Unification of Family and Flavor Symmetries Illustrated with SU(12)

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OUTLINE

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MOTIVATION

- Family and Flavor Symmetries are intimately related and remain much of a mystery today as to their precise structures.

Family symmetry relates particles within a family of quarks, leptons

Examples: SM: $\mathbf{SU(3)_c \times SU(2)_L \times U(1)_Y}$

GUTs: $\mathbf{SU(4) \times SU(2)_L \times SU(2)_R}$

$\mathbf{SU(5), SO(10), E_6}$

Flavor Symmetry relates families which appear to be replicas of each other.

Continuous: $\mathbf{SU(3), SU(2), U(1)}$

Discrete: $\mathbf{Z_2, Z_2 \times Z_2, ...}$

$\mathbf{S_3, A_4, S_4, \Delta(6n), T', ...}$

Latter groups became popular when the PMNS mixing matrix appeared to be close to TBM mixing.

In 2008 Werner Rodejohann and I raised issue whether TBM was an accidental or hidden symmetry. Appears to be badly broken.

- Conventional approach is to assume a direct product symmetry group $\mathbf{G}_{\mathbf{family}} \times \mathbf{G}_{\mathbf{flavor}}$. Problem arises due to fact there are too few IRs in GUTs for chiral fermion families:

$$\mathbf{SU(5) : 10, \overline{5};\ \ SO(10) : 16;\ \ E_6 : 27}$$

- Family and Flavor Unification requires a higher rank simple group. Some earlier attempts were based on $\mathbf{SO(18), SU(8), SU(9), SU(11)}$, but none were completely satisfactory.
- Here I describe an SU(12) model with interesting features that was constructed with the help of a Mathematica computer package called LieART written by Robert Feger and published in collaboration with Tom Kephart. This allows one to compute tensor products, branching rules, etc., and perform detailed searches for satisfactory models:
- An earlier version of the SU(12) model was published in PRD, but there were issues with some details which are corrected here.

INGREDIENTS OF A UNIFICATION GROUP

- SUSY SU(N) group must be large enough to assign matter families to individual IRs: N > 7. In fact, chiral SU(N) families can be found via AdS/CFT connection for $\mathbf{N \leq 14}$.
- Higher SU(N) GUT group replaces conventional GUT family groups.
- Symmetry breaking of SU(N) to smaller groups can occur in two ways:

$$\mathbf{SU(N) \rightarrow SU(N-1) \times U(1).... \rightarrow SU(5) \times U(1)'s}$$

one rank at a time with the help of SU(N) adjoint field;

or $\mathbf{SU(N) \rightarrow SU(5)}$ directly, preserving SUSY provided N is even, no SU(N) adjoint is present, and F-flat and D-flat conditions hold. Frampton and Kephart have shown such a dramatic reduction in rank is possible provided the sum of the Dynkin weights vanishes for the VEVs involved in lowering the rank. Such a case has been demonstrated by Kephart for N = 12.

- Matter fields must form an anomaly-free set of SU(N) and SU(5) IRs with 3 SU(5) families.
- SU(5) Higgs singlets must form SU(N) conjugate pairs to ensure D-flat directions, acquiring SU(5) VEVs at the SU(5) scale where the separation is given by $\epsilon \equiv \mathbf{M_{SU(5)}/M_{SU(N)} \sim 1/50}$
- An SU(5) adjoint 24 VEV should be present to break the SU(5) symmetry but not contained in an SU(N) adjoint which is assumed to be absent.
- One set or mixtures of two sets of Higgs doublets break the EW symmetry.
- Massive fermion pairs can be added at the SU(N) scale to be used in an effective operator approach.

SU(12) UNIFICATION MODELS

- SU(12) has 10 complex antisymmetric IRs:

$$\mathbf{12},\ \mathbf{66},\ \mathbf{220},\ \mathbf{495},\ \mathbf{792},\ (\mathbf{924}),\ \overline{\mathbf{792}},\ \overline{\mathbf{495}},\ \overline{\mathbf{220}},\ \overline{\mathbf{66}},\ \overline{\mathbf{12}}$$

  which can be represented by Young diagrams with 1 to 11 blocks stacked vertically. These representations contain no SU(5) exotics.

- One particularly small anomaly-free set of IRs which contains 3 chiral SU(5) families and pairs of fermions which become massive at the SU(5) scale is:

$$\mathbf{66} + \mathbf{2}(\mathbf{495}) + \overline{\mathbf{792}} + \mathbf{2}(\overline{\mathbf{220}}) + \mathbf{8}(\overline{\mathbf{12}})$$
$$\rightarrow \mathbf{3}(\mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}) + \mathbf{63}(\mathbf{10} + \overline{\mathbf{10}}) + \mathbf{84}(\mathbf{5} + \overline{\mathbf{5}}) + \mathbf{236}(\mathbf{1})$$

  given the $\mathbf{SU(12)} \rightarrow \mathbf{SU(5)}$ branching rules:

$$\begin{array}{rcrrrrr}
\mathbf{66} & \rightarrow & \mathbf{7}(\mathbf{5}) + & (\mathbf{10}) + & & + & \mathbf{21}(\mathbf{1}) \\
\mathbf{495} & \rightarrow & \mathbf{35}(\mathbf{5}) + & \mathbf{21}(\mathbf{10}) + & \mathbf{7}(\overline{\mathbf{10}}) + & (\overline{\mathbf{5}}) + & \mathbf{35}(\mathbf{1}) \\
\overline{\mathbf{792}} & \rightarrow & \mathbf{7}(\mathbf{5}) + & \mathbf{21}(\mathbf{10}) + & \mathbf{35}(\overline{\mathbf{10}}) + & \mathbf{35}(\overline{\mathbf{5}}) + & \mathbf{22}(\mathbf{1}) \\
\overline{\mathbf{220}} & \rightarrow & & (\mathbf{10}) + & \mathbf{7}(\overline{\mathbf{10}}) + & \mathbf{21}(\overline{\mathbf{5}}) + & \mathbf{35}(\mathbf{1}) \\
\overline{\mathbf{12}} & \rightarrow & & & & (\overline{\mathbf{5}}) + & \mathbf{7}(\mathbf{1})
\end{array}$$

