CLOCKWORK MECHANISM
A 4D/5D PERSPECTIVE ON BSM MODEL BUILDING

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AA, Dillon, "Clockwork Goldstone Bosons", arXiv:1612.04011

MPIK Heidelberg — November 26, 2018

VRIJE UNIVERSITEIT BRUSSEL

OUTLINE

- ✦ Introduction and motivation
- ✦ 4D clockwork mechanism
- ✦ 5D clockwork dimension
- ✦ Non-Abelian clockwork
- ✦ Summary
- ✦ Outlook: some possible directions for BSM model building

MOTIVATION

- Physical scales are related to the masses of particles

physics scale $\Lambda \sim \frac{m}{g}$ mass of particles / coupling

- Natural expectation (Dirac's Naturalness)

$$g \sim \mathcal{O}(1) \Longleftrightarrow \Lambda \approx m$$

e.g. Fermi scale $\Lambda_{\rm EW} \approx m_W$ since $g_{\rm EW} \sim \mathcal{O}(1)$

- Exception: *'t Hooft's Technical Naturalness*, i.e.

$g \ll 1$ is natural, if there is an enhanced symmetry when $g \to 0$

e.g. Weinberg op. scale $\Lambda_5 \gg m_\nu$ neutrino mass

(chiral symmetry)

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(chiral symmetry)

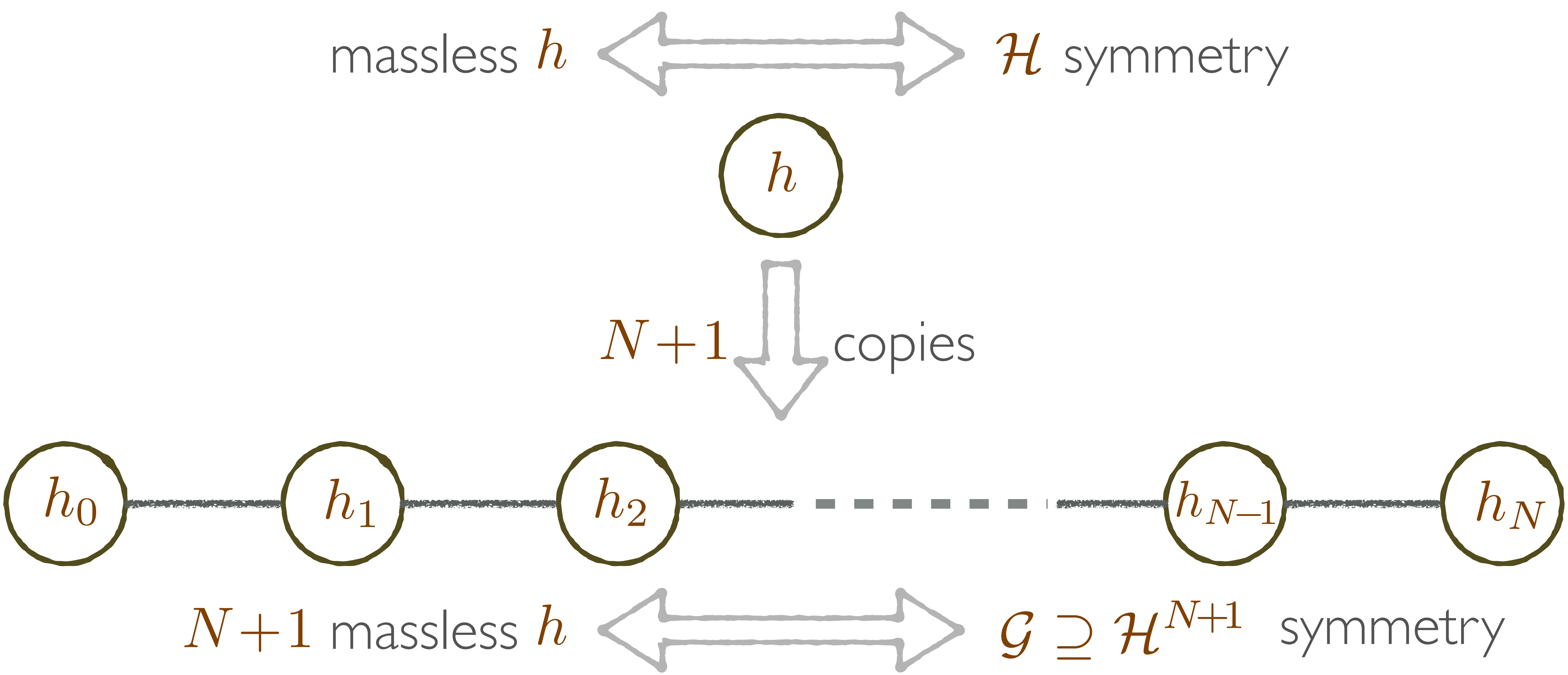
Can we have small couplings without large scales?

CLOCKWORK MECHANISM: BASIC IDEA

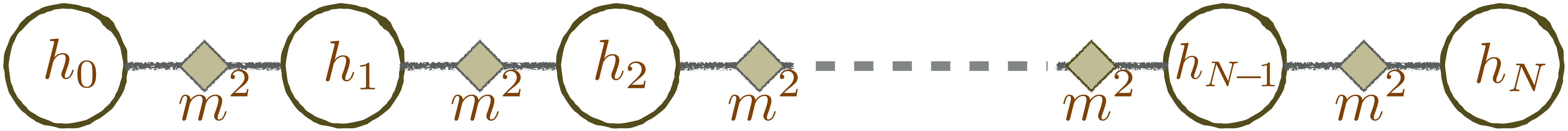
Choi-Im: 1511.00132
Kaplan-Rattazzi: 1511.01827

> Clockwork mechanism is a dynamical way to generate exponentially small couplings out of $\mathcal{O}(1)$ quantities in a fundamental theory.

✦ Imagine a particle h remains massless due to a symmetry $\mathcal{H}$



CLOCKWORK MECHANISM: BASIC IDEA



Now introduce an explicit soft breaking of $\mathcal{G}$ symmetry at a mass scale m by a mass-mixing links b/w nearest neighbors.

$N+1$ sites ⟹ N symmetry breaking links

N massive h' ⟺ $\mathcal{G}/\mathcal{H}'$ broken symmetry

massless h'_0 ⟺ $\mathcal{H}'$ unbroken symmetry

h'_0 is equally distributed along the chain.

This is a deconstruction of a flat extra dimension with a bulk scalar field

Arkani-Hamed, Cohen, Georgi: hep-th/0104005

CLOCKWORK MECHANISM: BASIC IDEA

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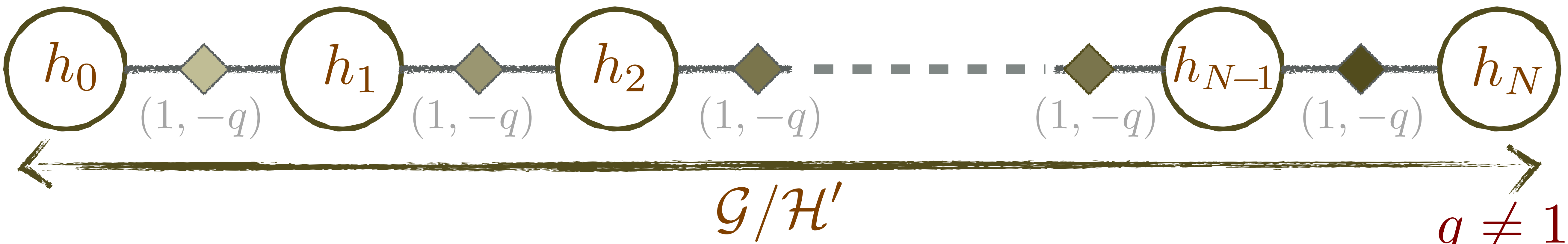
Giudice-McCullough: 1610.07962



$q \neq 1$

The key ingredient of the clockwork mechanism is that link-spurion interact with the nearest neighbor sites *asymmetrically*!

