Emergent Electroweak Gravity

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Outline

- Introduction
- Massive cosmological relics are quantum liquids (as opposed to a classical gas)
- Interactions of quantum liquids: all cosmological relics with $M \lesssim M_{\mathsf{Pl}}$ have propagating zero-sound
- Neutrino pairing: the Kohn-Luttinger effect
- The effective action: Emergent Electroweak Gravity
- Sundry considerations: relationship to BCS superconductivity, Renormalization, Gauge Invariance, Cutoff, Wenberg-Witten Theorem
- Conclusions

Introduction: Where are we?

The universe is not empty.

Even “vacuum” contains long wavelength neutrinos and photons.

To leading order we’re justified in ignoring them because

$$T_\nu, T_\gamma, p_{F\nu}, p_{F\gamma} << T_0, p_{F0} << M_W$$

.

Our field theories and experiments have accurately told us what lives at *high* energies ($W^\pm$, $Z^\pm$ and possibly H^0).

If I look at the scales that are known, the ratios of those scales seem to contain the Planck scale (M_Z^2/T_ν or M_Z^2/p_F).

Introduction: My Question

We never consider these ratios of scales. Why?

To my mind, this looks like a finite temperature/finite density theory with a small parameter ($p_F \simeq 10^{-3}$ eV) and a large parameter ($G_F^{-1/2} \simeq 10^{11}$ eV), and at each ratio of these scales, new dynamics arises.

Therefore I am led to the following question:

What is the Standard Model dynamics which arises proportional to: p_F, $p_F G_F$, $p_F^2 G_F^2$, $p_F^3 G_F$?

We *must* answer this question, because the answer may give corrections to gravitational dynamics, dark matter dynamics, the cosmological constant, and other anomalies such as Pioneer. Either these dynamics exists, or we should prove that it does not, before assuming that G_N, Λ (etc) are unrelated to the weak scale.

The Hierarchy Problem

Nearly every talk in particle physics (including this one) begins with the following magical incantation:

If we compute the one-loop corrections to the Higgs mass... we find...

$$\delta m_H^2 \propto M_{\mathsf{Pl}}^2$$

The argument goes: Any new matter or cutoff that couples to the Higgs introduces a correction proportional to that scale. Since we know gravity exists, therefore we know there is a scale above the Higgs mass that introduces this correction.

This argument has one flaw... Is M_{Pl} a scale? Is it a cutoff to field theory? Could M_{Pl} be a ratio of scales instead (e.g. $(G_F T)^{-1}$)

We have no idea!

This has been entertained e.g. by Sundrum (2003) "soft gravitons" and Sakharov (1967) "induced gravity" among many others.

Scales

The "scale" present in the problem is actually a coupling constant, G_N.

The scales at which gravity is tested include ~ 1 eV (CMB freeze-out) and ~ 1 MeV from Big Bang Nucleosynthesis (if we can ignore the ^{7}Li problem, and *no scale above that*.

It would be completely compatible with direct tests of gravity if gravity "turned off" at a scale $\gtrsim 1$ MeV

It is known that gravitational theories can emerge from field theory. e.g. "Analog Models", "Dumb Holes", [Sakharov, Liberati, Visser, etc] as well as a 3-dimensional gravity in ^{3}He [Volovik].

So let's make a radical assumption.

I don't believe in Gravity

The existing models for Gravity (String Theory, Loop Quantum Gravity) insist that somehow gravity is the most fundamental theory. Particle physics "accidentally" falls out of these theories.

This seems backwards to me. The Standard Model (along with Quantum Field Theory) is the most precise, most predictive theory we have ever had.

Since Theoretical Physics is a game of deciding which important theoretical or experimental result I'm going to ignore so that I can write my next paper, let's ignore gravity.

Let's pretend that I just discovered gravity. But I have at my disposal all of particle physics and the Standard Model (including possibly the Higgs).

What could there be in the vacuum that could cause these funny forces I just discovered? (photons, neutrinos)

Scales of the neutrino background

What scales do I know about? (note $p_F^3 = 3\pi^2 n$; $n = N/V$)

$p_F(\nu)$	2.34×10^{-4} eV	per flavor/anti
$\sqrt{\Delta m_{12}}$	8.94×10^{-3} eV	
$\sqrt{\Delta m_{23}}$	5.29×10^{-2} eV	
T_ν	1.68×10^{-4} eV	
$G_F^{-1/2}$	2.92×10^{12} eV	

What scales do I want to explain? (using p_F as representative of the low scale)

Λ	2.3×10^{-3} eV	$\mathcal{O}(p_F)$
$p_F(\chi)$	8.80×10^{-6} eV $\left(\frac{100\text{GeV}}{M_\chi}\right)$	$\mathcal{O}(p_F)$
$M_{\text{Pl}}^{-1} = \sqrt{G_N}$	$10^{-28} eV^{-1}$	$\mathcal{O}(p_F G_F)$
α_Λ	1.51×10^{-33} eV	$\mathcal{O}(p_F^3 G_F)$
α_{MOND}	2.63×10^{-34} eV	$\mathcal{O}(p_F^3 G_F)$
α_{Pioneer}	1.92×10^{-33} eV	$\mathcal{O}(p_F^3 G_F)$

Is this all a big coincidence?

Brief Review on the Cosmic Neutrino Background

Cosmic neutrinos decouple from the Big Bang plasma at a temperature around 2 MeV. At that time they have a thermal Fermi-Dirac distribution.