- Three SU(5) families can then be selected from among:

$$\begin{array}{llll} (\mathbf{10})\mathbf{66} = (\mathbf{2}+\mathbf{0}), & & (\overline{\mathbf{5}})\mathbf{495} = (\mathbf{4}+\mathbf{0}) \\ (\mathbf{10})\mathbf{495} = (\mathbf{2}+\mathbf{2}), & & (\overline{\mathbf{5}})\overline{\mathbf{792}} = (\mathbf{4}+\mathbf{3}) \\ (\mathbf{10})\overline{\mathbf{792}} = (\mathbf{2}+\mathbf{5}), & & (\overline{\mathbf{5}})\overline{\mathbf{220}} = (\mathbf{4}+\mathbf{5}) \\ (\mathbf{10})\overline{\mathbf{220}} = (\mathbf{2}+\mathbf{7}), & & (\overline{\mathbf{5}})\overline{\mathbf{12}} = (\mathbf{4}+\mathbf{7}) \end{array}$$

where (2 + 5) represents a column of 2 SU(5) boxes + 5 SU(12)/SU(5) boxes, as the SU(12) basis chosen places the SU(5) boxes at the top.

- Singlet Higgs conjugate pairs can be selected from among:

$$\begin{array}{llll} (\mathbf{1})\mathbf{12_H} = (\mathbf{0}+\mathbf{1}), & & (\mathbf{1})\overline{\mathbf{12}} = (\mathbf{5}+\mathbf{6}) \\ (\mathbf{1})\mathbf{66_H} = (\mathbf{0}+\mathbf{2}), & & (\mathbf{1})\overline{\mathbf{66}} = (\mathbf{5}+\mathbf{5}) \\ (\mathbf{1})\mathbf{220_H} = (\mathbf{0}+\mathbf{3}), & & (\mathbf{1})\overline{\mathbf{220}} = (\mathbf{5}+\mathbf{4}) \\ (\mathbf{1})\mathbf{495_H} = (\mathbf{0}+\mathbf{4}), & & (\mathbf{1})\overline{\mathbf{495}} = (\mathbf{5}+\mathbf{3}) \\ (\mathbf{1})\mathbf{792_H} = (\mathbf{0}+\mathbf{5}) \text{ or } (\mathbf{5}+\mathbf{0}), & & (\mathbf{1})\overline{\mathbf{792}} = (\mathbf{5}+\mathbf{2}) \text{ or } (\mathbf{0}+\mathbf{7}) \end{array}$$

- Higgs doublets which can get VEVs at the EW scale can be found among $(\mathbf{5})\mathbf{12},\ (\overline{\mathbf{5}})\overline{\mathbf{12}},\ (\mathbf{5})\overline{\mathbf{495}},\ (\overline{\mathbf{5}})\mathbf{495}$

- Renormalizable dim-4 operators can be formed from triplet vertices from which the Yukawa superpotential can be constuced.

- Effective higher-dimensional operators can be formed by inserting singlet Higgs and massive fermions in tree diagrams. With SUSY valid at the SU(5) scale, loop diagrams are highly suppressed.

- Massive fermion pairs at the SU(12) scale can be selected from
$$\mathbf{12} \times \overline{\mathbf{12}},\ \mathbf{66} \times \overline{\mathbf{66}},\ \mathbf{220} \times \overline{\mathbf{220}},\ \mathbf{495} \times \overline{\mathbf{495}},\ \mathbf{792} \times \overline{\mathbf{792}}$$

Particle Assignments for an Inverted Hierarchy Model

- Starting with the anomaly-free set $\mathbf{66 + 2(495) + \overline{792} + 2(\overline{220}) + 8(\overline{12})}$, assign SU(12) IRs for the three SU(5) 10 families defining the U matrix, the three $\overline{5}$ families defining the D matrix, and three singlet families for the DN and MN matrices.

- The model assignments chosen for illustration of Inverted Hierarchy are:

$$\begin{array}{rclrclrcl}
\mathbf{(10)495_1} & \supset & \mathbf{u_L, u^c_L, d_L, e^c_L} & \mathbf{(\overline{5})\overline{12}_1} & \supset & \mathbf{d^c_L, e_L, \nu_{eL}} & \mathbf{(1)\overline{792}_1} & \supset & \mathbf{N^c_{1L}} \\
\mathbf{(10)495_2} & \supset & \mathbf{c_L, c^c_L, s_L, \mu^c_L} & \mathbf{(\overline{5})\overline{12}_2} & \supset & \mathbf{s^c_L, \mu_L, \nu_{\mu L}} & \mathbf{(1)1_2} & \supset & \mathbf{N^c_{2L}} \\
\mathbf{66_3} & \supset & \mathbf{t_L, t^c_L, b_L, \tau^c_L} & \mathbf{(\overline{5})\overline{12}_3} & \supset & \mathbf{b^c_L, \tau_L, \nu_{\tau L}} & \mathbf{(1)\overline{220}_3} & \supset & \mathbf{N^c_{3L}}
\end{array}$$

with Higgs fields $\mathbf{(1)66_H, (1)\overline{66}_H, (1)792_H, (1)\overline{792}_H, (5)\overline{495}_H, (\overline{5})\overline{12}_H}$

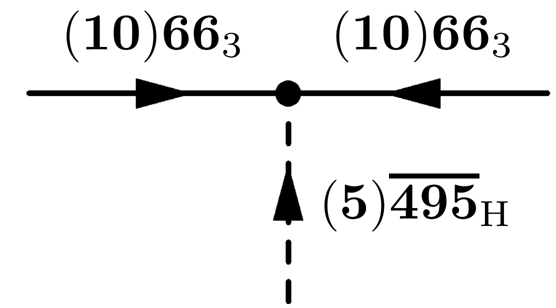
and a (24)5148_H which separates the D and L spectra and enables the breaking of SU(5) at the GUT scale,

and massive fermions $\mathbf{66, \overline{66}, 792, \overline{792}}$ at the SU(12) scale.