Zero-mode is smeared out along all sites asymmetrically

For $q > 1$, zero-mode h'_0 component in j^{th} site is

$$h_j \supset \frac{h'_0}{q^j}$$

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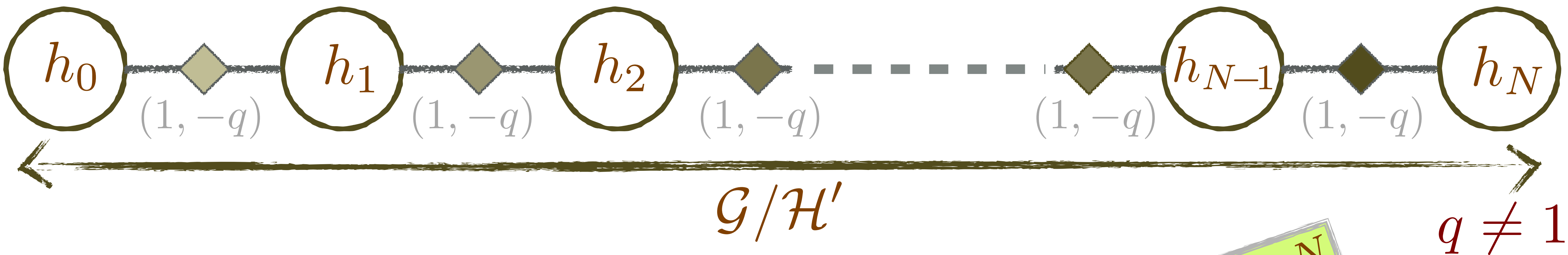

The key ingredient of the clockwork mech… …k-spurion interact with the nearest neighbor sit…

Hence anything coupled to Nth site will have q^{-N} suppressed coupling with the zero-mode h'_0 !

Zero-… …along all sites asymmetrically

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CLOCKWORK SCALAR

Choi-Im: 1511.00132
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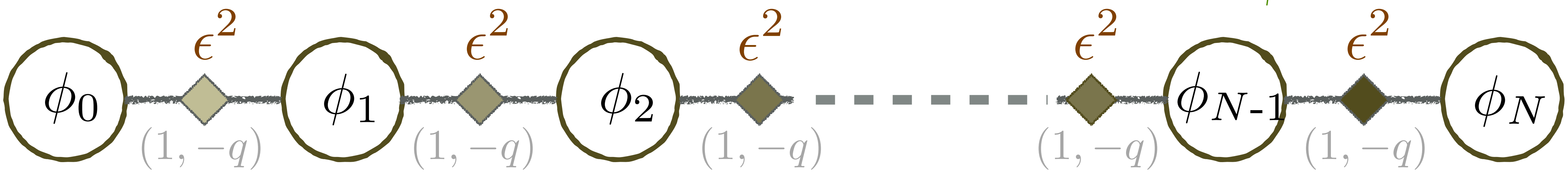

Complex scalar ϕ_j charged under $U(1)_j$ global symmetry at *j*-site!

$$\mathcal{L} = \sum_{j=0}^{N} \left(\partial_\mu \phi_j^\dagger \partial^\mu \phi_j - \mu^2 \phi_j^\dagger \phi_j - \frac{\lambda}{4} |\phi_j^\dagger \phi_j|^2 \right) + \epsilon^2 f^{3-q} \sum_{j=0}^{N-1} \left(\phi_j^\dagger \phi_{j+1}^q + \text{h.c.} \right)$$

explicit breaking $\epsilon \ll 1$

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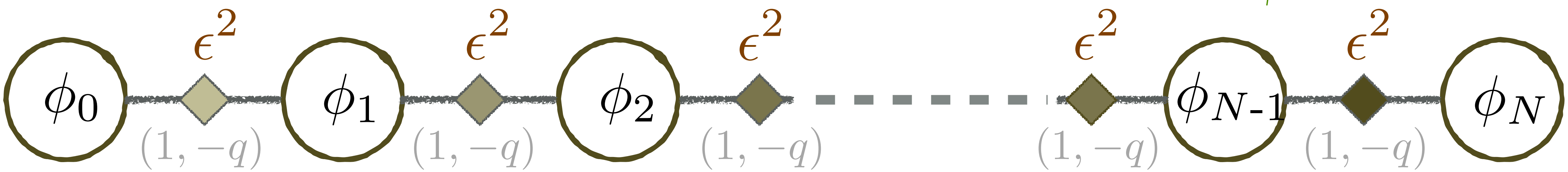

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$$\langle \phi_j \rangle \equiv f = \sqrt{\frac{-2\mu^2}{\lambda}}$$

explicit breaking $\epsilon \to 0$

Each site ϕ_j acquires VEV f and spontaneously break $U(1)_j$ symmetry.

Effective Field Theory below symmetry breaking scale f

$$\phi_j \to f\, U_j \equiv f\, e^{i\pi_j/\sqrt{2}f}$$

Non-linear sigma model

CLOCKWORK SCALAR

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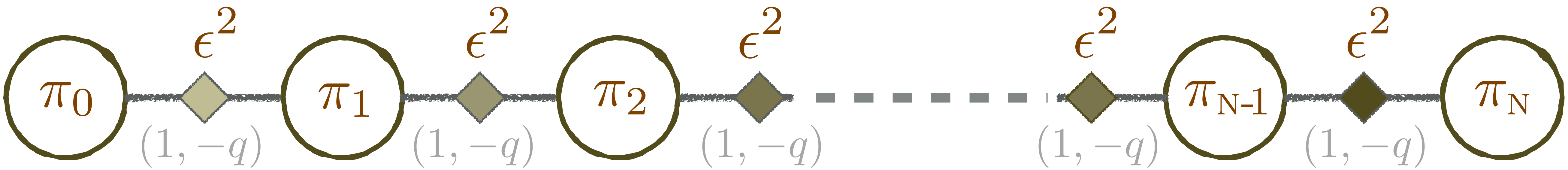


$$\mathcal{L} = f^2 \sum_{j=0}^{N} \partial_\mu U_j^\dagger \partial^\mu U_j + \epsilon^2 f^4 \sum_{j=0}^{N-1} \left(U_j^\dagger U_{j+1}^q + \text{h.c.} \right)$$

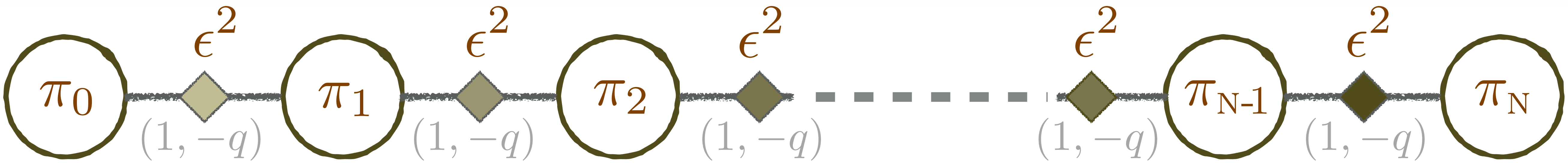
up to quadratic level

$$U_j \equiv e^{i\pi_j/\sqrt{2}f}$$

$$\mathcal{L}_\pi = \frac{1}{2} \sum_{j=0}^{N} \partial_\mu \pi_j^\dagger \partial^\mu \pi_j - \frac{1}{2}\epsilon^2 f^2 \sum_{j=0}^{N-1} (\pi_j - q\pi_{j+1})^2 + \mathcal{O}(\pi^4)$$

Mass mixing is the key feature!