As the universe expands, their density and temperature red-shift, leading to

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma = 1.95\text{K}; \qquad n_{\nu_i} = n_{\overline{\nu}_i} = \frac{3}{22} n_\gamma = \frac{56}{\text{cm}^3}$$

where T_γ and n_γ are the measured temperature and number density of CMB photons. Thus at least two species must be non-relativistic today. If neutrinos cluster gravitationally, the density is enhanced [Singh, Ma; Ringwald, Wong].

Due to large mixing, the flavor composition is equilibrated. All three mass eigenstates have equal densities. [Lunardini, Smirnov]

The asymmetry $\eta_\nu = (n_\nu - n_{\overline{\nu}})/n_\gamma$ is related to the baryon asymmetry $\eta_b = (n_b - n_{\overline{b}})/n_\gamma \simeq 10^{-10}$, so that any asymmetry can be neglected and we will assume $n_\nu = n_{\overline{\nu}}$.

What are neutrinos doing *today*

The dynamics of the neutrino background is given just by its kinetic term and self-interaction

$$\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\overline{\psi}\psi + \frac{g^2}{M_Z^2}\overline{\psi}\gamma^{\mu}\psi\overline{\psi}\gamma_{\mu}\psi$$

let us ignore the interactions for a few slides and concentrate on the kinetic term. It does 2 things:

- Gives rise to the 2 point function, transporting neutrinos in space

- Causes the expansion of the neutrino's wave packet

The latter effect is normally forgotten in QFT under the assumption that we have asymptotic localized particles. Is this a good assumption for a cosmological relic?

Wave packet expansion I/IV

Wave packets expand because different wave numbers move at different velocities in the presence of a mass or interaction. The wave number at $p = p_0 + \Delta p$ moves with velocity $v = (p_0 + \Delta p)/E$ while the wave number on the other side moves with velocity $v = (p_0 - \Delta p)/E$, and these wave numbers separate in space.

Thus the uncertainty of a wave packet evolves as

$$\Delta x(t)^2 = \Delta x_0^2 + \Delta v^2 t^2$$

In the relativistic case we must use

$$\Delta v = \frac{\Delta p}{E}(1 - v^2).$$

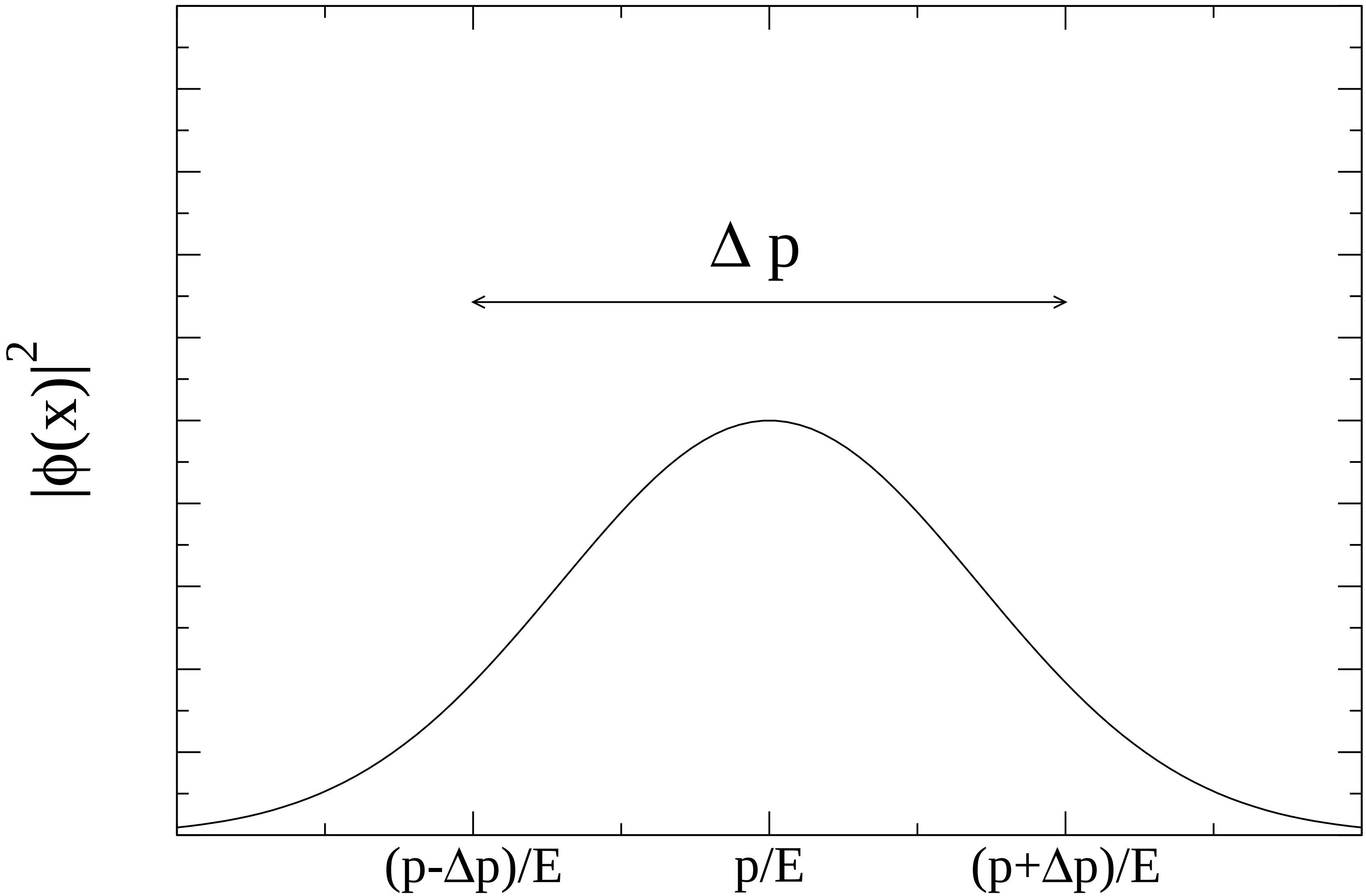
Assuming the uncertainty is given by the de Broglie wavelength

