Cascade Hierarchy and Grand Unified Theory

2nd Nov. 2009 @ MPIK

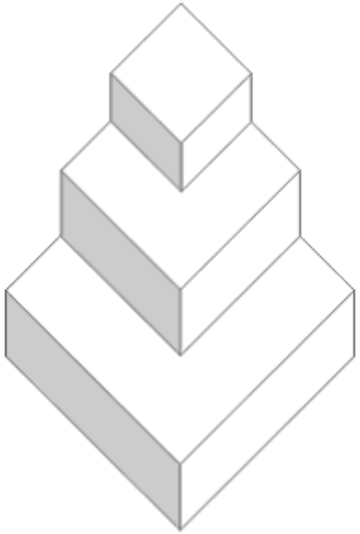
Ryo Takahashi (MPIK)

References

K. Kojima, H. Sawanaka and RT, [to appear]

N. Haba, RT, M. Tanimoto and K. Yoshioka, PRD78 (2008) 113002

Contents



cascade

1. Introduction

Standard Model (SM)

Fermion

Matters are composed of Quarks and Leptons.

Generation		1	2	3	Strong	Weak	EM Interaction
Quark	up-type	u	c	t	○	○	○ : $Q = +2/3$
	down-type	d	s	b	○	○	○ : $Q = -1/3$
Lepton	neutrino	ν_e	ν_μ	ν_τ	×	○	× : $Q = 0$
	charged lepton	e	μ	τ	×	○	○ : $Q = -1$

Gauge boson

Particles mediating the Strong, Weak and EM interactions:
Gluon, Weak boson and Photon

Higgs boson (undiscovered particle)

Electroweak symmetry breaking (EWSB)

1. Introduction

Standard Model (SM)

Fermion

Matters are composed of Quarks and Leptons.

Generation		1	2	3	Strong	Weak	EM Interaction
Quark	up-type	u	c	t	○	○	○ : $Q = +2/3$
	down-type	d	s	b	○	○	○ : $Q = -1/3$
Lepton	neutrino	ν_e	ν_μ	ν_τ	×	○	× : $Q = 0$
	charged lepton	e	μ	τ	×	○	○ : $Q = -1$

1. Introduction

Standard Model (SM)

Fermion

Matters are composed of Quarks and Leptons.

Generation		1	2	3	Strong	Weak	EM Interaction
Quark	up-type	u	c	t	○	○	○ : $Q = +2/3$
	down-type	d	s	b	○	○	○ : $Q = -1/3$
Lepton	neutrino	ν_e	ν_μ	ν_τ	×	○	× : $Q = 0$
	cherged lepton	e	μ	τ	×	○	○ : $Q = -1$

Quark masses :

$$(m_u, m_c, m_t) \sim (2 \times 10^{-3}, 1.3, 170) \text{ GeV}$$

$$(m_d, m_s, m_b) \sim (5 \times 10^{-3}, 0.1, 4) \text{ GeV}$$

Neutrino and Charged lepton masses :

$$\Delta m^2_{21} \sim 7.7 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{31} \sim 2.4 \times 10^{-3} \text{ eV}^2,$$

$$(m_e, m_\mu, m_\tau) \sim (5 \times 10^{-4}, 0.1, 1) \text{ GeV}$$

Mass matrix and Generation mixing

Fermion mass terms (with right-handed neutrino N_R)

$$-\mathcal{L}_m = \bar{f}_L M_f f_R + \bar{\nu}_L M_\nu^D N_R + N_R^T M_R N_R + h.c.$$

$$(f = u, d, e)$$

Quark sector :

$$V_{uL}^\dagger M_u V_{uR} = \mathrm{Diag}\{2 \times 10^{-3}, 1.3, 170\ \mathrm{GeV}\}$$

$$V_{dL}^\dagger M_d V_{dR} = \mathrm{Diag}\{5 \times 10^{-3}, 0.1, 4\ \mathrm{GeV}\}$$

$\Rightarrow V_{\mathrm{CKM}} \equiv V_{uL}^\dagger V_{dL}$: Mixing matrix for the quark sector

Lepton sector :

$$U_e^\dagger M_e U_{eR} = \mathrm{Diag}\{5 \times 10^{-4}, 0.1, 1\ \mathrm{GeV}\}$$

$$U_\nu^T M_\nu U_\nu = \mathrm{Diag}\{m_1, m_2, m_3\}, \quad M_\nu = -M_\nu^D M_R^{-1} (M_\nu^D)^T$$

$\Rightarrow U_{\mathrm{PMNS}} \equiv U_e^\dagger U_\nu$: Mixing matrix for the lepton sector

The structures of mass matrices determine the mixing matrices.

Generation mixing

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{12}s_{23}s_{13}e^{i\delta} - s_{12}c_{23} & -s_{12}s_{23}s_{13}e^{i\delta} + c_{12}c_{23} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13}e^{i\delta} + s_{12}s_{23} & -s_{12}c_{23}s_{13}e^{i\delta} - c_{12}s_{23} & c_{23}c_{13} \end{pmatrix}$$

Generation mixings of quarks : $V_{\rm CKM}V_{\rm CKM}^{\dagger} = 1 \quad (V_{\rm CKM} \equiv V_{uL}^{\dagger}V_{dL})$

$$\sin\theta_{12}^{q} \simeq 0.23$$

$$\sin\theta_{23}^{q} \simeq 0.041$$

⇒ Small Mixing

$$|\sin\theta_{13}^{q}e^{-i\delta}| \simeq 0.0036$$

Generation mixings of leptons : $U_{\rm PMNS}U_{\rm PMNS}^{\dagger} = 1 \quad (U_{\rm PMNS} \equiv U_{e}^{\dagger}U_{\nu})$

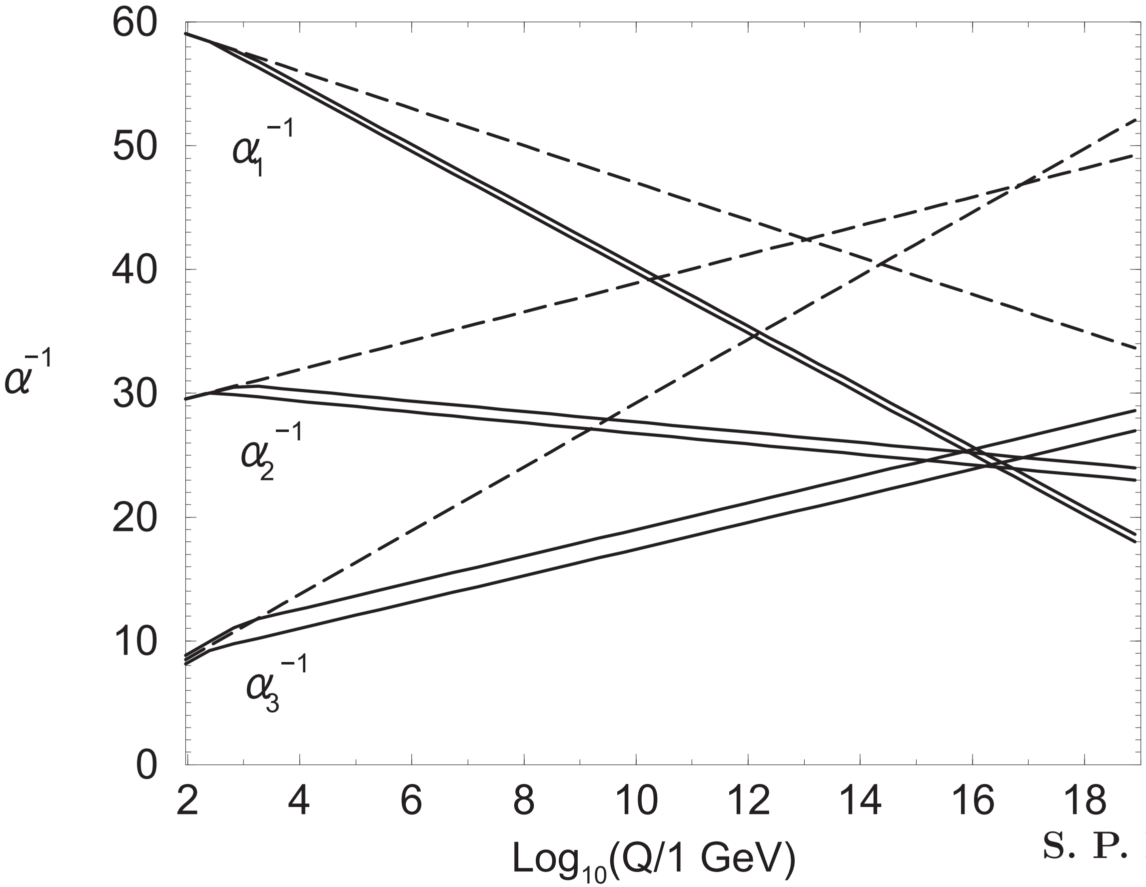
$|\sin\theta_{12}| \simeq \sqrt{0.32} \simeq 0.57$ ⇒ Large

