*E*₆ Grand Unified Theory and Spontaneous CP Violation Nobuhiro Maekawa (Nagoya Univ. KMI)

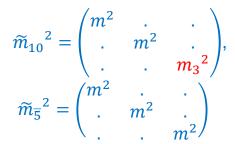
with M. Ishiduki, S.-G. Kim, K. Sakurai, K.I. Nagao, M.M. Nojiri, H. Kawase, T. Takayama

1. Introduction

- 2. *E*₆ Grand Unified Theory
- 3. Horizontal Symmetry
- 4. Spontaneous CP Violation
- 5. Summary and Discussion

Overview of this talk

- E_6 grand unified theory (GUT) Various hierarchies of quark and lepton masses and mixings can be explained. ($U_{e3} \sim \lambda$)
- *E*₆ GUT+non-Abelian horizontal symmetry The explanation is so natural that realistic Yukawa couplings can be realized after breaking the symmetries. Modified universal sfermion masses are predicted.
 - $1 m_3 \ll m$ is interesting.
 - a. light stop... weak scale stablity
 - b. heavy sfermion...suppress FCNC, CP
 - 2. All FCNC are sufficiently suppressed.
 - 3. Several are within reach of future exp.
 - 4. No signal in LHC is consistent with light stop!
 - 5. New type of SUSY CP problem looks serious.



Overview of this talk

*E*₆ GUT+horizontal sym. with sp. CP violation (Discrete sym. is introduced.)

Not only old type but also new type of SUSY CP problems can be solved.

Several bonus (light up quark, $V_{ub} \sim \lambda^4$, predictive power) Examining the neutrino sector New!

Same predictions as the usual E_6 GUT but non-trivial.

Introduction



Grand Unified Theories

- 2 Unifications
 - Gauge Interactions

 $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$ $SO(10) \supset SU(5)$ Matter

$$\begin{bmatrix} Q & U_R^c & E_R^c \\ 10 & + \\ 5 & + \\ 1 & - \\ 5 & - \\ 16 & - \\$$

Experimental supports for both unifications

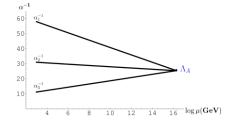
GUT is promising

Grand Unified Theories

Unification of gauge interactions quantitative evidence:

Non SUSY

SUSY GUT



• Unification of matters $(Y_u)_{ij} \mathbf{10}_i \mathbf{10}_j \mathbf{5}_H + (Y_d)_{ij} \mathbf{10}_i \overline{\mathbf{5}}_j \overline{\mathbf{5}}_H + (Y_v)_{ij} \overline{\mathbf{5}}_i \overline{\mathbf{5}}_j \mathbf{5}_H \mathbf{5}_H$ qualitative evidence:

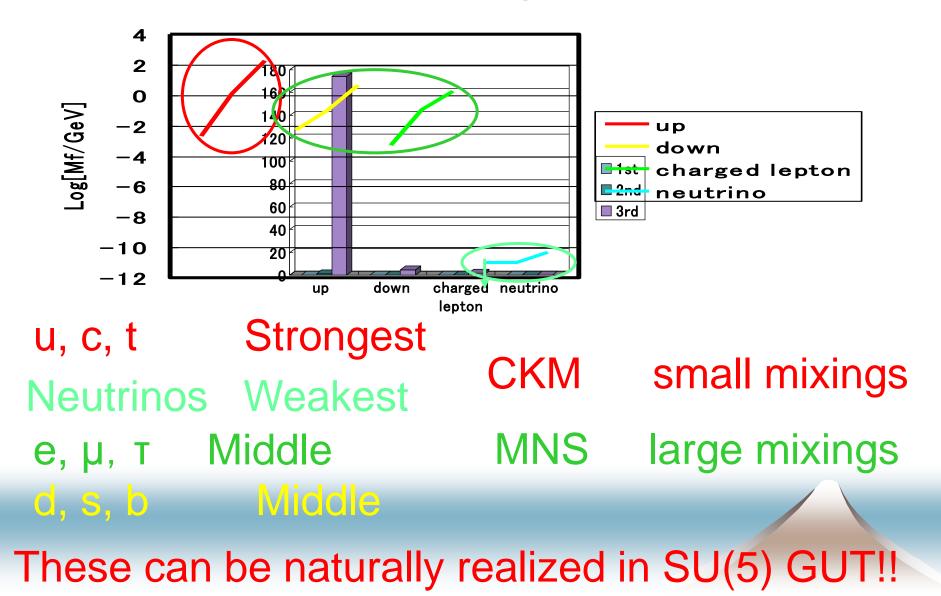
 $10_i(Q_i)$ have stronger hierarchy than $\overline{5}_i(L)$

hierarchies of masses and mixings

lepton >>quark (in hierarchies for mixings)

ups >> downs, electrons >> neutrinos (in mass hierarchies)

Masses & Mixings and GUT

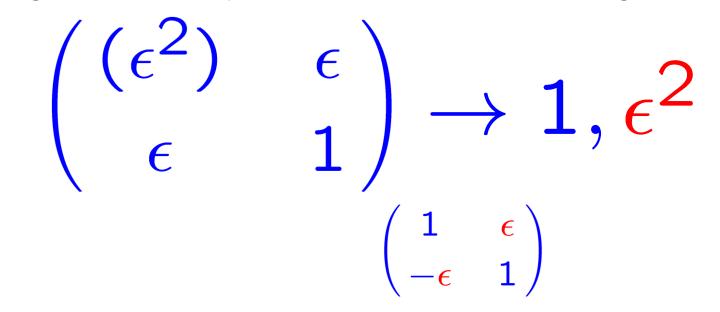


SU(5) SUSY GUT Albright-Barr Sato-Yanagida...

