Quantum gravity with purely virtual particles:

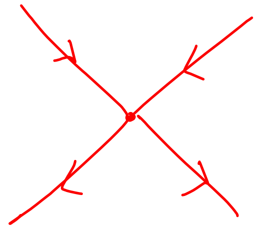
from quantum field theory

to primordial cosmology

Damiano Anselmi

The Fermi theory of the weak interactions is nonrenormalizable, because it contains a vertex with four fermions, muliplied by the Fermi constant, which has a negative dimension in units of mass

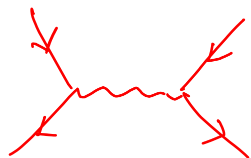
$$G_F(\bar{\psi}\psi)^2$$



$$[G_F] = -2$$

The problem can be solved by introducing massive intermediate bosons, Z and W

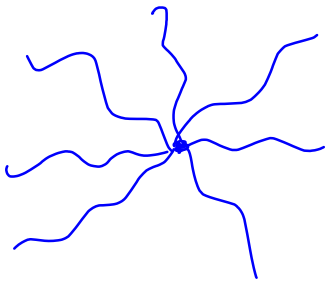
$$g\bar{\psi}A_\mu\psi$$



$$[g] = 0$$

Yang-Mills theory was formulated in 1954, but it remained in the shadow for long, because it was not known, at that time, how to give mass to the inermediate bosons. The spontaneous symmetry breaking mechanism and asymptotic freedom finally brought it to light

The Einstein-Hilbert action and the Starobinsky R+R^2 theory inflation are nonrenormalizable, like the Fermi theory. They contain vertices with infintely many external legs, and a coupling of negative dimension



$$[G_N] = -2$$

Again, the problem can be solved by introducing intermediate massive particles

Again, the particles are of a type not known before, but this time we need to change the very meaning of the word "particle" to gain renormalizability without losing unitarity

$$[g] = 0$$

Summary

- Purely virtual particles (fakeons): quantization prescription and comparison with alternative possibilities

- The quantum field theory of gravity

- Predictions in primordial cosmology:

    - fakeon projection on curved backgrounds

    - cosmic RG flow: fine structure constant, beta function, running coupling, resummation of leading and subleading logs...

    - perturbation spectra

    - r, $r + 8\, n_T$, tilts, running coefficients, etc.

The fakeon is a purely virtual particle

- It simulates a physical particle in many situations, but does not belong to the physical spectrum of asymptotic states

- It must be first introduced and projected away at the very end, to have unitarity "and keep everything local"

- Think of the Faddeev-Popov ghosts: they must be introduced to fix the gauge (while keeping the action local), but must be projected away at the end to recoved unitarity

- The key idea is a new quantization prescription for the poles of the free propagators, alternative to the Feynman i epsilon one

- The fakeon prescription/projection does not follow from a symmetry principle and it is not protected by that to all orders

- The fakeon can be proved to be consistent to all orders by means of a completely different approach

- The problem of quantum gravity is to make it renormalizable and unitary at the same time
- Higher derivatives lead to renormalizability, but violate unitarity (or so we are told)
- $$\frac{1}{(p^2-m_1^2)(p^2-m_2^2)} = \frac{1}{m_1^2-m_2^2}\left[\frac{1}{p^2-m_1^2} - \frac{1}{p^2-m_2^2}\right]$$

Unitarity $\quad : \quad S^{+} S = 1$

$$\sum_{|n\rangle \in V} \langle \overset{\text{out}}{a} | S^{+} \underbrace{|n\rangle\langle n|}^{\text{intermediate}} S | \overset{\text{in}}{b} \rangle = \langle \overset{\text{out}}{a} | \overset{\text{in}}{b} \rangle \qquad \sum_{|n\rangle \in V} |n\rangle\langle n| = \mathbb{1}$$

Pseudounitarity

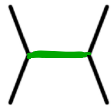
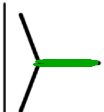
$$\sum_{|n\rangle \in W} \langle \overset{\text{out}}{a} | S^{+} \overbrace{|n\rangle (-1)^{\sigma_n} \langle n|}^{\text{intermediate}} S | \overset{\text{in}}{b} \rangle = \langle \overset{\text{out}}{a} | \overset{\text{in}}{b} \rangle$$

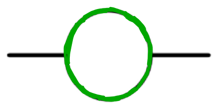
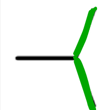
$$\sum_{|n\rangle \in W} |n\rangle (-1)^{\sigma_n} \langle n| = 1$$

$\sigma_n = 0$ for physical particle

$\sigma_n = 1$ for ghost

Diagrammatic version of the optical theorem:

$$2\,\mathrm{Im}\left[(-i)\right.$$  $$\left.\right] = \int d\Pi_f \left|\right.$$  $$\left.\right|^2$$

$$2\,\mathrm{Im}\left[(-i)\right.$$  $$\left.\right] = \int d\Pi_f \left|\right.$$  $$\left.\right|^2$$

"cutting equations"

Quantisation prescription

Propagator : $\pm \dfrac{1}{p^2 - m^2}$

Feynman : $\pm \dfrac{1}{p^2 - m^2 + i\epsilon}$ $\qquad \pm \delta(p^2 - m^2)$

Fakeon: 1) $\pm \dfrac{1}{2}\left[\dfrac{1}{p^2 - m^2 + i\epsilon} + \dfrac{1}{p^2 - m^2 - i\epsilon}\right]$

BUT BEWARE !! (= 0 on shell !) $\longrightarrow$ $0 \cdot \delta(p^2 - m^2)$

2) follow a certain procedure in loop diagrams

Bubble diagram

D. Anselmi, The quest for purely virtual quanta: fakeons versus Feynman-Wheeler particles, J. High Energy Phys. 03 (2020) 142, arXiv:2001.01942 [hep-th]

