Diffractive neutrino interactions: Breakdown of PCAC

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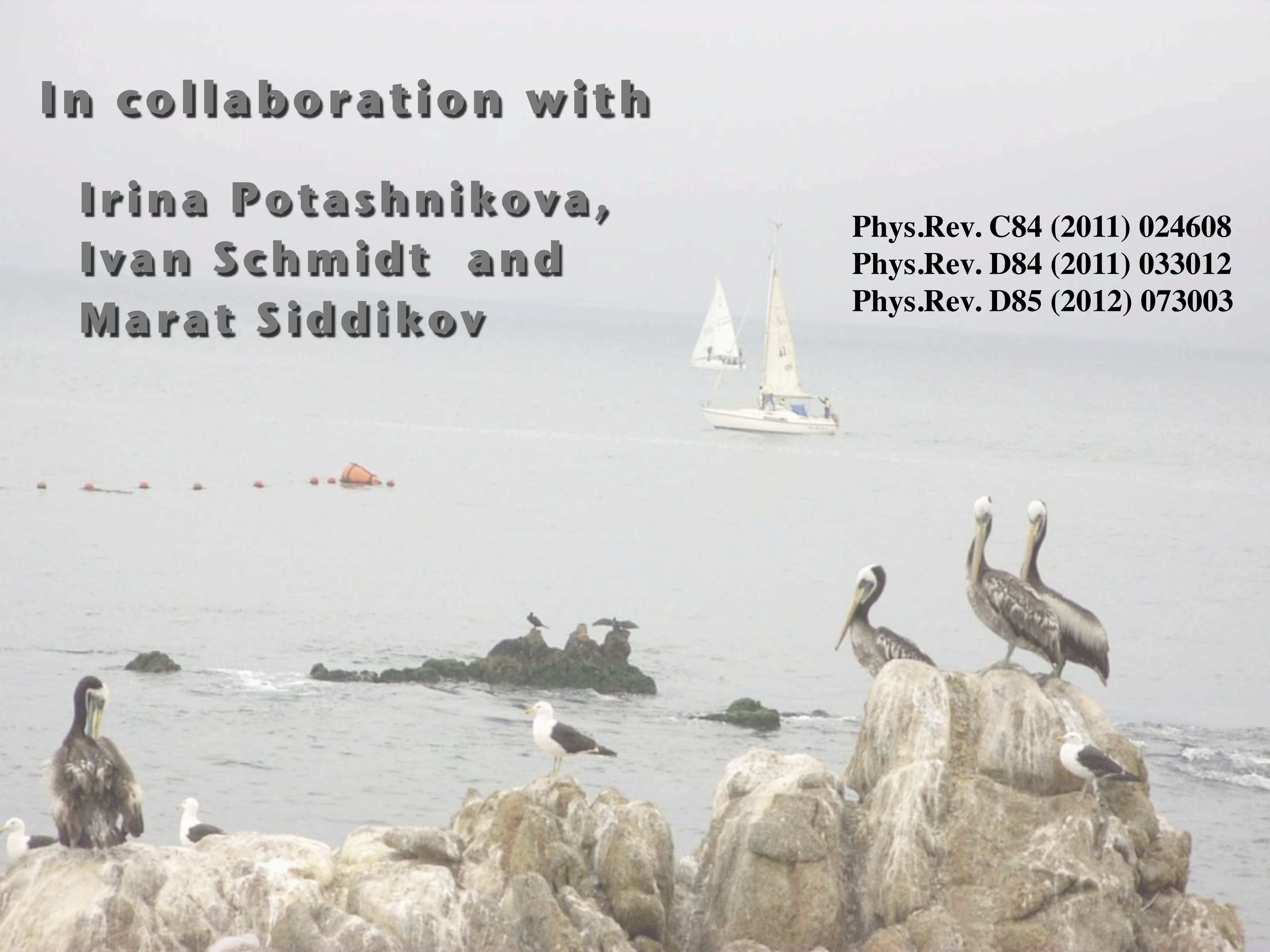


Outline

- Partial conservation of axial current: Goldstone mesons.
- Goldberger-Treiman relation: pion dominance?
- Hadronic properties of neutrinos at high energies.
- Tests of PCAC with high-energy neutrinos. The meaning of the Adler relation.
- Absorptive corrections to the PCAC relations in neutrino diffraction.
- Dramatic breakdown of PCAC in neutrino-nucleus interactions.

Conserved currents

In the chiral limit of massless quarks both the vector and axial currents are conserved:

$$\mathbf{q}_\mu \mathbf{V}_\mu = \mathbf{q}_\mu \left[\bar{q}(k')\,\gamma_\mu\, q(k)\right] = 0;$$

$$\mathbf{q}_\mu = \mathbf{k}'_\mu - \mathbf{k}_\mu$$

$$\mathbf{q}_\mu \mathbf{A}_\mu = \mathbf{q}_\mu \left[\bar{q}(k')\,\gamma_5\gamma_\mu\,\vec{\tau}\, q(k)\right] = 0;$$

Hadrons acquire large masses via the mechanism of spontaneous symmetry breaking. The hadronic currents are still conserved.

For the vector current this is rather obvious:

$$\mathbf{q}_\mu\, \mathbf{j}^{\mathbf{V}}_\mu = \mathbf{q}_\mu\, \bar{\mathbf{p}}(\mathbf{k}')\,\gamma_\mu\,\mathbf{n}(\mathbf{k}) = (\mathbf{m_n} - \mathbf{m_p})\bar{\mathbf{p}}\,\mathbf{n} = \mathbf{0}$$

(up to QED corrections)

Conserved currents

Conservation of axial current looks more problematic

$$\mathbf{q}_\mu\, \mathbf{j}^{\mathbf{A}}_\mu = \mathbf{q}_\mu\, \bar{\mathbf{p}}(\mathbf{k}')\, \gamma_\mu \gamma_5\, \mathbf{n}(\mathbf{k}) = (\mathbf{m_n} + \mathbf{m_p})\bar{\mathbf{p}}\, \gamma_5\, \mathbf{n} \neq \mathbf{0}$$

Nevertheless, in the general form,

$$\mathbf{j}^{\mathbf{A}}_\mu = \bar{\mathbf{p}}(\mathbf{k}')\, [\mathbf{g_A}\, \gamma_\mu \gamma_5 - \mathbf{g_p}\, \mathbf{q}_\mu \gamma_5]\, \mathbf{n}(\mathbf{k})$$

the axial current can be conserved if

$$\mathbf{g_P}(\mathbf{Q^2}) = \mathbf{g_A}(\mathbf{Q^2}) \frac{\mathbf{2m_N}}{\mathbf{Q^2}}$$

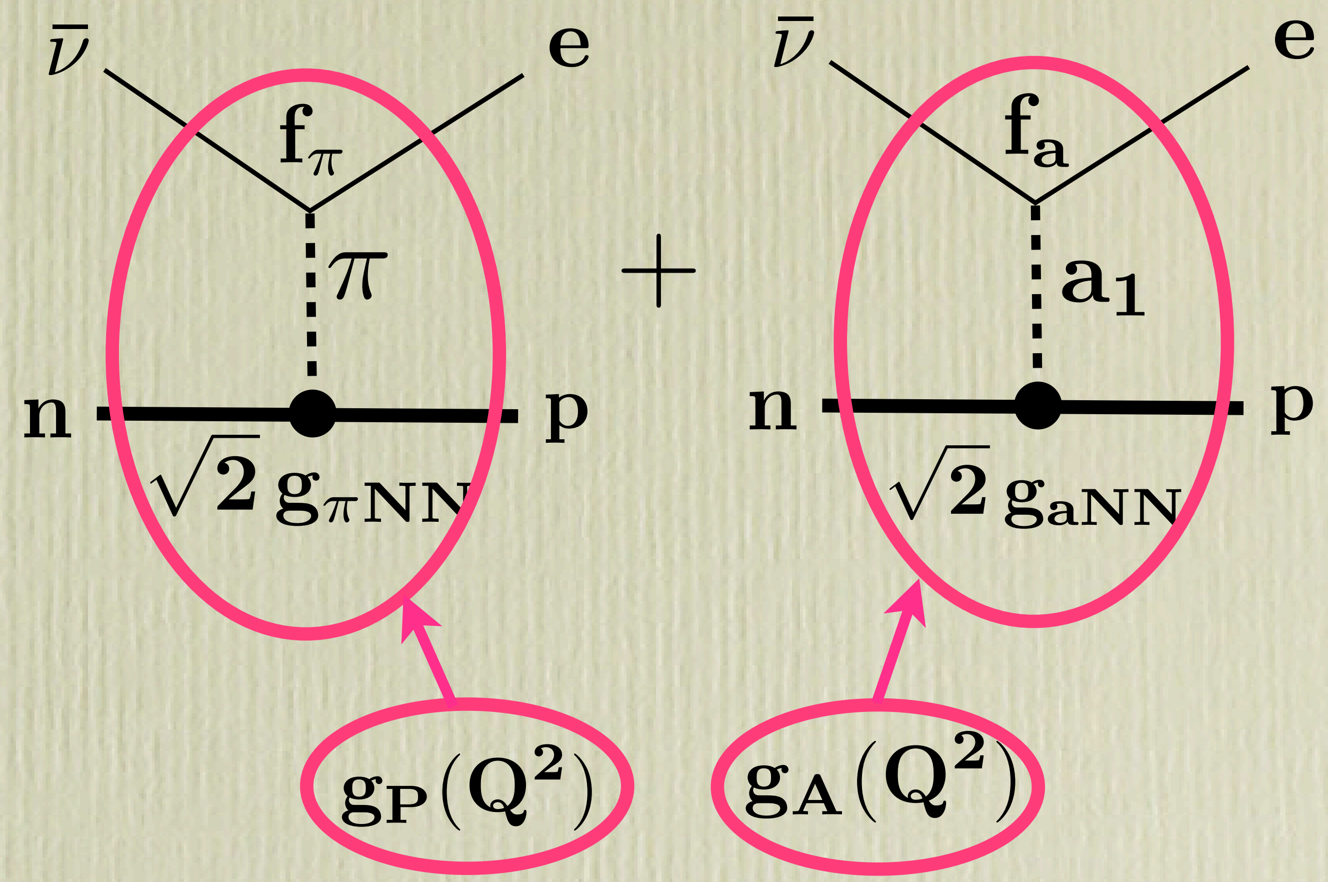
The pole behavior shows presence of a massless Goldstone particle

This proves the Goldstone theorem: spontaneous breaking of chiral symmetry generates massless particles identified with pions.