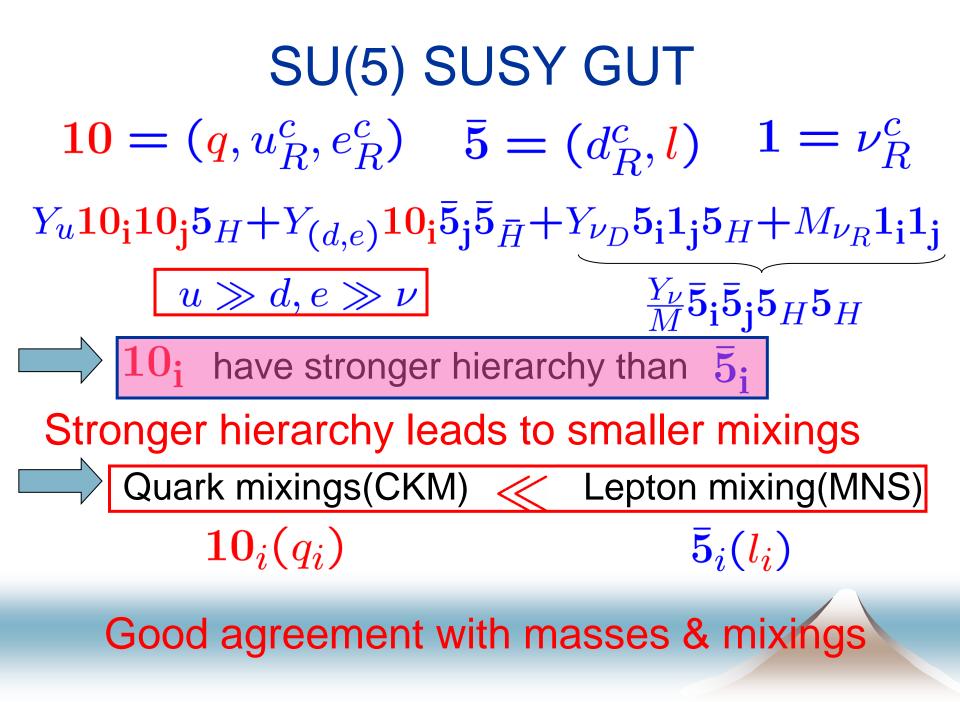
 $10 = (q, u_R^c, e_R^c) \quad \bar{5} = (d_R^c, l)$ $1 = \nu_{R}^{c}$ $Y_u \mathbf{10_i 10_j 5_H} + Y_{(d,e)} \mathbf{10_i \overline{5}_j \overline{5}_H} + Y_{\nu_D} \mathbf{\overline{5}_i 1_j 5_H} + M_{\nu_R} \mathbf{1_i 1_j}$ $u \gg d, e \gg \nu$ $\frac{Y_{\nu}}{M}\overline{\mathbf{5}}_{\mathbf{i}}\overline{\mathbf{5}}_{\mathbf{j}}\mathbf{5}_{H}\mathbf{5}_{H}$ 10_i have stronger hierarchy than 5_i Stronger hierarchy leads to smaller mixings Quark mixings(CKM) <</th>Lepton mixing(MNS) $10_{i}(q_{i})$ $\overline{5}_i(l_i)$

Mass hierarchy and mixings

Stronger hierarchy leads to smaller mixings



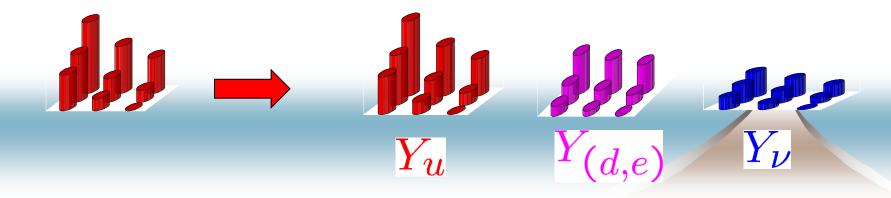
Stronger hierarchy - Smaller mixings



E₆ Grand Unified Theory

The assumption in SU(5) GUT 10_i have stronger hierarchy than $\overline{5_i}$ can be derived.

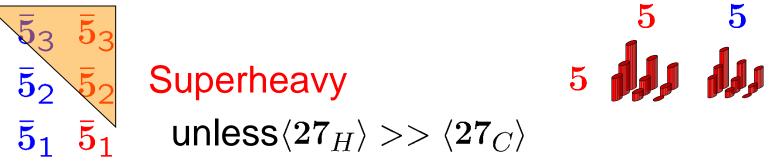
Various Yukawa hierarchies can be induced from one Yukawa hierarchy in E_6 GUT.