Feynman

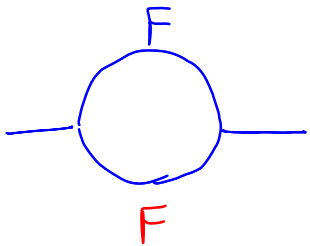
$$\frac{1}{p^2-m^2+i\epsilon}$$

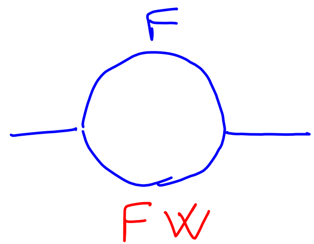
Feynman-Wheeler

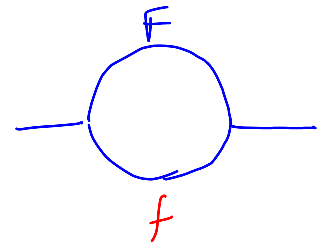
$$\frac{1}{2}\left(\text{Feynman}+\text{Feynman}^*\right)$$

Fakeon

$$\left.\frac{1}{p^2-m^2}\right|_f$$







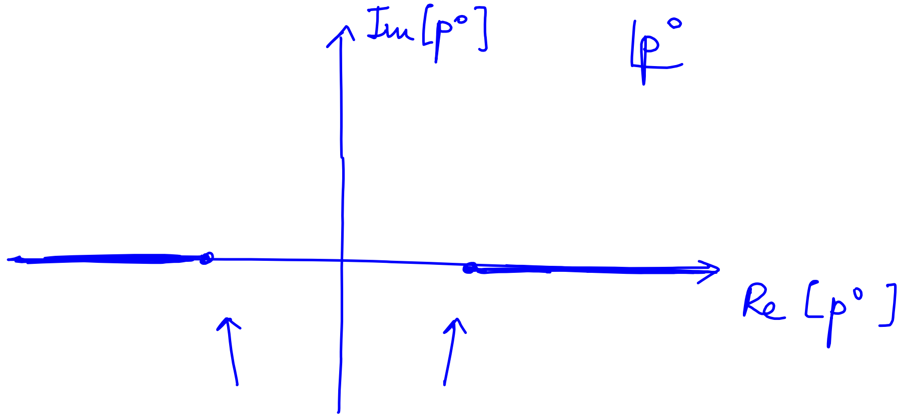
$$\int\frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{(p-k)^2-m_1^2+i\epsilon}\,\frac{1}{k^2-m_2^2\pm i\epsilon}$$

!!!

In formulas Feynman-Feynman

$$\int_0^1 dx \ln\left[-p^2 x(1-x) + m_1^2 x + m_2^2(1-x) - i\epsilon\right]$$

Analytic structure:

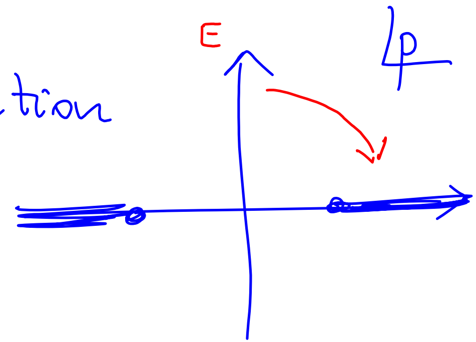


$p^2 = (m_1+m_2)^2$: threshold

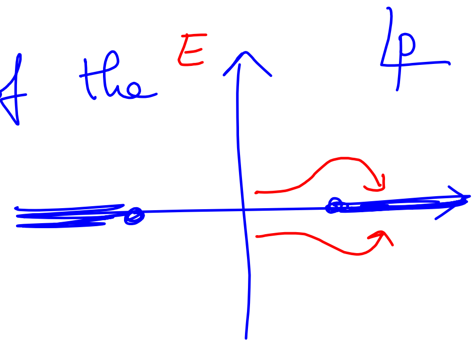
Feynman - fakeon

1) Start from the Euclidean version

2) Initiate the Wick rotation



3) Complete the Wick rotation nonanalytically by means of the average continuation

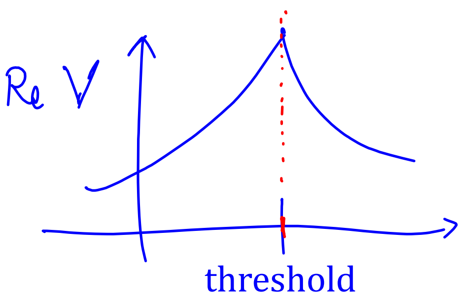


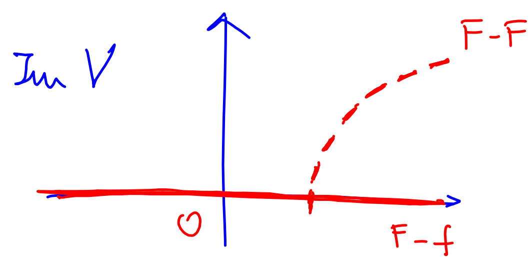
In formulas Feynman-Feynman

$$V \equiv \int_0^1 dx \ln\left[-p^2 x(1-x) + m_1^2 x + m_2^2(1-x) - i\epsilon\right]$$

becomes Feynman-fakeon (= fakeon-fakeon)

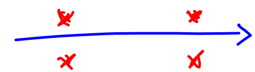
$$V = \frac{1}{2}\int_0^1 dx \ln\left[\left(-p^2 x(1-x) + m_1^2 x + m_2^2(1-x)\right)^2\right]$$



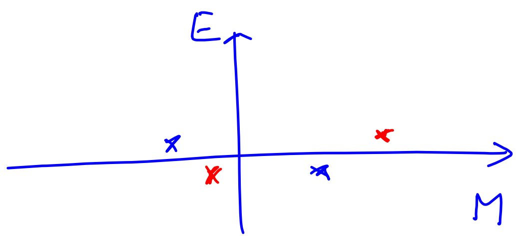


F - F : From Euclidean or directly in Minkowsky

F - FW : directly in Minkowsky

$\mathcal{P}\,\frac{1}{p^2-m^2}$: 