Goldberger-Treiman conspiracy

PCAC leads to a miraculous relation between the quantities having very different origin

$$\sqrt{2}\mathbf{m_N}\,\mathbf{g_A}(0) = \mathbf{f}_\pi\,\mathbf{g}_{\pi\mathbf{NN}}$$



It is tempting to interpret the Goldberger-Treiman relation in terms of pion pole dominance.

However, the pion pole does not contribute to the β-decay, because the lepton current is conserved (up to the electron mass).

$$\Gamma(\pi \to \bar{\nu}\mathbf{e}) \;\propto\; \mathbf{m_e^2}$$

The axial-vector formfactor $\mathbf{g_A}(\mathbf{Q^2})$ represents the contribution of heavy states, which are related to the pion term via PCAC.

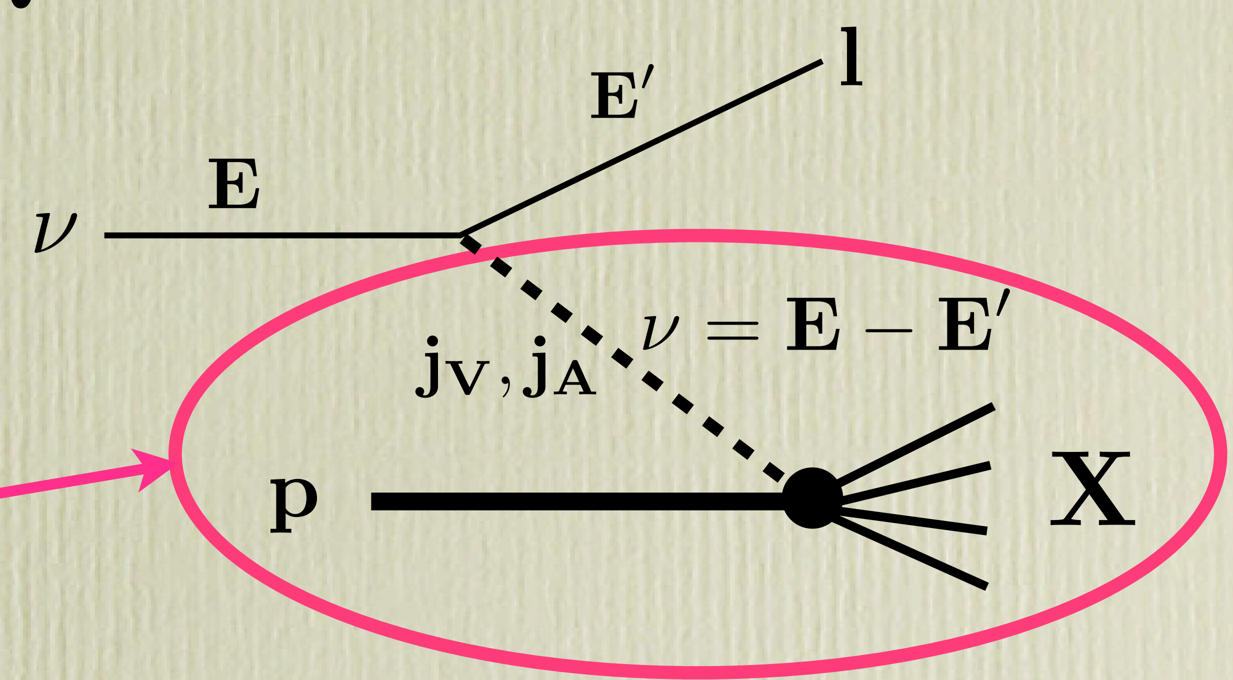
Hadronic properties of neutrinos

Although the non-trivial GT relation is well confirmed by data on neutron decay and muon capture, the PCAC hypothesis should be tested thoroughly in other processes.

The Fock components of a high-energy neutrino at low scale are dominated by the axial-vector hadronic fluctuations, since the vector term vanishes at $Q^2 \to 0$ due to CVC.

$$\nu + p \to l + X$$

$$M = \frac{G}{\sqrt{2}} l_\mu (V_\mu + A_\mu)$$



$$l_\mu = \bar{l}(k')\,\gamma_\mu(1+\gamma_5)\,\nu(k)$$

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LOW Q^2, HIGH ν NEUTRINO PHYSICS

(CVC, PCAC, HADRON DOMINANCE)

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Adler relation

$$\left|\overline{\mathbf{M}}\right|^2_{Q^2\to 0} = \frac{\mathbf{G}^2}{2}\,\mathbf{L}_{\mu\nu}\,\mathbf{A}_{\mu\nu}\,; \qquad \mathbf{L}_{\mu\nu}(\mathbf{Q}^2 \to \mathbf{0}) = \mathbf{2}\,\frac{\mathbf{E}_\nu(\mathbf{E}_\nu - \nu)}{\nu^2}\,\mathbf{q}_\mu \mathbf{q}_\nu$$

At $\mathbf{Q^2 \to 0}$ the vector current contribution and the transverse part of the axial term vanish, only $\sigma_{\mathbf{L}}^{\mathbf{A}}$ survives, and

PCAC: $\mathbf{q}_\mu\,\mathbf{j}^{\mathbf{A}}_\mu = \mathbf{m}^2_\pi\,\phi_\pi$

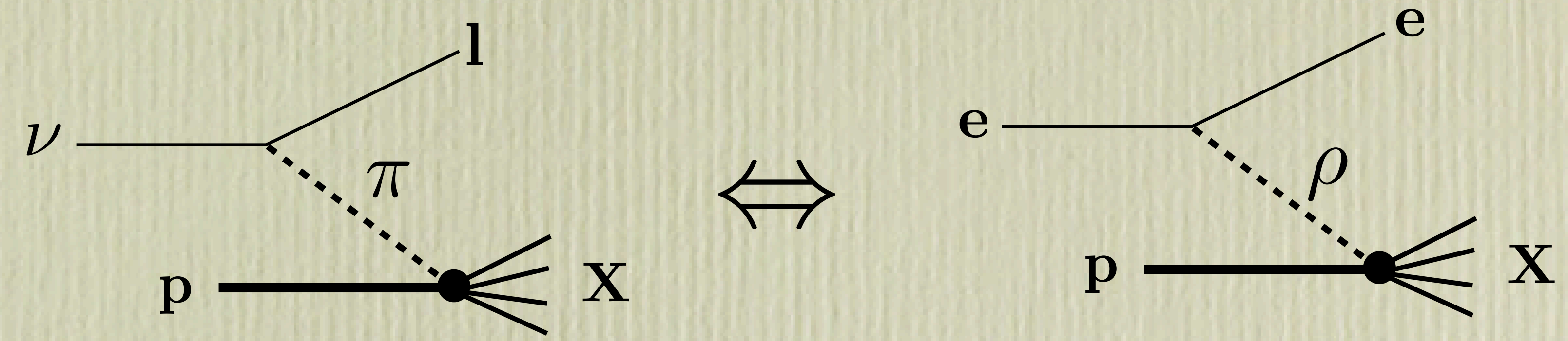
$$\mathbf{q}_\mu\,\mathbf{A}_{\mu\nu}\,\mathbf{q}_\nu = \frac{\mathbf{1}}{\nu}\,\mathbf{f}_\pi\,\sigma(\pi\mathbf{p} \to \mathbf{X})$$

leading to the Adler relation:

$$\left.\frac{\mathbf{d}^2\sigma(\nu\mathbf{p} \to \mathbf{l}\,\mathbf{X})}{\mathbf{d}\mathbf{Q}^2\,\mathbf{d}\nu}\right|_{\mathbf{Q}^2=\mathbf{0}} = \frac{\mathbf{G}^2}{2\pi^2}\,\mathbf{f}^2_\pi\,\frac{\mathbf{E}-\nu}{\mathbf{E}\nu}\,\sigma(\pi\mathbf{p} \to \mathbf{X})$$

Pion dominance?

In analogy to the vector dominance model it is tempting to interpret the Adler relation as a manifestation of pion dominance.