EFFECTIVE OPERATOR APPROACH

- Note that there is only one unique assignment for U33 which yields both SU(5) and SU(12) singlets for the top quark 3-point vertex:

$$\begin{aligned}\mathbf{U33:}\quad & \mathbf{(10)66_3.(5)\overline{495}_H.(10)66_3} \\ & = \mathbf{(2+0).(1+7).(2+0)} \\ & = \mathbf{(4+0).(1+7) = (5+7)} \\ & \sim \mathbf{(1)1}\end{aligned}$$



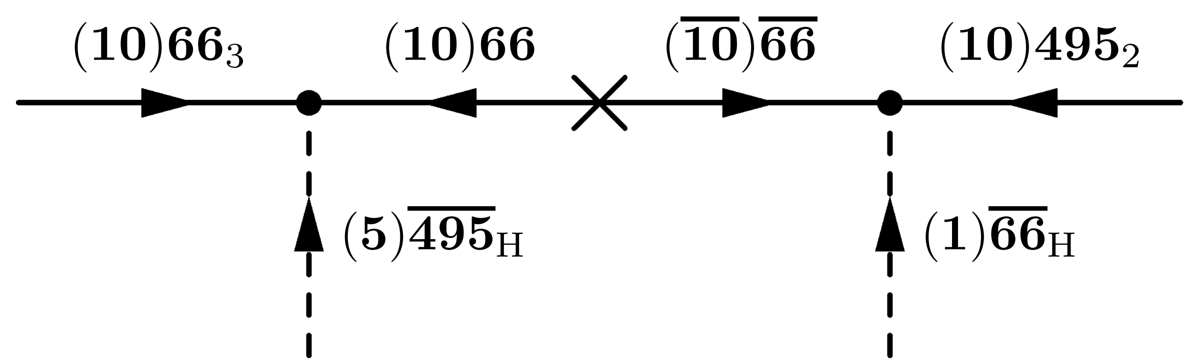
$$\mathbf{(10)66_3.(5)\overline{495}_H.(10)66_3}$$

with Yukawa coupling $\mathbf{h^u_{33}}$ and VEV $\mathbf{v_u}$

$$\sim \mathbf{h^u_{33} v_u u^T_{3L} u^c_{3L}}$$

- Other matrix elements arise from higher order diagrams with massive fermions integrated out. Each additional vertex contributes a factor of $\epsilon = \frac{\langle \mathbf{1} \rangle_{\mathbf{SU(5)}}}{\mathbf{M_{SU(12)}}} \sim \frac{\mathbf{1}}{\mathbf{50}}$ so the higher order diagrams are quite suppressed.

For example



$$\mathbf{U32:}\quad \sim \mathbf{h^u_{32} \epsilon v_u u^T_{3L} u^c_{2L}}$$

$$\mathbf{U32:}\quad \mathbf{(10)66_3.(5)\overline{495}_H.(10)66 \times (\overline{10})\overline{66}.(1)\overline{66}_H.(10)495_2}$$

UpType Diagrams:

Dim 4: **U33:** $(10)66_3.(5)\overline{495}.(10)66_3$

Dim 5: **U13:** $(10)495_1.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66_3$

U31: $(10)66_3.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)\overline{66}.(10)495_1$

U23: $(10)495_2.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66_3$

U32: $(10)66_3.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)\overline{66}.(10)495_2$

Dim 6: **U11:** $(10)495_1.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)\overline{66}.(10)495_1$

U12: $(10)495_1.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)\overline{66}.(10)495_2$

U21: $(10)495_2.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)\overline{66}.(10)495_1$

U22: $(10)495_2.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(5)\overline{495}.(10)66\times(\overline{10})\overline{66}.(1)\overline{66}.(10)495_2$

DownType Diagrams:

Dim 4: **D31:** $(10)66_3.(\overline{5})\overline{12}.(\overline{5})\overline{12}_1$

D32: $(10)66_3.(\overline{5})\overline{12}.(\overline{5})\overline{12}_2$

D33: $(10)66_3.(\overline{5})\overline{12}.(\overline{5})\overline{12}_3$

Dim 5: **D11:** $(10)495_1.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(\overline{5})\overline{12}.(\overline{5})\overline{12}_1$

D12: $(10)495_1.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(\overline{5})\overline{12}.(\overline{5})\overline{12}_2$

D21: $(10)495_2.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(\overline{5})\overline{12}.(\overline{5})\overline{12}_1$

D13: $(10)495_1.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(\overline{5})\overline{12}.(\overline{5})\overline{12}_3$

D22: $(10)495_2.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(\overline{5})\overline{12}.(\overline{5})\overline{12}_2$

D23: $(10)495_2.(1)\overline{66}.(\overline{10})\overline{66}\times(10)66.(\overline{5})\overline{12}.(\overline{5})\overline{12}_3$

Dirac Diagrams:

Dim 5:

DN11: $(\overline{5})\overline{12}_1.(5)\overline{495}.(1)792\times(1)\overline{792}.(1)\overline{66}.(1)\overline{792}_1$

DN12: $(\overline{5})\overline{12}_1.(5)\overline{495}.(1)792\times(1)\overline{792}.(1)792.(1)1_2$

DN21: $(\overline{5})\overline{12}_2.(5)\overline{495}.(1)792\times(1)\overline{792}.(1)\overline{66}.(1)\overline{792}_1$

DN31: $(\overline{5})\overline{12}_3.(5)\overline{495}.(1)792\times(1)\overline{792}.(1)\overline{66}.(1)\overline{792}_1$

DN22: $(\overline{5})\overline{12}_2.(5)\overline{495}.(1)792\times(1)\overline{792}.(1)792.(1)1_2$

DN32: $(\overline{5})\overline{12}_3.(5)\overline{495}.(1)792\times(1)\overline{792}.(1)792.(1)1_2$

Dim 6:

DN13: $(\overline{5})\overline{12}_1.(5)\overline{495}.(1)792\times(1)\overline{792}.(1)\overline{66}.(1)\overline{792}\times(1)792.(1)\overline{66}.(1)\overline{220}_3$

DN23: $(\overline{5})\overline{12}_2.(5)\overline{495}.(1)792\times(1)\overline{792}.(1)\overline{66}.(1)\overline{792}\times(1)792.(1)\overline{66}.(1)\overline{220}_3$

DN33: $(\overline{5})\overline{12}_3.(5)\overline{495}.(1)792\times(1)\overline{792}.(1)\overline{66}.(1)\overline{792}\times(1)792.(1)\overline{66}.(1)\overline{220}_3$