CLOCKWORK MECHANISM

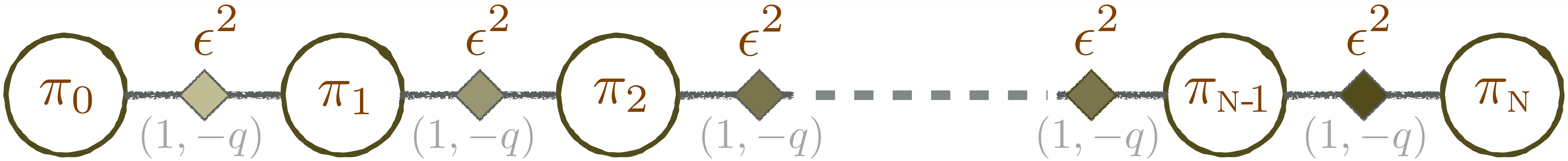


$$\mathcal{L} = \frac{1}{2}\sum_{j=0}^{N} \partial_\mu \pi_j^\dagger \partial^\mu \pi_j + \frac{1}{2}\sum_{i,j=0}^{N} \pi_i M_{ij}^2 \pi_j + \mathcal{O}(\pi^4)$$

$$M_{ij}^2 = \epsilon^2 f^2 \begin{pmatrix} 1 & -q & 0 & \cdots & & 0 \\ -q & 1+q^2 & -q & \cdots & & 0 \\ 0 & -q & 1+q^2 & \cdots & & 0 \\ \vdots & \vdots & \vdots & \ddots & & \vdots \\ & & & & 1+q^2 & -q \\ 0 & 0 & 0 & \cdots & -q & q^2 \end{pmatrix}$$

One zero-mode and N massive modes!

CLOCKWORK MECHANISM



$$\mathcal{L} = \frac{1}{2}\sum_{j=0}^{N} \partial_\mu \pi_j^\dagger \partial^\mu \pi_j + \frac{1}{2}\sum_{i,j=0}^{N} \pi_i M_{ij}^2 \pi_j + \mathcal{O}(\pi^4)$$

- Mass Eigenbasis

$$\pi_j = \sum_{k=0}^{N} \mathcal{O}_{jk} \pi'_k$$

$$\mathcal{O}^T M^2 \mathcal{O} = \mathrm{diag}(\underbrace{m^2_{\pi'_0}}_{=0}, \underbrace{m^2_{\pi'_1}, \cdots, m^2_{\pi'_N}}_{\sim \epsilon^2 f^2})$$

- Key observation of the clockwork mechanism is

$$\mathcal{O}_{j0} \sim \frac{1}{q^j}, \quad \mathcal{O}_{jk} \sim 1 \quad \Longrightarrow \quad \pi_j \sim \frac{\pi'_0}{q^j} + \sum_{k=1}^{N} \mathcal{O}_{jk} \pi'_k$$

EXPONENTIALLY SMALL COUPLINGS

- Couple external fields (SM) to the Nth-site

$$\mathcal{L} = \frac{1}{16\pi^2}\frac{\pi_N}{f}G_{\mu\nu}\widetilde{G}^{\mu\nu}$$

in the mass eigenbases $\quad \pi_N \sim \frac{\pi'_0}{q^N} + \sum_{k=1}^{N} \pi'_k$

$$\mathcal{L} \approx \frac{1}{16\pi^2}\left(\frac{\pi'_0}{q^N f} + \sum_{k=1}^{N}\frac{\pi'_k}{f}\right)G_{\mu\nu}\widetilde{G}^{\mu\nu}$$

- Note that π'_0 interactions are q^{-N} suppressed!

$$f_0 \equiv q^N f \gg f$$

- If π'_0 is the QCD axion then the PQ symmetry breaking scale f can be as low as the electroweak scale!

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in the mass eigenbases $\pi_N \sim \pi'_0 \; \ldots \; N$

$$\mathcal{L} \approx \frac{1}{\ldots} \Big(\ldots \; {}_{k=1} \ldots \frac{\ldots}{f} \Big) G_{\mu\nu}\widetilde{G}^{\mu\nu}$$

Hence clockwork mechanism generates exponential hierarchies out of $\mathcal{O}(1)$ parameters!

- Note ...eractions are q^{-N} suppressed!

$$f_0 \equiv q^N f \gg f$$

- If π'_0 is the QCD axion then the PQ symmetry breaking scale f can be as low as the electroweak scale!

CLOCKWORK INTERACTIONS

AA, Dillon:1612.04011

✦ Clockwork potential for pseudo-Goldstone bosons is

$$V(\pi) = -\epsilon^2 f^4 \sum_{j=0}^{N-1} \left(U_j^\dagger U_{j+1}^q + \text{h.c.} \right)$$

up to quartic level ⇓

$$U_j \equiv e^{i\pi_j/\sqrt{2}f}$$

$$V(\pi) = \sum_{j=0}^{N-1} \left[\frac{\epsilon^2 f^2}{2} (\pi_j - q\pi_{j+1})^2 - \frac{\epsilon^2}{2(4!)} (\pi_j - q\pi_{j+1})^4 \right]$$

✦ Clockwork potential in the mass eigenbasis:

$$V(\pi') = \sum_{j=0}^{N-1} \left[\frac{\epsilon^2 f^2}{2} R_{jk_1} R_{jk_2} \pi'_{k_1} \pi'_{k_2} - \frac{\epsilon^2}{2(4!)} R_{jk_1} R_{jk_2} R_{jk_3} R_{jk_4} \pi'_{k_1} \pi'_{k_2} \pi'_{k_3} \pi'_{k_4} \right]$$

$$R_{jk} \equiv \mathcal{O}_{jk} - q\mathcal{O}_{j+1,k} \qquad R_{jk_1} R_{jk_2} = m_j^2 \delta_{k_1 k_2} \qquad R_{j0} = 0$$

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up to quartic level

U … $\sqrt{2} f$

$$V(\pi) = \sum_{j=0}^{N-1} \Bigg[\frac{\epsilon^2 f^2}{2} (\pi \;\cdots\; \frac{}{2(4!)} (\pi_j - q\pi_{j+1})^4 \Bigg]$$

Zero-mode has no interactions due to clockwork shift-symmetry but are there any accidental symmetries among the clockwork gears?

✦ Cl… in the mass eigenbasis:

$$V(\pi \;\cdots \sum_{j=0} \Bigg[\frac{\epsilon^2 f^2}{2} R_{jk_1} R_{jk_2} \pi'_{k_1} \pi'_{k_2} - \frac{\epsilon^2}{2(4!)} R_{jk_1} R_{jk_2} R_{jk_3} R_{jk_4} \pi'_{k_1} \pi'_{k_2} \pi'_{k_3} \pi'_{k_4} \Bigg]$$

$R_{jk} \equiv \mathcal{O}_{jk} - q\mathcal{O}_{j+1,k}$ $\qquad R_{jk_1} R_{jk_2} = m_j^2 \delta_{k_1 k_2}$ $\qquad R_{j0} = 0$

CLOCKWORK PARITY

AA, Dillon:1612.04011

- Clockwork interactions have a very non-trivial feature, they are "only" allowed for

$$k_1 \pm k_2 \pm k_3 \pm \ldots \pm k_d = 0$$

by choosing either $+$ or $-$ for each term.

Analogue to KK-parity selection rules in extra dimensional models!