$$\Delta x_0 = \lambda/p = \lambda/\sqrt{3mkT}$$

allows us to derive the condition for a quantum liquid with $t = 0$ (or equivalently $\Delta p = 0$) from $\Delta x > n^{-1/3}$:

$$T < \frac{n^{2/3}\lambda^2}{3mk}$$

Wave packet expansion II/IV



Wave packet expansion II/IV

The "quantum liquid" condition is:

$$\Delta x \gg n^{-1/3}.$$

The opposite limit is the "classical gas" limit, and is the limit used by scattering theory (particles are localized):

$$\Delta x \ll n^{-1/3} \sim b.$$

where b is the impact parameter in scattering theory. The temperature condition is valid only if scattering occurs sufficiently often that the time dependence of the wave packet can be neglected:

$$\tau \ll \frac{\Delta x}{\Delta v} = E\frac{\Delta x}{\Delta p}$$

where $\tau = (\sigma n v)^{-1}$ is the mean time between collisions. This holds for atomic and nuclear matter at the densities usually considered.

Notice that the other assumption $\Delta p = E\Delta v = 0$ implies $\Delta x = \infty$ and vacuum calculations are not appropriate. they must be done at finite density. (i.e. we're in a momentum eigenstate but there is *no empty space*)

Wave packet expansion conclusion

Putting everything together using $t = \tau$:

$$\frac{1}{p^2} + \frac{(1-v^2)^2}{\sigma^2 n^2} > \frac{1}{\lambda^2 n^{2/3}}. \tag{1}$$

If we can neglect the first term, which is valid for decoupled relics, we obtain the quantum liquid criterion for weakly coupled relics:

$$\sigma < \frac{\lambda(1-v^2)}{n^{2/3}}. \tag{2}$$

This is very (very very) well satisfied for both relic neutrinos and dark matter ($\sigma \simeq 10^{-56}\mathrm{eV}^{-2}$, $n^{-2/3} \simeq 10^{-8}\mathrm{eV}^{-2}$). This means:

1) We have to worry about the dynamics of a quantum liquid for any massive cosmological relic (dark matter, at least 2 flavors of neutrinos)

2) We need to worry about quantum liquid dynamics of massless relics (lightest neutrino, axions, photons) too, because $T \sim n^{-1/3}$ and the low-momentum components of the distribution function are a quantum liquid.

Scales

This interactions of cosmic neutrinos are a theory of contact interactions in a quantum liquid at finite density and zero temperature. The fundamental parameters are the Fermi momentum p_F and G_F.

Let us examine the effective range expansion of neutrino self-scattering to get an idea of the scales:

$$k \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} k^2 l_0 + \ldots$$

where $a = \sqrt{\sigma_{\nu\nu}/4\pi} \simeq T_\nu G_F$ is the s-wave scattering length and $l_0 = \sqrt{G_F}$ is the range of the potential. Thus we have the approximation regime $a \ll l_0$.

This is the *opposite* approximation regime to atomic and nuclear finite density systems, BEC's, and BCS superconductivity, so one must be careful when applying results from those fields, and we want to take $a \to 0$.

Therefore, the leading dynamics occurs due to this p-wave term.

Dismiss some Bad Ideas

The leading interactions at finite density come in at $\mathcal{O}(p_F^2 G_F^2)$, but one can also write $T_\nu^2 G_F^2$ which is the same order. Is it relevant?

The combination $T_\nu^2 G_F^2$ is the self-interaction cross section of neutrinos. This would seem to be a hydrodynamic theory. However then one has to confront the flux. The inverse mean free path of a neutrino is

$$\lambda^{-1} = (\sigma n)^{-1} = T_\nu^2 p_F^3 G_F^2 \simeq \mathcal{O}(p_F^5 G_F^2)$$

and much larger than the horizon size, and the interaction rate is too low to be interesting.

One can also write $m_\nu^2 G_F^2$ but this would only arise in conjunction with p_F or T_ν.

$T_\gamma^2 G_F^2$ doesn't make sense.

The finite density theory of photons has a very small self-interaction (smaller than the neutrino self-interactions).

What's different about a quantum liquid?

We have non-zero density everywhere. Particles are not isolated or localized.

$\Rightarrow$ Contact operators have expectation values in "vacuum".

This means that those contact operators can define propagating composite degrees of freedom.

This is *zero-sound*.

Just as with a BEC (cooper pair), it is the attractive interactions that define the propagating modes.

This is also index of refraction (forward-scattering) physics, which is important when there is no scattering! (look through a plate of glass)

What's different about a quantum liquid?

Dark Matter with an attractive self-interaction would form a superfluid at high densities, in exactly the same way as a BCS superconductor. The long wavelength fluctuations of the condensate are a goldstone boson corresponding to the spontaneous breaking of the global $U(1)$ fermion number conservation. This condensate is also the zero sound above T_C.

The required self-interaction is a point operator which originates by integrating out heavy fields (e.g. squarks, higgses, sleptons, etc).