Majorana Diagrams:

Dim 4:

MN11: $(1)\overline{792}_1.(1)\overline{66}.(1)\overline{792}_1$

MN12: $(1)\overline{792}_1.(1)792.(1)1_2$

MN21: $(1)1_2.(1)792.(1)\overline{792}_1$

Dim 5:

MN13: $(1)\overline{792}_1.(1)\overline{66}.(1)\overline{792}\times(1)792.(1)\overline{66}.(1)\overline{220}_3$

MN31: $(1)\overline{220}_3.(1)\overline{66}.(1)792\times(1)\overline{792}.(1)\overline{66}.(1)\overline{792}_1$

MN22: $(1)1_2.(1)66.(1)\overline{66}\times(1)66.(1)\overline{66}.(1)1_2$

MN22: $(1)1_2.(1)792.(1)\overline{792}\times(1)792.(1)\overline{792}.(1)1_2$

MN23: $(1)1_2.(1)\overline{66}.(1)66\times(1)\overline{66}.(1)792.(1)\overline{220}_3$

MN23: $(1)1_2.(1)792.(1)\overline{792}\times(1)792.(1)\overline{66}.(1)\overline{220}_3$

MN32: $(1)\overline{220}_3.(1)\overline{66}.(1)792\times(1)\overline{792}.(1)792.(1)1_2$

MN32: $(1)\overline{220}_3.(1)792.(1)\overline{66}\times(1)66.(1)\overline{66}.(1)1_2$

Dim 6:

MN33: $(1)\overline{220}_3.(1)\overline{66}.(1)792\times(1)\overline{792}.(1)\overline{66}.(1)\overline{792}\times(1)792.(1)\overline{66}.(1)\overline{220}_3$

MASS MATRICES: Leading Order Terms for IH Model

- Dropping the prefactors:

$$\mathbf{M_U} \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon^2 & \epsilon \\ \epsilon & \epsilon & 1 \end{pmatrix} \mathbf{v_u}, \qquad \mathbf{M_D} \sim \begin{pmatrix} \epsilon & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon \\ 1 & 1 & 1 \end{pmatrix} \mathbf{v_d}$$

$$\mathbf{M_{DN}} \sim \begin{pmatrix} \epsilon & \epsilon & \epsilon^2 \\ \epsilon & \epsilon & \epsilon^2 \\ \epsilon & \epsilon & \epsilon^2 \end{pmatrix} \mathbf{v_u}, \qquad \mathbf{M_{MN}} \sim \begin{pmatrix} 1 & 1 & \epsilon \\ 1 & \epsilon & \epsilon \\ \epsilon & \epsilon & \epsilon^2 \end{pmatrix} \mathbf{\Lambda_R}$$

$$\mathbf{M_L} \sim \mathbf{M_D^T}, \quad \mathbf{M}_\nu = -\mathbf{M_{DN}} \mathbf{M}^{-1} \mathbf{M_{DN}^T} \sim \begin{pmatrix} \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \end{pmatrix} \frac{\mathbf{v_u^2}}{\mathbf{\Lambda_R}}$$

- $\mathbf{M_U}, \mathbf{M_{MN}}, \mathbf{M}_\nu$ are symmetric, $\mathbf{M_D}, \mathbf{M_L}, \mathbf{M_{DN}}$ doubly lopsided. Note that $\mathbf{M}_\nu$ has a mild hierarchy.

PHENOMENOLOGICAL FIT TO DATA PROCEDURE

- 30 data points are attempted to be fit by the model. They are the 9 quark and charged lepton masses, the 3 neutrino mass squared differences, the 9 CKM quark and 9 PMNS neutrino mixing matrix elements.

- The fit parameters include up to 6 U, 6 MN, 9 D, 9 DN prefactors and Λ_R or a maximum of 31 real parameters. Initially the prefactors are chosen randomly around +1 or -1 in the ranges [0.7, 1.3] or [-1.3, -0.7]. If some of the prefactors do not appear in the lowest order of the light neutrino mass matrix, they are not included in the fit parameters, hence the number may be smaller than 30.
  The scale factor taken is $\epsilon = (1/6.5)^2 = 0.0237$

- The quark sector is fit first with sets of random numbers and signs used as initial values for the quark prefactors. A model emerging from the fit is accepted if the chi-squared drops below a set value. Then all possible Majorana assignments are tried with prefactors set randomly again, etc. For a given model, fits for both NH and IH are attempted.

Example of Inverted Hierarchy Model

SU(5) Level:	$63\,(\overline{\mathbf{10}}) + 66\,(\mathbf{10}) + 87\,(\overline{\mathbf{5}}) + 84\,(\mathbf{5}) + 239\,(\mathbf{1})$
Fermions:	$(\mathbf{10})\mathbf{495}_1, (\mathbf{10})\mathbf{495}_2, (\mathbf{10})\mathbf{66}_3, (\overline{\mathbf{5}})\overline{\mathbf{12}}_1, (\overline{\mathbf{5}})\overline{\mathbf{12}}_2, (\overline{\mathbf{5}})\overline{\mathbf{12}}_3, (\mathbf{1})\overline{\mathbf{792}}_1, (\mathbf{1})\mathbf{1}_2, (\mathbf{1})\overline{\mathbf{220}}_3$
Higgs:	$(\mathbf{5})\overline{\mathbf{495}}, (\overline{\mathbf{5}})\overline{\mathbf{12}}, (\mathbf{1})\mathbf{66}, (\mathbf{1})\overline{\mathbf{66}}, (\mathbf{1})\mathbf{792}, (\mathbf{1})\overline{\mathbf{792}}$
M. Fermions:	$\mathbf{66}, \overline{\mathbf{66}}, \mathbf{792}, \overline{\mathbf{792}}$
Mass Matrices:	$\begin{pmatrix} 2 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}_U, \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}_D, \begin{pmatrix} 1 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 1 & 2 \end{pmatrix}_{DN}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 2 \end{pmatrix}_{MN}, \begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{pmatrix}_{\text{Light Neutrino}}$ in terms of powers of epsilon