F - f : from Euclidean



F - FW:

$$\Sigma'(p) = \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{1}{(p-k)^2 - m_1^2 + i\epsilon} \frac{1}{k^2 - m_2^2 - i\epsilon}$$

careful !

$$= -\int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{1}{(p-k)^2 - m_1^2 + i\epsilon} \frac{1}{-k^2 + m_2^2 + i\epsilon}$$

$$= -\int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{1}{\left[i\epsilon - (1-2x)k^2 + p^2 \frac{x(1-x)}{1-2x} - m_1^2 x + m_2^2(1-x)\right]^2},$$

$$= U(p^2 + i\epsilon, m_1^2, m_2^2) - U(p^2 - i\epsilon, m_2^2, m_1^2)$$

$$i(4\pi)^2 U(p^2, m_1^2, m_2^2) = \frac{v_+}{2p^2}\left(\ln \frac{4\Lambda^2}{m_2^2} - z \ln \frac{1+z}{1-z}\right)$$

$\left(\ln \Lambda = \frac{2}{\varepsilon}\right)$

non local divergence!

$$z = \frac{\sqrt{u_+ u_-}}{v_+}, \qquad u_\pm = (m_1 \pm m_2)^2 - p^2, \qquad v_\pm = p^2 \mp m_1^2 \pm m_2^2$$

Renormalizability and locality

Counterterms :

Feynman	Feynman-Wheeler	Fakeon
$\dfrac{i \ln \Lambda^2}{4\pi^2}$	$\dfrac{i \ln \Lambda^2}{4\pi^2} \dfrac{m_1^2 - m_2^2}{p^2}$	$\dfrac{i \ln \Lambda^2}{4\pi^2}$
local	non local	local

Stability : Thresholds

Feynman

$$p^2 = (m_1 + m_2)^2$$

Feynman-Wheeler

$$p^2 = (m_1 - m_2)^2$$

Fakeon

$$p^2 = (m_1 + m_2)^2$$

(fake)

Unitarity:

$$2\,\mathrm{Im}(-i\mathcal{M}) = \frac{\sqrt{p^2-(m_1+m_2)^2}\sqrt{p^2-(m_1-m_2)^2}}{8\pi p^2} \quad \text{times:}$$

$\Theta(p^2-(m_1+m_2)^2)$ $\qquad$ $-\Theta((m_1-m_2)^2-p^2)$ $\qquad$ 0

Feynman $\qquad$ Feynman-Wheeler $\qquad$ Fakeon

What about the Lee-Wick models?

Reformulated, they

become something

D. Anselmi and M. Piva, A new formulation of Lee-Wick quantum field theory, J. High Energy Phys. 06 (2017) 066, arXiv:1703.04584 [hep-th];

D. Anselmi and M. Piva, Perturbative unitarity in Lee-Wick quantum field theory, Phys. Rev. D 96 (2017) 045009 arXiv:1703.05563 [hep-th].

totally different from what Lee and Wick intended

LW never conceived the idea of projecting states away consistently

Their approach is more like "living with ghosts"

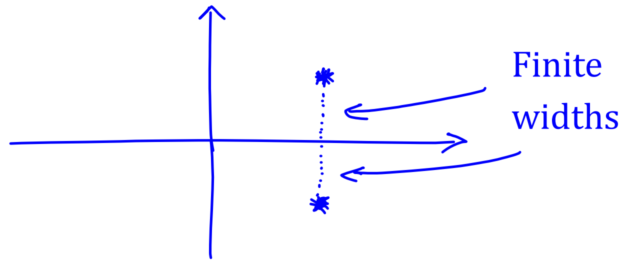
Hawking, Hertog, "Living with ghosts", PRD 65 (2002) 103515 arXiv:hep-th/0107088

NO THANKS!

The muons are a good example why this procedure is wrong

But there was another problem:

LW models apparently need pairs of complex conjugate poles



This requires MORE higher-derivatives

$$\int \left(R + R^2 + R\Box R\right)\sqrt{-g}$$

but then gravity is superrenormalizable (unlike the gauge interactions of the standard model) and NOT UNIQUE

The new models do not have this problem and with a few tricks they can handle poles on the real axis (zero widths at the tree level)

D. Anselmi, On the quantum field theory of the gravitational interactions, J. High Energy Phys. 06 (2017) 086, arXiv: 1704.07728 [hep-th]

QG becomes unique

D. Anselmi, Fakeons and Lee-Wick models, J. High Energy Phys. 02 (2018) 141 arXiv:1801.00915 [hep-th].

All-order theorems here

Quantum gravity

Consider the (renormalizable) higher-derivative Lagrangian

$$S_{\mathrm{geom}}(g,\Phi) = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - \frac{R^2}{6m_\phi^2}\right)$$

If quantized as usual, this theory has a ghost

K.S. Stelle, Renormalization of higher derivative quantum gravity, Phys. Rev. D 16 (1977) 953.

Solution: quantize the would-be ghost as a purely virtual particle

D. Anselmi, On the quantum field theory of the gravitational interactions, J. High Energy Phys. 06 (2017) 086, arXiv: 1704.07728 [hep-th]

IMPORTANT: the action

$$S_{\text{geom}}(g,\Phi)=-\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R+\frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}-\frac{R^2}{6m_\phi^2}\right)$$

is NOT the classical limit of the quantum field theory of gravity, because it is unprojected

It should be regarded more or less like a gauge-fixed action: it is useful to derive simple Feynman rules, but it contains things (e.g. the Faddeev-Popov ghosts) that should be dropped to do physics

The classical limit is obtained by CLASSICIZING the quantum field theory of gravity, since the fakeon is purely quantum

D. Anselmi, Fakeons, microcausality and the classical limit of quantum gravity, Class. and Quantum Grav. 36 (2019) 065010, arXiv:1809.05037 [hep-th]

D. Anselmi, Fakeons and the classicization of quantum gravity: the FLRW metric, J. High Energy Phys. 04 (2019) 61, arXiv:1901.09273 [gr-qc]

The triplet of quantum gravity

$h_{\mu\nu}$ = graviton, fluctuation of the metric

ϕ = inflaton, mass m_ϕ

$\chi_{\mu\nu}$ = fakeon, spin 2, mass m_χ

Renormalization constants, beta functions, widths, absorptive parts, dressed propagators, checks of the optical theorem...