However, neutrinos do not fluctuate to pions because of conservation of the lepton current $\mathbf{q}_\mu \, \mathbf{l}_\mu = \mathbf{0}$

$$\mathbf{A}_\mu = \frac{\mathbf{f}_\pi \, \mathbf{q}_\mu}{\mathbf{Q^2 + m_\pi^2}} \, \mathbf{T}(\pi \mathbf{p} \to \mathbf{X}) + \frac{\mathbf{f}_{\mathbf{a_1}}}{\mathbf{Q^2 + m_{a_1}^2}} \, \mathbf{T}_\mu(\mathbf{a_1 p} \to \mathbf{X}) + \ldots$$

The pion pole contains factor $\mathbf{q}_\mu$ and does not contribute

The contribution of heavy states

$\mathbf{A}_\mu$ can be conserved only if these two terms are related and cancel each other in $\mathbf{q}_\mu \mathbf{A}_\mu$

Piketty-Stodolsky paradox

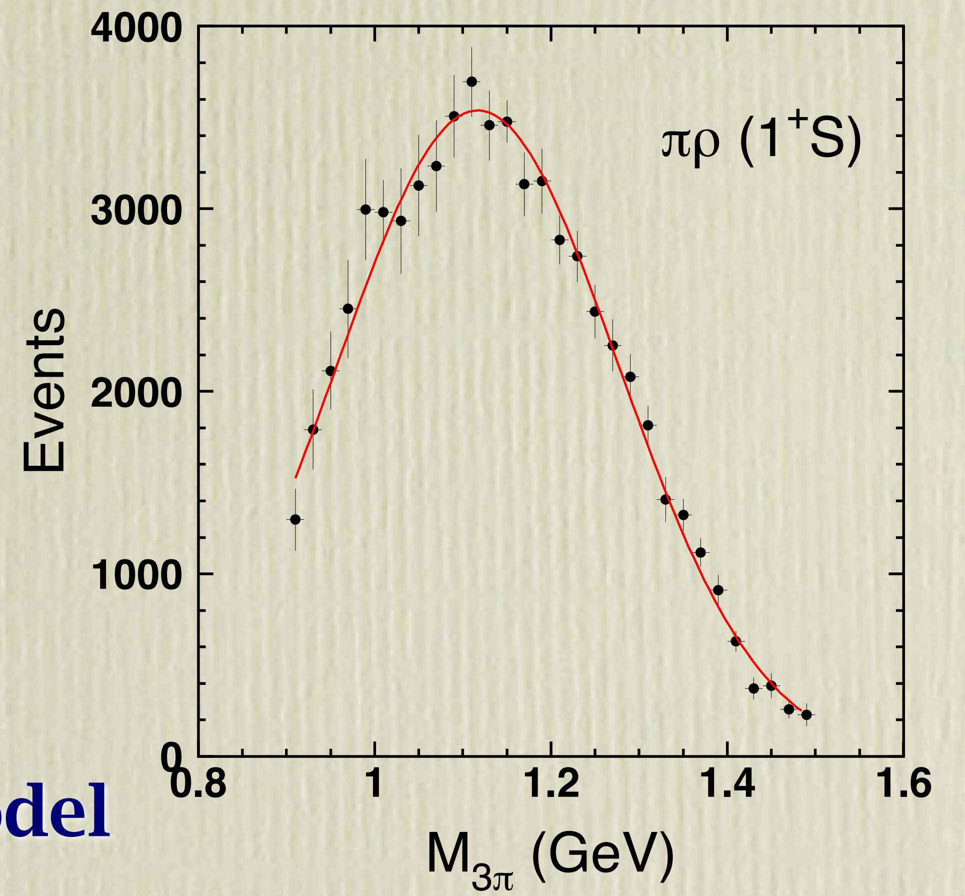
The first failure of PCAC

The Adler relation says that the combined contribution of all heavy axial states at $Q^2 \to 0$ should miraculously reproduce the pion pole.

This unusual connection was challenged by Piketty & Stodolsky, who assumed a_1 pole dominance. Then PCAC leads to the relation $\boxed{\sigma_{\rm diff}(\pi p \to a_1 p) \approx \sigma_{\rm el}(\pi p \to \pi p)}$

which contradicts data by factor ~20 (!)

The problem is relaxed after inclusion of the ϱπ cut and other diffractive excitations into the dispersion relation. Indeed, the relation $\sigma_{\rm diff}(\pi p \to X p) \approx \sigma_{\rm el}(\pi p \to \pi p)$ does not contradict data.



The ϱπ cut can be represented by an effective pole $\tilde{a}_1$, so we arrive at a two-pole $(\pi + \tilde{a}_1)$ model

Incurable PS puzzle

Even if this relation $\boxed{\sigma_{\text{diff}}(\pi p \to X p) \approx \sigma_{\text{el}}(\pi p \to \pi p)}$

were accurate, it cannot hold for ever.

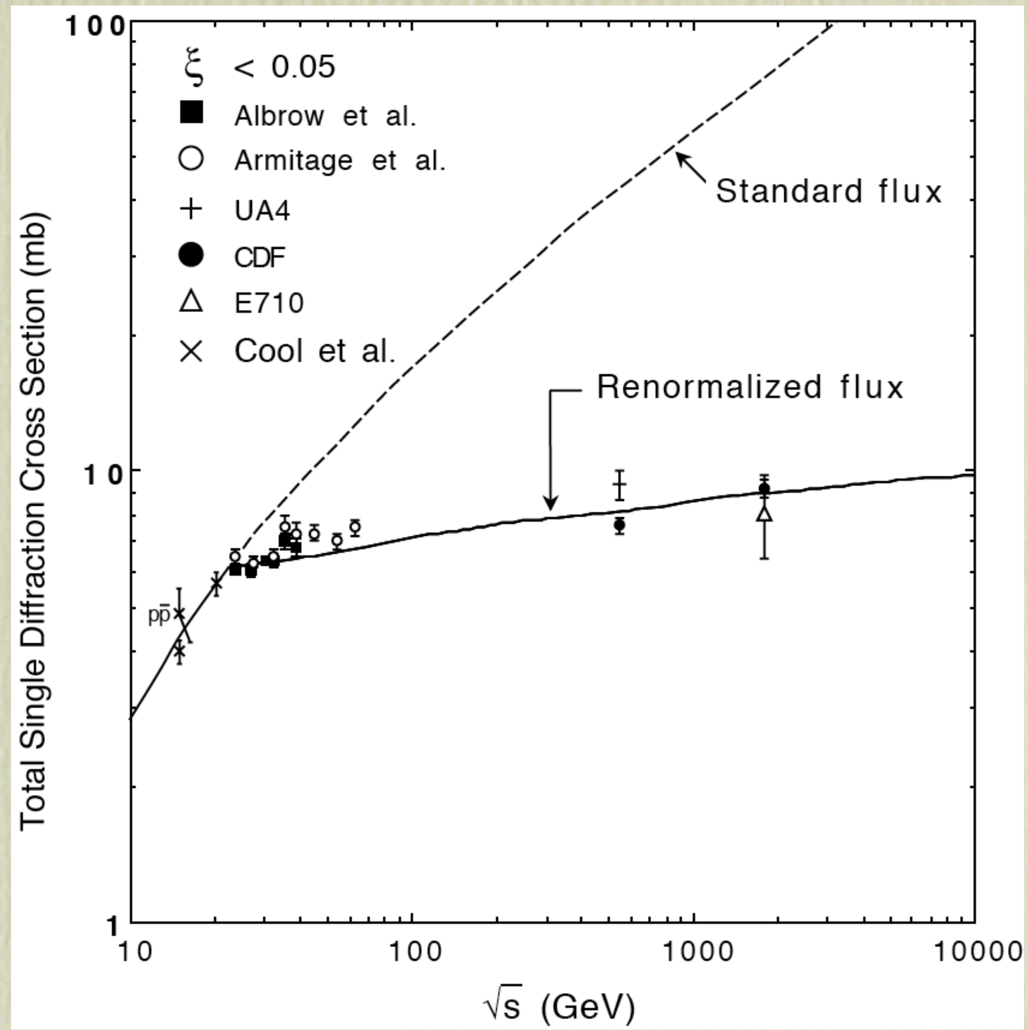
E.g. at very high energies in the Froissart regime

$$\frac{\sigma_{\text{el}}}{\sigma_{\text{tot}}} \longrightarrow \frac{1}{2}$$

while

$$\frac{\sigma_{\text{diff}}}{\sigma_{\text{tot}}} \longrightarrow \frac{\text{const}}{\ln(s)}$$

Diffraction is suppressed by absorptive corrections, while elastic cross section is enhanced.