The attractive self-interaction need not be large. Any *infinitecimal* attractive coupling is sufficient to create zero-sound. Also, we only need something that competes with gravity in strength for it to be important, and $M_W \ll M_{\mathrm{Pl}}$, so attractive self-interactions imply zero-sound that interacts substantially stronger than gravity.

Neutrinos have *repulsive* self-interactions [Caldi, Chodos, '99]

Repulsive zero-sound: the Kohn-Luttinger Effect

There are only 2 options for the self-interactions: attractive or repulsive. Does a quantum fluid with repulsive interactions remain "normal" when $\Delta x \gg n^{-1/3}$?

This was answered in '65 by Kohn and Luttinger, and the answer is *no*.

Given a point-like interaction, the tree interaction scales as e^{-l^4} with partial wave number l.

The one loop correction has a new infrared divergence on the Fermi surface however, and gives rise to a correction

$$\delta V(\cos\theta) = -|V(\theta = 0)|^2 \frac{m p_F}{16\pi^2}(1 - \cos\theta)\log(1 - \cos\theta)$$

Kohn-Luttinger Effect

Let us expand this correction in partial waves

$$V_l = \int_{-1}^{1} d(\cos\theta) P_l(\cos\theta) V(\cos\theta)$$

$$V_l = (-1)^{l+1} \frac{m p_F}{4\pi^2} \frac{|V(\theta = 0)|^2}{l^4}.$$

Notice this is *attractive* for odd l.

The leading term occurs due to tree level t-channel Z exchange.

The conclusion of Kohn and Luttinger is that for fermions with any sufficiently weak interaction mediated by heavy fields, they must undergo a superfluid transition if the temperature becomes low enough (at fixed density), because this correction scales as l^{-4} while the direct (tree level) interaction scales as e^{-l^4}.

In other words, there *always* exists a superfluid state for a cold, weakly coupled Fermi gas. And therefore, there also exists *zero sound* at weak coupling.

Let us decompose the previous result into (non-relativistic) diagrams:

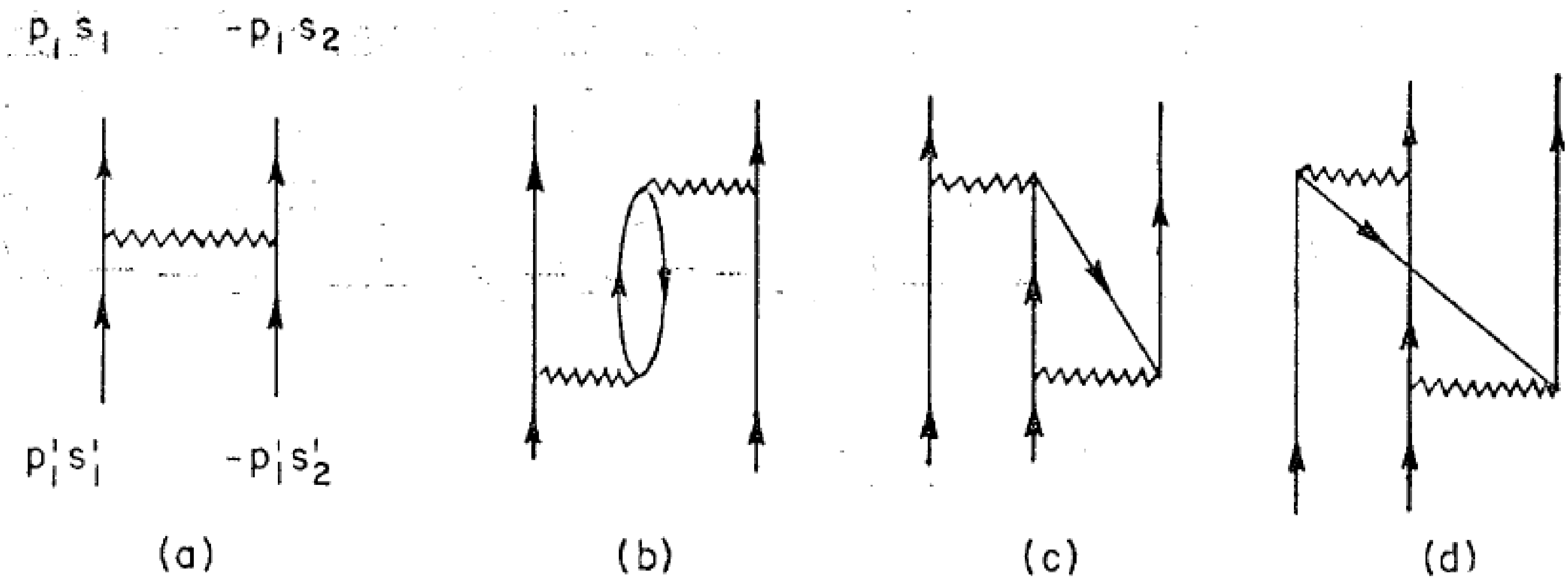


Diagram (a) is finite and repulsive. Diagrams (b) and (c) have logarithmic singularities at $\cos\theta = 1$. Diagram (d) has a logarithmic singularity at $\cos\theta = -1$.

Diagram (d) is an *exchange* interaction. That is, the propagating neutrino and the background neutrino change places.

Kohn-Luttinger, ctd...