D. Anselmi and M. Piva, The ultraviolet behavior of quantum gravity, J. High Energ. Phys. 05 (2018) 27, arXiv:1803.07777 [hep-th]

D. Anselmi and M. Piva, Quantum gravity, fakeons and microcausality, J. High Energy Phys. 11 (2018) 21, arXiv:1806.03605 [hep-th]

D. Anselmi, E. Bianchi and M. Piva, Predictions of quantum gravity in inflationary cosmology: effects of the Weyl-squared term, J. High Energy Phys. 07 (2020) 211, arXiv:2005.10293 [hep-th]

D. Anselmi, Cosmic inflation as a renormalization-group flow: the running of power spectra in quantum gravity, arXiv:2007.15023 [hep-th]

D. Anselmi, High-order corrections to inflationary perturbation spectra in quantum gravity, arXiv: 2010.04739 [hep-th]

For cosmology, we use

$$S_{\mathrm{QG}} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R+\frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)+\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(D_\mu\phi D^\mu\phi-2V(\phi)\right),$$

$$V(\phi)=\frac{3m_\phi^2}{32\pi G}\left(1-\mathrm{e}^{\phi\sqrt{16\pi G/3}}\right)^2$$

Starobinsky potential

The Friedmann equations do not change

$$\dot{H}=-4\pi G\dot{\phi}^2, \qquad H^2=\frac{4\pi G}{3}\left(\dot{\phi}^2+2V(\phi)\right), \qquad \ddot{\phi}+3\frac{\dot{a}}{a}\dot{\phi}=-V'(\phi),$$

$$H=\dot{a}/a$$

Define the coupling ("fine structure constant" of inflation)

$$\alpha=\sqrt{\frac{4\pi G}{3}}\frac{\dot{\phi}}{H}=\sqrt{-\frac{\dot{H}}{3H^2}}$$

Then the Friedmann equations become

$$\dot{\alpha} = m_\phi \sqrt{1-\alpha^2} - H(2+3\alpha)\left(1-\alpha^2\right)$$

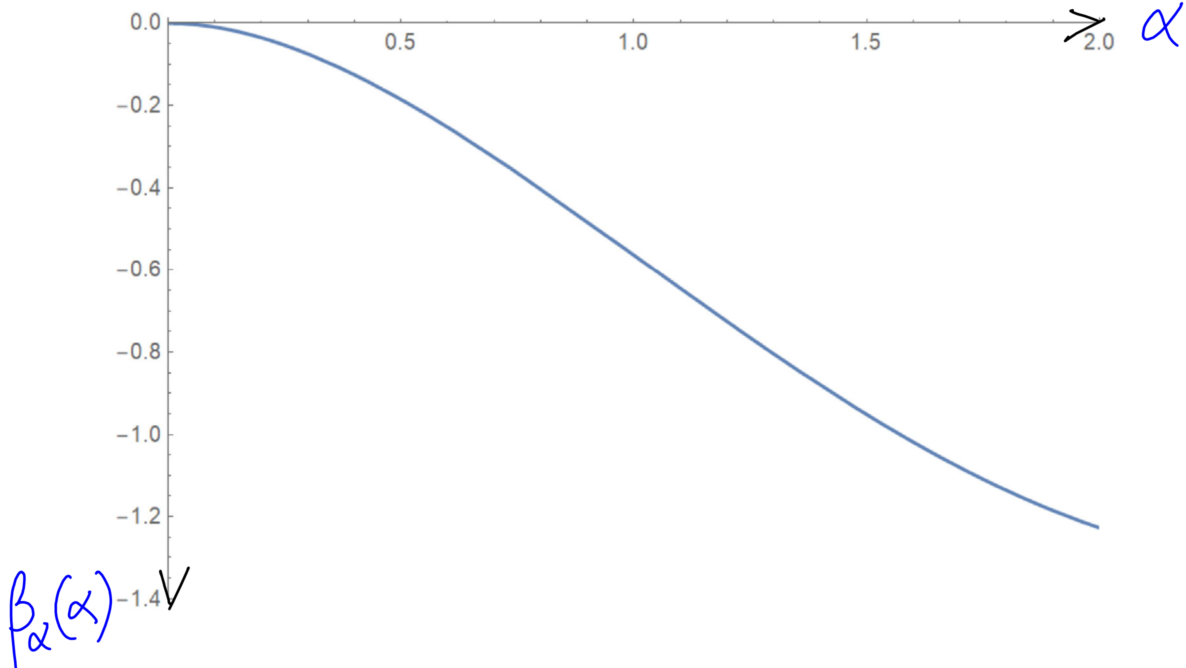
In conformal time $\tau = -\int_t^{+\infty} \frac{\mathrm{d}t'}{a(t')}$

We can introduce the beta function $\beta_\alpha = \frac{d\alpha}{d\ln|\tau|}$ of the

"cosmic RG flow"