$$\begin{array}{ccc} E_{6} & \text{Unification} \\ \begin{array}{c} \text{Guisey-Ramond-Sikivie,} \\ \text{Aichiman-Stech, Shafi,} \\ \text{Barbieri-Nanopoulos,} \\ \text{Bando-Kugo,...} \end{array} \\ 27_{i} = \mathbf{16}_{i}[\mathbf{10}_{i} + \mathbf{\bar{5}}_{i} + \mathbf{1}_{i}] + \mathbf{10}_{i}[\mathbf{5}_{i} + \mathbf{\bar{5}}_{i}] + \mathbf{1}_{i}[\mathbf{1}_{i}] \\ (i = 1, 2, 3) & \underbrace{(\mathbf{16}_{C})} & \underbrace{(\mathbf{1}_{H})} \end{array} \\ \end{array} \\ \text{Three of six } \mathbf{\bar{5}} \text{ become superheavy after the breaking} \\ E_{6} \xrightarrow{\langle \mathbf{1}_{H} \rangle} & SO(\mathbf{10}) \xrightarrow{\langle \mathbf{16}_{C} \rangle} & SU(\mathbf{5}) \\ W = Y^{H} \mathbf{27}_{i} \mathbf{27}_{j} \langle \mathbf{27}_{H} \rangle + Y^{C} \mathbf{27}_{i} \mathbf{27}_{j} \langle \mathbf{27}_{C} \rangle \\ \text{Once we fix } Y^{H}, Y^{C}, \langle \mathbf{27}_{H} \rangle, \langle \mathbf{27}_{C} \rangle, \\ \text{three light modes of six } \mathbf{\bar{5}} \text{ are determined.} \end{array}$$

Milder hierarchy for $\overline{5}_i(l)$ Bando-N.M. 01 N.M, T. Yamashita 02

 $\bullet \overline{5}$ fields from 27_3 become superheavy.

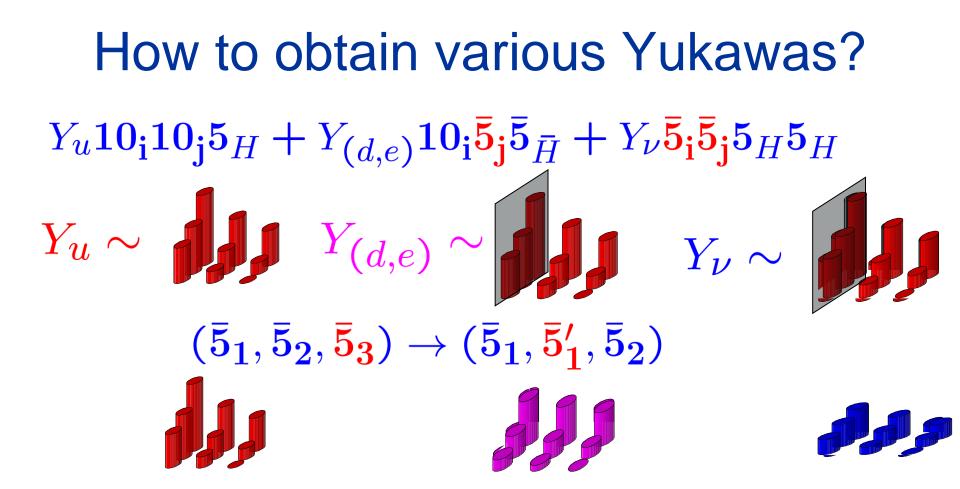


• Light modes $(\overline{5}_1, \overline{5}_1, \overline{5}_2)$ have smaller Yukawa couplings and milder hierarchy than $(10_1, 10_2, 10_3)$

 $Y_{\nu_D}, Y_d << Y_u$ •Larger mixings in lepton sector than in quark sector.

•Small $\tan\beta$

•Small neutrino Dirac masses } Suppressed radiative LFV



$$\begin{aligned} & \text{SO(10) GUT relations} \quad Y_{d} = Y_{e}^{T} = Y_{u} = Y_{\nu_{D}} \\ & \stackrel{10_{1}}{} \begin{pmatrix} 10_{2} & 10_{3} & 11 & 12 & 13 \\ \lambda^{5} & \lambda^{4} & \lambda^{2} \\ 10_{3} \begin{pmatrix} \lambda^{5} & \lambda^{4} & \lambda^{2} \\ \lambda^{3} & \lambda^{2} & 1 \end{pmatrix} \Big|_{\lambda^{2}}^{\lambda} Y_{\nu_{D}} \sim \frac{5}{5}_{1} \begin{pmatrix} \lambda^{6} & \lambda^{5} & \lambda^{3} \\ \lambda^{5.5} & \lambda^{4.5} & \lambda^{2.5} \\ \lambda^{5} & \lambda^{4} & \lambda^{2} \end{pmatrix} \Big]_{\lambda^{0.5}}^{\lambda_{0.5}} \\ & \frac{(5_{1}, 5_{1}, 5_{2})}{(5_{1}, 5_{1}, 5_{2})} & 5_{1} + \lambda^{\Delta} 5_{3} & (\Delta = 3 - r) \\ & \frac{(5_{1}, 5_{1}, 5_{2})}{(\lambda^{5} & \lambda^{4.5} & \lambda^{2} \\ 10_{3} \begin{pmatrix} \lambda^{6} & \lambda^{5.5} & \lambda^{3} \\ \lambda^{5} & \lambda^{4.5} & \lambda^{2} \\ \lambda^{3} & \lambda^{2.5} & 1 \end{pmatrix}} \Big|_{\lambda^{2}}^{\lambda} & \sum_{mail tan \beta} \\ & \sum_{\lambda^{0.5} & \lambda^{0.5}}^{\lambda^{0.5} & \lambda^{3}} \\ & V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^{3} \\ \lambda & 1 & \lambda^{2} \\ \lambda^{3} & \lambda^{2} & 1 \end{pmatrix}} & V_{MNS} \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \end{aligned}$$