- We find an accidental discrete $Z_2^{\text{odd}} \times Z_2^{\text{even}}$ symmetry in the interaction terms, under which

$$\pi'_{k\text{-odd}} \xrightarrow[Z_2^{\text{even}}]{Z_2^{\text{odd}}} \mp \pi'_{k\text{-odd}} \qquad \pi'_{k\text{-even}} \xrightarrow[Z_2^{\text{even}}]{Z_2^{\text{odd}}} \pm \pi'_{k\text{-even}}$$

e.g. interactions terms of the following form are forbidden

$$\pi'_1 \pi'_1 \pi'_1 \pi'_2, \quad \pi'_1 \pi'_1 \pi'_2 \pi'_3, \text{etc.}$$

GAUGED CLOCKWORK SCALAR

AA, Dillon:1612.04011

In UV completions, gauge symmetries are preferred over global symmetries. Here, we provide *a clockwork scalar from gauge symmetries*.

Each site has a $U(1)_j$ gauge symmetry with A_j^μ gauge boson. Complex scalar fields ϕ_j are the link fields.

$$A_\mu^j \rightarrow A_\mu^j - \frac{1}{g}\partial_\mu \alpha_j(x)$$

Gauge Transformations:

$$\phi_0 \rightarrow e^{-iq\alpha_1(x)}\phi_0,$$

$$\phi_j \rightarrow e^{i(\alpha_j(x)-q\alpha_{j+1}(x))}\phi_j, \quad (j = 1\ldots, N-1)$$

$$\phi_N \rightarrow e^{i\alpha_N(x)}\phi_N$$

GAUGED CLOCKWORK SCALAR

AA, Dillon: 1612.04011

All ϕ_j acquire VEVs and spontaneously break $U(1)_j$ gauge symmetry

$N+1$ Goldstone bosons ⟺ N gauge bosons

$$\mathcal{M}^2_{A,ij} = m_A^2 \begin{pmatrix} 1+q^2 & -q & \cdots & 0 & 0 \\ -q & 1+q^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1+q^2 & -q \\ 0 & 0 & \cdots & -q & 1+q^2 \end{pmatrix}_{N\times N}$$

$$\mathcal{M}^2_{\pi,ij} = \xi m_A^2 \begin{pmatrix} q^2 & -q & 0 & \cdots & 0 & 0 \\ -q & 1+q^2 & -q & \cdots & 0 & 0 \\ 0 & -q & 1+q^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1+q^2 & -q \\ 0 & 0 & 0 & \cdots & -q & 1 \end{pmatrix}_{N+1\times N+1}$$

No zero-mode

One zero-mode

N gauge bosons acquire mass by the Higgs mechanism, i.e. "eating" N GBs

GAUGED CLOCKWORK SCALAR

AA, Dillon:1612.04011

1 massless Goldstone boson remains uneaten: *Accidental* $U(1)_{\rm CW}$

A clockwork scalar with gauge bosons as *Gears!*

Application

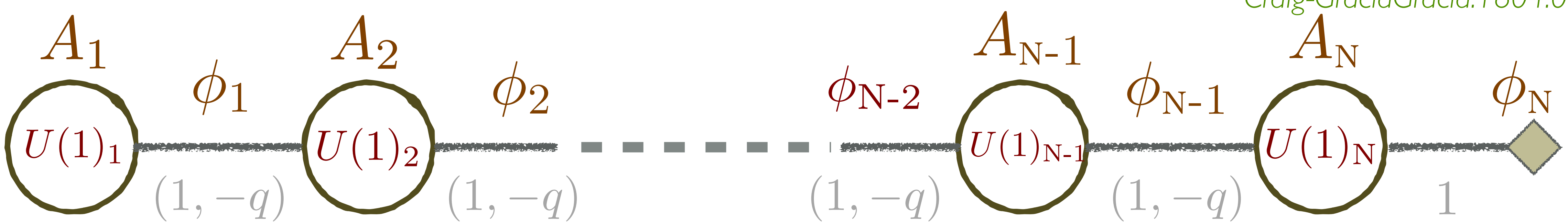
Gauged clockwork scalar as an axion where Pecci-Quinn symmetry is the accidental clockwork symmetry!

Coy-Frigerio-Ibe:1706.04529
Bonnefoy-Dudas-Pokorski:1804.01112

GAUGED CLOCKWORK: A LIGHT VECTOR

Lee:1708.03564

Craig-GraciaGracia:1804.01112



N Goldstone bosons ⟺ N Gauge symmetries

N gauge bosons acquire mass by the Higgs mechanism, i.e. "eating" N GBs

A very light clockwork (zero-mode) vector boson with heavy vector *gears!*

GAUGED CLOCKWORK: A MASSLESS VECTOR

Giudice-McCullough: 1610.07962

$N-1$ Goldstone bosons ⟺ N Gauge symmetries

A massless clockwork vector boson
with heavy vector *gears!*

LARGE N LIMIT: A CLOCKWORK DIMENSION

- In large N limit the discrete site clockwork model can be thought of as a deconstruction of an extra dimension

$$ds^2 = \underbrace{a^2(z)}_{\text{warped factor}}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

where $-L \leq z \leq L$ and the geometry is S_1/Z_2 orbifold.

- Consider a bulk scalar field

$$S = \int d^4x \int_0^L dz \sqrt{g}\left(\frac{1}{2}g^{MN}\partial_M\phi\partial_N\phi\right),$$

$$= \int d^4x \int_0^L dz \left(\frac{1}{2}a^3(z)\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}a^3(z)\partial_z\phi\partial^z\phi\right),$$

rescaling $\phi \to a^{-3/2}(z)\phi$

$$S = \int d^4x \int_0^L dz \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}a^3(z)\left(\partial_z a^{-3/2}\phi\right)^2\right)$$

DECONSTRUCTION OF CLOCKWORK DIMENSION

- Let us discretize the extra dimension:

$$z \to j\ell \qquad \text{where } \ell = L/N \text{ is the lattice spacing } (j = 0, 1, 2, \cdots, N)$$

$$\int_0^L dz \to \sum_{j=0}^{N-1} \ell \quad \text{and} \quad \frac{df(z)}{dz} \to \frac{f_{j+1}(\ell) - f_j(0)}{\ell} \quad \text{with} \quad f(j\ell) \equiv f_j$$

- Discretized scalar action is

$$S = \int d^4x \left(\frac{1}{2} \sum_{j=0}^{N} (\partial_\mu \phi_j)^2 + \frac{1}{\ell^2} \sum_{j=0}^{N-1} \left(a^{-3/2}(0)\phi_j - a^{-3/2}(\ell)\phi_{j+1} \right)^2 \right)$$

which can be parametrized as for $a(z) = e^{-\frac{2}{3}kz}$

$$S = \int d^4x \left(\frac{1}{2} \sum_{j=0}^{N} (\partial_\mu \phi_j)^2 + m_\phi^2 \sum_{j=0}^{N-1} \left(\phi_j - q_\phi \phi_{j+1} \right)^2 \right)$$

$$q_\phi \equiv e^{k\ell} \quad \text{and} \quad m_\phi \equiv 1/\ell$$

CLOCKWORK MASS SPECTRUM

Giudice-McCullough: 1610.07962

- Clockwork scalar theory

$$\mathcal{L} = \frac{1}{2}\sum_{j=0}^{N} \partial_\mu \phi_j \partial^\mu \phi_j - \frac{1}{2}\sum_{i,j=0}^{N} \phi_i \mathcal{M}^2_{\phi,ij}\phi_j$$