Quark and Charged Lepton Pheno after fit:
$\epsilon = 0.024$, $v_u = 173.951$ GeV,, $v_d = 4.11718$ GeV, $\chi^2_q = 0.239$

Up–type masses	Down–type masses	Lepton Masses	CKM Matrix
$m_u = 2.2$ MeV $m_c = 600.$ MeV $m_t = 166.$ GeV	$m_d = 2.71$ MeV $m_s = 90.7$ MeV $m_b = 2.32$ GeV	$m_e = 2.71$ MeV $m_\mu = 90.7$ MeV $m_\tau = 2.32$ GeV	$\begin{pmatrix} 0.974 & 0.227 & 0.003 \\ -0.227 & 0.973 & 0.042 \\ 0.007 & -0.042 & 0.999 \end{pmatrix}$

Quark and Charged Lepton Pheno measured:

Up–type masses	Down–type masses	Lepton Masses	CKM Matrix
$m_u = 2.2$ MeV $m_c = 600.$ MeV $m_t = 166$ GeV	$m_d = 3.8$ MeV $m_s = 75.$ MeV $m_b = 2.78$ GeV	$m_e = 0.501$ MeV $m_\mu = 104.$ MeV $m_\tau = 1.75$ GeV	$\begin{pmatrix} 0.974 & 0.225 & 0.003 \\ -0.225 & 0.973 & 0.041 \\ 0.009 & -0.040 & 0.999 \end{pmatrix}$

Neutrino Pheno after fit:
$\Lambda_R = -9.2\times10^{12}$GeV, $\chi^2_\nu = 0.239$

Mass Differences	Mixing Angles	Phase	MNS Matrix	Heavy–Neutrino Masses	Light–Neutrino Masses
$\Delta_{21} = 7.5\times10^{-5}\text{eV}^2$ $\Delta_{31} = -2.4\times10^{-3}\text{eV}^2$ $\Delta_{32} = -2.5\times10^{-3}\text{eV}^2$	$\sin^2(\theta_{12}) = 0.306$ $\sin^2(\theta_{23}) = 0.42$ $\sin^2(\theta_{13}) = 0.0228$	$\delta = 0$	$\begin{pmatrix} 0.824 & 0.547 & -0.151 \\ -0.503 & 0.580 & -0.641 \\ -0.263 & 0.603 & 0.753 \end{pmatrix}$	$M_1 = -2.56\times10^9$ GeV $M_2 = -3.31\times10^{12}$ GeV $M_3 = -1.47\times10^{13}$ GeV	$m_1 = 48.9$ meV $m_2 = 49.7$ meV $m_3 = 0.022$ meV

Neutrino Pheno measured:

Mass Differences	Mixing Angles	Phase	MNS Matrix
$\lvert\Delta_{21}\rvert = 7.6\times10^{-5}\text{eV}^2$ $\lvert\Delta_{31}\rvert = 2.4\times10^{-3}\text{eV}^2$ $\lvert\Delta_{32}\rvert = 2.4\times10^{-3}\text{eV}^2$	$\sin^2(\theta_{12}) = 0.306$ $\sin^2(\theta_{23}) = 0.42$ $\sin^2(\theta_{13}) = 0.0228$	$\delta = 0$	$\begin{pmatrix} 0.824 & 0.547 & -0.151 \\ -0.503 & 0.580 & -0.641 \\ -0.263 & 0.603 & 0.753 \end{pmatrix}$

Mass Matrices:

$$M_u=\begin{pmatrix} -0.96\,\epsilon^2 & -0.9\,\epsilon^2 & 1.2\,\epsilon \\ -0.9\,\epsilon^2 & 1.1\,\epsilon^2 & 2.1\,\epsilon \\ 1.2\,\epsilon & 2.1\,\epsilon & -0.95 \end{pmatrix}, M_d=\begin{pmatrix} 0.44\,\epsilon & -0.27\,\epsilon & 0.26\,\epsilon \\ 0.77\,\epsilon & -0.33\,\epsilon & 0.15\,\epsilon \\ -0.028 & 0.17 & -0.54 \end{pmatrix}, M_l=\begin{pmatrix} 0.44\,\epsilon & 0.77\,\epsilon & -0.028 \\ -0.27\,\epsilon & -0.33\,\epsilon & 0.17 \\ 0.26\,\epsilon & 0.15\,\epsilon & -0.54 \end{pmatrix},$$

$$M_{MN}=\begin{pmatrix} 1.2 & -0.76 & 0.95\,\epsilon \\ -0.76 & \epsilon\,h^{mn}_{22} & -0.47\,\epsilon \\ 0.95\,\epsilon & -0.47\,\epsilon & 0.21\,\epsilon^2 \end{pmatrix}, M_{DN}=\begin{pmatrix} 1.5\,\epsilon & 1.2\,\epsilon & -1.4\,\epsilon^2 \\ -0.82\,\epsilon & -2.3\,\epsilon & -2.2\,\epsilon^2 \\ 0.85\,\epsilon & 0.61\,\epsilon & -1.7\,\epsilon^2 \end{pmatrix}, M^{LO}_\nu=\frac{v^2}{\Lambda_R}\begin{pmatrix} 17.\,\epsilon^2 & -2.5\,\epsilon^2 & 13.\,\epsilon^2 \\ -2.5\,\epsilon^2 & 26.\,\epsilon^2 & 3.2\,\epsilon^2 \\ 13.\,\epsilon^2 & 3.2\,\epsilon^2 & 11.\,\epsilon^2 \end{pmatrix}=\frac{v^2}{\Lambda_R}\begin{pmatrix} 0.0094 & -0.0014 & 0.0072 \\ -0.0014 & 0.015 & 0.0018 \\ 0.0072 & 0.0018 & 0.0061 \end{pmatrix},$$

$$M^{Full}_\nu=\frac{v^2}{\Lambda_R}\begin{pmatrix} \frac{\epsilon^2\,(11.\,\epsilon-7.4)}{1.\,\epsilon-0.44} & \frac{\epsilon^2\,(8.7\,\epsilon+1.1)}{1.\,\epsilon-0.44} & \frac{\epsilon^2\,(11.\,\epsilon-5.6)}{1.\,\epsilon-0.44} \\ \frac{\epsilon^2\,(8.7\,\epsilon+1.1)}{1.\,\epsilon-0.44} & \frac{\epsilon^2\,(4.1\,\epsilon-11.)}{1.\,\epsilon-0.44} & \frac{\epsilon^2\,(7.6\,\epsilon-1.4)}{1.\,\epsilon-0.44} \\ \frac{\epsilon^2\,(11.\,\epsilon-5.6)}{1.\,\epsilon-0.44} & \frac{\epsilon^2\,(7.6\,\epsilon-1.4)}{1.\,\epsilon-0.44} & \frac{\epsilon^2\,(10.\,\epsilon-4.8)}{1.\,\epsilon-0.44} \end{pmatrix}=\frac{v^2}{\Lambda_R}\begin{pmatrix} 0.0096 & -0.0018 & 0.0073 \\ -0.0018 & 0.015 & 0.0017 \\ 0.0073 & 0.0017 & 0.0061 \end{pmatrix}$$