$|\sin\theta_{23}| \simeq \sqrt{0.50} \simeq 0.71$ ⇒ Maximal Mixing

$|\sin\theta_{13}| \leq \sqrt{0.05} \simeq 0.22$ ⇒ Small

How can we realize these mixing angles? ⇒ Mass matrix

SUSY-GUT



S. P. Martin, [hep-ph/9709356]

In SUSY-GUT:

unification of Quark & Lepton ⇒ unification of mass matrices,

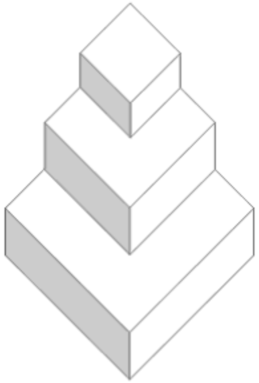
- **look for mass matrices realising experimentally observed values**
- **construct flavour theory leading to the mass matrices**

  ⇒ focus on "*Cascade Hierarchy*"

2. Cascade Hierarchy

Cascade (mass) matrix

$$M_{\mathrm{cas}} = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \delta \ll \lambda \ll 1$$



("cascade hierarchy" by Dorsner and Barr, (2001))

(mass) Eigenvalues : $m_1 : m_2 : m_3 \sim \delta : \lambda : 1$

Mixing angles :

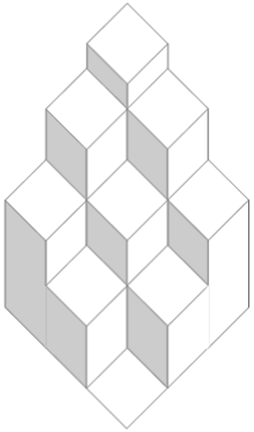
$$\theta_{12} \sim \frac{\delta}{\lambda}, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta, \quad \left[\theta_{ij} \sim \frac{m_i}{m_j}\right]$$

- The cascade mass matrix leads to small mixing angles.
- It can lead to large mixing angles (LMA) of the lepton sector in the (type-I) seesaw. Haba, RT, Tanimoto, Yoshioka, (2008)

2. Cascade Hierarchy

Waterfall (mass) matrix

$$M_{\rm wat} = \begin{pmatrix} \delta^2 & \delta\lambda & \delta \\ \delta\lambda & \lambda^2 & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \delta \ll \lambda \ll 1$$



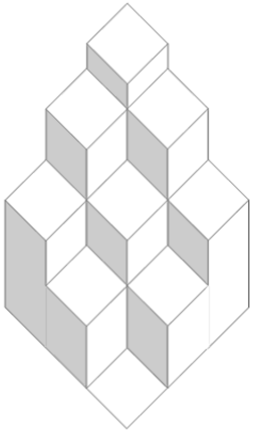
(mass) Eigenvalues : $m_1 : m_2 : m_3 \sim \delta^2 : \lambda^2 : 1$

Mixing angles :

$$\theta_{12} \sim \frac{\delta}{\lambda}, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta, \qquad \left[\theta_{ij} \sim \sqrt{\frac{m_i}{m_j}}\right]$$

2. Cascade Hierarchy

Waterfall (mass) matrix



$$M_{\text{wat}} = \begin{pmatrix} \delta^2 & \delta\lambda & \delta \\ \delta\lambda & \lambda^2 & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \delta \ll \lambda \ll 1$$

(mass) Eigenvalues : $m_1 : m_2 : m_3 \sim \delta^2 : \lambda^2 : 1$

Mixing angles :

$$\theta_{12} \sim \frac{\delta}{\lambda}, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta, \qquad \left[\theta_{ij} \sim \sqrt{\frac{m_i}{m_j}} \right]$$

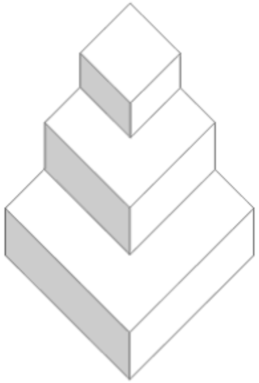
- This cannot lead to the LMA of lepton sector even in the seesaw.
- If $M_d \simeq M_{\text{wat}}$, it cannot even reproduce the θ^q_{ij} :

  $\theta^q_{12} \sim \sqrt{m_{d_1}/m_{d_2}}$, $\theta^q_{23} \sim m_{d_2}/m_{d_3}$, $\theta^q_{13} \sim \theta^q_{12}\theta^q_{23}$

2. Cascade Hierarchy

Cascade (mass) matrix

$$M_{\text{cas}} = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \delta \ll \lambda \ll 1$$



("cascade hierarchy" by Dorsner and Barr, (2001))

(mass) Eigenvalues : $m_1 : m_2 : m_3 \sim \delta : \lambda : 1$

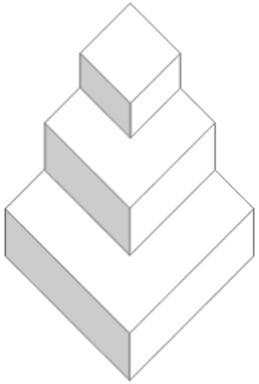
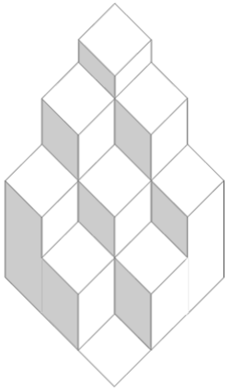
Mixing angles :

$$\theta_{12} \sim \frac{\delta}{\lambda}, \quad \theta_{23} \sim \lambda, \quad \theta_{13} \sim \delta, \quad \left[\theta_{ij} \sim \frac{m_i}{m_j}\right]$$

Quark sector :

$$\theta_{12}^q \sim \sqrt{m_{d_1}/m_{d_2}}, \theta_{23}^q \sim m_{d_2}/m_{d_3}, \theta_{13}^q \sim \theta_{12}^q \theta_{23}^q$$

2. Cascade Hierarchy

	Cas. 		**Wat.** 	
M_ν^D	○		×	
M_d	×		×	

2. Cascade Hierarchy

If M_d is taken as the cascade form, θ_{23}^q can be reproduced but θ_{12}^q and θ_{13}^q cannot be reproduced.

If M_d is taken as the waterfall form, θ_{12}^q can be reproduced but θ_{23}^q and θ_{13}^q cannot be reproduced.

2. Cascade Hierarchy

If M_d is taken as the cascade form, θ^q_{23} can be reproduced but θ^q_{12} and θ^q_{13} cannot be reproduced.

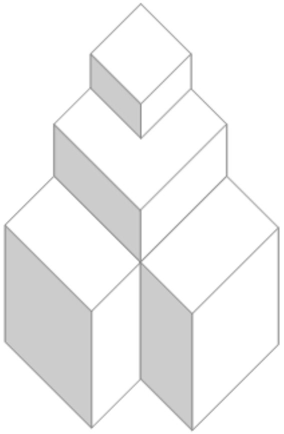
If M_d is taken as the waterfall form, θ^q_{12} can be reproduced but θ^q_{23} and θ^q_{13} cannot be reproduced.

Let us mix the cascade with the waterfall.