$$m^2_{\phi'_0} = 0 \qquad m^2_{\phi'_n} \simeq m^2$$

$$q_\phi = e^{\frac{kL}{N}}$$

duality

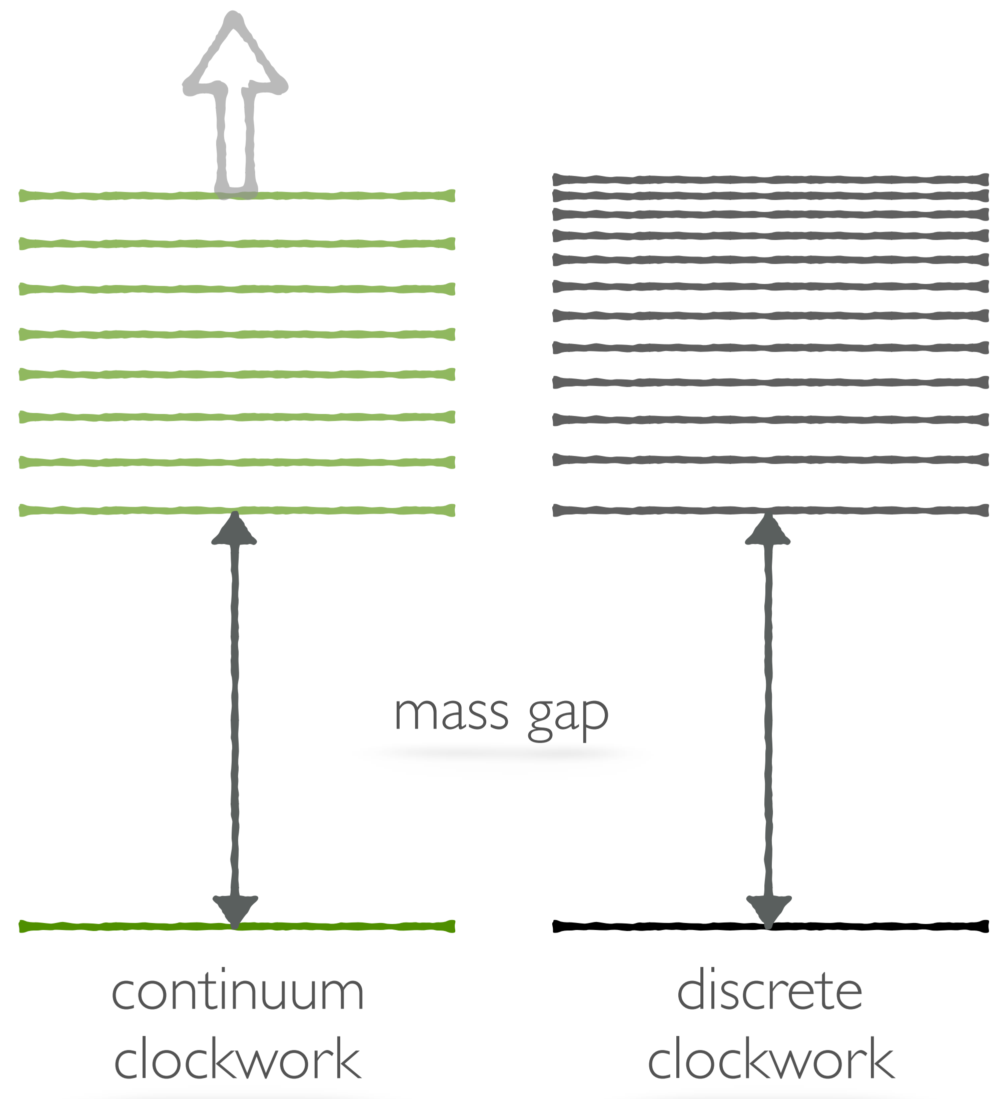
- Linear Dilaton Geometry!

$$ds^2 = e^{-\frac{4}{3}kz}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

Mass spectrum in the Large N limit

$$m^2_{\phi'_0} = 0 \qquad m^2_{\phi'_n} \simeq k^2 + \frac{n^2}{L^2}$$

KK-modes $(n = 1, 2, \cdots, N)$



mass spectra

5D GAUGED CLOCKWORK SCALAR

AA, Dillon:1612.04011

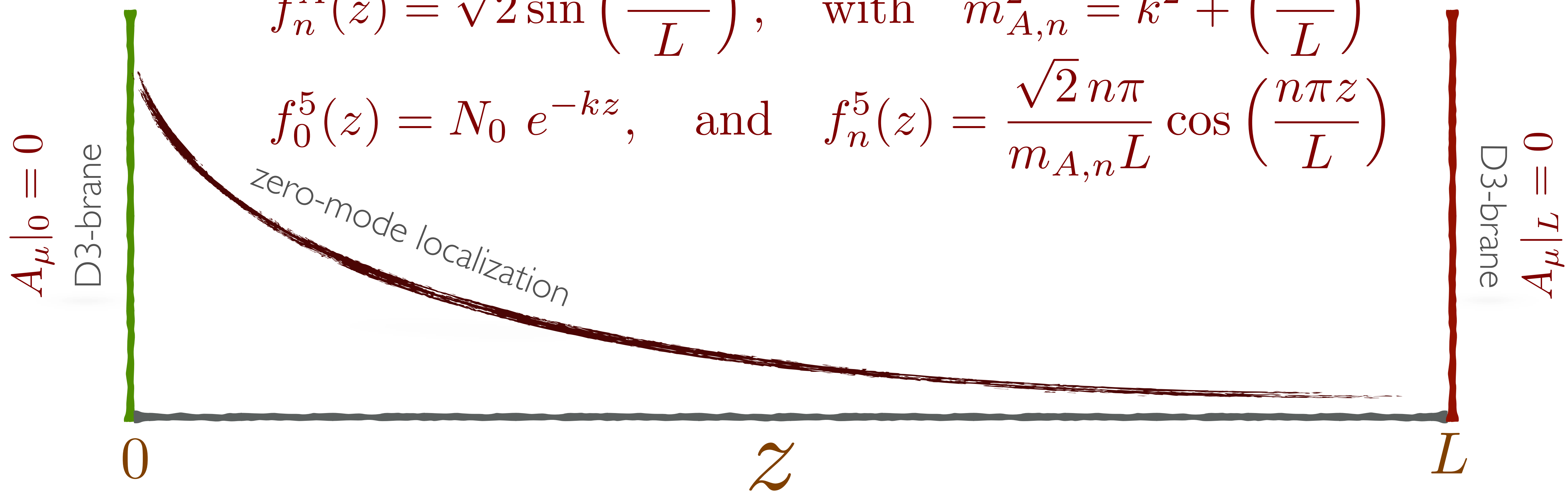
Linear Dilaton Geometry in 5D

$$S_A = \int d^4x \int_0^L dz\ e^{2kz} \left[-\frac{1}{4} F^{MN} F_{MN} + \frac{1}{2\xi} \left(\partial_\mu A^\mu + \xi e^{-2kz} \partial_5 (e^{2kz} A_5) \right)^2 \right]$$

Bulk gauge boson: $A_M = (A_\mu, A_5)$

$$f_n^A(z) = \sqrt{2} \sin\left(\frac{n\pi z}{L}\right), \quad \text{with} \quad m_{A,n}^2 = k^2 + \left(\frac{n\pi}{L}\right)^2$$

$$f_0^5(z) = N_0\ e^{-kz}, \quad \text{and} \quad f_n^5(z) = \frac{\sqrt{2}\, n\pi}{m_{A,n} L} \cos\left(\frac{n\pi z}{L}\right)$$



Zero-mode is localized towards one of the brane and its coupling on the other brane would be exponentially suppressed!