The Fermi surface in this case is at $2p_F$ rather than at p_F as in the case of BCS superconductivity. This is because the particles running in the loops are what cause the divergence, and there are 2 of them

The coefficient of this new attractive term on the Fermi surface is

$$\tau\frac{g_Z^4 E_F p_F}{4\pi^2 M_Z^4} \simeq G_N$$

Because this condensate occurs in the p-wave, these s-wave condensates do not occur

$$\psi\psi; \qquad \psi^\dagger\overline{\sigma}^a\psi$$

the contact interaction for the latter is related to the former by a Fierz transformation. The first is a Cooper pair. The second could only form in the presence of a particle-antiparticle gas (i.e. impossible for electrons, baryons).

The actual condensate contains a derivative.

Kohn-Luttinger, ctd...

In traditional condensed matter (attractive interaction) BEC's, the condensate only exists for momentum modes near the Fermi surface, $p \sim p_F$ and it is only momentum modes near the Fermi surface that are long-lived.

For the Kohn-Luttinger effect, to the unusual kinematics (it's a ladder diagram), one can always keep both internal fermion propagators on the Fermi surface, for *any* value of the external momentum. (in the low energy limit $p \ll M_Z$)

That is to say a new double-pole develops in the $\nu\overline{\nu} \to \nu\overline{\nu}$ scattering amplitude, regardless of the external momentum.

Poles in scattering amplitudes are the hallmark of a bound state.

Some Calculational details *

One can regard this problem as zero-temperature and finite density.

Temperature effects only affect cross sections and are down by $T^2 p_F^3 G_F^2$ which is much smaller than leading $p_F^2 G_F^2$ we're interested in.

The poles that occur due to finite density occur *regardless of the form of the distribution function*. The system is definitely out of equilibrium anyway.

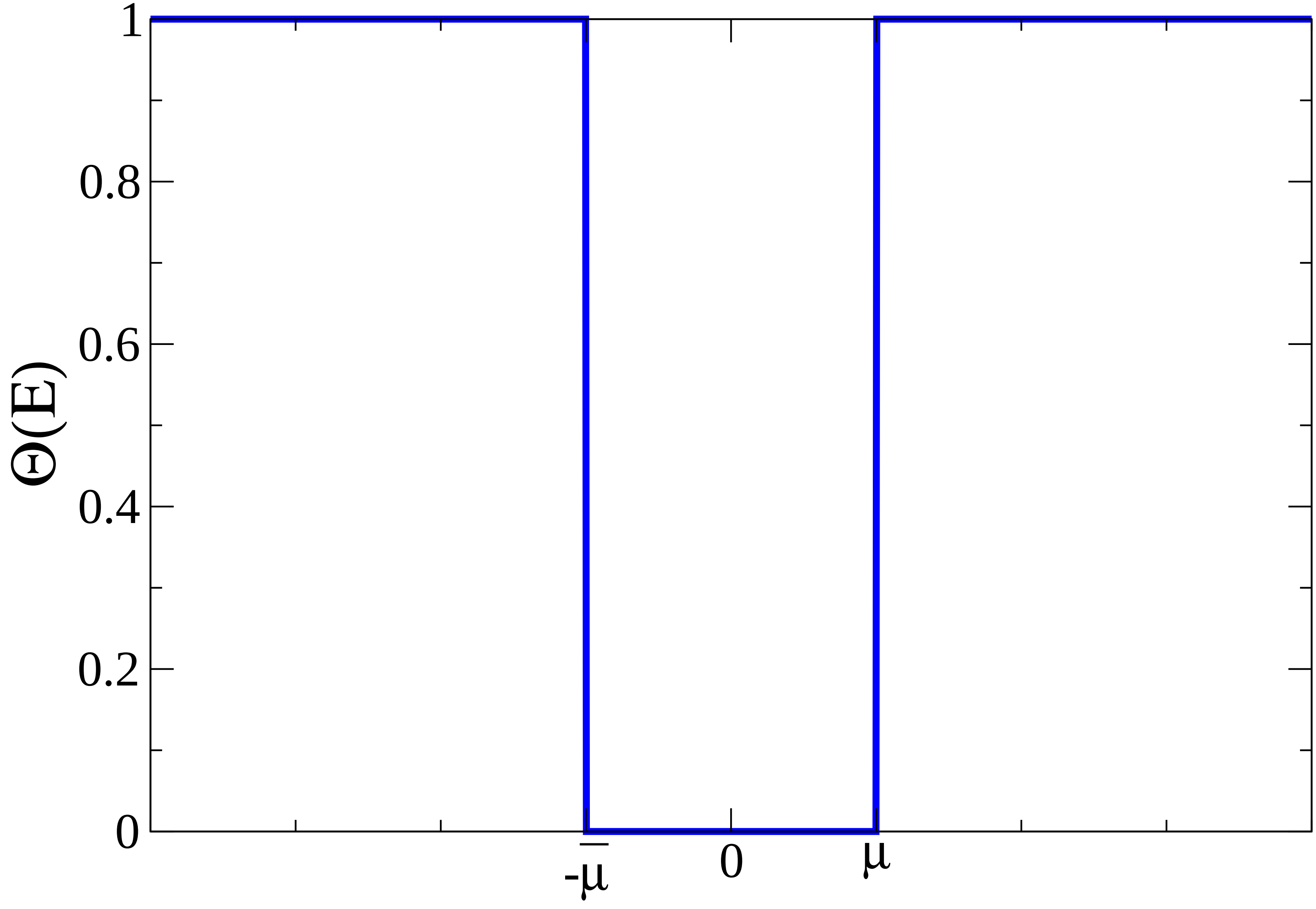
Then one can write the fermion propagator as:

$$\begin{aligned} S_F(p) &= \Theta(\mu - E)\frac{i}{\not{p} - m + i\epsilon} + \Theta(\overline{\mu} + E)\frac{i}{\not{p} - m - i\epsilon} \\ &= \frac{i}{\not{p} - m + i\epsilon} - \left(\frac{i}{\not{p} - m + i\epsilon} - \frac{i}{\not{p} - m - i\epsilon}\right)(\Theta(E - \mu) - \Theta(\overline{\mu} + E)) \end{aligned}$$

We're going to Pauli-block some of the momentum modes from the loop integral.