2. Cascade Hierarchy

Hybrid Cascade (H.C.) matrix

$$M_{\text{hyb}} = \begin{pmatrix} \epsilon & \delta & \delta \\ \delta & \lambda & \lambda \\ \delta & \lambda & 1 \end{pmatrix} v, \quad \epsilon \ll \delta \ll \lambda \ll 1$$



(mass) Eigenvalues : $m_1 : m_2 : m_3 \sim \delta^2/\lambda : \lambda : 1$

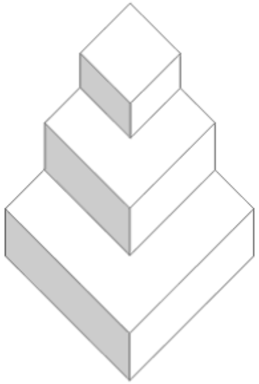
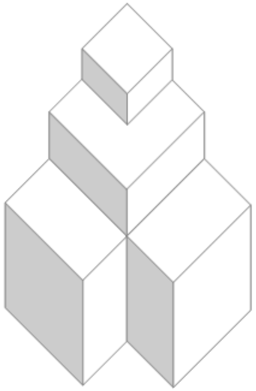
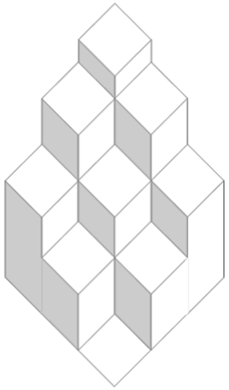
Mixing angles :

$$\theta_{12} \sim \frac{\delta}{\lambda} \sim \sqrt{\frac{m_1}{m_2}}, \quad \theta_{23} \sim \lambda \sim \frac{m_2}{m_3}, \quad \theta_{13} \sim \delta,$$

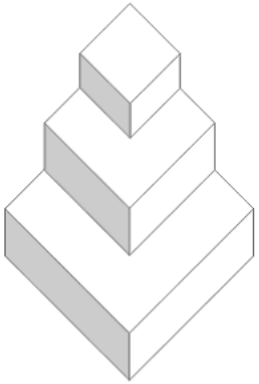
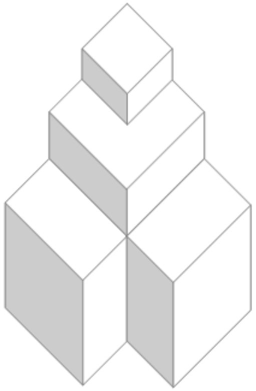
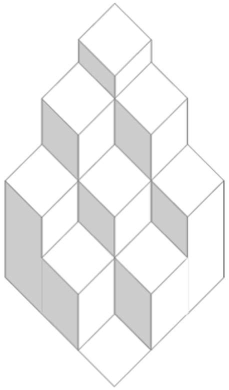
Quark sector :

$$\theta^q_{12} \sim \sqrt{\frac{m_{d_1}}{m_{d_2}}}, \quad \theta^q_{23} \sim \frac{m_{d_2}}{m_{d_3}}, \quad \theta^q_{13} \sim \theta^q_{12}\theta^q_{23}$$

2. Cascade Hierarchy

	Cas. 	H.C. 	Wat. 	
M_ν^D	○	×	×	
M_d	×	○	×	

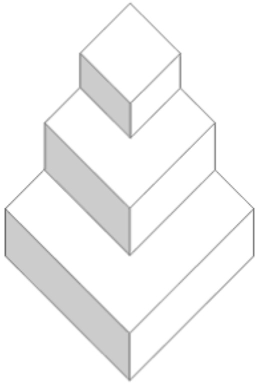
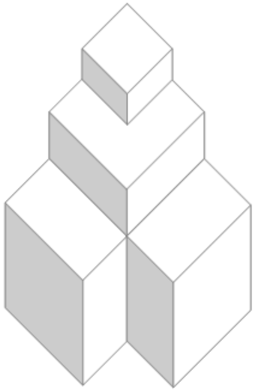
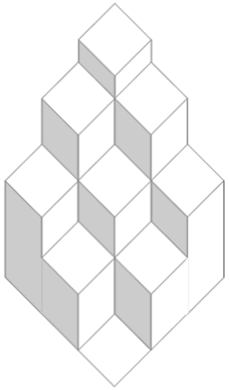
2. Cascade Hierarchy

	Cas.	H.C.	Wat.	
				$\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
M_ν^D	○	×	×	×
M_e				○
M_d	×	○	×	
M_R				○

- M_ν^D =Cas. can lead to LMA in the basis of diagonal M_e & M_R

Haba, RT, Tanimoto, Yoshioka, PRD 78 (2008) 113002

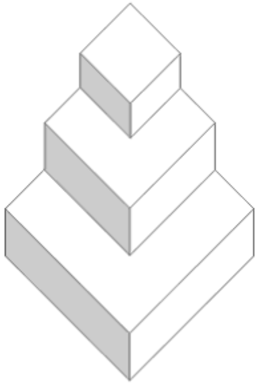
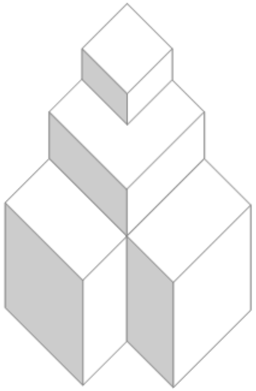
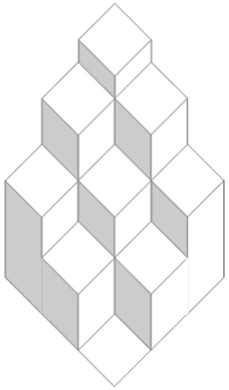
2. Cascade Hierarchy

	Cas.	H.C.	Wat.	
				$\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
M_ν^D	○	×	×	×
M_e	○			○
M_d	×	○	×	
M_R	○			○

- M_ν^D =Cas. is also OK even if M_e & M_R =Cas..

Haba, RT, Tanimoto, Yoshioka, PRD 78 (2008) 113002

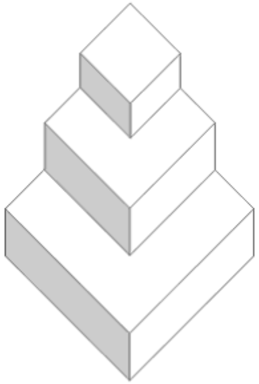
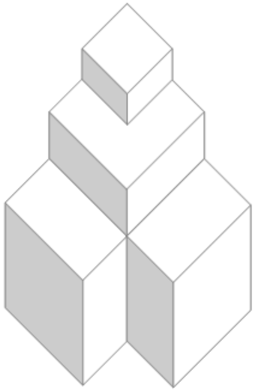
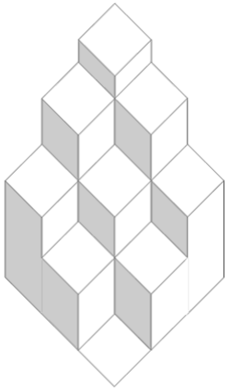
2. Cascade Hierarchy

	Cas. 	H.C. 	Wat. 	Small Mixing or $\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
M_ν^D	○	×	×	×
M_e	○	○	○	○
M_d	×	○	×	
M_R	○	○	○	○

- M_ν^D =Cas. is OK as long as M_e & M_R lead to only small mixings.

Kojima, Sawanaka, RT, [to appear]

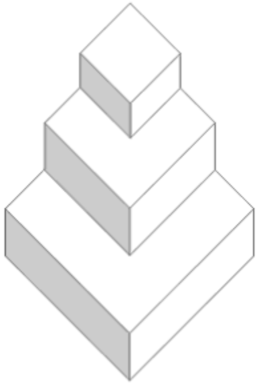
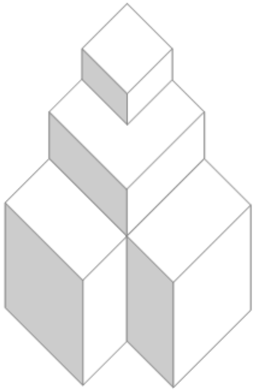
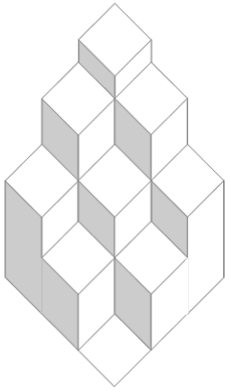
2. Cascade Hierarchy

	Cas.	H.C.	Wat.	Small Mixing or
				$\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
M_ν^D	○	×	×	×
M_e	○	○	○	○
M_d	×	○	×	×
M_u	○	○	○	○
M_R	○	○	○	○

- V_{CKM} is almost determined by M_d: M_u is correction.