Right-handed neutrinos

$$W = \frac{Y^{XY}}{\Lambda} 27_i 27_j \langle \overline{27}_X \rangle \langle \overline{27}_Y \rangle$$

$$X, Y = \overline{H}, \overline{C}$$

$$M_R = Y^{XY} \frac{\langle \overline{27}_X \rangle \langle \overline{27}_Y \rangle}{\Lambda}$$

• The same hierarchy $Y^{XY} \sim Y^H \sim Y^C$

$$\begin{split} M_{\nu} &= Y_{\nu_D} M_R^{-1} Y_{\nu_D}^T \sim \begin{pmatrix} \lambda^2 & \lambda^{1.5} & \lambda \\ \lambda^{1.5} & \lambda & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix} \frac{\langle H_u \rangle^2 \Lambda}{\langle 1_H \rangle^2} \\ \frac{\Delta m_{solar}^2}{\Delta m_{atm}^2} \sim \frac{m_{\nu_{\mu}}^2}{m_{\nu_{\tau}}^2} \sim \lambda^2 \end{split}$$
 LMA for solar neutrino problem

1st Summary

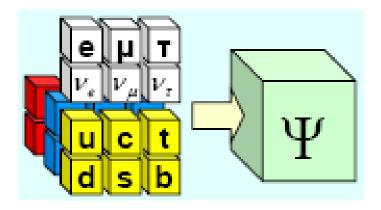
 E_6 unification explains why the lepton sector has larger mixings than the quark sector.(Large $U_{e3} \sim \lambda$)

Suppressed radiative LFV

Small Y_{ν_D} Small $\tan \beta$ A basic Yukawa hierarchy $Y \sim Y_u$ The other Yukawa hierarchies $Y_u \sim \int Y_{(d,e)} \sim \int Y_\nu \sim \int Y_\nu \sim \int$ Hierarchy of 10_i is stronger than that of $\overline{5}_i$

Three $\mathbf{5_i}$ come from the first 2 generation of $\mathbf{27_1}, \mathbf{27_2}$

Horizontal symmetry



All three generation quark and leptons can be unified into a single (or two) field(s)

By breaking the horizontal symmetry, realistic quark and lepton masses and mixings can be obtained.

Peculiar sfermion mass spectrum is predicted.

$$\widetilde{m}_{10}^{2} = \begin{pmatrix} m^{2} & . & . \\ . & m^{2} & . \\ . & . & m_{3}^{2} \end{pmatrix}, \quad \widetilde{m}_{\overline{5}}^{2} = \begin{pmatrix} m^{2} & . & . \\ . & m^{2} & . \\ . & . & m^{2} \end{pmatrix}$$

Horizontal Symmetry

Dine-Kagan-Leigh Pomarol-Tommasini Barbieri-Hall…

- •Origin of Yukawa hierarchy •Semi-universal sfermion masses to suppress FCNC $\Phi_a, \Phi_3, H_u, H_d(\Phi = Q, U, D, L, E, N)$
- The 1st 2 generation have universal sfermion masses.
- Large top Yukawa coupling

 $U(2)_{H} \longrightarrow U(1)_{H} \longrightarrow X$ $\langle \bar{F}^{a} \rangle / \Lambda \sim \epsilon \quad \langle A^{ab} \rangle / \Lambda \sim \epsilon'$ $V \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' & 0 & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 + \epsilon^{2} & \epsilon \\ 0 & \epsilon & O(1) \end{pmatrix} \tilde{m}^{2}$

 $Y_u \sim Y_d \sim Y_e \sim Y_{\nu}$? Not sufficient to suppress FCNC

Large neutrino mixings and FCNC

• The universal sfermion masses only for the 1st 2 generation $\overline{5}$ do not suppress FCNC sufficiently

Universality for all three generations is required!

E_6 unification solves these problems

• Various Yukawa hierarchies can be obtained from one basic hierarchy.

Breaking $U(2)_H \longrightarrow SU(2)_H \longrightarrow X$ $\lambda \Lambda \qquad \lambda^2 \Lambda$

gives the basic hierarchical structure.

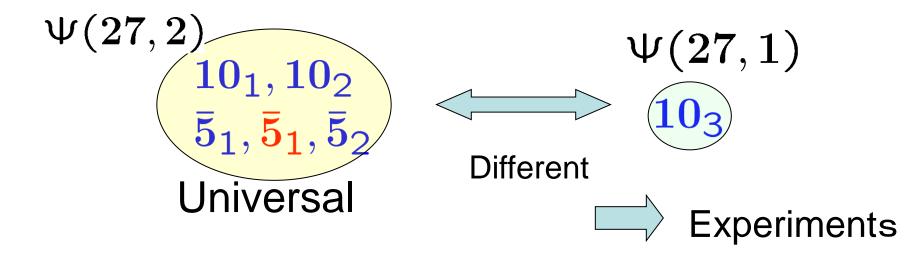
• All the three light ${ar 5}$ fields come from $\Psi(27,2)$

and therefore have universal sfermion masses.

Important for suppressing FCNC sufficiently, because the mixings of V_{Ξ} are large.