MATRICES Giving Best Fit for This Inverted Hierarchy Model

$$\mathbf{M_U} = \begin{pmatrix} -0.96\epsilon^2 & -0.9\epsilon^2 & 1.2\epsilon \\ -0.9\epsilon^2 & 1.1\epsilon^2 & 2.1\epsilon \\ 1.2\epsilon & 2.1\epsilon & -0.95 \end{pmatrix} \mathbf{v_u}, \quad \mathbf{M_D} = \begin{pmatrix} 0.44\epsilon & -0.27\epsilon & 0.26\epsilon \\ 0.77\epsilon & -0.33\epsilon & 0.15\epsilon \\ -0.028 & 0.17 & -0.54 \end{pmatrix} \mathbf{v_d}$$

$$\mathbf{M_{DN}} = \begin{pmatrix} 1.5\epsilon & 1.2\epsilon & -1.4\epsilon^2 \\ -0.82\epsilon & -2.3\epsilon & -2.2\epsilon^2 \\ 0.85\epsilon & 0.61\epsilon & -1.7\epsilon^2 \end{pmatrix} \mathbf{v_u}, \quad \mathbf{M_{MN}} = \begin{pmatrix} 1.2 & -0.76 & 0.95\epsilon \\ -0.76 & \epsilon h_{22}^{mn} & -0.47\epsilon \\ 0.95\epsilon & -0.47\epsilon & 0.21\epsilon^2 \end{pmatrix} \mathbf{\Lambda_R}$$

$$\mathbf{M}_\nu = \begin{pmatrix} 0.0096 & -0.0018 & 0.0073 \\ -0.0018 & 0.015 & 0.0017 \\ 0.0073 & 0.0017 & 0.0061 \end{pmatrix} \frac{\mathbf{v_u^2}}{\mathbf{\Lambda_R}}$$

- All prefactors except one are within $\mathcal{O}(\mathbf{0.1}-\mathbf{10})$ of unity as a result of the Monte Carlo fitting procedure.

Particle Assignments for a Normal Hierarchy Model

- Starting with the anomaly-free set $\mathbf{66 + 2(495) + \overline{792} + 2(\overline{220} + 8(\overline{12})}$, assign SU(12) IRs for the three SU(5) 10 families defining the U matrix, the three $\overline{5}$ families to define the D matrix, and three singlet families for the DN and MN matrices.

- The model assignments chosen for illustration of Normal Hierarchy are:

$$\begin{array}{llllllllll}
\mathbf{(10)\overline{792}_1} & \supset & \mathbf{u_L, u^c_L, d_L, e^c_L} & \mathbf{(\overline{5})\overline{12}_1} & \supset & \mathbf{d^c_L, e_L, \nu_{eL}} & \mathbf{(1)1_1} & \supset & \mathbf{N^c_L} \\
\mathbf{(10)\overline{220}_2} & \supset & \mathbf{c_L, c^c_L, s_L, \mu^c_L} & \mathbf{(\overline{5})\overline{12}_2} & \supset & \mathbf{s^c_L, \mu_L, \nu_{\mu L}} & \mathbf{(1)\overline{220}_2} & \supset & \mathbf{N^c_L} \\
\mathbf{(10)66_3} & \supset & \mathbf{t_L, t^c_L, b_L, \tau^c_L} & \mathbf{(\overline{5})\overline{12}_3} & \supset & \mathbf{b^c_L, \tau_L, \nu_{\tau L}} & \mathbf{(1)1_3} & \supset & \mathbf{N^c_L}
\end{array}$$

with Higgs fields $\mathbf{(1)495_H,\ (1)\overline{495}_H,\ (1)792_H,\ (1)\overline{792}_H,\ (5)\overline{495}_H,\ (\overline{5})\overline{12}_H}$

and a (24)5148_H which separates the D and L spectra and enables the breaking of SU(5) at the GUT scale

and massive fermions $\mathbf{66, \overline{66}, 792, \overline{792}}$ at the SU(12) scale.

Example of a Normal Hierarchy Model

SU(5) Level:	$63\,(\overline{\mathbf{10}}) + 66\,(\mathbf{10}) + 87\,(\overline{\mathbf{5}}) + 84\,(\mathbf{5}) + 239\,(\mathbf{1})$
Fermions:	$(\mathbf{10})\overline{\mathbf{792}}_1, (\mathbf{10})\overline{\mathbf{220}}_2, (\mathbf{10})\mathbf{66}_3, (\overline{\mathbf{5}})\mathbf{12}_1, (\overline{\mathbf{5}})\overline{\mathbf{12}}_2, (\overline{\mathbf{5}})\overline{\mathbf{12}}_3, (\mathbf{1})\mathbf{1}_1, (\mathbf{1})\overline{\mathbf{220}}_2, (\mathbf{1})\mathbf{1}_3$
Higgs:	$(\mathbf{5})\overline{\mathbf{495}}, (\overline{\mathbf{5}})\overline{\mathbf{12}}, (\mathbf{1})\mathbf{495}, (\mathbf{1})\overline{\mathbf{495}}, (\mathbf{1})\mathbf{792}, (\mathbf{1})\overline{\mathbf{792}}$
M. Fermions:	$\mathbf{66}, \overline{\mathbf{66}}, \mathbf{792}, \overline{\mathbf{792}}$
Mass Matrices:	$\begin{pmatrix} 2 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}_U, \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}_D, \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}_{DN}, \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}_{MN}, \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}_{\text{Light Neutrino}}$

in terms of powers of epsilon

Quark and Charged Lepton Pheno after fit:
$\epsilon = 0.024$, $v_u = 173.951$ GeV,,
$v_d = 4.11718$ GeV,
$\chi_q^2 = 0.239$