Kojima, Sawanaka, RT, [to appear]

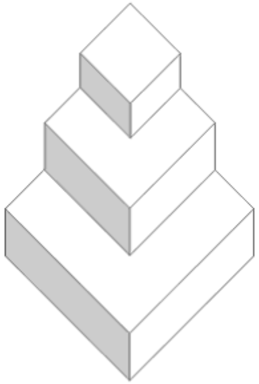
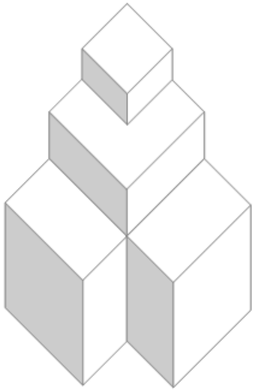
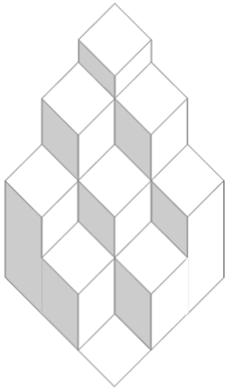
2. Cascade Hierarchy

	Cas. 	H.C. 	Wat. 	Small Mixing or $\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
M_ν^D	○	×	×	×
M_e	○	○	○	○
M_d	×	○	×	×
M_u	○	○	○	○
M_R	○	○	○	○

- **It would seem that we can explain the masses and mixings of the quark and lepton sectors in terms of only Cas. & H.C. hierarchies$\cdots$.**

Kojima, Sawanaka, RT, [to appear]

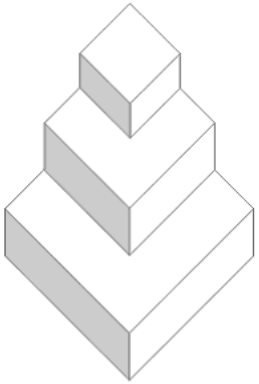
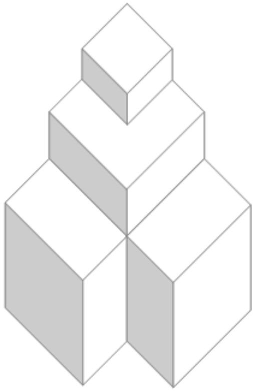
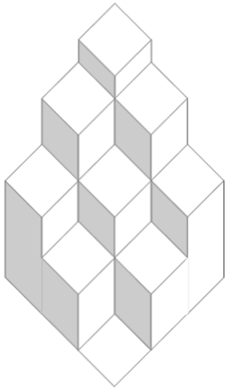
2. Cascade Hierarchy

	Cas. 	H.C. 	Wat. 	Small Mixing or $\begin{pmatrix} \ast & 0 & 0 \\ 0 & \ast & 0 \\ 0 & 0 & \ast \end{pmatrix}$
M_ν^D	○	×	×	×
M_e	○	○	○	○
M_d	×	○	×	×
M_u	○	○	○	○
M_R	○	○	○	○

- **SUSY-GUT ?**

Kojima, Sawanaka, RT, [to appear]

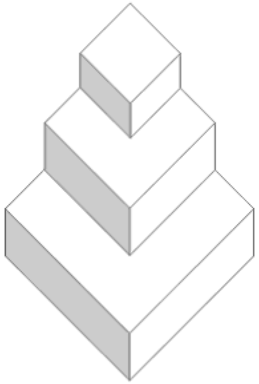
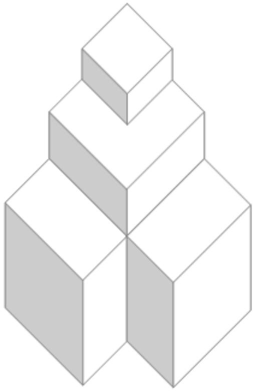
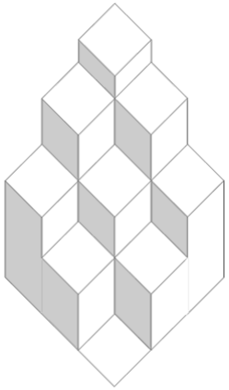
2. Cascade Hierarchy

	Cas. 	H.C. 	Wat. 	Small Mixing or $\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
M_ν^D	○	×	×	×
M_e	×	○	×	×
M_d	×	○	×	×
M_u	○	○	○	○
M_R	○	○	○	○

- **For example, $M_e \simeq M_d^T$ in the $SU(5)$ GUT.**

Kojima, Sawanaka, RT, [to appear]

2. Cascade Hierarchy

	Cas. 	H.C. 	Wat. 	Small Mixing or $\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$
M_ν^D	○	×	×	×
M_e	×	○	×	×
M_d	×	○	×	×
M_u	○	○	○	○
M_R	○	○	○	○

- **Motivation to study (H.) Cas. in $SU(5)$ GUT.**

Kojima, Sawanaka, RT, [to appear]

3. Lepton Sector

Tri-bimaximal generation mixing

Harrison, Perkins, Scott (2002)

Experiments

$$U_{\mathrm{TB}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \Rightarrow \begin{array}{l} \sin^2\theta_{12} = \frac{1}{3} \\ \sin^2\theta_{23} = \frac{1}{2} \\ \sin^2\theta_{13} = 0 \end{array} \Leftrightarrow \begin{array}{l} \sin^2\theta_{12} \simeq 0.32 \simeq \frac{1}{3} \\ \sin^2\theta_{23} \simeq 0.5 = \frac{1}{2} \\ \sin^2\theta_{13} \lesssim 0.05 \end{array}$$

- ν_2 : tri-maximal mixture of $\nu_e, \ \nu_\mu, \ \nu_\tau$
- ν_3 : bi -maximal mixture of $\nu_\mu, \ \nu_\tau$

$$\boxed{U_{\mathrm{TB}} \simeq U_{\mathrm{MNS}}^{\mathrm{exp}}}$$

$$\Downarrow$$

- Motivation to look for hidden flavour structure

- Flavour symmetry: e.g. Discrete symmetry; S_3, A_4,,,
- Texture analysis: e.g. "Cascade matrix",,,

3. Lepton Sector

Structure of neutrino mass matrix leading to TB mixing

$$\begin{aligned} M_\nu &= U_{\rm TB}^* \cdot {\rm Diag}\{m_1, m_2, m_3\} \cdot U_{\rm TB}^\dagger \\ &= \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \end{aligned}$$

Cascade matrix (seesaw mechanism : $M_\nu \simeq M_\nu^D M_R^{-1} (M_\nu^D)^T$)

Dirac mass matrix

Majorana mass matrix

$$M_\nu^D = \begin{pmatrix} \delta_1 & \delta_2 & \delta_3 \\ \delta_2 & \lambda_1 & \lambda_2 \\ \delta_3 & \lambda_2 & 1 \end{pmatrix} v, \quad (\delta_i \ll \lambda_j \ll 1)$$

$$M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

$$M_\nu = \frac{v^2}{M_1}\begin{pmatrix} \delta_1^2 & \delta_1\delta_2 & \delta_1\delta_3 \\ \delta_1\delta_2 & \delta_2^2 & \delta_2\delta_3 \\ \delta_1\delta_3 & \delta_2\delta_3 & \delta_3^2 \end{pmatrix} + \frac{v^2}{M_2}\begin{pmatrix} \delta_2^2 & \delta_2\lambda_1 & \delta_2\lambda_2 \\ \delta_2\lambda_1 & \lambda_1^2 & \lambda_1\lambda_2 \\ \delta_2\lambda_2 & \lambda_1\lambda_2 & \lambda_2^2 \end{pmatrix} + \frac{v^2}{M_3}\begin{pmatrix} \delta_3^2 & \delta_3\lambda_2 & \delta_3 \\ \delta_3\lambda_2 & \lambda_2^2 & \lambda_2 \\ \delta_3 & \lambda_2 & 1 \end{pmatrix}$$

3. Lepton Sector

Structure of neutrino mass matrix leading to TB mixing

$$\begin{aligned} M_\nu &= U_{\rm TB}^* \cdot {\rm Diag}\{m_1, m_2, m_3\} \cdot U_{\rm TB}^\dagger \\ &= \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \end{aligned}$$

Cascade matrix (seesaw mechanism : $M_\nu \simeq M_\nu^D M_R^{-1} (M_\nu^D)^T$)

Dirac mass matrix

Majorana mass matrix

$$M_\nu^D = \begin{pmatrix} \delta_1 & \delta_2 & \delta_3 \\ \delta_2 & \lambda_1 & \lambda_2 \\ \delta_3 & \lambda_2 & 1 \end{pmatrix} v, \ (\delta_i \ll \lambda_j \ll 1)$$

$$M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

$$M_\nu = \frac{v^2}{M_1}\begin{pmatrix} \delta_1^2 & \delta_1\delta_2 & \delta_1\delta_3 \\ \delta_1\delta_2 & \delta_2^2 & \delta_2\delta_3 \\ \delta_1\delta_3 & \delta_2\delta_3 & \delta_3^2 \end{pmatrix} + \frac{v^2}{M_2}\begin{pmatrix} \delta_2^2 & \delta_2\lambda_1 & \delta_2\lambda_2 \\ \delta_2\lambda_1 & \lambda_1^2 & \lambda_1\lambda_2 \\ \delta_2\lambda_2 & \lambda_1\lambda_2 & \lambda_2^2 \end{pmatrix} + \frac{v^2}{M_3}\begin{pmatrix} \delta_3^2 & \delta_3\lambda_2 & \delta_3 \\ \delta_3\lambda_2 & \lambda_2^2 & \lambda_2 \\ \delta_3 & \lambda_2 & 1 \end{pmatrix}$$