Discussion

- Extension to U(3)_H is straightforward.
 All three generation quarks and leptons are unified into a single multiplet (27, 3)
- If it is local, D term must be cared.
- Peculiar sfermion spectrum



 Any mechanisms for the basic hierarchy. Extra dimension Stringy calculation **Froggatt-Nielsen mechanism** (Anomalous U(1)) • E₆ Higgs sector (Doublet-triplet splitting) Anomalous U(1) N.M.-Yamashita 02,03 Generic interactions with O(1) coefficients. Orbifold breaking $E_6 \rightarrow SU(3)^3$

2nd Summary

- In E₆GUT, one basic hierarchy for Yukawa couplings results in various hierarchical structures for quarks and leptons including larger neutrino mixings.
- Horizontal symmetry can easily reproduce the basic hierarchy, and suppress FCNC naturally.
- The simpler unification of quarks and leptons explains the more questions.

 E_6 3 × 27 \implies larger neutrino mixings

 $E_6 \times \begin{cases} U(2)_H & \mathbf{2}(27, 2+1) \\ U(3)_H & \mathbf{1}(27, 3) \end{cases} \text{ SUSY flavor problem}$

Predictions of E6 GUT +horizontal symmetry

Flavor physics : Kim-N.M.-Matsuzaki-Sakurai-Yoshikawa 06,08

LHC physics : Kim-N.M.-Nagao-Nojiri-Sakurai 09 Sakurai-Takayama 11

 m_3 must be around the weak scale, because of the stability of the weak scale, while m can be taken larger.

10:
$$\begin{pmatrix} m^2 & m^2 \\ m^2 & m^2 \\ m^2 & m^2 \end{pmatrix}$$
 5: $\begin{pmatrix} m^2 & m^2 \\ m^2 & m^2 \\ m^2 & m^2 \end{pmatrix}$

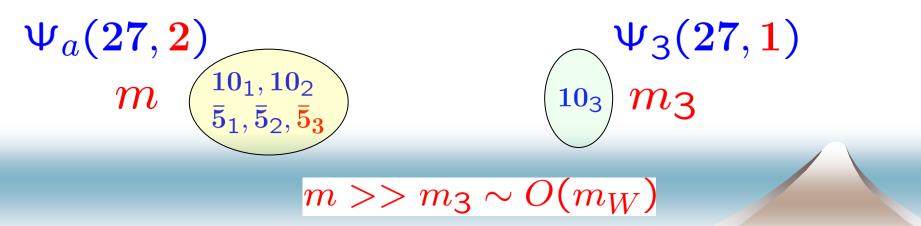
Non universal SUSY breaking

Universal sfermion masses for 5 fields $\delta_{\overline{\mathbf{5}}} \sim \delta_{d_R^c} \sim \delta_l \sim 0$ Non universality for 10 fields $\delta_{10} \sim \delta_q \sim \delta_{u_R} \sim \delta_{e_R}$ $\sim V_{CKM}^{\dagger} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_{CKM} \sim \begin{pmatrix} \lambda^{0} & \lambda^{3} & \lambda^{3} \\ \lambda^{5} & \lambda^{4} & \lambda^{2} \\ \lambda^{3} & \lambda^{2} & 1 \end{pmatrix}$ $V_q \sim V_{u_B} \sim V_{e_B} \sim V_{CKM}$

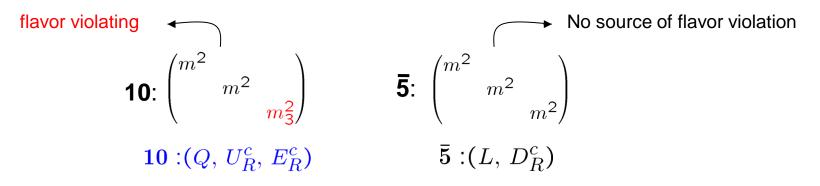
• Weak scale stability requires $m_3^2 \sim O((100 \text{GeV})^2)$ but almost no constraint for m_0

Structures suppressing FCNC for $\overline{5}_i$

- Small Yukawa couplings
- \bullet Small tan β
- Universal sfermion masses for $\overline{5}_{i}$
- *m* can increase without destabilizing the weak scale. (Effective SUSY)



How does FCNC processes take place in this model?



For example, for the right-handed charged slepton sector,

$$\tilde{e}_{R}^{\dagger} \begin{pmatrix} m^{2} & & \\ & m^{2} & \\ & & m_{3}^{2} \end{pmatrix} \tilde{e}_{R} \rightarrow \tilde{e}_{R}^{\dagger} V^{\dagger} \begin{pmatrix} m^{2} & & \\ & m^{2} & \\ & & m_{3}^{2} \end{pmatrix} V \tilde{e}_{R} = \tilde{e}_{R}^{\dagger} \tilde{m}_{\tilde{e}_{R}}^{2} \tilde{e}_{R}$$

Since 10 contains Q, the form of unitary matrix V is CKM-like. We can parametrize it with Cabibbo angle λ .

$$V \sim \begin{pmatrix} 1 & \lambda & \lambda^{\circ} \\ \lambda & 1 & \lambda^{2} \\ \lambda^{3} & \lambda^{2} & 1 \end{pmatrix} \qquad \lambda = 0.22 \qquad \qquad \Delta m^{2} = (m_{3}^{2} - m^{2})$$

$$\tilde{m}_{\tilde{e}_{R}}^{2} = \tilde{m}_{\tilde{q}_{L}}^{2} = \tilde{m}_{\tilde{u}_{R}}^{2} = V^{\dagger} \begin{pmatrix} m^{2} & m^{2} \\ m^{2} & m^{2} \\ m^{2} & m^{2} \end{pmatrix} V \sim \begin{pmatrix} m^{2} & \Delta m^{2} \lambda^{5} & \Delta m^{2} \lambda^{3} \\ \Delta m^{2} \lambda^{5} & m^{2} & \Delta m^{2} \lambda^{2} \\ \Delta m^{3} \lambda^{3} & \Delta m^{2} \lambda^{2} & m^{2} \end{pmatrix}$$