Absorption effects in $\nu p \to l\pi p$

Absorptive factor for the diffractive amplitude at impact parameter b

has the eikonal form $e^{-\mathrm{Im}\, f^{\pi p}(b)}$, **where** $\mathrm{Im}\, f^{\pi p}(b) = \dfrac{\sigma_{tot}^{\pi p}}{4\pi B_{el}^{\pi p}}\, e^{-b^2/2B_{el}^{\pi p}}$

$$\sigma_{diff}^{\pi p}(b) = [\sigma_{diff}^{\pi p}(b)]_0\, e^{-2\mathrm{Im}\, f^{\pi p}(b)} \qquad = \sigma_{el}^{\pi p}(b)\, e^{-2\mathrm{Im}\, f^{\pi p}(b)}$$

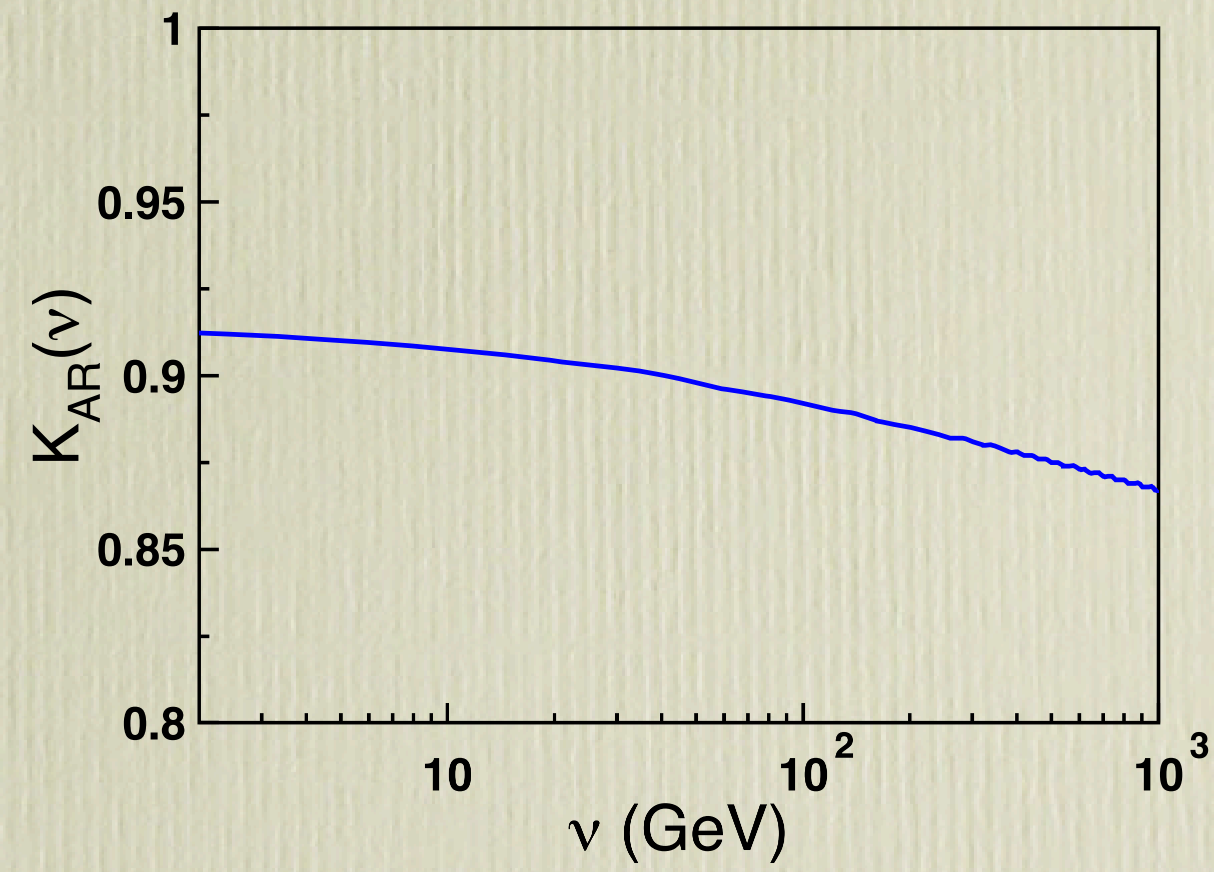
absorption

Adler relation

$$\sigma_{el}(b) \equiv \frac{d\sigma_{el}}{d^2b} = \left|1 - e^{-\mathrm{Im}\, f(b)}\right|^2$$

$$K_{AR} \equiv \frac{\sigma(\nu p \to l\pi p)}{\sigma_{AR}(\nu p \to l\pi p)} = \frac{\sigma_{diff}^{\pi p}}{\sigma_{el}^{\pi p}}$$

$$= \frac{\int d^2b\, \left|1 - e^{-\mathrm{Im}\, f^{\pi p}(b)}\right|^2 e^{-2\mathrm{Im}\, f^{\pi p}(b)}}{\int d^2b\, \left|1 - e^{-\mathrm{Im}\, f^{\pi p}(b)}\right|^2}$$



Nuclear targets

Most of neutrino experiments have been and will be performed on nuclear targets

The absorptive corrections to neutrino-nucleus interactions are tremendously enhanced.

On heavy nuclei the PCAC (Adler) condition, $\sigma^{\pi\mathbf{A}}_{\mathbf{diff}} \approx \sigma^{\pi\mathbf{A}}_{\mathbf{el},}$ is severely broken: diffraction vanishes, while the elastic cross section saturates at the maximal value allowed the unitarity bound.

$$\sigma^{\pi\mathbf{A}}_{\mathbf{diff}} \propto \mathbf{A}^{\mathbf{1/3}} \qquad \Longleftrightarrow \qquad \sigma^{\pi\mathbf{A}}_{\mathbf{el}} \propto \mathbf{A}^{\mathbf{2/3}}$$

- **Absorption enhances elastic, but suppresses inelastic diffraction.**

Thus, the Adler relation is incurable: diffractive diagonal and off-diagonal amplitudes cannot be universally related, since they are affected by absorptive corrections differently.

Diffractive neutrino-production of pions

Diffractive pion production on a nucleus may be coherent

$$\nu + \mathbf{A} \to \mathbf{l} + \pi + \mathbf{A}$$

(the nucleus remains intact)

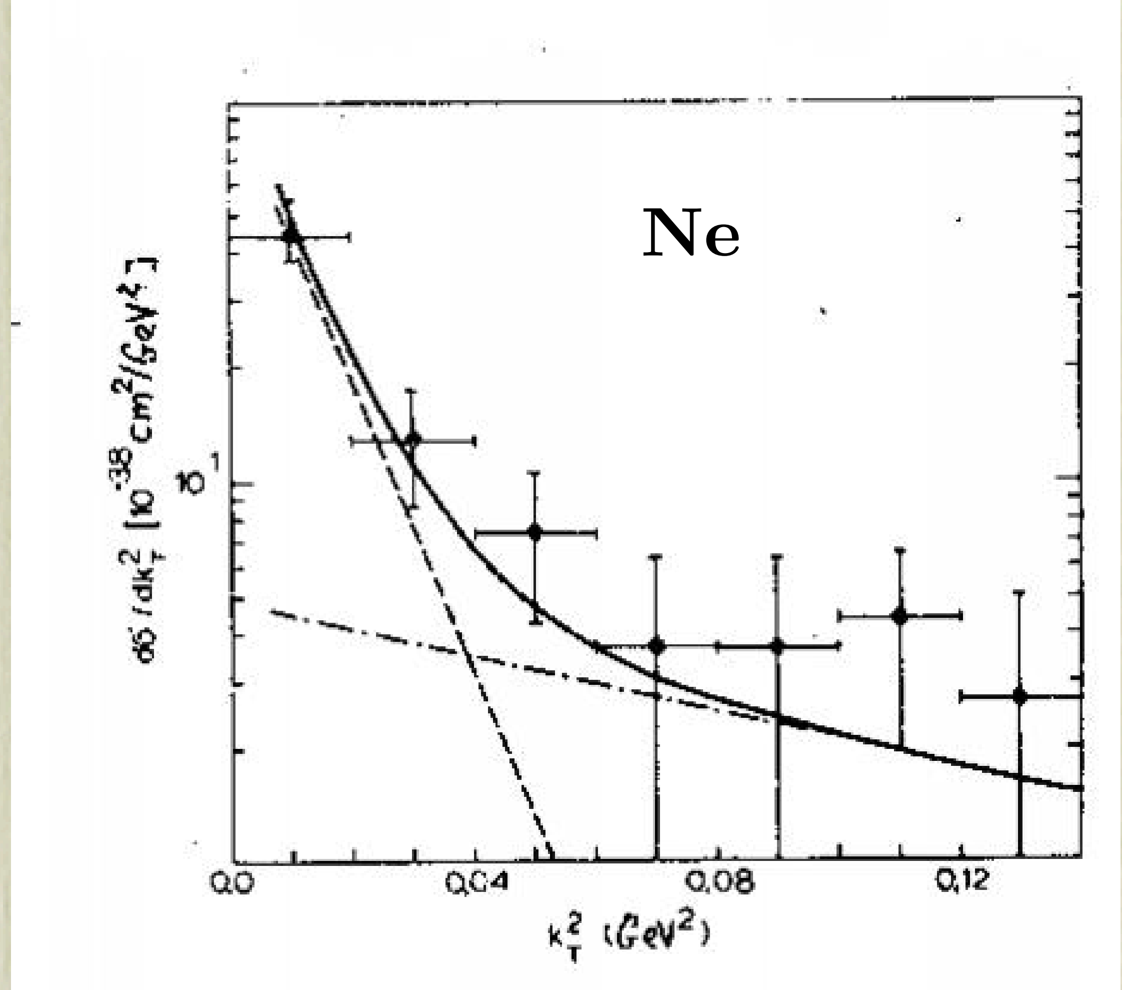
or incoherent

$$\nu + \mathbf{A} \to \mathbf{l} + \pi + \mathbf{A}^*$$

(the nucleus decays to fragments without particle production)

The two processes have very different p_T distributions, which help to separate them (statistically)

They also have different energy and Q dependences. Much can be learned from our experience with nuclear effects for vector current.