3. Lepton Sector

Structure of neutrino mass matrix leading to TB mixing

$$M_\nu = U^*_{\text{TB}} \cdot \text{Diag}\{m_1, m_2, m_3\} \cdot U^\dagger_{\text{TB}}$$
$$= \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

Cascade matrix (seesaw mechanism : $M_\nu \simeq M_\nu^D M_R^{-1} (M_\nu^D)^T$)

Dirac mass matrix

$$M_\nu^D = \begin{pmatrix} \delta_1 & \delta_2 & \delta_3 \\ \delta_2 & \lambda_1 & \lambda_2 \\ \delta_3 & \lambda_2 & 1 \end{pmatrix} v, \quad (\delta_i \ll \lambda_j \ll 1)$$

Majorana mass matrix

$$M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

$$M_\nu = \frac{v^2}{M_1}\begin{pmatrix} \delta_1^2 & \delta_1\delta_2 & \delta_1\delta_3 \\ \delta_1\delta_2 & \delta_2^2 & \delta_2\delta_3 \\ \delta_1\delta_3 & \delta_2\delta_3 & \delta_3^2 \end{pmatrix} + \frac{v^2}{M_2}\begin{pmatrix} \delta_2^2 & \delta_2\lambda_1 & \delta_2\lambda_2 \\ \delta_2\lambda_1 & \lambda_1^2 & \lambda_1\lambda_2 \\ \delta_2\lambda_2 & \lambda_1\lambda_2 & \lambda_2^2 \end{pmatrix} + \frac{v^2}{M_3}\begin{pmatrix} \delta_3^2 & \delta_3\lambda_2 & \delta_3 \\ \delta_3\lambda_2 & \lambda_2^2 & \lambda_2 \\ \delta_3 & \lambda_2 & 1 \end{pmatrix}$$

$$\delta_1 = \delta_2 = \delta_3 \equiv \delta, \quad \lambda_1 = -\lambda_2 \equiv \lambda$$

3. Lepton Sector

Structure of neutrino mass matrix leading to TB mixing

$$\begin{aligned} M_\nu &= U_{\rm TB}^* \cdot {\rm Diag}\{m_1, m_2, m_3\} \cdot U_{\rm TB}^\dagger \\ &= \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \end{aligned}$$

Cascade matrix (seesaw mechanism : $M_\nu \simeq M_\nu^D M_R^{-1} (M_\nu^D)^T$)

Dirac mass matrix

Majorana mass matrix

$$M_\nu^D = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} v, \quad (\delta \ll \lambda \ll 1)$$

$$M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

$$M_\nu = \frac{v^2}{M_1}\delta^2 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{v^2}{M_2}\lambda^2 \begin{pmatrix} \delta^2/\lambda^2 & \delta/\lambda & -\delta/\lambda \\ \delta/\lambda & 1 & -1 \\ -\delta/\lambda & -1 & 1 \end{pmatrix} + \frac{v^2}{M_3}\begin{pmatrix} \delta^2 & -\delta\lambda & \delta \\ -\delta\lambda & \lambda^2 & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix}$$

$$\delta_1 = \delta_2 = \delta_3 \equiv \delta, \quad \lambda_1 = -\lambda_2 \equiv \lambda$$

3. Lepton Sector

Structure of neutrino mass matrix leading to TB mixing

$$\begin{aligned} M_\nu &= U_{\rm TB}^* \cdot {\rm Diag}\{m_1, m_2, m_3\} \cdot U_{\rm TB}^\dagger \\ &= \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \end{aligned}$$

Cascade matrix (seesaw mechanism : $M_\nu \simeq M_\nu^D M_R^{-1} (M_\nu^D)^T$)

Dirac mass matrix

Majorana mass matrix

$$M_\nu^D = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} v, \ \ (\delta \ll \lambda \ll 1)$$

$$M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

$$M_\nu = \frac{v^2}{M_3}\begin{pmatrix} \delta^2 & -\delta\lambda & \delta \\ -\delta\lambda & \lambda^2 & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} + \frac{v^2}{M_1}\delta^2 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{v^2}{M_2}\lambda^2 \begin{pmatrix} \delta^2/\lambda^2 & \delta/\lambda & -\delta/\lambda \\ \delta/\lambda & 1 & -1 \\ -\delta/\lambda & -1 & 1 \end{pmatrix}$$

$$\delta_1 = \delta_2 = \delta_3 \equiv \delta, \quad \lambda_1 = -\lambda_2 \equiv \lambda$$

3. Lepton Sector

Structure of neutrino mass matrix leading to TB mixing

$$\begin{aligned} M_\nu &= U_{\rm TB}^* \cdot {\rm Diag}\{m_1, m_2, m_3\} \cdot U_{\rm TB}^\dagger \\ &= \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \end{aligned}$$

Cascade matrix (seesaw mechanism : $M_\nu \simeq M_\nu^D M_R^{-1} (M_\nu^D)^T$)

Dirac mass matrix

$$M_\nu^D = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} v, \ \ (\delta \ll \lambda \ll 1)$$

Majorana mass matrix

$$M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

$$M_\nu = \frac{v^2}{M_3}\begin{pmatrix} \delta^2 & -\delta\lambda & \delta \\ -\delta\lambda & \lambda^2 & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} + \frac{v^2}{M_1}\delta^2 \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{v^2}{M_2}\lambda^2 \begin{pmatrix} \delta^2/\lambda^2 & \delta/\lambda & -\delta/\lambda \\ \delta/\lambda & 1 & -1 \\ -\delta/\lambda & -1 & 1 \end{pmatrix}$$

$$|m_1| \ll |m_{2,3}|, \quad |\delta m_3| \ll |\lambda m_2|$$

Cascade matrix (seesaw mechanism)

Dirac mass matrix

Majorana mass matrix

$$M_\nu^D = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} v, \quad (\delta \ll \lambda \ll 1) \qquad M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

Mass eigenvalues

$$(m_1,\ m_2,\ m_3) = \left(\frac{v^2}{6M_3},\ \frac{3\delta^2 v^2}{M_1} + \frac{v^2}{3M_3},\ \frac{2\lambda^2 v^2}{M_2} + \frac{v^2}{2M_3} \right)$$