Non decoupling feature of this model (in lepton flavor violation)

$$\tilde{m}_{\tilde{e}_R}^2 \sim \begin{pmatrix} m^2 & \Delta m^2 \lambda^5 & \Delta m^2 \lambda^3 \\ \Delta m^2 \lambda^5 & m^2 & \Delta m^2 \lambda^2 \\ \Delta m^3 \lambda^3 & \Delta m^2 \lambda^2 & m_3^2 \end{pmatrix} \qquad \begin{array}{l} \lambda = 0.22 \\ \Delta m^2 \lambda^2 & \Delta m^2 \lambda^2 \\ \Delta m^2 \lambda^2 & m_3^2 \end{pmatrix} \qquad \Delta m^2 = (m_3^2 - m_3^2) \\ \end{array}$$

• By picking up the 3-2 element, the size of $\tau \rightarrow \mu$ transition rate is order λ^2

$$\tau \to \mu \gamma \qquad \qquad \tilde{\tau}_R \xrightarrow{(\tilde{m}_{\tilde{e}_R}^2)_{32}} \tilde{\mu}_R \approx \frac{1}{m_3^2} \Delta m^2 \lambda^2 \frac{1}{m^2} \longrightarrow \frac{\lambda^2}{m_3^2}$$

• For $\mu \rightarrow e\gamma$, there are two passes to change the flavor $\mu \rightarrow e$. Both they are order λ^5

$$\mu \to e\gamma \qquad \underbrace{\tilde{\mu}_R}_{\tilde{e}_R} \underbrace{(\tilde{m}_{\tilde{e}_R}^2)_{21}}_{\tilde{e}_R} \approx \frac{1}{m^2} \Delta m^2 \lambda^5 \frac{1}{m^2} \longrightarrow 0$$

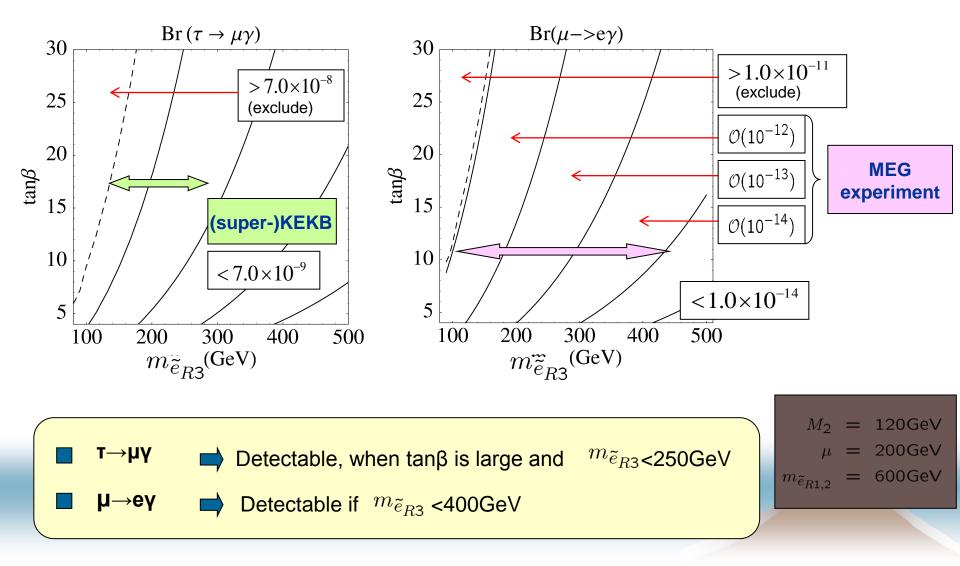
$$\underbrace{\tilde{\mu}_R}_{\tilde{\mu}_R} \underbrace{(\tilde{m}_{\tilde{e}_R}^2)_{23}}_{\tilde{\tau}_R} \underbrace{(\tilde{m}_{\tilde{e}_R}^2)_{31}}_{\tilde{e}_R} e_R \approx \frac{1}{m^2} \underbrace{(\Delta m^2)^2 \lambda^5}_{m_3^2} \frac{1}{m^2} \longrightarrow \frac{\lambda^5}{m_3^2}$$

If we raise overall SUSY scale m ...

$$m^2 \longrightarrow \infty$$

Propagator suppression from 1 or 2 generation becomes stronger, but mass difference Δm^2 increase. As a result, **both transition rate remain finite, and don't decouple!**

Can we discover the LFV at the future experiments?

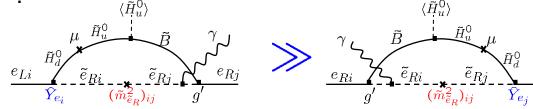


This model says that final state lepton tends to be right-handed. Final state lepton has different chirality from initial one.