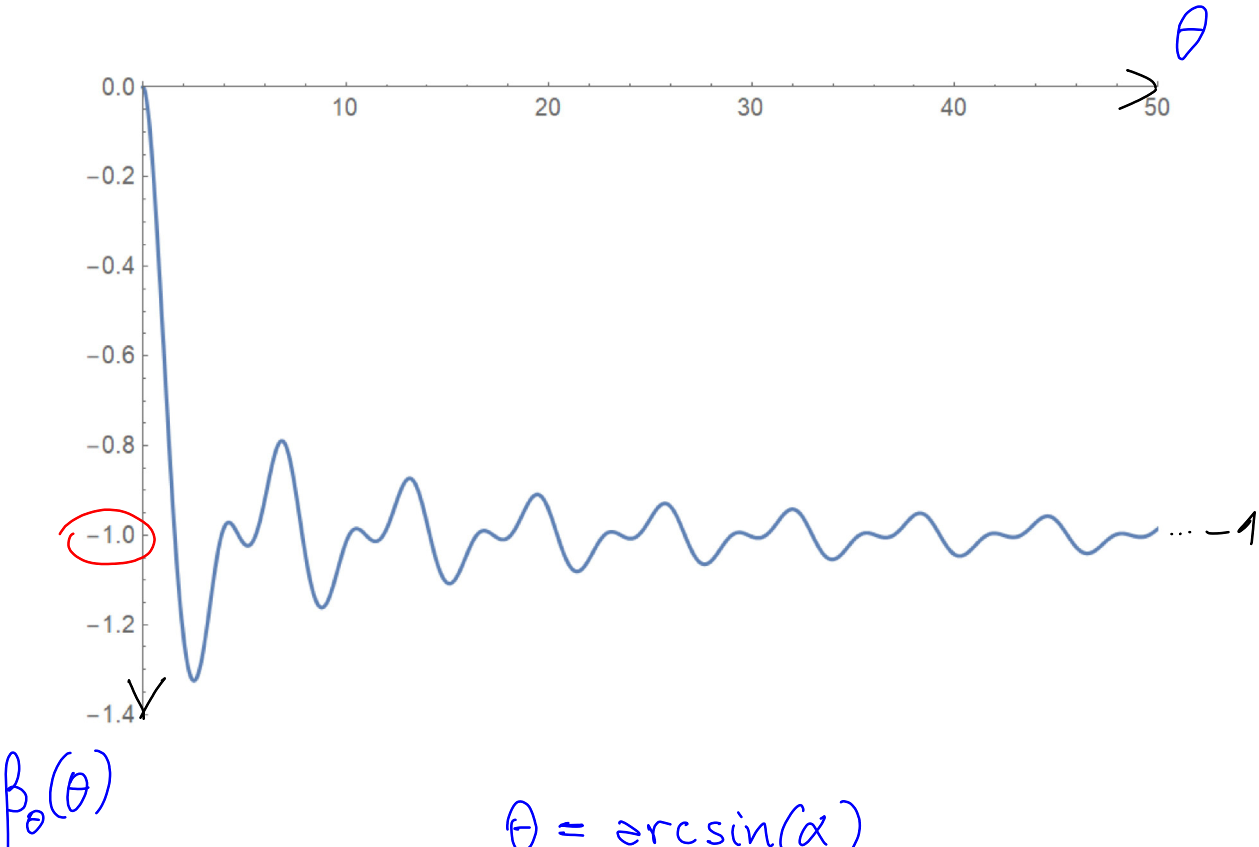
$$\beta_\alpha = -2\alpha^2 \left[1 + \frac{5}{6}\alpha + \frac{25}{9}\alpha^2 + \frac{383}{27}\alpha^3 + \frac{8155}{81}\alpha^4 + \frac{72206}{81}\alpha^5 + \frac{2367907}{243}\alpha^6 + \mathcal{O}(\alpha^7)\right]$$

D. Anselmi, Cosmic inflation as a renormalization-group flow: the running of power spectra in quantum gravity, arXiv:2007.15023 [hep-th]



$$\beta_\alpha(\alpha) = -2\alpha^2 + O(\alpha^3)$$

[like pure QCD]



$\theta = \arcsin(\alpha)$

Leading log running coupling

$$\alpha = \frac{\alpha_k}{1 + 2\alpha_k \ln(-k\tau)}$$

Running coupling to the NNLL order

$$\alpha = \frac{\alpha_k}{\lambda}\left(1 - \frac{5\alpha_k}{6\lambda}\ln\lambda\right)\left[1 + \frac{25\alpha_k^2}{12\lambda^2}\left(1 - \lambda - \frac{\ln\lambda}{3}(1 - \ln\lambda)\right)\right]$$

$$\lambda \equiv 1 + 2\alpha_k \ln\eta, \; \underline{\eta = -k\tau}$$

RG equations:

Spectra: $\mathcal{P}(k) = \tilde{\mathcal{P}}(\alpha_k)$ in the superhorizon limit

$\alpha_k = \alpha(1/k) =$ running coupling

Expansion in leading and subleading logs

$$\alpha(1/\kappa) = \alpha\left(\alpha_*, \ln \kappa/\kappa_*\right)$$

κ_* = pivot scale = $0.05\ \mathrm{Mpc}^{-1}$

α_* = pivot coupling = $0.0087 \pm 0.0090 \sim \frac{1}{115}$

Planck collaboration, Planck 2018 results. X. Constraints on inflation, arXiv:1807.06211 [astro-ph.CO].

Expansion in powers of α_* , but

the product $\alpha_* \ln \frac{\kappa}{\kappa_*}$ is considered of order zero

and treated exactly

Results

Effects of the fakeon $\chi_{\mu\nu}$ everywhere

$$\mathcal{P}_T(k) = \frac{4m_\phi^2 \zeta G}{\pi}\left[1 - 3\zeta\alpha_k\left(1 + 2\alpha_k\gamma_M + 4\gamma_M^2\alpha_k^2 - \frac{\pi^2\alpha_k^2}{3}\right) + \frac{\zeta^2\alpha_k^2}{8}(94 + 11\xi)\right.$$
$$\left. + 3\gamma_M\zeta^2\alpha_k^3(14+\xi) - \frac{\zeta^3\alpha_k^3}{12}(614 + 191\xi + 23\xi^2) + \mathcal{O}(\alpha_k^4)\right]$$

$$\mathcal{P}_\mathcal{R}(k) = \frac{Gm_\phi^2}{12\pi\alpha_k^2}\left[1 + (5 - 4\gamma_M)\alpha_k + \left(4\gamma_M^2 - \frac{40}{3}\gamma_M + \frac{7}{3}\pi^2 - \frac{67}{12} - \underbrace{\frac{\xi}{2}F_\mathrm{s}(\xi)}\right)\alpha_k^2 + \mathcal{O}(\alpha_k^3)\right]$$