Characteristic time scales

$$t_c^\pi = \frac{2\nu}{m_\pi^2 + Q^2} \gg t_c^A = \frac{2\nu}{Q^2 + m_A^2}$$

Controls the interference between pions produced in different points.

The $\tilde{a}_1$-fluctuation lifetime

Correspondingly, there are three energy regimes

I

$$\nu < (Q^2 + m_\pi^2) R_A$$

$$\nu < 500\,\mathrm{MeV}$$

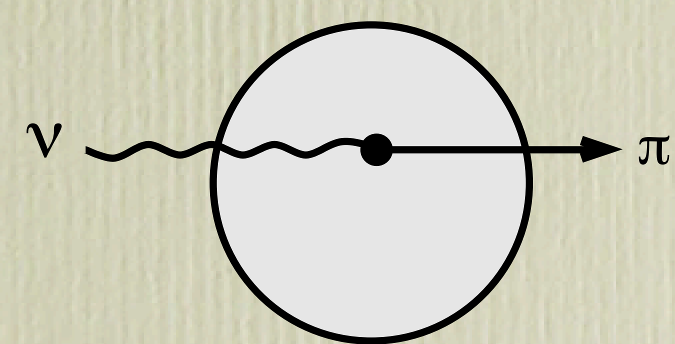
Coherent production is suppressed

II

$$(Q^2 + m_\pi^2) R_A < \nu < (Q^2 + m_A^2) R_A$$

$$0.5 < \nu < 40\,\mathrm{GeV}$$

Production is coherent, but without shadowing

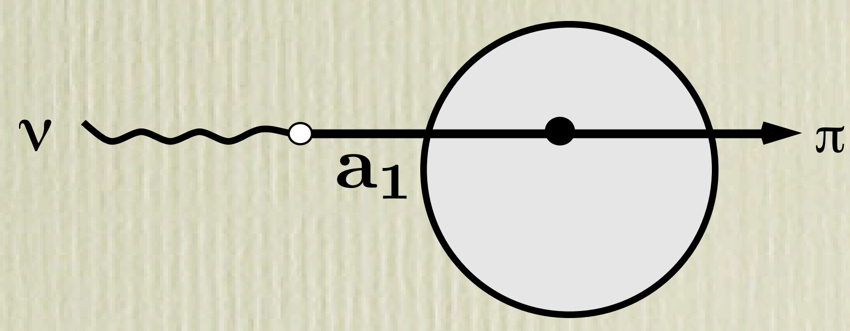


III

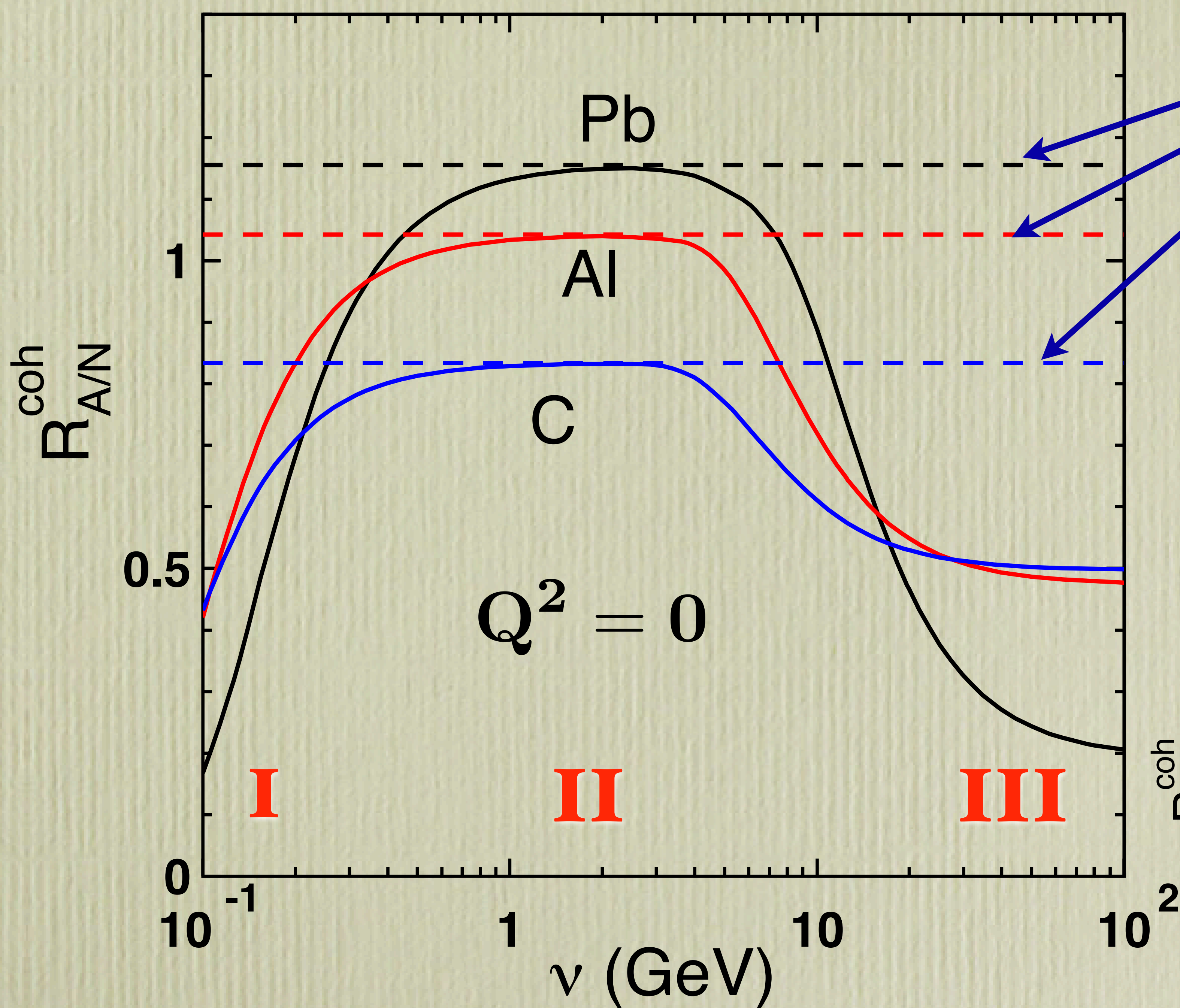
$$\nu > (Q^2 + m_A^2) R_A$$

$$\nu > 40\,\mathrm{GeV}$$

Maximal shadowing

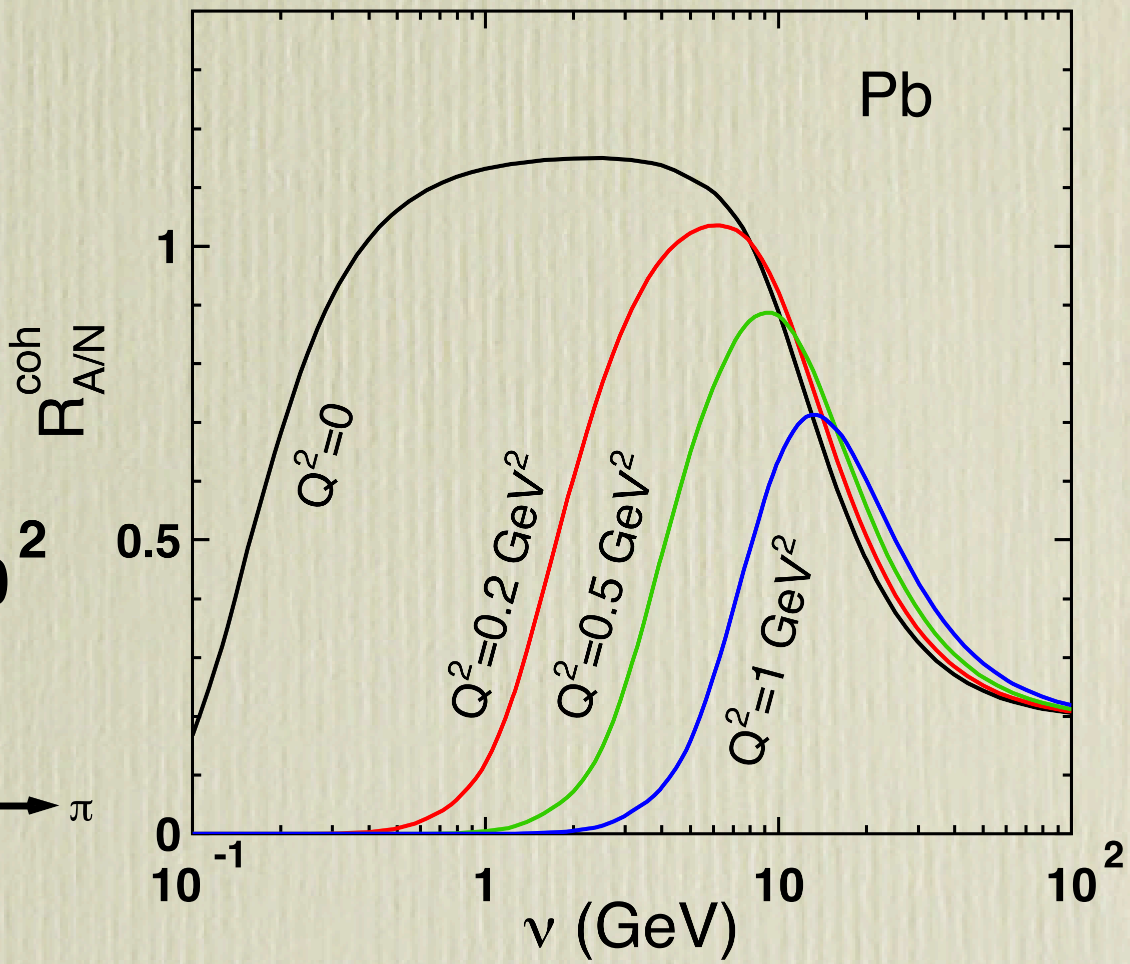


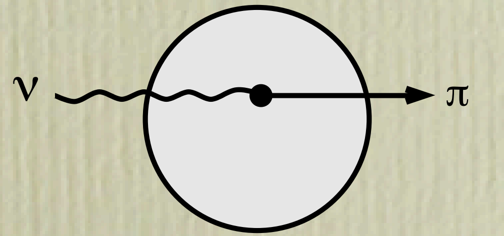
Coherent production of pions: $\nu A \to l \pi A$