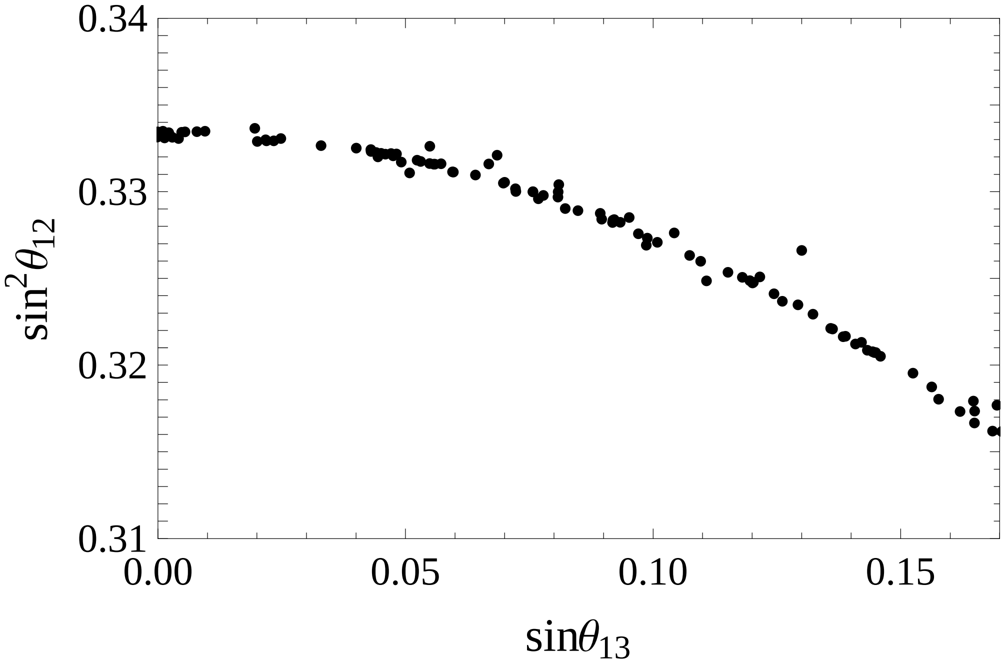
Mixing angles

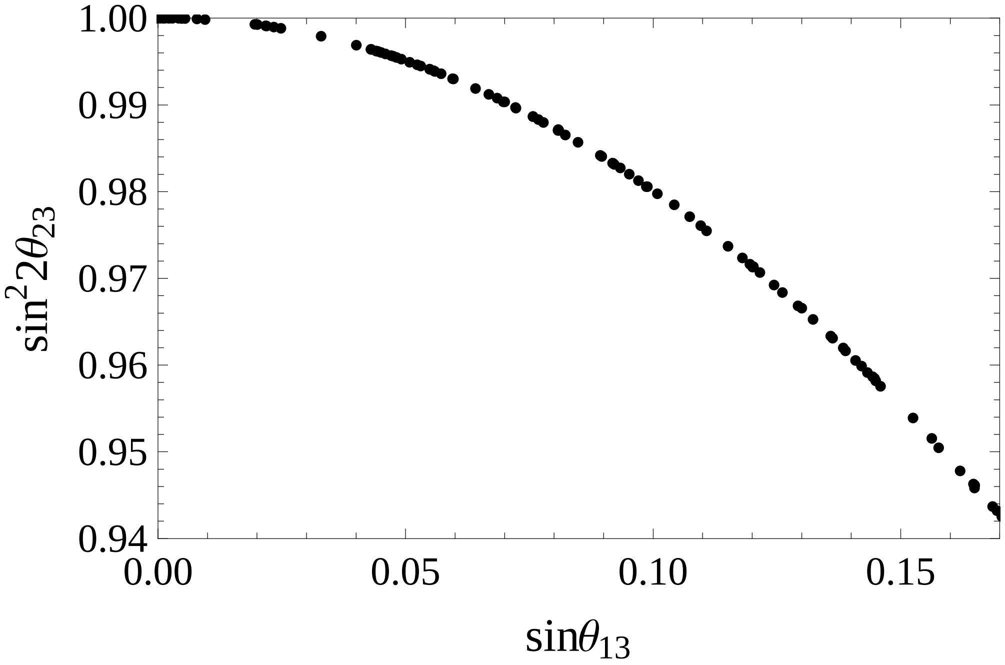
$$\sin^2\theta_{12} = \left| \frac{1}{\sqrt{3}} - \mathcal{O}\left(\frac{m_1}{m_2}\right) \right|^2$$

$$\sin^2\theta_{23} = \left| \frac{-1}{\sqrt{2}} + \mathcal{O}\left(\frac{m_1}{m_3}\right) + \mathcal{O}\left(\frac{\delta}{\lambda}\frac{m_2}{m_3}\right) \right|^2$$

$$\sin^2\theta_{13} = \left| \mathcal{O}\left(\frac{\delta}{\lambda}\right) + \mathcal{O}\left(\frac{m_1 m_2}{m_3^2}\right) \right|^2$$

Mixing angles (numerical analyses)





Cascade matrix (seesaw mechanism)

Dirac mass matrix **Majorana mass matrix**

$$M_\nu^D = \begin{pmatrix} \delta & \delta & \delta \\ \delta & \lambda & -\lambda \\ \delta & -\lambda & 1 \end{pmatrix} v, \ (\delta \ll \lambda \ll 1) \qquad M_R = \begin{pmatrix} M_1 & & \\ & M_2 & \\ & & M_3 \end{pmatrix}$$

Mass eigenvalues

$$(m_1, \ m_2, \ m_3) = \left(\frac{v^2}{6M_3}, \ \frac{3\delta^2 v^2}{M_1} + \frac{v^2}{3M_3}, \ \frac{2\lambda^2 v^2}{M_2} + \frac{v^2}{2M_3} \right)$$

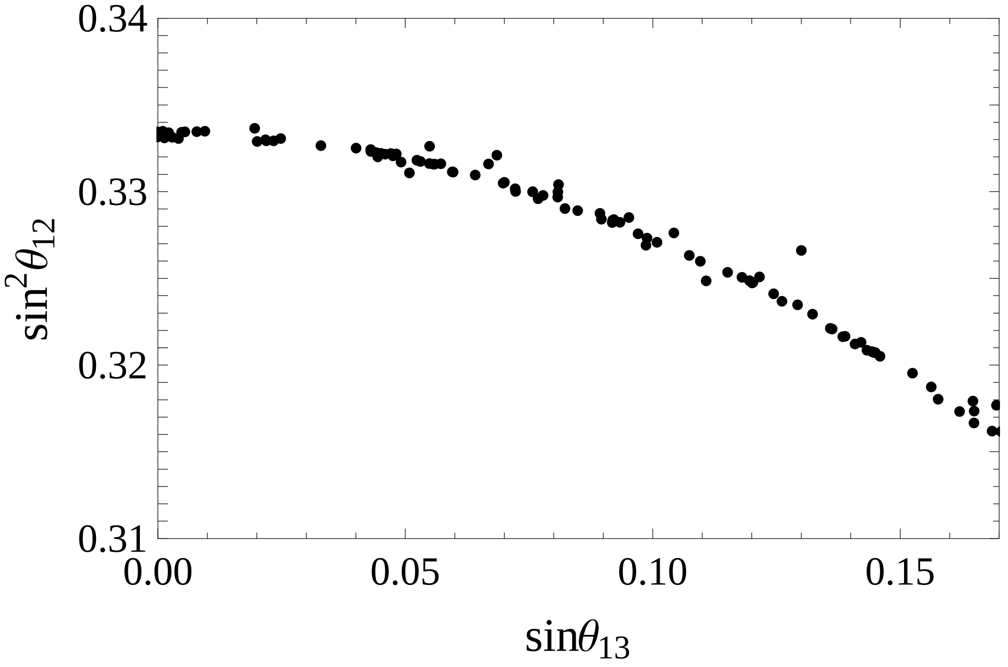
Mixing angles

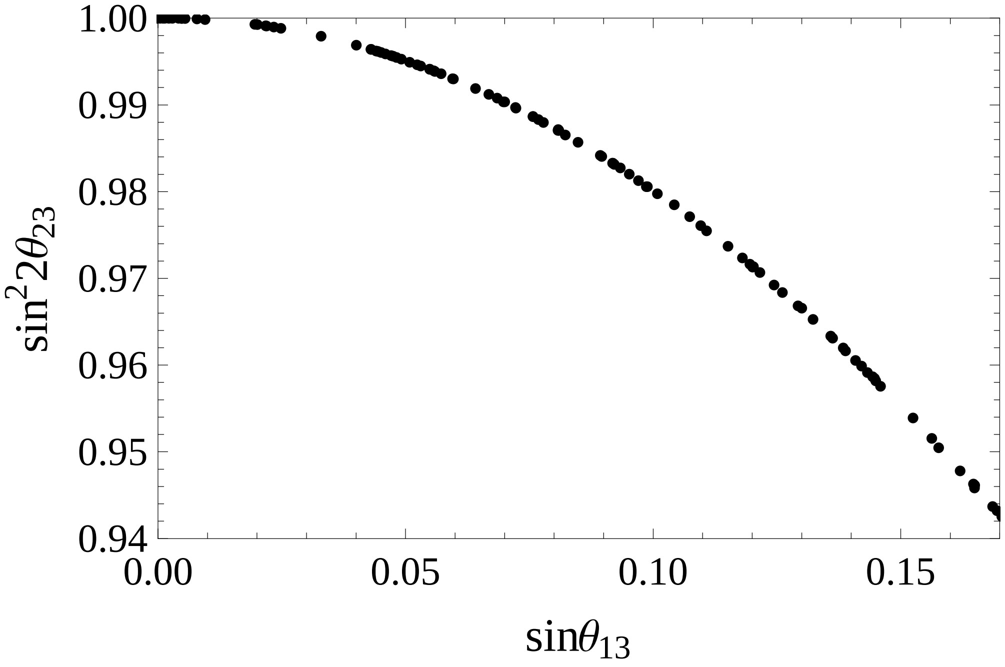
$$\sin^2\theta_{12} = \left| \frac{1}{\sqrt{3}} - \mathcal{O}\left(\frac{m_1}{m_2}\right) \right|^2$$

$$\sin^2\theta_{23} = \left| \frac{-1}{\sqrt{2}} + \mathcal{O}\left(\frac{m_1}{m_3}\right) + \mathcal{O}\left(\frac{\delta}{\lambda}\frac{m_2}{m_3}\right) \right|^2$$

$$\sin^2\theta_{13} = \left| \mathcal{O}\left(\frac{\delta}{\lambda}\right) + \mathcal{O}\left(\frac{m_1 m_2}{m_3^2}\right) \right|^2$$

Mixing angles (numerical analyses)