First effect of the fakeon

where

$$\xi = \frac{m_\phi^2}{m_\chi^2}, \qquad \zeta = \left(1 + \frac{\xi}{2}\right)^{-1}, \qquad \tilde{\gamma}_M = \gamma_M - \frac{i\pi}{2}, \qquad \gamma_M = \gamma_E + \ln 2,$$

$$F_\mathrm{s}(\xi) = 1 + \frac{\xi}{4} + \frac{\xi^2}{8} + \frac{\xi^3}{8} + \frac{7\xi^4}{32} + \frac{19}{32}\xi^5 + \frac{295}{128}\xi^6 + \frac{1549}{128}\xi^7 + \frac{42271}{512}\xi^8 + \mathcal{O}(\xi^9)$$

"Dynamical" tensor-to-scalar ratio

$$r(k) = \frac{\mathcal{P}_T(k)}{\mathcal{P}_\mathcal{R}(k)}$$

Tilts

$$n_T = -\beta_\alpha(\alpha_k)\frac{\partial \ln \mathcal{P}_T}{\partial \alpha_k}, \qquad n_\mathcal{R} - 1 = -\beta_\alpha(\alpha_k)\frac{\partial \ln \mathcal{P}_\mathcal{R}}{\partial \alpha_k}$$

Running coefficients

$$\frac{\mathrm{d}^n n_T}{\mathrm{d}\ln k^n} = \left(-\beta_\alpha(\alpha_k)\frac{\partial}{\partial \alpha_k}\right)^n n_T, \qquad \frac{\mathrm{d}^n n_\mathcal{R}}{\mathrm{d}\ln k^n} = \left(-\beta_\alpha(\alpha_k)\frac{\partial}{\partial \alpha_k}\right)^n n_\mathcal{R}$$

$$n_T = -6\left[1 + 4\gamma_M\alpha_k + (12\gamma_M^2 - \pi^2)\alpha_k^2\right]\zeta\alpha_k^2 + \left[24 + 3\xi + 4(31 + 2\xi)\gamma_M\alpha_k\right]\zeta^2\alpha_k^3 - \frac{1}{8}(1136 + 566\xi + 107\xi^2)\zeta^3\alpha_k^4 + \mathcal{O}(\alpha_k^5),$$

$\xi = \frac{m_\phi^2}{m_\chi^2}$, $\zeta = \frac{2m_\chi^2}{m_\phi^2 + 2m_\chi^2}$

$$n_\mathcal{R} - 1 = -4\alpha_k + \frac{4\alpha_k^2}{3}(5 - 6\gamma_M) - \frac{2\alpha_k^3}{9}(338 - 90\gamma_M + 72\gamma_M^2 - 42\pi^2 + \underbrace{9\xi F_\mathrm{s}}) + \mathcal{O}(\alpha_k^4)$$

$$r + 8n_T = -192\zeta\alpha_k^3 + 8(202\zeta + 65\xi\zeta - 144\gamma_M - 8\pi^2 + 3\xi F_\mathrm{s})\zeta\alpha_k^4 + \mathcal{O}(\alpha_k^5)$$

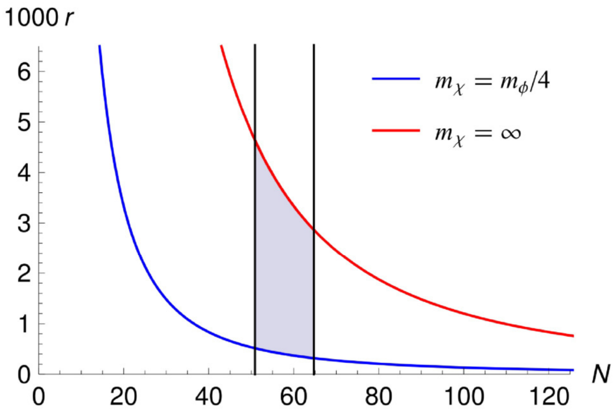
$0 \cdot \alpha_k^2$

A coincidence makes the lowest order of this quantity vanish,
but it is definitely not zero

D. Anselmi, E. Bianchi and M. Piva, Predictions of quantum gravity in inflationary cosmology: effects of the Weyl-squared term, J. High Energy Phys. 07 (2020) 211 arXiv:2005.10293 [hep-th]

The consistency of the fakeon projection on a curved background puts a lower bound on m_χ :