Up–type masses	Down–type masses	Lepton Masses	CKM Matrix
$m_u = 2.2$ MeV $m_c = 600.$ MeV $m_t = 166.$ GeV	$m_d = 2.71$ MeV $m_s = 90.7$ MeV $m_b = 2.32$ GeV	$m_e = 2.71$ MeV $m_\mu = 90.7$ MeV $m_\tau = 2.32$ GeV	$\begin{pmatrix} 0.974 & 0.227 & 0.003 \\ -0.227 & 0.973 & 0.042 \\ 0.007 & -0.042 & 0.999 \end{pmatrix}$

Quark and Charged Lepton Pheno measured:

Up–type masses	Down–type masses	Lepton Masses	CKM Matrix
$m_u = 2.2$ MeV $m_c = 600.$ MeV $m_t = 166$ GeV	$m_d = 3.8$ MeV $m_s = 75.$ MeV $m_b = 2.78$ GeV	$m_e = 0.501$ MeV $m_\mu = 104.$ MeV $m_\tau = 1.75$ GeV	$\begin{pmatrix} 0.974 & 0.225 & 0.003 \\ -0.225 & 0.973 & 0.041 \\ 0.009 & -0.040 & 0.999 \end{pmatrix}$

Neutrino Pheno after fit:
$\Lambda_R = 2.2\times10^{14}$GeV,
$\chi_\nu^2 = 0.239$

Mass Differences	Mixing Angles	Phase	MNS Matrix	Heavy–Neutrino Masses	Light–Neutrino Masses
$\Delta_{21} = 7.5\times10^{-5}\text{eV}^2$ $\Delta_{31} = 2.5\times10^{-3}\text{eV}^2$ $\Delta_{32} = 2.4\times10^{-3}\text{eV}^2$	$\sin^2(\theta_{12}) = 0.306$ $\sin^2(\theta_{23}) = 0.42$ $\sin^2(\theta_{13}) = 0.0228$	$\delta = 0$	$\begin{pmatrix} 0.824 & 0.547 & -0.151 \\ -0.503 & 0.580 & -0.641 \\ -0.263 & 0.603 & 0.753 \end{pmatrix}$	$M_1 = 4.1\times10^{12}$ GeV $M_2 = 6.11\times10^{12}$ GeV $M_3 = 1.37\times10^{13}$ GeV	$m_1 = 1.01$ meV $m_2 = 8.7$ meV $m_3 = 49.7$ meV

Neutrino Pheno measured:

Mass Differences	Mixing Angles	Phase	MNS Matrix
$\lvert\Delta_{21}\rvert = 7.6\times10^{-5}\text{eV}^2$ $\lvert\Delta_{31}\rvert = 2.4\times10^{-3}\text{eV}^2$ $\lvert\Delta_{32}\rvert = 2.4\times10^{-3}\text{eV}^2$	$\sin^2(\theta_{12}) = 0.306$ $\sin^2(\theta_{23}) = 0.42$ $\sin^2(\theta_{13}) = 0.0228$	$\delta = 0$	$\begin{pmatrix} 0.824 & 0.547 & -0.151 \\ -0.503 & 0.580 & -0.641 \\ -0.263 & 0.603 & 0.753 \end{pmatrix}$

Mass Matrices:

$$M_u=\begin{pmatrix} -0.97\,\epsilon^2 & -0.93\,\epsilon^2 & 1.2\,\epsilon \\ -0.93\,\epsilon^2 & 1.1\,\epsilon^2 & 2.1\,\epsilon \\ 1.2\,\epsilon & 2.1\,\epsilon & -0.95 \end{pmatrix}, M_d=\begin{pmatrix} 0.42\,\epsilon & -0.4\,\epsilon & 0.092\,\epsilon \\ 0.72\,\epsilon & -0.46\,\epsilon & -0.088\,\epsilon \\ -0.07 & 0.32 & -0.46 \end{pmatrix}, M_l=\begin{pmatrix} 0.42\,\epsilon & 0.72\,\epsilon & -0.07 \\ -0.4\,\epsilon & -0.46\,\epsilon & 0.32 \\ 0.092\,\epsilon & -0.088\,\epsilon & -0.46 \end{pmatrix},$$

$$M_{MN}=\begin{pmatrix} 1.5\,\epsilon & -1.2\,\epsilon & 0.85\,\epsilon \\ -1.2\,\epsilon & \epsilon^2\,h_{22}^{mn} & 0.075\,\epsilon \\ 0.85\,\epsilon & 0.075\,\epsilon & 1.6\,\epsilon \end{pmatrix}, M_{DN}=\begin{pmatrix} 1.\,\epsilon & 1.5\,\epsilon & 0.88\,\epsilon \\ -0.044\,\epsilon & 1.1\,\epsilon & -1.1\,\epsilon \\ -2.\,\epsilon & -1.8\,\epsilon & 2.\,\epsilon \end{pmatrix}, M_\nu^{LO}=\frac{v^2}{\Lambda_R}\begin{pmatrix} 2.2\,\epsilon & 2.8\,\epsilon & -7.5\,\epsilon \\ 2.8\,\epsilon & 1.\,\epsilon & -3.5\,\epsilon \\ -7.5\,\epsilon & -3.5\,\epsilon & 9.\,\epsilon \end{pmatrix}=\frac{v^2}{\Lambda_R}\begin{pmatrix} 0.052 & 0.065 & -0.18 \\ 0.065 & 0.024 & -0.083 \\ -0.18 & -0.083 & 0.21 \end{pmatrix},$$