Parameter Independent Relation

$$\frac{1}{9}\left(\sin^2\theta_{23}-\frac{1}{2}\right)-\frac{r}{4}\left(\sin^2\theta_{12}-\frac{1}{3}\right)-\frac{\sqrt{2}\,r}{27}\sin\theta_{13} \;=\; 0$$

$$\left(r \equiv \Delta m^2_{21}/\Delta m^2_{31} \ll 1\right)$$

Realisation of cascade Dirac mass matrix in $U(1)$ flavour theory

	L_1	L_2	L_3	R_1	R_2	R_3	ϕ_1	ϕ_2	ϕ_3
$U(1)_f$	$2m+1$	1	0	$2m+1$	1	0	$-2m-3$	-2	-1

(m : positive integer)

$$M_D = \begin{pmatrix} \frac{\phi_1 \phi_2^{m-1} \phi_3}{\Lambda^{m+1}} & \frac{\phi_2^{m+1}}{\Lambda^{m+1}} & \frac{\phi_2^m \phi_3}{\Lambda^{m+1}} \\ \frac{\phi_2^{m+1}}{\Lambda^{m+1}} & \frac{\phi_2}{\Lambda} & \frac{\phi_3}{\Lambda} \\ \frac{\phi_2^m \phi_3}{\Lambda^{m+1}} & \frac{\phi_3}{\Lambda} & 1 \end{pmatrix} v$$

$$\Downarrow$$

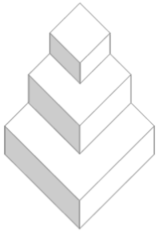
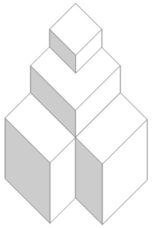
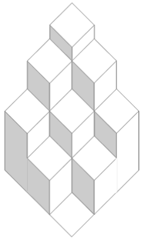
$$\Downarrow \ \langle \phi_1 \rangle \simeq \langle \phi_2 \rangle \simeq \langle \phi_3 \rangle \equiv \lambda \Lambda$$

$$\Downarrow$$

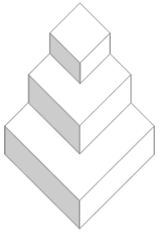
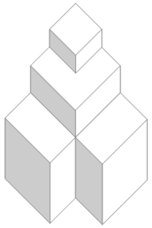
$$M_D \simeq \begin{pmatrix} \lambda^{m+1} & \lambda^{m+1} & \lambda^{m+1} \\ \lambda^{m+1} & \lambda & \lambda \\ \lambda^{m+1} & \lambda & 1 \end{pmatrix} v, \qquad \delta = \lambda^{m+1}$$

Charged lepton sector

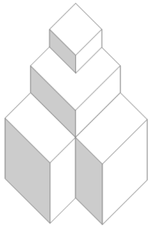
If we do not consider a high energy theory (e.g. $SU(5)$ GUT,,,),

$M_e =$ , , , $\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$, $\left(\text{Small Mix.} \right)$.

If we focus on cascade(-like) hierarchy,

$M_e =$ , .

For example, if we choose $SU(5)$ GUT as a high energy theory,

$M_e =$ .

In any case, effects from the charged lepton sector should be corrections for the lepton mixing angles ($\simeq U_{\rm TB}$).

Charged lepton sector

$$M_e = \begin{pmatrix} \delta_1^e & \delta_2^e & \delta_3^e \\ \delta_2^e & \lambda_1^e & \lambda_2^e \\ \delta_3^e & \lambda_2^e & 1 \end{pmatrix} v, \qquad \begin{cases} \delta_1^e \sim \delta_2^e \sim \delta_3^e \sim \mathcal{O}(\delta^e) \\ \lambda_1^e \sim \lambda_2^e \sim \mathcal{O}(\lambda^e) \end{cases}$$

$$|\lambda^e| \simeq \frac{|m_\mu|}{|m_\tau|} \simeq 6 \times 10^{-2}$$

$$|\delta^e| \simeq \frac{|m_e|}{|m_\tau|} \simeq 3 \times 10^{-4}$$

Effects on mixing angles

$$\sin^2\theta_{12} \simeq \left| \frac{1}{\sqrt{3}} - \mathcal{O}\left(\frac{m_1}{m_2}\right) \underbrace{- \frac{1}{\sqrt{3}} \frac{m_e}{m_\mu}}_{< 1\%} \right|^2 \Rightarrow F_{e12}\left(\delta_e, \lambda_e, \frac{\delta_e}{\lambda_e}\right)$$

$$\sin^2\theta_{23} \simeq \left| \frac{-1}{\sqrt{2}} + \mathcal{O}\left(\frac{m_1}{m_3}\right) + \mathcal{O}\left(\frac{\delta}{\lambda} \frac{m_2}{m_3}\right) \underbrace{- \frac{1}{\sqrt{2}} \frac{m_\mu}{m_\tau}}_{\sim 6\%} \right|^2 \Rightarrow F_{e23}\left(\delta_e, \lambda_e, \frac{\delta_e}{\lambda_e}\right)$$

$$\sin^2\theta_{13} \simeq \left| \mathcal{O}\left(\frac{\delta}{\lambda}\right) + \mathcal{O}\left(\frac{m_1 m_2}{m_3^2}\right) \underbrace{+ \frac{1}{\sqrt{2}} \frac{m_e}{m_\mu}}_{\lesssim 1\%} \right|^2 \Rightarrow F_{e13}\left(\delta_e, \lambda_e, \frac{\delta_e}{\lambda_e}\right)$$

Right-handed neutrino sector

Effects from the right-handed neutrino sector should be also tiny as well as the charged lepton case :

$$\sin^2\theta_{12} \simeq \left| \frac{1}{\sqrt{3}} - \mathcal{O}\left(\frac{m_1}{m_2}\right) + F_e\left(\delta_{e12}, \lambda_e, \frac{\delta_e}{\lambda_e}\right) + F_{R12}\left(\delta_R, \lambda_R, \frac{\delta_R}{\lambda_R}\right) \right|^2$$

$$\sin^2\theta_{23} \simeq \left| \frac{-1}{\sqrt{2}} + \mathcal{O}\left(\frac{m_1}{m_3}\right) + \mathcal{O}\left(\frac{\delta}{\lambda}\frac{m_2}{m_3}\right) + F_{e23}\left(\delta_e, \lambda_e, \frac{\delta_e}{\lambda_e}\right) + F_{R23}\left(\delta_R, \lambda_R, \frac{\delta_R}{\lambda_R}\right) \right|^2$$

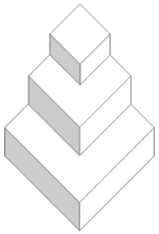
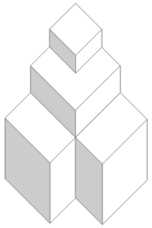
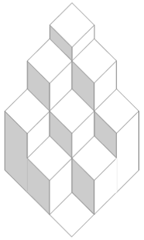
$$\sin^2\theta_{13} \simeq \left| \mathcal{O}\left(\frac{\delta}{\lambda}\right) + \mathcal{O}\left(\frac{m_1 m_2}{m_3^2}\right) + F_{e13}\left(\delta_e, \lambda_e, \frac{\delta_e}{\lambda_e}\right) + F_{R13}\left(\delta_R, \lambda_R, \frac{\delta_R}{\lambda_R}\right) \right|^2$$

4. Quark Sector

Up-type quark

Effects from the up sector on the structure of CKM matrix become small because the magnitude of mass hierarchy is large :

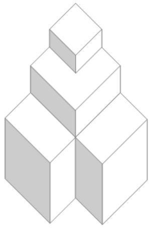
$$V_{uL}^{\dagger} M_u V_{uR} = \mathrm{Diag}\{2 \times 10^{-3}, 1.3, 170\ \mathrm{GeV}\}$$

$M_u =$ , , , $\begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$, $\left(\text{Small Mix.} \right)$

Down-type quark

The structure of mass matrix for the down sector almost determines the structure of CKM matrix :

$$V_{dL}^{\dagger} M_d V_{dR} = \mathrm{Diag}\{5 \times 10^{-3}, 0.1, 4\ \mathrm{GeV}\}$$

$M_d =$ 

5. Embedding (H.)Cas. textures into $SU(5)$ GUT

- $SU(5)$ GUT

GUT relation : $M_e \simeq M_d^T \;\Rightarrow\; M_e$ should be H.C. form

- Mass matrices with (hyb.) cascade hierarchies in $SU(5)$ GUT :

$$M_{\nu D} \simeq \begin{pmatrix} \delta_\nu & \delta_\nu & \delta_\nu \\ \delta_\nu & \lambda_\nu & \lambda_\nu \\ \delta_\nu & \lambda_\nu & 1 \end{pmatrix} \xi_\nu v_u, \quad |\delta_\nu| \ll |\lambda_\nu| \ll 1 : \text{Cas.}$$

$$M_d^T \simeq M_e \simeq \begin{pmatrix} \epsilon_d & \delta_d & \delta_d \\ \delta_d & \lambda_d & \lambda_d \\ \delta_d & \lambda_d & 1 \end{pmatrix} \xi_d v_d, \quad |\epsilon_d| \ll |\delta_d| \ll |\lambda_d| \ll 1 : \text{H.C.}$$