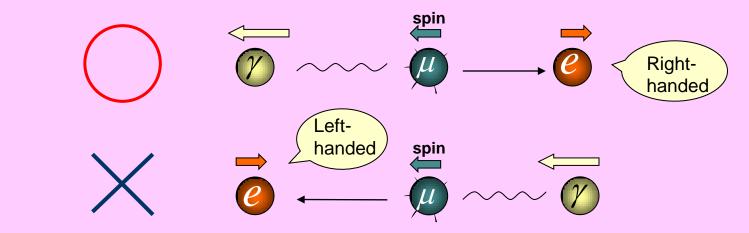
$$e_{i} \xrightarrow{p} e_{j} \xrightarrow{p'} e_{j} \quad T = \epsilon^{*\mu} \bar{u}_{i}(p) i \sigma_{\mu\nu} q^{\nu} (A_{L}^{ij} P_{L} + A_{R}^{ij} P_{R}) u_{j}(p')$$

Opposite from MSSM+ ν_R

• Intermediate state must be right-handed to pick up $\tilde{m}_{\tilde{e}_R}^2$ the .



How can we see this feature experimentally?

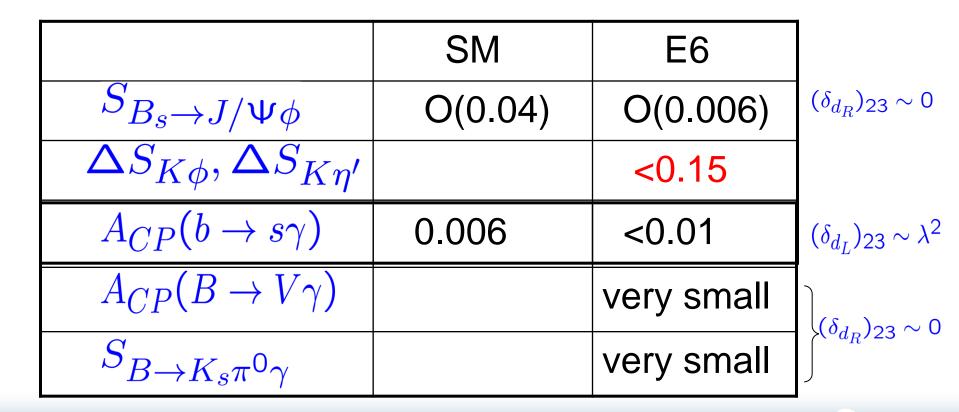


It is possible to check this feature experimentally by measuring the angular distribution of final state lepton.

Predictions (Quark sector)

- The maginitudes are $\tilde{m}_{\tilde{d}_L}^2 = \tilde{m}_{\tilde{u}_L}^2 = \tilde{m}_{\tilde{u}_R}^2 \sim \begin{pmatrix} m^2 & \Delta m^2 \lambda^5 & \Delta m^2 \lambda^3 \\ \Delta m^2 \lambda^5 & m^2 & \Delta m^2 \lambda^2 \\ \Delta m^3 \lambda^3 & \Delta m^2 \lambda^2 & m_3^2 \end{pmatrix}$
 - the RGE effects in the universal mass case.
- New CP phases!!
 - The CP violation in B meson system may be detectable

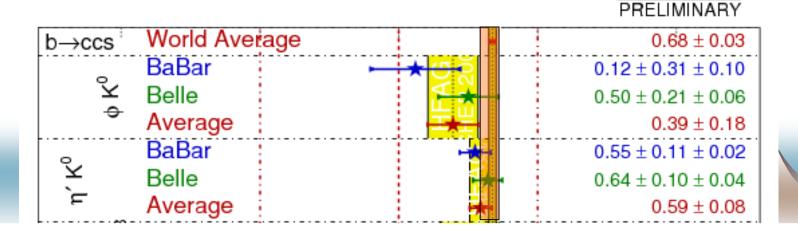
CP violation in B meson



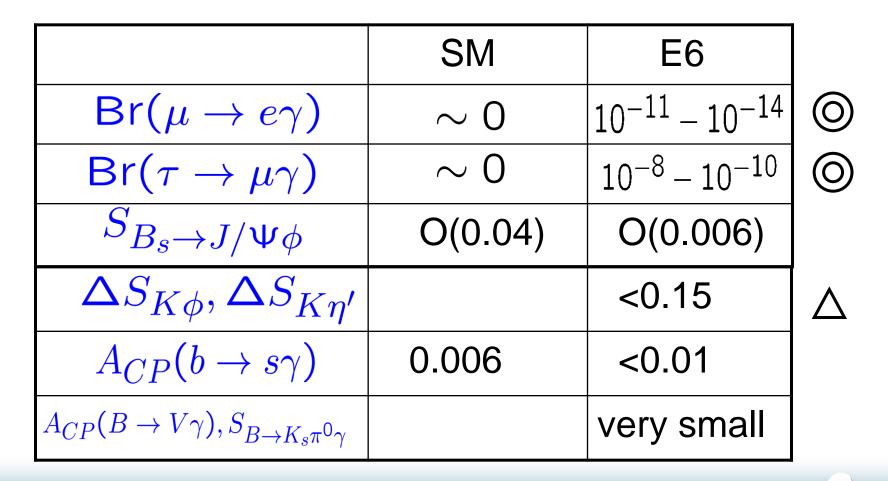
For $\tan\beta \sim 10$

 $B_d \rightarrow \phi Ks, \eta' Ks$

 $\Delta S_{\phi K_s}^{SUSY}, \Delta S_{\eta'K_s}^{SUSY} \sim O(0.1) \text{ is possible.}$ Gluino contribution is decoupled. Chargino contribution is not decoupled. in the limit $m >> m_3$ O(0.1) deviation in B factory may be confirmed in SuperB factory. $\sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}})$