5D GAUGED CLOCKWORK SCALAR

AA, Dillon:1612.04011

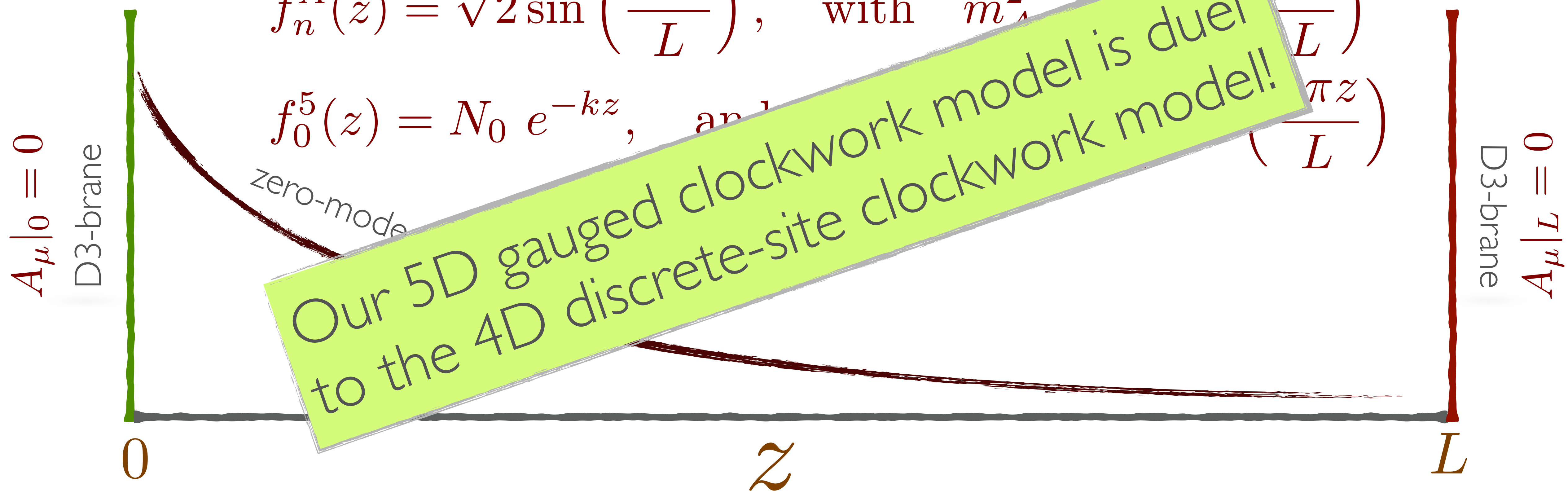
Linear Dilaton Geometry in 5D

$$S_A = \int d^4x \int_0^L dz\ e^{2kz} \left[-\frac{1}{4} F^{MN} F_{MN} + \frac{1}{2\xi} \left(\partial_\mu A^\mu + \xi e^{-2kz} \partial_5 (e^{2kz} A_5) \right)^2 \right]$$

Bulk gauge boson: $A_M = (A_\mu, A_5)$

$$f_n^A(z) = \sqrt{2} \sin\left(\frac{n\pi z}{L}\right), \quad \text{with} \quad m^2_{A} \ \ldots \left(\frac{n\pi}{L}\right)^2$$

$$f_0^5(z) = N_0\ e^{-kz}, \quad \text{an}\ldots \left(\frac{n\pi z}{L}\right)$$



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NON-ABELIAN CLOCKWORK

AA, Dillon:1612.04011

Let us consider a non-abelian global group $\mathcal{G}$ at each site, which spontaneous breaks to $\mathcal{H}$ at scale f, giving $h^{\hat{a}}$ Goldstones!

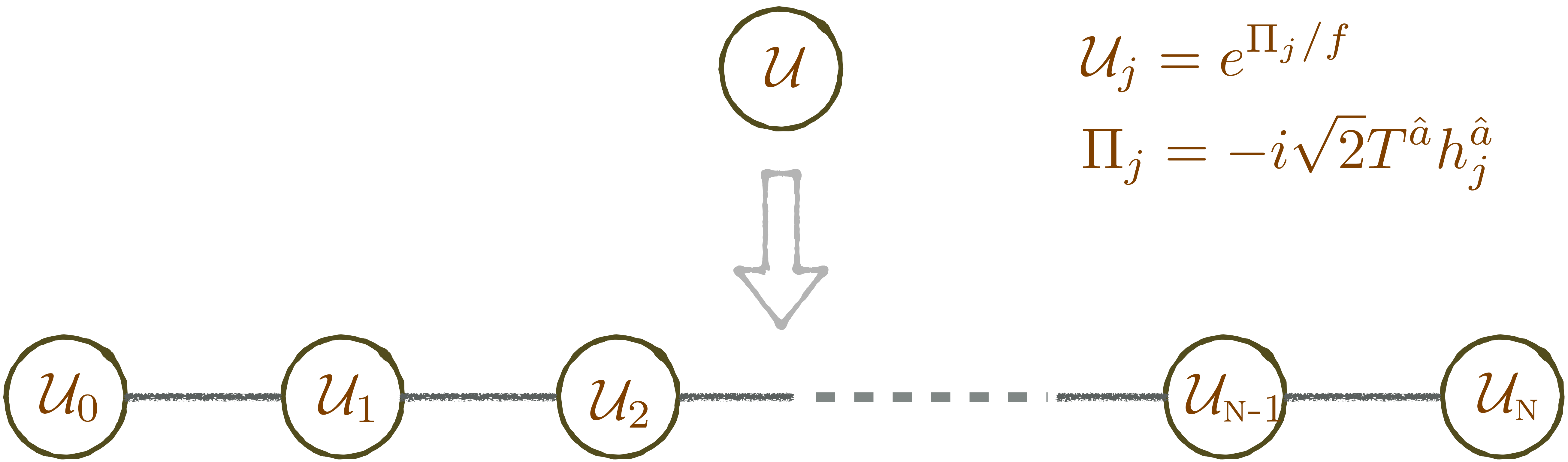
$$\mathcal{G}/\mathcal{H} = SO(N)/SO(N-1)$$

massless $h^{\hat{a}}$ ⟺ $\mathcal{H}$ symmetry

$$\Sigma_j = \Sigma_j^v \mathcal{U}_j$$

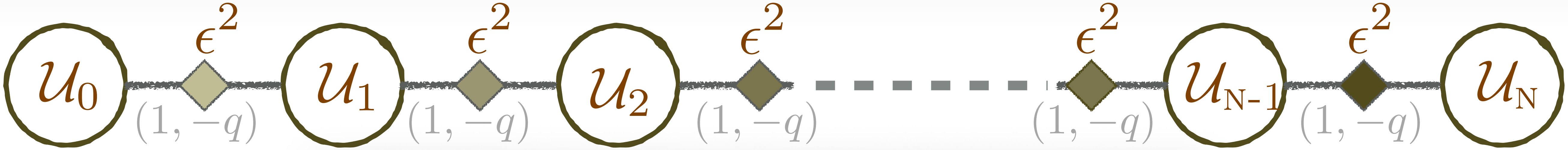
$$\mathcal{U}_j = e^{\Pi_j/f}$$

$$\Pi_j = -i\sqrt{2}T^{\hat{a}}h_j^{\hat{a}}$$



$N+1$ massless $h^{\hat{a}}$ ⟺ $\mathcal{G}^{N+1} \supseteq \mathcal{H}^{N+1}$ symmetry

CLOCKWORK COMPOSITE HIGGS



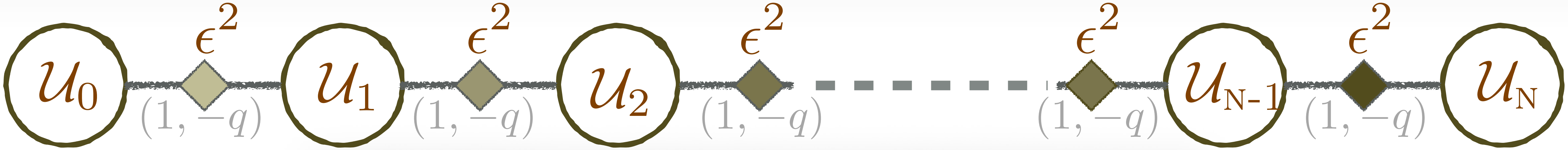
$$\mathcal{L}_{\rm CW} = \frac{\epsilon^2 f_\pi^4}{4} \sum_{j=0}^{N-1} \mathrm{Tr}\left[\mathcal{U}_j^\dagger \mathcal{U}_{j+1}^q + \mathrm{h.c.}\right]$$

$\Downarrow$

$$\mathcal{H}^{N+1}/\mathcal{H}'$$

$q \neq 1$???