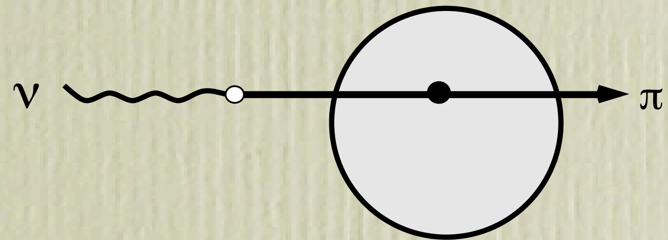


Adler relation

$$R^{coh}_{A/N}(\nu, Q^2) = \frac{d\sigma(\nu A \to l\pi A)/dQ^2\, d\nu}{A\, d\sigma(\nu N \to l\pi N)/dQ^2\, d\nu}$$

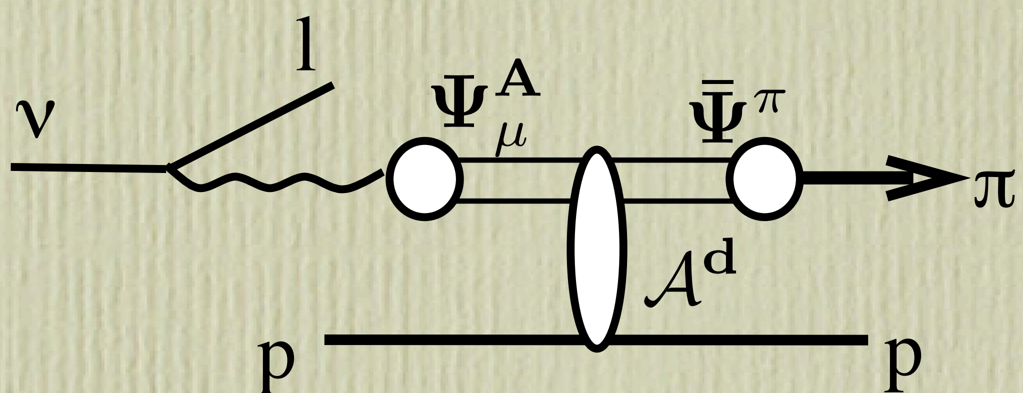






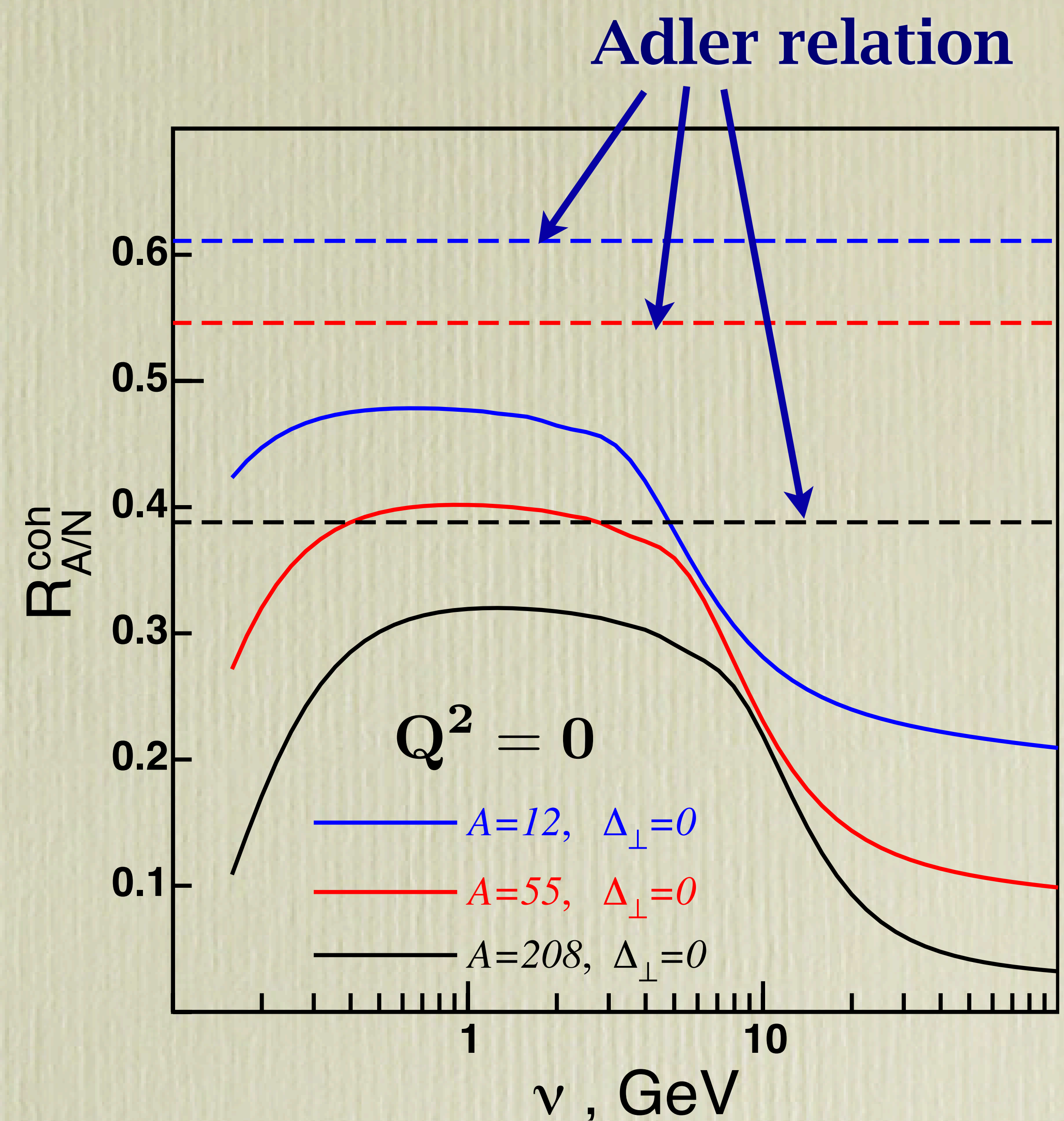
Color dipoles: more of PCAC breaking

$\nu \mathbf{p} \to \mathbf{l}(\bar{\mathbf{q}}\mathbf{q})\mathbf{p} \to \mathbf{l}\pi\mathbf{p}$



$$\nu \frac{d^3\sigma_{\nu p\to\mu\pi p}}{d\nu\, dt\, dQ^2} = \frac{G_F^2\, L_{\mu\nu} \left(W_\mu^A\right)^* W_\nu^A}{32\pi^3 m_N^2 E_\nu^2 \sqrt{1+Q^2/\nu^2}}$$

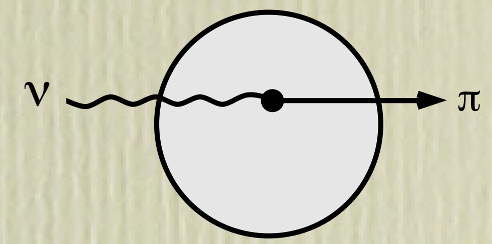
$$W_\mu^A\left(s,\Delta,Q^2\right) = \int_0^1 d\beta\, d^2r\, \bar\Psi^\pi(\beta,r)\, \mathcal{A}^d(\beta,r;\Delta)\, \Psi_\mu^A(\beta,r)$$



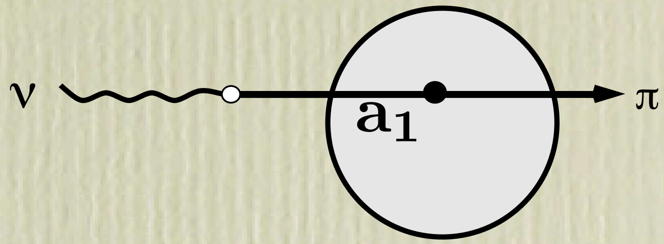
- The dipole amplitude $\mathcal{A}^d$ is fitted to photoproduction and DIS data.
- The light-cone $\bar{q}q$ distribution amplitudes Ψ_μ^A, Ψ^π are calculated in the instanton vacuum model