* Bob McElrath and Aleksi Vuorinen, to appear

The zero-temperature distribution function



More Calculational Details

As long as the momentum modes that get Pauli blocked have $p < M_Z^2/T$, then we don't care *which* momentum modes are blocked, and it's equivalent to consider a degenerate distribution $\Theta(\mu - E)$.

The number of modes that are blocked is defined by the density parameter, $p_F = (3\pi n)^{1/3}$ or $E_F = \mu(T=0) = \sqrt{m^2 + p_F^2}$.

This is *almost* equivalent to putting in a chemical potential. A chemical potential μ is a Lagrange multiplier which forces conservation of $N = n_f - n_{\overline{f}}$: $\mu^\alpha \overline{\psi} \gamma_\alpha \psi$. In the rest frame, $\mu^\alpha = (\mu, \vec{0})$.

This is only appropriate in equilibrium where particle-antiparticle pairs are quickly annihilated.

For relic neutrinos and dark matter, we need to *separately conserve* n_ν and $n_{\overline{\nu}}$, necessitating two "chemical potentials" μ and $\overline{\mu}$ (but remember $(n_\nu - n_{\overline{\nu}})/(n_\nu + n_{\overline{\nu}}) \sim 10^{-10}$).

Yet More Calculational Details: Renormalization

One might consider doing a Taylor expansion around $q = 0$ on the gauge boson propagator which would generate $(E^a_\mu)^2$. Since this is an irrelevant operator, it has a polynomial running anyway, and we can absorb Lorentz-invariant functions like q^2 into the definition of G_F or g_Z^2/M_Z^2.

If we choose to renormalize at the scale $q^2 = p_F^2$, we can choose that at that scale, the *only* operator that appears is

$$\frac{g_Z^2}{M_Z^2}\chi^\dagger\overline{\sigma}^a\chi\chi^\dagger\overline{\sigma}_a\chi$$

Then at one-loop we generate

$$-\tau\frac{g_Z^4 E_F p_F}{4\pi^2 M_Z^4}\int_{xy}\left[(1-\eta_\nu)E^{a\dagger}_\mu E^\mu_a + \eta_\nu A^\dagger_\mu A^\mu\right].$$

which are clearly proportional to the renormalization scale p_F (and would disappear if we renormalize around $q = 0$!)

We are not at zero temperature or density, and if we renormalize around $q = 0$ we miss important physics. . .

Identify the neutrino condensate

Let us examine the possible quasi-particles containing one derivative:

$$A_\mu(x,y) = \frac{i}{2p_F}\Big(\tilde\partial_\mu\chi(x)\epsilon\chi(y) - \chi(x)\epsilon\tilde\partial_\mu\chi(y)\Big)$$
$$E^a_\mu(x,y) = \frac{i}{2p_F}\Big(\tilde\partial_\mu\chi^\dagger(x)\overline{\sigma}^a\chi(y) - \chi^\dagger(x)\overline{\sigma}^a\tilde\partial_\mu\chi(y)\Big)$$

These arise from integrating out the Z and including the 1-loop corrections from the previous slide(s). The 4-point interactions are

$$A^\dagger_\mu A^\mu; \qquad E^{a\dagger}_\mu E^\mu_a$$

these are the *same* interaction (related to each other by a Fierz transformation). The derivative is

$$\tilde\partial_\mu = (\partial_t, v\vec\partial)$$

reflecting the fact that the dispersion relation for these states is $E = vp$ with $v < c$ (there is an index of refraction). The interaction terms are therefore

$$-\tau\frac{g^4 E_F p_F}{4\pi^2 M_Z^4}A^\dagger_\mu A^\mu; \qquad -\tau\frac{g^4 E_F p_F}{4\pi^2 M_Z^4}E^{a\dagger}_\mu E^\mu_a$$

these are clearly tachyonic mass terms.

The A_μ condensate is a particle-particle (or antiparticle-antiparticle condensate) while E^a_μ is a particle-antiparticle condensate. Therefore the original interaction can be rewritten:

$$-\tau\frac{g_Z^4 E_F p_F}{4\pi^2 M_Z^4}\int_{xy}\left[(1-\eta_\nu)E^{a\dagger}_\mu E^\mu_a + \eta_\nu A^\dagger_\mu A^\mu\right].$$

where

$$\eta_\nu = \frac{n_\nu - n_{\overline{\nu}}}{n_\nu + n_{\overline{\nu}}}$$

is the asymmetry between neutrinos and anti-neutrinos. After the phase transition has occured, the original Fermi gas is described by momentum distribution functions for A_μ and E^a_μ, in addition to one for free fermions.

Because $\eta_\nu \simeq 10^{-10}$ (it is related to the baryon-to-photon ratio), the dominant condensate is E^a_μ, and I will neglect A_μ hereafter.

Introduce the condensate into the action (A)

There are two nice ways to introduce the composite into the action.