$$\frac{m_\phi}{4} < m_\chi < \infty$$

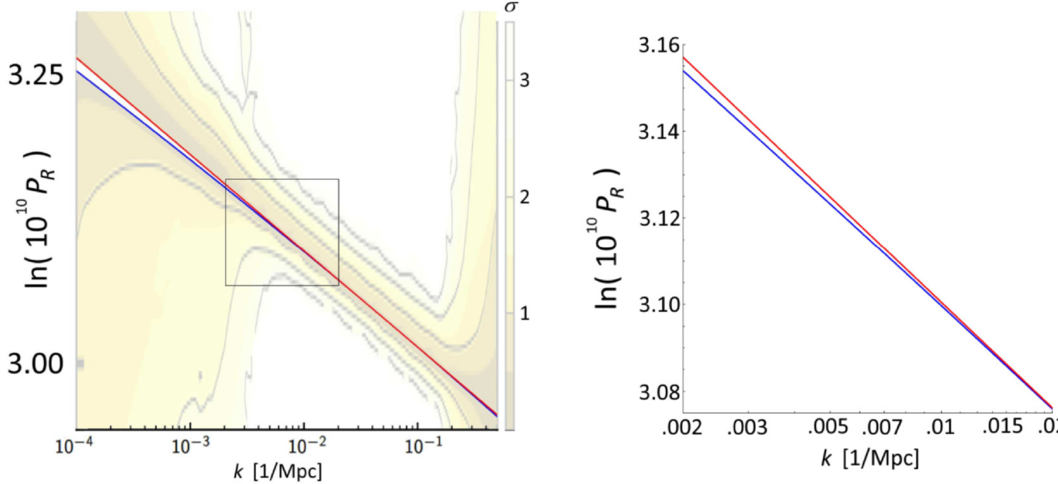


Allowed values of the tensor-to-scalar ratio r

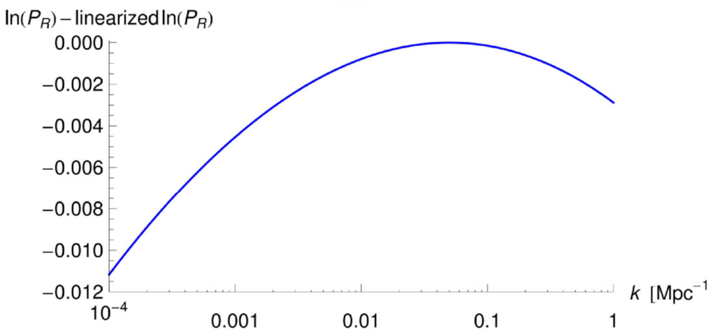
$$0.4 \lesssim 1000r \lesssim 3,$$

$$-0.4 \lesssim 1000n_T \lesssim -0.05$$

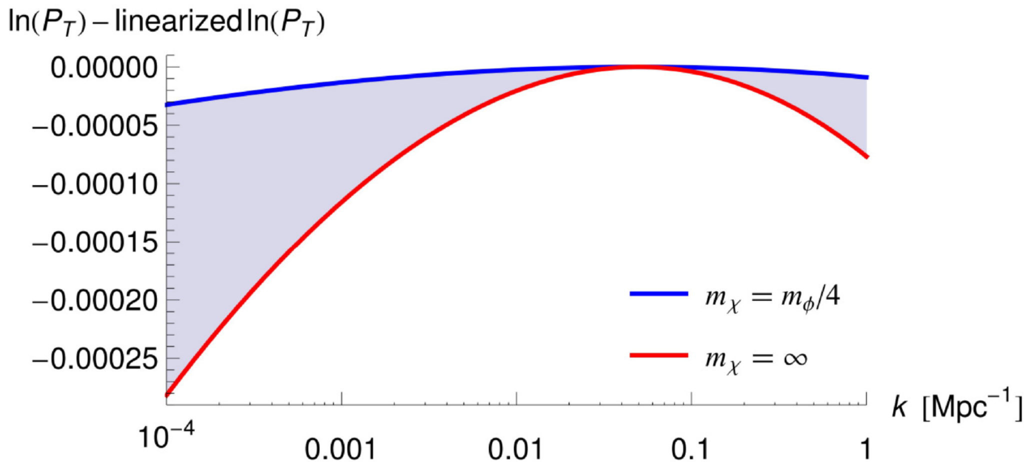
for $N = 60$



D. Anselmi, Cosmic inflation as a renormalization-group flow: the running of power spectra in quantum gravity, arXiv:2007.15023



Running of the scalar spectrum



Running of the tensor spectrum

Derivation (tensor spectrum)

$$S_{\rm QG} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left(R + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right) + \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left(D_\mu\phi D^\mu\phi - 2V(\phi)\right)$$

Expand with

$$g_{\mu\nu} = \mathrm{diag}(1,-a^2,-a^2,-a^2) - 2a^2\left(u\delta^1_\mu\delta^1_\nu - u\delta^2_\mu\delta^2_\nu + v\delta^1_\mu\delta^2_\nu + v\delta^2_\mu\delta^1_\nu\right),$$

u=u(t,z)

v=v(t,z)

The quadratic Lagrangian is

$$(8\pi G)\frac{\mathcal{L}_{\rm t}}{a^3} = \underbrace{\dot{u}^2 - \frac{k^2}{a^2}u^2}_{\text{Starobinsky limit}} - \frac{1}{m_\chi^2}\underbrace{\left[\ddot{u}^2 - 2\left(H^2 - \frac{3}{2}\alpha^2 H^2 + \frac{k^2}{a^2}\right)\dot{u}^2 + \frac{k^4}{a^4}u^2\right]}_{\text{HD fakeon correction}}$$

$m_\chi \to \infty$ $\to$ Mukhanov-Sasaki equation $\to$ spectrum

Elimination of the higher derivatives by means of an extra field U:

$$\mathcal{L}_t' = \mathcal{L}_t + \frac{a^3}{8\pi G m_\chi^2}\left[m_\chi^2\sqrt{\gamma}U - \ddot{u} - 3H\left(1 - \frac{4\alpha^2 H^2}{m_\chi^2\gamma}\right)\dot{u} - fu\right]^2$$

$$f = m_\chi^2\gamma + \frac{k^2}{a^2} + \frac{\alpha^2 H^2}{m_\chi^2\gamma}\left(3m_\chi^2 - 12H^2 + 24\alpha H^2 - \frac{2\alpha^2 H^2(17m_\chi^2 - 38H^2)}{m_\chi^2\gamma}\right) \qquad \gamma = 1 + 2\frac{H^2}{m_\chi^2}$$