CLOCKWORK COMPOSITE HIGGS



$$\mathcal{L}_{\text{CW}} = \frac{\epsilon^2 f_\pi^4}{4} \sum_{j=0}^{N-1} \text{Tr}\left[\mathcal{U}_j^\dagger \mathcal{U}_{j+1}^q + \text{h.c.}\right]$$

$$\mathcal{H}^{N+1}/\mathcal{H}'$$

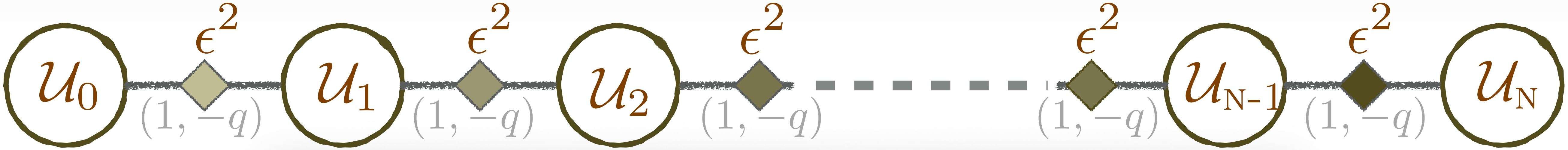
$$q \neq 1 \;???$$

- ✦ Clockwork interaction terms breaks all symmetries (perhaps not bad*) but, …
- ✦ Explicit breaking terms cannot be motivated by spurion analysis!

see also Craig et al.:1704.07831

**Csaki et al.:1811.06019, applied in Linear Dilaton Geometry*

CLOCKWORK COMPOSITE HIGGS



$$\mathcal{L}_{\rm CW} = \frac{\epsilon^2 f_\pi^4}{4} \sum_{j=0}^{N-1} \mathrm{Tr}\left[e^{(\Pi_j - q\Pi_{j+1})/f} + \mathrm{h.c.}\right]$$

$$\mathcal{H}^{N+1}/\mathcal{H}'$$

- Clockwork interaction terms do keep $\mathcal{H}'$ unbroken but …
- Explicit breaking terms cannot be motivated by linearized spurion analysis in a complete UV theory!

NON-ABELIAN CLOCKWORK

AA, Dillon:1612.04011

Let us consider a vector-like global group $\mathcal{G}$ at each site, which spontaneous breaks to $\mathcal{H}$ at scale f, giving $h^{\hat{a}}$ Goldstones!

$$\mathcal{G}_j/\mathcal{H}_j = SU(\mathcal{N})_L^j \times SU(\mathcal{N})_R^j / SU(\mathcal{N})_V^j$$

$$\Phi_j \to L_j \Phi_j R_j^\dagger$$

$$\mathcal{L}_{\rm CW} = \epsilon^2 f_\pi^{3-q} \sum_{j=0}^{N-1} \mathrm{Tr}\left(\Phi_j^T \Phi_{j+1}^q + \mathrm{h.c.}\right)$$

For $q \neq 1$ all the $\mathcal{G}$ symmetries are explicitly broken in such a way that preserves the diagonal subgroup. $L_j = R_j = L_{j+1} = R_{j+1}$

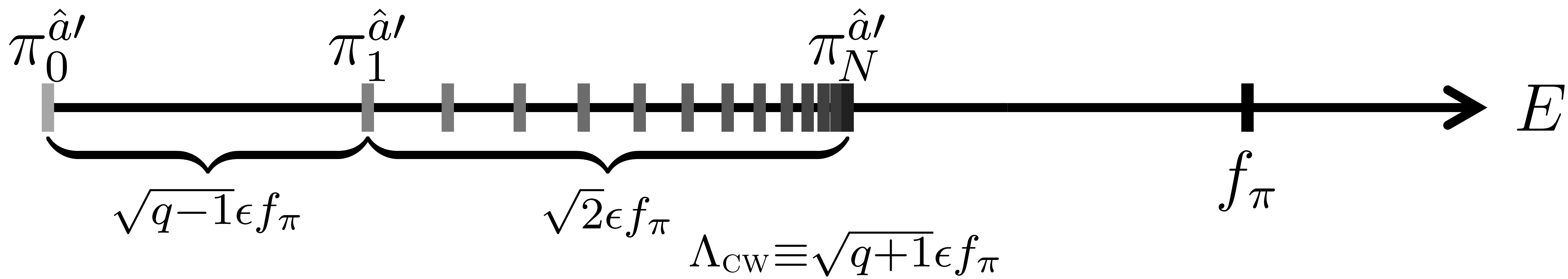
No exact Goldstone modes!

NON-ABELIAN CLOCKWORK

AA, Dillon:1612.04011

✦ Potential for pseudo-Goldstone modes is

$$V(\Pi) = \sum_{j=0}^{N-1} \Big[\epsilon^2 f^2 \mathrm{Tr}\left((\Pi_j - q\Pi_{j+1})^2\right) - \frac{\epsilon^2}{6}\mathrm{Tr}\left((\Pi_j - q\Pi_{j+1})^4\right) - \frac{\epsilon^2 q^2}{3}\mathrm{Tr}\left(\Pi_j[\Pi_j, \Pi_{j+1}]\Pi_{j+1}\right) \; + \; \mathrm{h.c.}\Big]$$



After clockwork rotations at tree-level there is a massless multiplet. However, quartic interactions would generate small mass!

$$m^2_{\Pi'_0} \sim \frac{\epsilon^2 q^2}{16\pi^2} m^2_{\rm gears} \ll m^2_{\rm gears}$$

NON-ABELIAN CLOCKWORK

AA, Dillon:1612.04011

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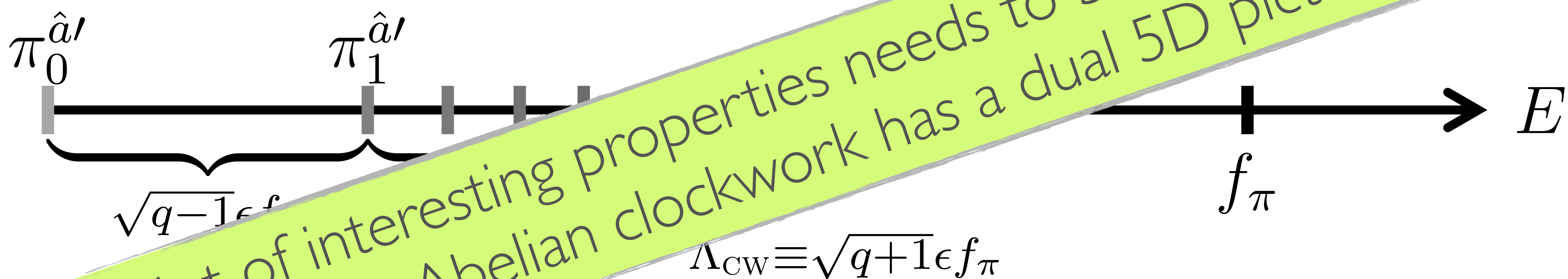
$$- \frac{\epsilon^2 q^2}{3}\mathrm{Tr}\left(\Pi_j[\Pi_j, \Pi_{j+1}]\Pi_{j+1}\right) \ + \ \mathrm{h.c.}\Big]$$



A lot of interesting properties needs to be explored!!!
e.g. do non-Abelian clockwork has a dual 5D picture???