(A) The Hubbard Stratonivich transformation (a.k.a. Saddle Point integration):

Multiply the action by

$$const = \int \mathcal{D}[X_\mu^a] \exp\left\{\lambda \frac{i}{\hbar} \int d^4x d^4y \, X_\mu^a(x,y) X_\nu^{\dagger b}(x,y) \eta_{ab} \eta^{\mu\nu}\right\}$$

where

$$X_\mu^a = E_\mu^a - \frac{i}{2p_F}\left(\chi^\dagger \overline{\sigma}^a \tilde{\partial}_\mu \chi - \tilde{\partial}_\mu \chi^\dagger \overline{\sigma}^a \chi\right)$$

This removes the induced quartic term in favor of E_μ^a, leaving the only a kinetic and tree level interaction term for ψ.

Note that this transformation is only valid if the effective coupling $\lambda < 0$ (attractive). Were $\lambda > 0$ this would not be a gaussian integral, and this transformation would be invalid.

(B) By Legendre Transformation

Following the "Nonequilibrium Quantum Field Theory" (a.k.a. 2PI) developed by Cornwall, Jackiw, and Tomboulis, one can insert a pair current N^a_μ. First let us note that

$$\frac{\delta\Gamma}{\delta E^a_\mu} = E^{a\dagger}_\mu + \epsilon^a_\mu = 0 \tag{3}$$

where $\epsilon^a_\mu = \delta^a_\mu \delta^4(x-y)$, which comes from the fermion's kinetic term:

$$\int_x \chi^\dagger \partial_\mu \overline{\sigma}_\mu \chi = \int_{xy} E^a_\mu \delta^\mu_a \delta^4(x-y)$$

Therefore we will need to shift E^a_μ, so that the equations of motion for E^a_μ are quadratic. Thus we have a generating functional:

$$W[\eta, N^a_\mu] = -i\hbar \ln \int \mathcal{D}[\chi] \exp\left\{\frac{i}{\hbar}\left[S[\chi] + \int_x \chi\eta + \frac{1}{2}\int_{xy} N^a_\mu (E^\mu_a + \epsilon^\mu_a)\right]\right\}. \tag{4}$$

Neutrino condensate symmetry breaking

The effective potential has an enhanced symmetry: $SO(3,1)_{\text{space}}$ (greek indices) $\times SO(3,1)_{\text{spin}}$ (roman indices):

$$\int_x \chi^\dagger \overline{\sigma}^a \chi \chi^\dagger \overline{\sigma}_a \chi - \int_{xy} E_\mu^{a\dagger} E_a^\mu - \int_{xy} A^{\mu\dagger} A_\mu$$

The only possible symmetry breaking terms $(\chi^\dagger \overline{\sigma}^\mu \partial_\mu \chi)^n \mathcal{O}$ are removed by the leading order equations of motion (and generate higher-order interactions for the zero-sound).

This leaves the kinetic term as defining the order parameter for the breaking of this enhanced symmetry back down to a single $SO(3,1)$. The effective potential is:

$$-\tau \frac{g_Z^4 E_F p_F}{4\pi^2 M_Z^4} E_\mu^a E_a^\mu + \frac{\alpha}{4!} \epsilon^{abcd} \epsilon_{\mu\nu\sigma\rho} E_a^\mu E_b^\nu E_c^\sigma E_d^\rho + \frac{\beta}{4!} (E_\mu^a E_a^\mu)^2$$

which clearly indicates that the zero-sound goldston is only properly described with an expectation value $\langle E_\mu^a \rangle$.

Lorentz Symmetry Breaking

The expectation value for E^a_μ has a simpler interpretation in terms of the stress tensor for a massless fermion:

$$\langle \tau^{\mu\nu} \rangle = \frac{1}{2} \langle E^a_\lambda \rangle \left[\delta^\nu_a \eta^{\lambda\mu} + \delta^\mu_a \eta^{\lambda\nu} + 2\delta^\lambda_a \eta^{\mu\nu} \right]$$

The Lorentz symmetry is actually two symmetries, spacetime and spin:

$$L_{\mu\nu} = i(x_\mu \partial_\nu - x_\nu \partial_\mu); \qquad S_{ab} = \frac{i}{2}(\gamma_a \gamma_b - \gamma_b \gamma_a)$$

the neutrino transforms as a scalar $(0,0)$ under the first group and a spinor $(\frac{1}{2},0)$ under the second group.

The Lagrangian is not symmetric under both groups. In particular, the kinetic terms for both fermions and weak bosons may relate the two groups to one another.

$$\overline{\psi} \gamma^\mu \partial_\mu \psi; \qquad (\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

Mass terms and four point interactions are symmetric under both groups.

Lorentz Symmetry Breaking at low energy

At low energies we may integrate out the Z boson, resulting in a tower of 4-point (and higher) operators. However gauge dependence may then appear in the 4-point operators. Let us examine the gauge boson propagator in the R_ξ gauge:

$$D_{\mu\nu}(q) = -i\frac{\eta_{\mu\nu}}{q^2 - M^2} + i(1-\xi)\frac{q^\mu q^\nu}{(q^2 - M^2)(q^2 - \xi M^2)}.$$

The gauge dependence of the second term gets cancelled in physical processes by the goldstone boson contribution.

The gauge parameter ξ shuffles the longitudinal portion of the gauge boson between the goldstone and the 3-dof gauge boson.

However, a *Majorana* particle cannot couple to the Z^0's goldstone boson. The gauge dependence vanishes by the Ward identity. Any $q^\mu q^\nu$ cannot contribute to physical processes (the same as in QED).

Therefore, WLOG, we may take $\xi = 1$ (Feynman-t'Hooft gauge). The couplings of a Majorana neutrino are entirely *transverse*.