Diagonalization of the de Sitter limit:

$$u = \frac{U+V}{\sqrt{\gamma}}$$

$$(8\pi G)\frac{\mathcal{L}_t'}{a^3} = \underline{\dot{U}^2 - \frac{hk^2}{a^2}U^2 - \frac{9}{8}m_\phi^2\xi\zeta\alpha^2\left[1 - \frac{\zeta\alpha}{6}(40 - 7\xi) + \frac{\zeta^2\alpha^2}{144}(2800 - 3806\xi - 497\xi^2)\right]U^2}$$
$$-\dot{V}^2 + \left[m_\chi^2 + \frac{m_\phi^2}{2}(1 - 3\alpha) + \frac{k^2}{a^2}\right]V^2 + \frac{3m_\phi^2\zeta\alpha^2}{2}\left(1 - \xi + \frac{4\xi\zeta k^2}{m_\phi^2 a^2}\right)UV$$
$$-\frac{3m_\phi^2\zeta^2\alpha^3}{4}\left[6 - 22\xi + \xi^2 + (6 - 7\xi - 2\xi^2)\frac{4\xi\zeta k^2}{m_\phi^2 a^2}\right]UV \qquad V(U)=O(\alpha^2)$$

$$\xi = \frac{m_\phi^2}{m_\chi^2}, \qquad \zeta = \left(1 + \frac{\xi}{2}\right)^{-1}, \qquad h = 1 - 3\xi\zeta^2\alpha^2 + \frac{3\xi\zeta^3\alpha^3}{2}(6 - 7\xi - 2\xi^2) + \mathcal{O}(\alpha^4).$$

Fakeon projection:

insert the ansatz $$V=\alpha^2\left(v_1+v_2\alpha\right)U+v_3\alpha^3\dot{U}+\mathcal{O}(\alpha^4)$$ v_1, v_2, v_3 unknown constants

into the *V* equation of motion. Using the *U* equation of motion, we find

$$V=-\frac{3\xi\zeta^2\alpha^2}{4}\left[1-\xi-(6-19\xi-2\xi^2)\frac{\zeta\alpha}{2}\right]U+\frac{3(1-\xi)\xi^2\zeta^3\alpha^3}{m_\phi}\dot{U}+\mathcal{O}(\alpha^4)$$

projected Mukhanov-Sasaki action :

$$S_{\rm t}^{\rm prj} = \frac{1}{2}\int \mathrm{d}\eta \left(w'^2 - h w^2 + 2\frac{w^2}{\eta^2} + \sigma_{\rm t}\frac{w^2}{\eta^2} \right)$$

mass "renormalization"

where

$$w = \frac{aU\sqrt{k}}{\sqrt{4\pi G}}, \qquad \sigma_{\rm t} = 9\zeta\alpha^2 + \frac{3\zeta^2\alpha^3}{2}(32+43\xi) + \zeta^3\alpha^4 F_{\rm t}(\xi) + \mathcal{O}(\alpha^5)$$

$$F_{\rm t}(\xi) = 364 + \frac{4037}{8}\xi + \frac{6145}{16}\xi^2 + \frac{81}{2}\xi^3$$

$$h = 1 - 3\xi\zeta^2\alpha^2 + \frac{3\xi\zeta^3\alpha^3}{2}(6 - 7\xi - 2\xi^2) + \mathcal{O}(\alpha^4)$$

Making the change of variables $\eta = -k\tau \longrightarrow \tilde{\eta}(\eta)$

with $\tilde{\eta}'(\eta) = \sqrt{h(\eta)},\ \tilde{\eta}(0) = 0$

we obtain the action

$$\tilde{S}_{\mathrm{t}}^{\mathrm{prj}} = \frac{1}{2}\int \mathrm{d}\tilde{\eta}\left(\tilde{w}'^2 - \tilde{w}^2 + \frac{2\tilde{w}^2}{\tilde{\eta}^2} + \tilde{\sigma}_{\mathrm{t}}\frac{\tilde{w}^2}{\tilde{\eta}^2}\right)$$

with

$$\tilde{w}(\tilde{\eta}(\eta)) = h(\eta)^{1/4}w(\eta), \qquad \tilde{\sigma}_{\mathrm{t}} = \frac{\tilde{\eta}^2(\sigma_{\mathrm{t}}+2)}{\eta^2 h} + \frac{\tilde{\eta}^2}{16h^3}\left(4hh'' - 5h'^2\right) - 2$$

The Bunch-Davies vacuum condition in these variables is the standard one:

$$\tilde{w}(\tilde{\eta}) \simeq \frac{\mathrm{e}^{i\tilde{\eta}}}{\sqrt{2}} \qquad \text{for } \tilde{\eta} \to \infty,$$

THEN,

- Go back to the original variables, η , $w(\eta)$ ...
- Work out the Bunch-Davies vacuum conditions for those
- Solve the Muk.-Sas. equation for $w(\eta) = w_0 + \alpha_k w_1 + \alpha_k^2 w_2 + \cdots$
- Go back to U through the w definition $$w = \frac{aU\sqrt{k}}{\sqrt{4\pi G}},$$
- Find $V(U)$ through the fakeon projection $V = \alpha^2 (v_1 + v_2\alpha) U + v_3\alpha^3 \dot{U} + \mathcal{O}(\alpha^4)$
- Find u through the de Sitter diagonalization $$u = \frac{U + V}{\sqrt{\gamma}} \qquad \gamma = 1 + 2\frac{H^2}{m_\chi^2}$$
- Finally, work out the spectrum $\mathcal{P}_T$ from $\langle u \ u \rangle$

Conclusions

The constraints originated by high-energy physics (locality, renormalizability and unitarity) allow us to overcome the arbitrariness of classical theories and formulate a basically unique quantum field theory of gravity, which contains just two parameters more than the Einstein theory, which are the mass of the inflaton and the mass of the spin-2 fakeon.

Other constraints coming from cosmology allow us to derive a condition that binds the two masses. Altogether, the constraints from cosmology and those from high-energy physics produce a very predictive theory, which could be tested in the forthcoming years thanks to inflation.

We have worked out the spectra, the tensor-to-scalar ratio, the tilts, the running coefficients and the combination r+8nT to the next-to-next-to-leading log order. Thanks to the "cosmic" renormalization-group flow, we have resummed the leading and subleading logs and expressed all quantities as power series in the running inflationary fine structure constant $\alpha \sim 1/115$.

Hopefully, primordial cosmology will turn into an arena for precision tests of quantum gravity, which will experience the same level of success the standard model of particle physics reached in the past decades!!!