$$M_\nu^{Full}=\frac{v^2}{\Lambda_R}\begin{pmatrix} \frac{(-0.77\,\epsilon-3.4)\,\epsilon}{1.\,\epsilon-1.6} & \frac{(0.32\,\epsilon-4.3)\,\epsilon}{1.\,\epsilon-1.6} & \frac{(0.49\,\epsilon+12.)\,\epsilon}{1.\,\epsilon-1.6} \\ \frac{(0.32\,\epsilon-4.3)\,\epsilon}{1.\,\epsilon-1.6} & \frac{(-1.\,\epsilon-1.6)\,\epsilon}{1.\,\epsilon-1.6} & \frac{\epsilon\,(3.\,\epsilon+5.5)}{1.\,\epsilon-1.6} \\ \frac{(0.49\,\epsilon+12.)\,\epsilon}{1.\,\epsilon-1.6} & \frac{\epsilon\,(3.\,\epsilon+5.5)}{1.\,\epsilon-1.6} & \frac{(-12.\,\epsilon-14.)\,\epsilon}{1.\,\epsilon-1.6} \end{pmatrix}=\frac{v^2}{\Lambda_R}\begin{pmatrix} 0.053 & 0.066 & -0.18 \\ 0.066 & 0.024 & -0.085 \\ -0.18 & -0.085 & 0.22 \end{pmatrix}$$

MATRICES Giving Best fit for This Normal Hierarchy Model

$$\mathbf{M_U} = \begin{pmatrix} -0.97\epsilon^2 & -0.93\epsilon^2 & 1.2\epsilon \\ -0.93\epsilon^2 & 1.1\epsilon^2 & 2.1\epsilon \\ 1.2\epsilon & 2.1\epsilon & -0.95 \end{pmatrix} \mathbf{v_u}, \quad \mathbf{M_D} = \begin{pmatrix} 0.42\epsilon & -0.4\epsilon & 0.092\epsilon \\ 0.72\epsilon & -0.46\epsilon & -0.088\epsilon \\ -0.07 & 0.32 & -0.46 \end{pmatrix} \mathbf{v_d}$$

$$\mathbf{M_{DN}} = \begin{pmatrix} 1.0\epsilon & 1.5\epsilon & 0.88\epsilon \\ -0.044\epsilon & 1.1\epsilon & -1.1\epsilon \\ -2.0\epsilon & -1.8\epsilon & 2.0\epsilon \end{pmatrix} \mathbf{v_u}, \quad \mathbf{M_{MN}} = \begin{pmatrix} 1.5\epsilon & -1.2\epsilon & 0.85\epsilon \\ -1.2\epsilon & h_{22}^{mn}\epsilon & 0.075\epsilon \\ 0.85\epsilon & 0.075\epsilon & 1.6\epsilon \end{pmatrix} \mathbf{\Lambda_R}$$

$$\mathbf{M}_\nu = \begin{pmatrix} 0.053 & 0.066 & -0.18 \\ 0.066 & 0.024 & -0.085 \\ -0.18 & -0.085 & 0.22 \end{pmatrix} \frac{\mathbf{v_u^2}}{\mathbf{\Lambda_R}}$$

- Several prefactors are outside the $\mathcal{O}(\mathbf{0.1} - \mathbf{10})$ range of unity as a result of the Monte Carlo fitting procedure.

SUMMARY and DISCUSSION OF RESULTS

- Unified SU(12) SUSY GUT models obtained from a simple SU(12) anomaly-free set of IRs containing 3 SU(5) chiral families under the direct breaking of SU(12) $\rightarrow$ SU(5) $\rightarrow$ SM.
- For this purpose an effective theory approach was used to determine leading order tree-level diagrams for dim-(4+n) matrix elements in powers of $\epsilon^{\mathbf{n}}$ where epsilon is the ratio of the SU(5) to SU(12) scale. Computer looping over the allowed (SU(5))SU(12) families was carried out to best fit the known data with prefactors of $\mathcal{O}(\mathbf{1})$.
- Although the number of prefactors is comparable to the number of data points, we find the fitting procedure is consistent with the known data. It also allows us to say something about the unknowns incorporated in the model fits:
  neutrino mass hierarchy and masses, the favored octant for the atmospheric mixing angle, and the CP-violating phase.

- Running over all the possible family assignments in the chosen anomaly-free SU(12) set $\mathbf{66 + 2(495) + \overline{792} + 2(\overline{220}) + 8(\overline{12})}$ we find

  8 IH and 7 NH models with the atmospheric neutrino angle in the first octant, and

  8 IH and 2 NH models with the angle in the second octant.

- For all the well-fitted models, a (real) leptonic phase of $\delta_{\mathbf{CP}} = \mathbf{0}^\circ$ is obtained indicating a preference for the 1st or 4th quadrant.

- While there is some preference for the inverted hierarchy, the masses for all such models are found to be equal to the following:

$$\mathbf{m_2 = 49.7}\ \mathrm{meV},\ \mathbf{m_1 = 48.9}\ \mathrm{meV},\ \mathbf{m_3 \leq 1.0}\ \mathrm{meV}$$

  Hence the sum of the light neutrino masses is about 0.1 eV, which is just slightly less than the present 0.17 eV cosmological bound.

- For four of the models, the lightest RH neutrino mass is equal to zero, suggesting a very light RH sterile neutrino or a heavier one which may be observed at the LHC.

Dynkin Indices Test for SU(12) --> SU(5)

- Use the Superpartner VEVs of the chiral fields $\overline{\mathbf{792}},\ \mathbf{495},\ \mathbf{495},\ \overline{\mathbf{12}},\ \overline{\mathbf{12}},\ \overline{\mathbf{12}}$

$$
\begin{array}{lcl}
\mathbf{v}_{\{12,11,10,9,8\}} & \rightarrow & [\mathbf{10000-100000}] \\
\mathbf{v}^{\{12,11,10,9\}} & \rightarrow & [\mathbf{-1000100000}] \\
\mathbf{v}^{\{9,8,7,6\}} & \rightarrow & [\mathbf{000-10001000}] \\
\mathbf{v}_{6} & \rightarrow & [\mathbf{000001-10000}] \\
\mathbf{v}_{7} & \rightarrow & [\mathbf{0000001-1000}] \\
\mathbf{v}_{9} & \rightarrow & [\mathbf{0001-1000000}] \\
 & \boldsymbol{\Sigma} & [\mathbf{00000000000}]
\end{array}
$$

which implies SUSY is unbroken, sum of Dynkin weights vanishes.