M_X)\\ &= (1+\epsilon_3)\,\alpha_3(M_X)\ .\end{aligned}$$

where

$$\epsilon_1 = \frac{\epsilon_2}{3} = -\frac{\epsilon_3}{2} = \frac{\sqrt{2}}{5\sqrt{\pi}}\frac{c\eta}{\sqrt{\alpha_G}}\frac{M_X}{\hat{M}_{\rm Pl}} \qquad \eta = \sqrt{1+\frac{N}{12\pi}}$$

- One can show that this gravitationally interaction between the Higgs bosons of the grand unified theory and the force carriers of the grand unified theory leads to an uncertainty which is bigger than the two loop effect when it comes to the unification of the gauge coupling

- E.g. the uncertainty in α_1 at the unification scale due to the new operator is of the order of 2.1% whereas the two loop correction is of the order of 1.7%

- Furthermore, for many values of *c* and *N*, there is no solution to the unification condition.

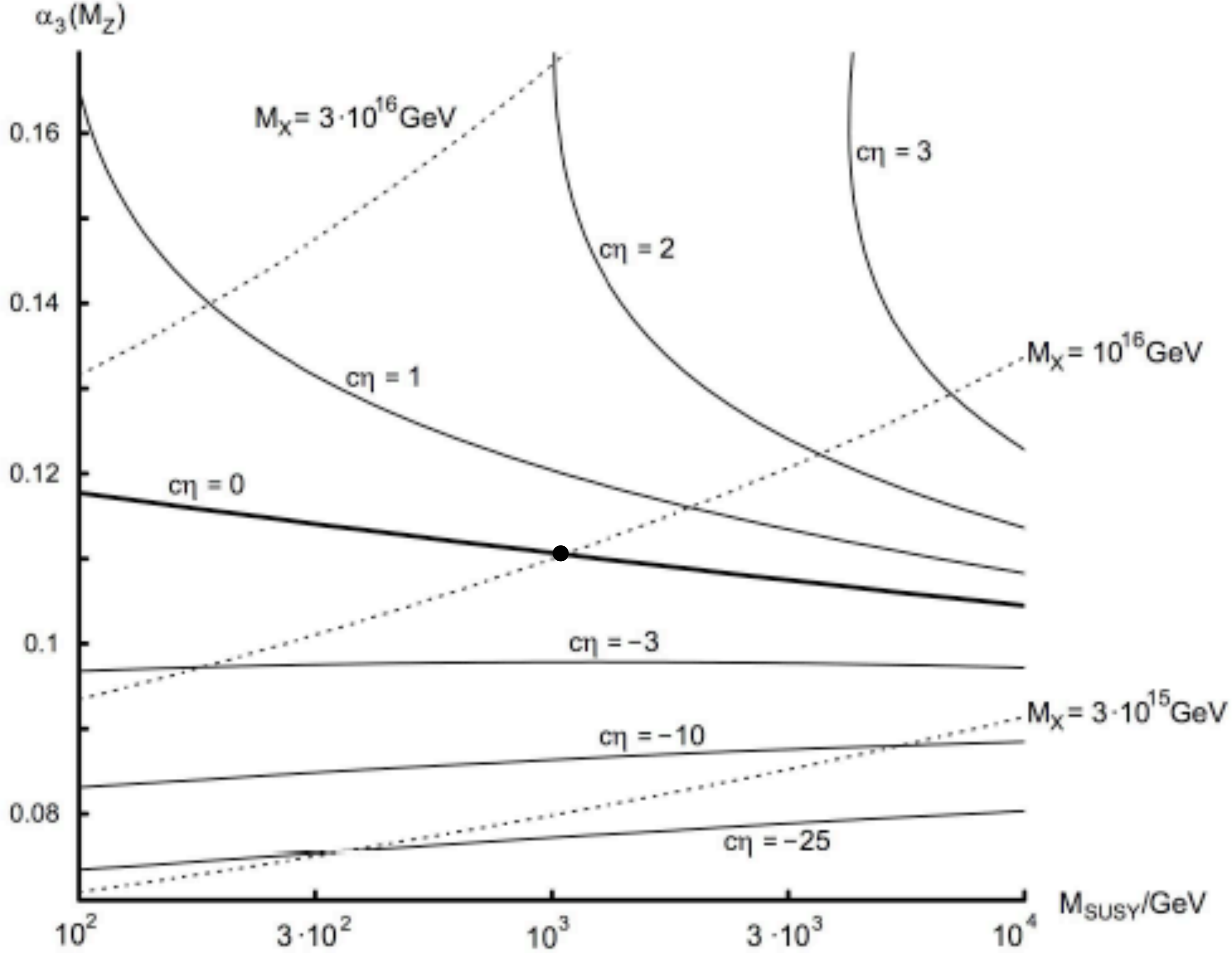
- On the other hand, we can easily unify models which apparently don't work out (like the standard model) without the need for low energy new physics.

- NB: there are similar operators affecting the unification of fermion masses: $\frac{c}{\hat{\mu}_\star}\bar{\Psi}\phi\Psi H + h.c.$

XC, S. Hsu & D. Reeb

Usual solution: $\alpha_3(M_Z)$=0.117, M_{SUSY}=1 TeV

$$\eta = \sqrt{1 + \frac{N}{12\pi}}$$



LEP does not favor supersymmetric unification!!!

Conclusions

- We have discussed a conservative effective action for quantum gravity within usual QFTs such as the standard model or GUT.

- EFT techniques lead to predictions which can be confronted to data.

- We have seen some universal features of quantum gravity: the Planck scale is dynamical, space-time becomes non-local at that scale & strong dynamics at the Planck scale.

- This progress in quantum gravity enables phenomenological applications, e.g. dark matter.
- New model building tools in early universe cosmology
- Applications to gravitational waves
- Applications to grand unified theories
- One cannot ignore quantum gravity, even at low energy!

Thanks for your attention!