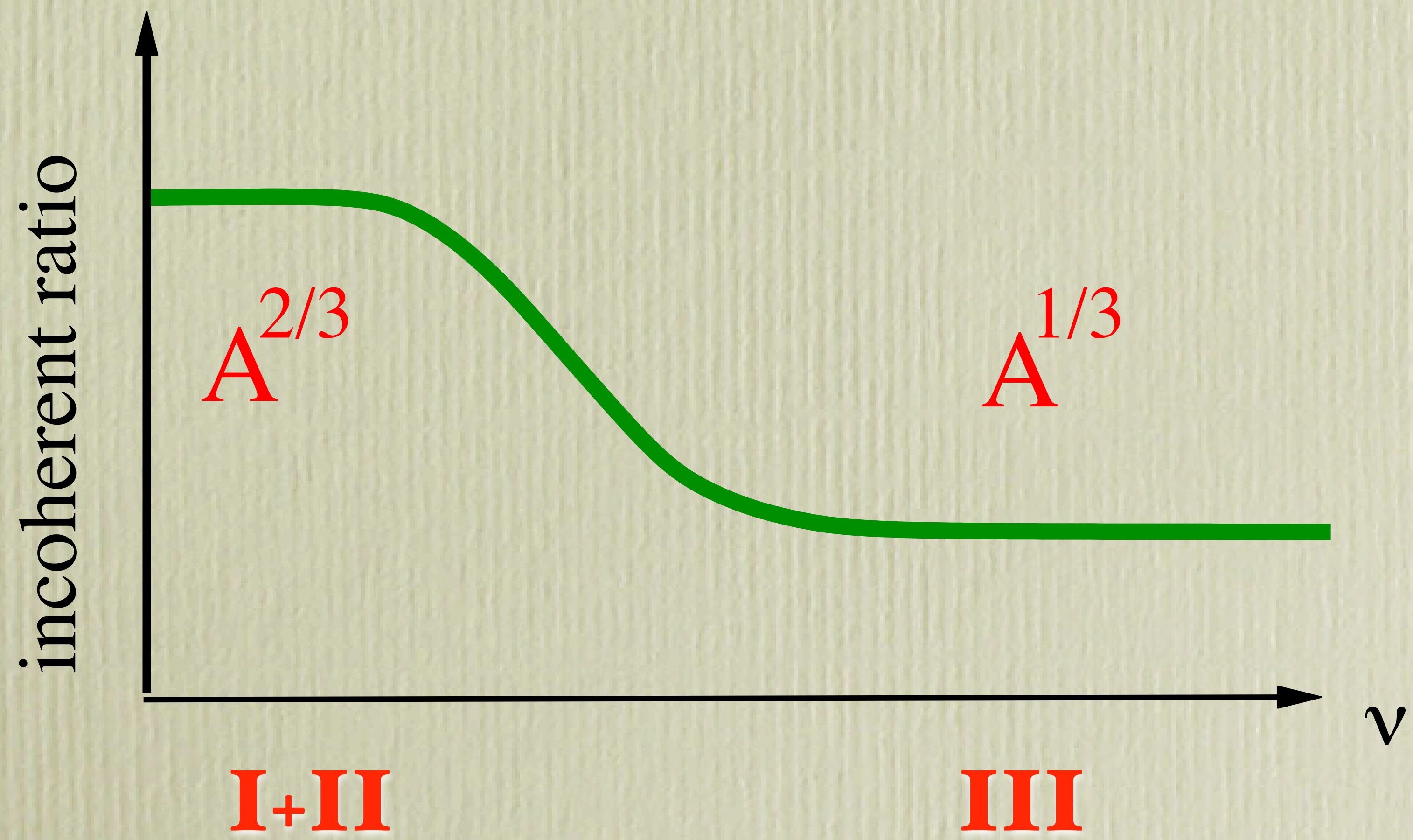
Incoherent production of pions

Regime **I+II,** final state attenuation: $A^{2/3}$



Regimes **III** both initial and final state attenuation: $A^{1/3}$

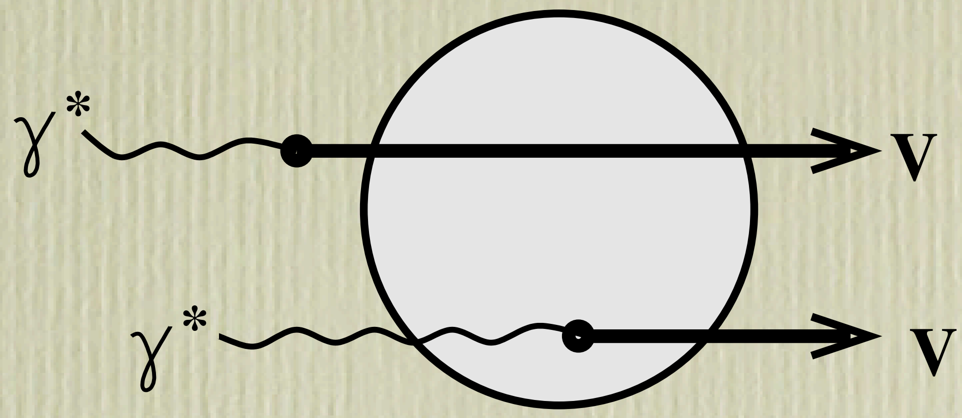




Diffractive electro-production

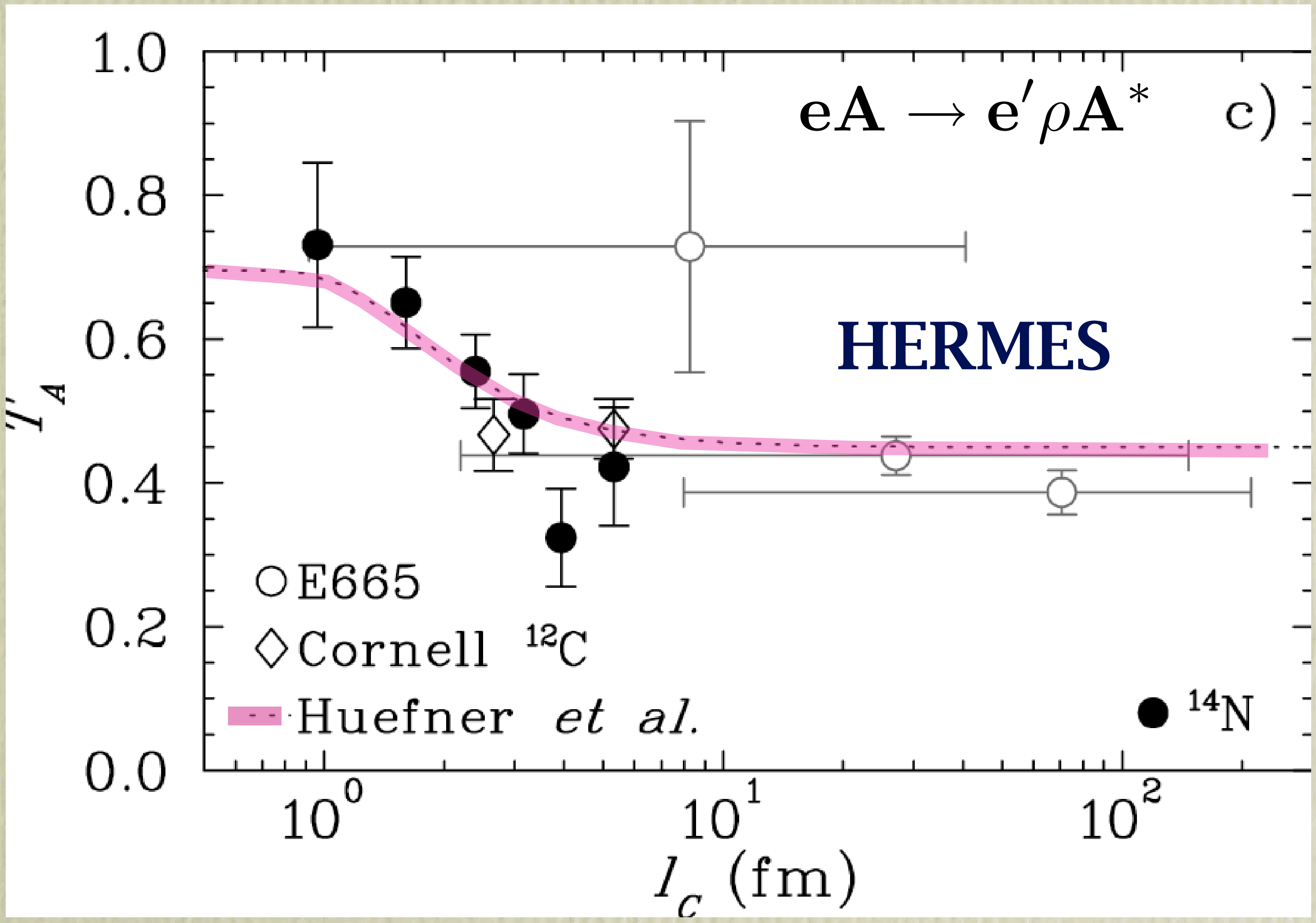
The important time scale,

$$t_c^{\rho} = \frac{2\,\nu}{m_\rho^2 + Q^2}$$



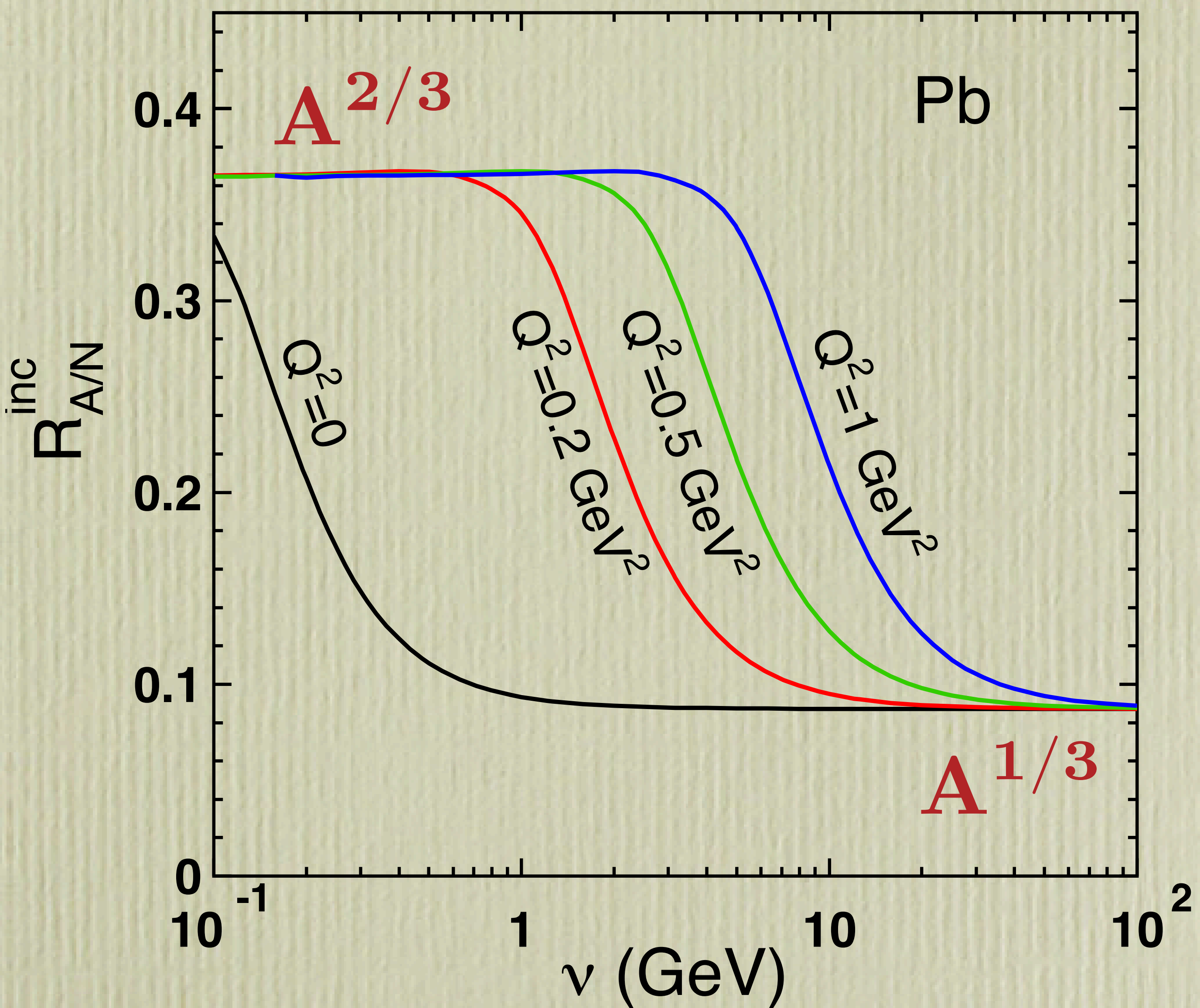
$t_c >> R_A$ $\quad A^{1/3}$

$t_c << R_A$ $\quad A^{2/3}$

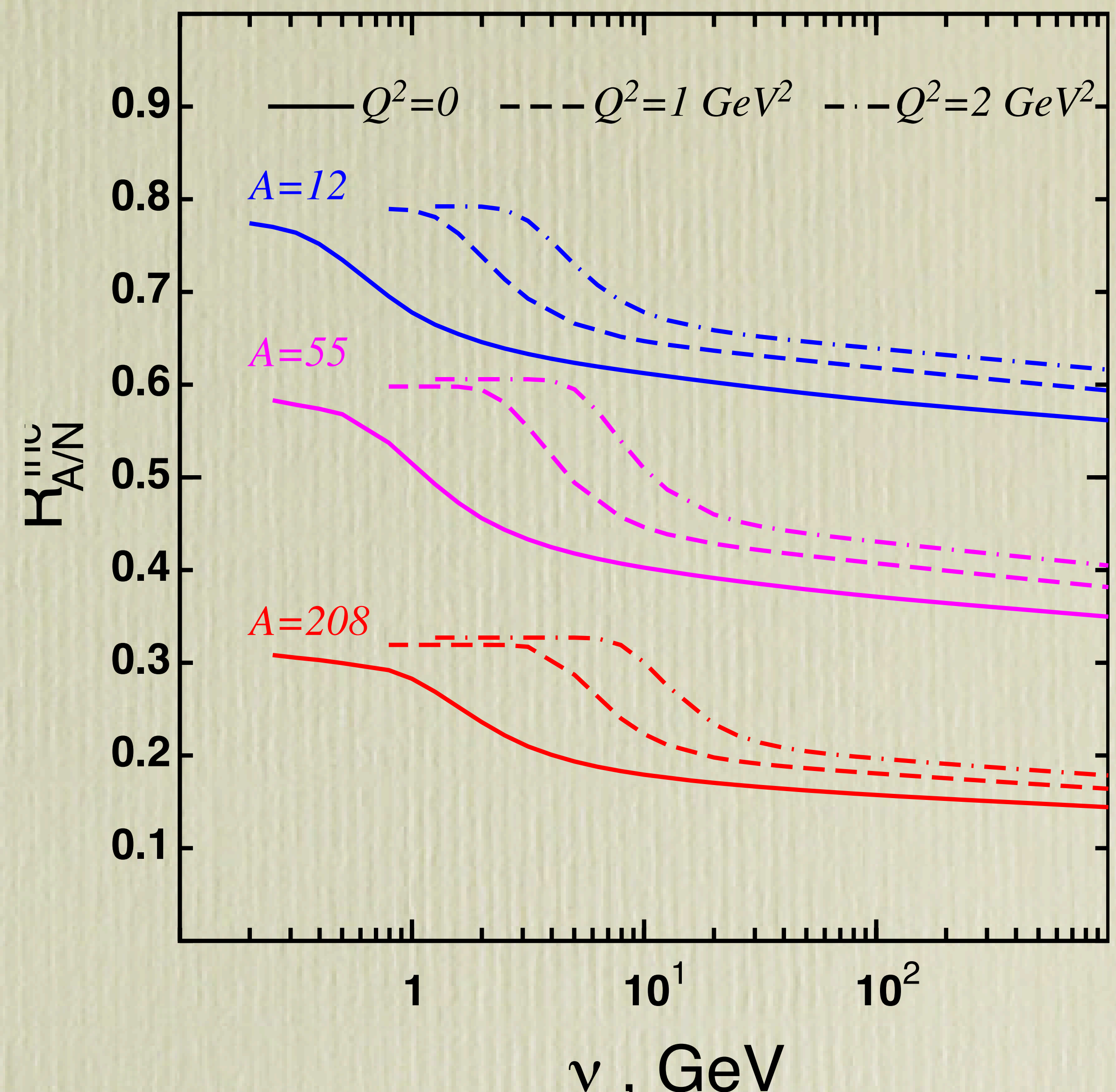


Incoherent production of pions: $\nu \mathbf{A} \to \mathbf{l}\pi \mathbf{A}^*$

2-channel model



Dipole description



As function of energy $\sigma(\nu \mathbf{A} \to \mathbf{l}\pi \mathbf{A}^*) \propto \mathbf{A}^{2/3} \Rightarrow \mathbf{A}^{1/3}$

Adler relation: $\sigma(\nu \mathbf{A} \to \mathbf{l}\pi \mathbf{A}^*) \propto \mathbf{A}^{1/3}$

is restored at high energies !

Summary

- PCAC imposes a miraculous relation between the quantities of very different physical origin, g_A, f_π, $g_{\pi NN}$. The Goldberger-Treiman relation is not a result of pion exchange, which is suppressed in β-decay and muon capture. This is a result of a miraculous link between light and heavy states.

- Diffractive interactions of high-energy neutrinos open new opportunities for testing the nontrivial constraints imposed by PCAC. Although a neutrino cannot emit a pion, the Adler relation naively looks like a result of pion dominance.

- In the diffractive neutrino-production of pions PCAC establishes a link between diagonal and off-diagonal amplitudes, which cannot be correct, because both are strongly and differently affected by the absorption.

- The Adler relation for coherent neutrino-production of pions on nuclei is broken at all energies, but especially at high energies. On the contrary, in incoherent production the Adler relation is broken at low, but is restored at high energies.