Lorentz Symmetry Breaking at low energy

The gauge boson propagator is then

$$D_{\mu\nu}(q) = -i\frac{\eta_{\mu\nu}}{q^2 - M^2}$$

The metric in the numerator is both the metric of $SO(3,1)_{spin}$ and $SO(3,1)_{space}$.

Integrating out the Z boson generates a tower of point-like operators with external, transverse neutrinos. e.g. at leading order

$$\frac{\chi^\dagger(x)\overline{\sigma}^a\chi(x)\chi^\dagger(y)\overline{\sigma}^b\chi(y)\eta_{ab}}{q^2 - M^2 + i\epsilon} \tag{5}$$

here we take the η_{ab} to be the metric of $SO(3,1)_{spin}$.

All interaction terms are $SO(3,1) \times SO(3,1)$ symmetric! (roman a, b, c for spin and greek μ, ν, λ indices for space)

A possibly-irrelevant aside

This problem is entirely real.

A Weyl fermion can be represented as a four-component real spinor.

There exists a real representation of four-component the gamma matrices.

The price one pays for this is that the signature of the metric is $(-,+,+,+)$. One cannot get $(+,-,-,-)$.

So we carry around i's, daggers and complex conjugates everywhere, solely so that it can flip the signs of masses for us.

The Weinberg Witten Theorem

Weinberg and Witten (1980) told us that for any massless spin 2 object with a conserved Lorentz covariant stress tensor, its self-scattering matrix elements are zero.

This is generally used to "rule-out" a composite graviton, and indeed it does rule out a meson-like composite graviton.

However the theory of neutrino zero-sound is *NOT* Lorentz covariant. The fundamental theory is, but p_F breaks it! This results in the following Lorentz-breaking objects*

	value today	flat space (WW) limit
$\langle E_\mu^a \rangle$	$\mathcal{O}(10^{-3})$ eV	0
$n = v/c$	$1 - G_F^2 p_F^4 \simeq 1$	1
p_F	$\mathcal{O}(10^{-3})$ eV	0
G_N	$\mathcal{O}(p_F^2 G_F^2)$	0
M_{Pl}	$\mathcal{O}(1/p_F G_F)$	∞

* Alejandro Jenkins and Bob McElrath, to appear

Relationship to BCS Superconductivity

The BCS transition temperature is exponentially suppressed for weak interactions, and is proportional to

$$T_C \sim p_F e^{-1/p_F^2 G_F} \sim 10^{-4}\text{eV} e^{-10^{31}}$$

for s-wave attractive interactions, which is incredibly tiny, and much lower than $T_\nu \sim 1.9$K. This implies a line in the temperature-density phase diagram for the superfluid transition, and the superfluid phase only exists for $p_F \sim M_W$.

So the universe today is well above the transition temperature.

The "cooper pair" therefore is unstable and has a width proportional to the mean free path of its constituents.

For electrons, this means that cooper pairs are not a good description above T_C, because their mean free path is small. However here, the mean free path is $\lambda \sim T_\nu^2 p_F^3 G_F^2$ is so small that the cooper pair can be considered a stable state on timescales short compared to the mean time between collisions.

The Cutoff

This theory has a cutoff when the neutrino becomes strongly interacting enough to destroy its quantum liquid state.

For the condensed theory to be valid, scattering must not occur often enough to destroy the condensate. Scattering becomes strong again when the CM energy puts the Z on pole. For a probe with energy E, this occurs when

$$E = M_Z^2/T_\nu \simeq M_{Pl}$$

Therefore, in the *lab* frame, this low-energy effective gravitational description of the relic neutrinos is valid throughout the range of energies we have explored.

In particular, the weakly coupled gravitational description is valid for nucleosynthesis, CMB freezeout, and "black hole" event horizons.

This Quasi-Particle is a Graviton

We already know what a $SO(3,1)$ bi-vector is: the vierbein (tetrad):

$$g_{\mu\nu}(x,y) = E^a_\mu(x,y) E^b_\nu(x,y) \eta_{ab}$$

This field has an internal global $SO(3,1)$ symmetry due to the spin Lorentz invariance.

This is different from the first-order (Palatini) formulation of gravity (which uses a *local* internal Lorentz symmetry).

Thus the fermion spin dependence is not a gauge symmetry, but is a physical observable in this theory. The spin distribution of the fermion gives rise to *Torsion*.

Such a theory was explored by Hebecker and Wetterich [2003; Wetterich 2003, 2004]. They conclude that the addition of torsion, due to a global, rather than local Lorentz symmetry is at present *unobservable*.

This theory differs from that of Hebecker and Wetterich due to the presence of the $SO(3,1) \times SO(3,1)$ symmetry breaking structure, and the associated metric $\eta_{\mu\nu}$. (e.g. they don't have $(E^a_\mu)^2$ or $(E^a_\mu)^4$)

Conclusions

Dark matter and neutrino cosmology must take into account the fact that decoupled matter is a quantum liquid today, when $\sigma \lesssim n^{-2/3}$.

The Standard Model contains a graviton and gravi-photon: they are acoustic quasi-particles ("zero sound") in the Cosmic Neutrino Background.

While this may not be the theory of gravity everyone was looking for, we *must* take it into account. The Standard Model is well tested, cosmic neutrinos certainly must exist.

This theory may also contain the keys to galactic rotation curves, neutrino mass, and cosmic expansion, at the next order in $\sqrt{p_F^2 G_F}$.

This theory is *supremely testable* and *falsifiable* (unlike other gravity theories). We can make W's, Z's, and neutrinos. It contains zero free parameters.