Afte[illegible] rotations at tree-level there is a massless multiplet.
How[illegible]ver, quartic interactions would generate small mass!

$$m^2_{\Pi_0'} \sim \frac{\epsilon^2 q^2}{16\pi^2} m^2_{\rm gears} \ll m^2_{\rm gears}$$

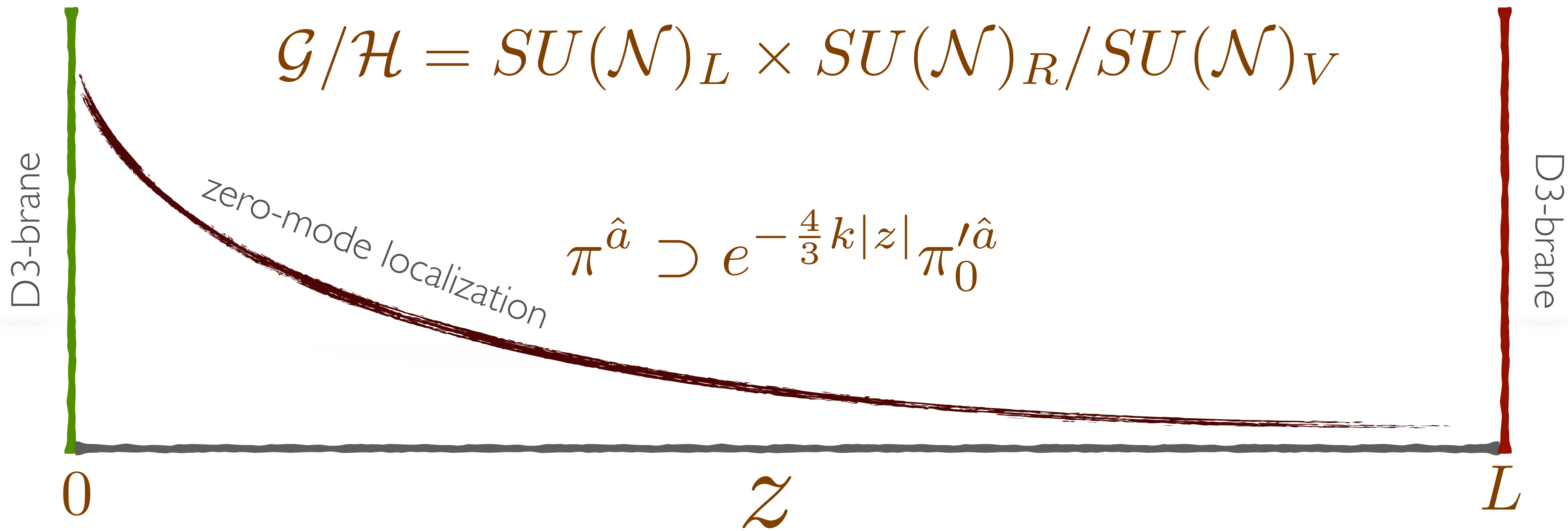
DECONSTRUCTING NON-ABELIAN CLOCKWORK

AA, Dillon: 1612.04011

Linear Dilaton Geometry in 5D

$$\mathcal{G}/\mathcal{H} = SU(\mathcal{N})_L \times SU(\mathcal{N})_R / SU(\mathcal{N})_V$$

$$\pi^{\hat{a}} \supset e^{-\frac{4}{3}k|z|} \pi_0'^{\hat{a}}$$



Zero-mode is localized towards one of the brane and hence its coupling on the other brane would be exponentially suppressed!

DECONSTRUCTING NON-ABELIAN CLOCKWORK

AA, Dillon:1612.04011

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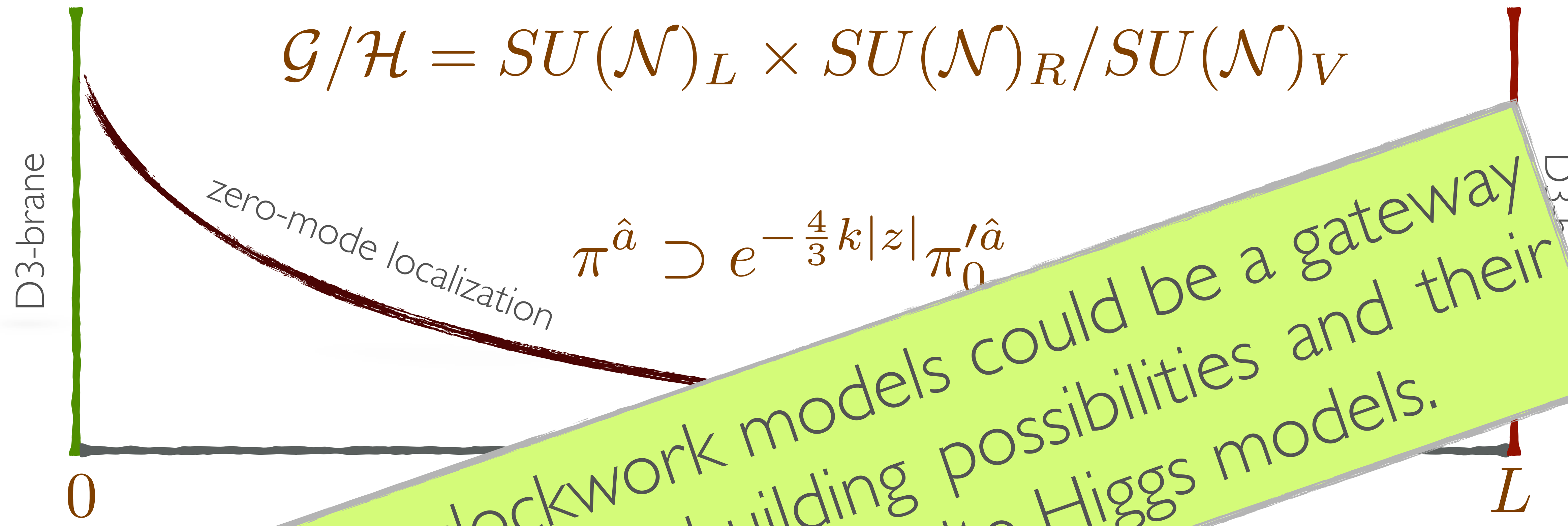

$$\pi^{\hat{a}} \supset e^{-\frac{4}{3}k|z|}\pi_0'^{\hat{a}}$$

Zer ... ards one of the brane and hence its coup ... ther brane would be exponentially suppressed!

Non-Abelian clockwork models could be a gateway to interesting model building possibilities and their applications, including composite Higgs models.

SUMMARY

- Clockwork mechanism is engineered to generate exponentially small (or large) quantities in theories which have $\mathcal{O}(1)$ parameters!

- It provides a concrete example where scales much above Planck scale can be reached without invoking quantum gravity or breaking down of EFT.

- Large N limit of the discrete clockwork model correspond to an extra dimension with "linear dilaton geometry".

- We extended the clockwork mechanism to Abelian gauged and non-Abelian models.

OUTLOOK

Many new model building possibilities!

- (secluded) Dark matter from clockwork dynamics.
  Hambye-Teresi-Tytgat:1612.06411, Kim-McDonald:1709.04105, Goudelis-Mohan-Sengupta:1807:06642
- Flavor and neutrino physics with clockwork mechanics.
  Ibarra-Kushwaha-Vempati:1711.02070, Patel:1711.05393, Alonso-Carmona-Dillon-Kamenik-Camalich-Zupan:1807:06642
- Phenomenology of the clockwork gears/KK-modes.
  Giudice-Kats-McCullough-Torre-Urbano:1711.08437
- Clockwork inflation/cosmology.
  Kehagias-Riotto:1611.03316, Long:1803.07086
- Continuum Naturalness (top-partners are "unparticles"!)
  Csaki et al.:1811.06019
- "Dictionary" between discrete and continuum clockwork models.