— —

$$M_u \simeq \begin{pmatrix} \epsilon_u & \delta_u & \delta_u \\ \delta_u & \lambda_u & \lambda_u \\ \delta_u & \lambda_u & 1 \end{pmatrix} v_u, \quad \begin{cases} |\epsilon_u| = |\delta_u| \ll |\lambda_u| \ll 1 & : \text{Cas.} \\ |\epsilon_u| \ll |\delta_u| \ll |\lambda_u| \ll 1 & : \text{H.C.} \end{cases}$$

$$M_R \simeq \begin{pmatrix} \epsilon_R & \delta_R & \delta_R \\ \delta_R & \lambda_R & \lambda_R \\ \delta_R & \lambda_R & 1 \end{pmatrix} M, \quad \begin{cases} |\epsilon_R| = |\delta_R| \ll |\lambda_R| \ll 1 & : \text{Cas.} \\ |\epsilon_R| \ll |\delta_R| \ll |\lambda_R| \ll 1 & : \text{H.C.} \end{cases}$$

One parameter fit of cascade hierarchies

When we consider a realisation in $U(1)$ flavour theory ($\langle\phi_1\rangle \simeq \langle\phi_2\rangle \simeq \cdots \equiv \lambda\Lambda$), we can describe all cascading mass matrices by only one "*seed* (λ)" : $\lambda = \lambda_c \simeq 0.23$

$$M_d^T \simeq M_e \simeq \begin{pmatrix} \lambda_c^{k_d+3} & \lambda_c^3 & \lambda_c^3 \\ \lambda_c^3 & \lambda_c^2 & \lambda_c^2 \\ \lambda_c^3 & \lambda_c^2 & 1 \end{pmatrix} \lambda_c^{q_d} v_d \quad \Rightarrow k_d \geq 1,\ q_d = \begin{cases} 0 : \text{large } \tan\beta \\ 1 : \text{moderate } \tan\beta \\ 2 : \text{small } \tan\beta \end{cases}$$

⇒ q_d can be determied by $\tan\beta$. There is a lower baund on k_d.

$$M_\nu^D \simeq \begin{pmatrix} \lambda_c^{k_{\nu 1}} & \lambda_c^{k_{\nu 1}} & \lambda_c^{k_{\nu 1}} \\ \lambda_c^{k_{\nu 1}} & \lambda_c^{k_{\nu 2}} & \lambda_c^{k_{\nu 2}} \\ \lambda_c^{k_{\nu 1}} & \lambda_c^{k_{\nu 2}} & 1 \end{pmatrix} \lambda_c^{q_\nu} v_u,$$

⇒ Structures of neutrino mass matrices are evaluated by Δm_{21}^2, $|\Delta m_{31}^2|$, and the PMNS mixing angles.

$$M_u \simeq \begin{pmatrix} \lambda_c^{k_u+5(7)} & \lambda_c^{5(7)} & \lambda_c^{5(7)} \\ \lambda_c^{5(7)} & \lambda_c^3 & \lambda_c^3 \\ \lambda_c^{5(7)} & \lambda_c^3 & 1 \end{pmatrix} v_u \quad \Rightarrow \text{For H.C. (Cas.) } k_u \geq 2$$

⇒ It is interesting to understand the generation mixings and phenomenologies for the quarks/leptons in terms of λ_c unit.

6. Summary

Quark sector

$$(m_u, m_c, m_t) \sim (2 \times 10^{-3}, 1.3, 170) \text{ GeV}$$
$$(m_d, m_s, m_b) \sim (5 \times 10^{-3}, 0.1, 4) \text{ GeV}$$
$$\sin\theta_{12} \simeq 0.23$$
$$\sin\theta_{23} \simeq 0.041 \qquad \Rightarrow \text{Small Mixing}$$
$$|\sin\theta_{13} e^{-i\delta}| \simeq 0.0036$$

Lepton sector

$$\Delta m^2_{21} \sim 7.7 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{31} \sim 2.4 \times 10^{-3} \text{ eV}^2,$$
$$(m_e, m_\mu, m_\tau) \sim (5 \times 10^{-4}, 0.1, 1) \text{ GeV}$$
$$|\sin\theta_{12}| \simeq \sqrt{0.32} \simeq 0.57 \qquad \Rightarrow \text{Large}$$
$$|\sin\theta_{23}| \simeq \sqrt{0.50} \simeq 0.71 \qquad \Rightarrow \text{Maximal Mixing}$$
$$|\sin\theta_{13}| \leq \sqrt{0.05} \simeq 0.22 \qquad \Rightarrow \text{Small}$$

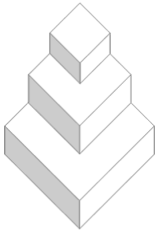
Comprehensive understanding of mixings/masses
for quark/lepton sectors!

6. Summary

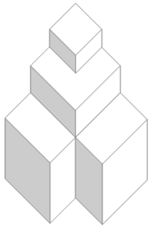
Comprehensive understanding of mixings/masses for quark/lepton sectors!

$\Downarrow$

Cascading Matrices



and



For example, in the $SU(5)$ GUT, $M_e \simeq M_d^T$.

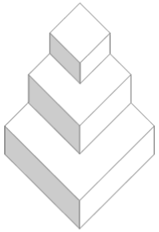
K. Kojima, H. Sawanaka and RT, [to appear]

6. Summary

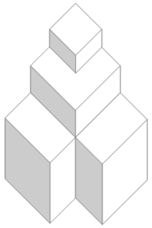
Comprehensive understanding of mixings/masses
for quark/lepton sectors!

$\Downarrow$

Cascading Matrices



and



For example, in the $SU(5)$ GUT, $M_e \simeq M_d^T$.

K. Kojima, H. Sawanaka and RT, [to appear]

$\Downarrow$

$\sim$ Cascading Matrices & SUSY-GUT $\sim$
We may understand the mixings/masses
for quark/lepton sectors in terms of λ_c unit.

6. Summary

- ***Past Work***

  Lepton sector described by cascade texture

  ⇒ Phenomenologies: LFV, Leptogenesis, and CP violation

  Haba, RT, Tanimoto, Yoshioka, PRD78 (2008) 113002, 0804.4055 [hep-ph].

- ***Present Work***

  Cascade Hierarchy and $SU(5)$ SUSY GUT

  Kojima, Sawanaka, RT, 09xx.xxxx [hep-ph].

6. Summary

- ***Past Work***

  Lepton sector described by cascade texture

  ⇒ Phenomenologies: LFV, Leptogenesis, and CP violation

  Haba, RT, Tanimoto, Yoshioka, PRD78 (2008) 113002, 0804.4055 [hep-ph].

- ***Present Work***

  Cascade Hierarchy and $SU(5)$ SUSY GUT

  Kojima, Sawanaka, RT, 09xx.xxxx [hep-ph].

- ***Future Work***

  Phenomenology in $SU(5)$ SUSY GUT with Cascade Hierarchy

  Construct other Cascade Models

6. Summary

- ***Past Work***

  Lepton sector described by cascade texture

  ⇒ Phenomenologies: LFV, Leptogenesis, and CP violation

  Haba, RT, Tanimoto, Yoshioka, PRD78 (2008) 113002, 0804.4055 [hep-ph].

- ***Present Work***

  Cascade Hierarchy and $SU(5)$ SUSY GUT

  Kojima, Sawanaka, RT, 09xx.xxxx [hep-ph].

- ***Future Work***

  Phenomenology in $SU(5)$ SUSY GUT with Cascade Hierarchy

  ??, RT, 10xx.xxxx [hep-ph].

  Construct other Cascade Models

  ??, RT, 10xx.xxxx [hep-ph].

Would you like to calculate (construct) them with me?

6. Summary

- ***Past Work***

  Lepton sector described by cascade texture

  ⇒ Phenomenologies: LFV, Leptogenesis, and CP violation

  Haba, RT, Tanimoto, Yoshioka, PRD78 (2008) 113002, 0804.4055 [hep-ph].

- ***Present Work***

  Cascade Hierarchy and $SU(5)$ SUSY GUT

  Kojima, Sawanaka, RT, 09xx.xxxx [hep-ph].

- ***Future Work***

  Phenomenology in $SU(5)$ SUSY GUT with Cascade Hierarchy

  ??, RT, 10xx.xxxx [hep-ph].

  Construct other Cascade Models

  ??, RT, 10xx.xxxx [hep-ph].

Thank you very much!