Summary table of E6 predictions



Discussions

- Strictly speaking, $\delta_{\overline{5}} \neq 0$ $\delta_{LR} \neq 0$ when $U(2)_H$ e.g. $\delta_{\overline{5}} \sim \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^{3.5} \\ \lambda^4 & \lambda^3 & \lambda^{2.5} \\ \lambda^{3.5} & \lambda^{2.5} & \lambda^2 \end{pmatrix}$
 - This can be consistent with the experiments, but the predictions can be changed. If we take $m_0 >> m_3$, this model dependent parts can be neglected. No weak scale instability!!

LHC signatures

 $m_0 >> m_3 \sim M_{1/2}$ Kim-N.M.-Nagao-Nojiri-Sakurai 09 Sakurai-Takayama 11

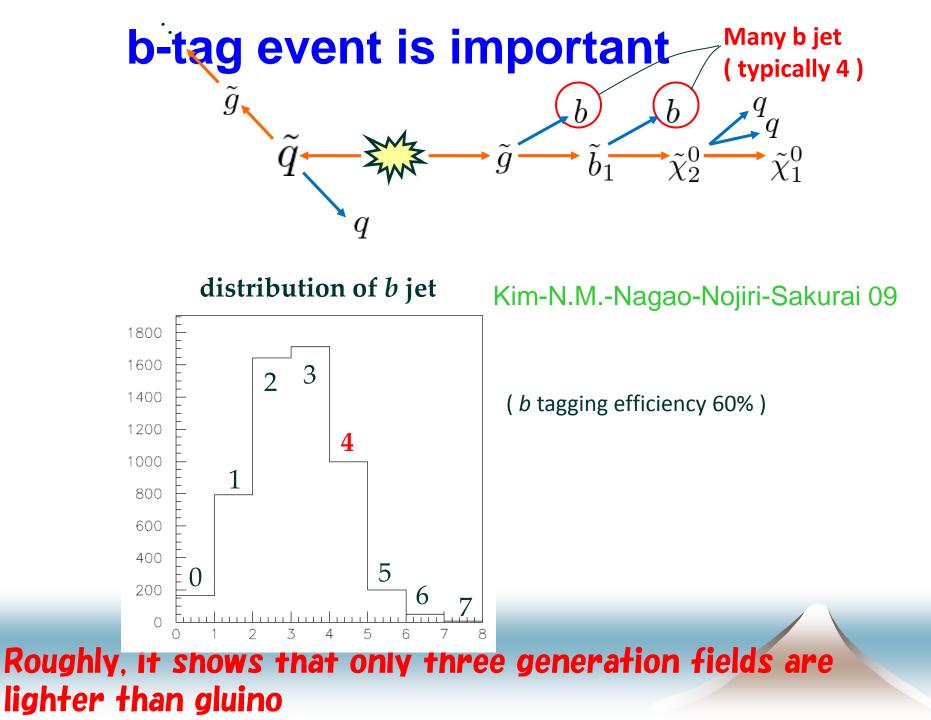
stop, sbottom, gluino....light the other squark..... heavy

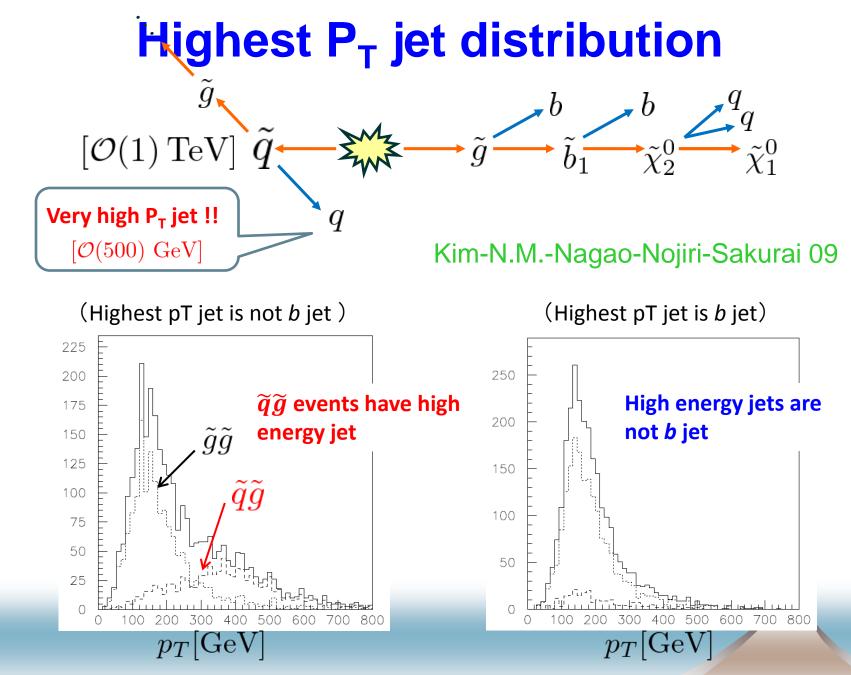
production rate becomes lower

$$p + p \rightarrow \tilde{g} + \tilde{g}, \tilde{g} + \tilde{q}, \tilde{q} + \tilde{\tilde{q}}$$

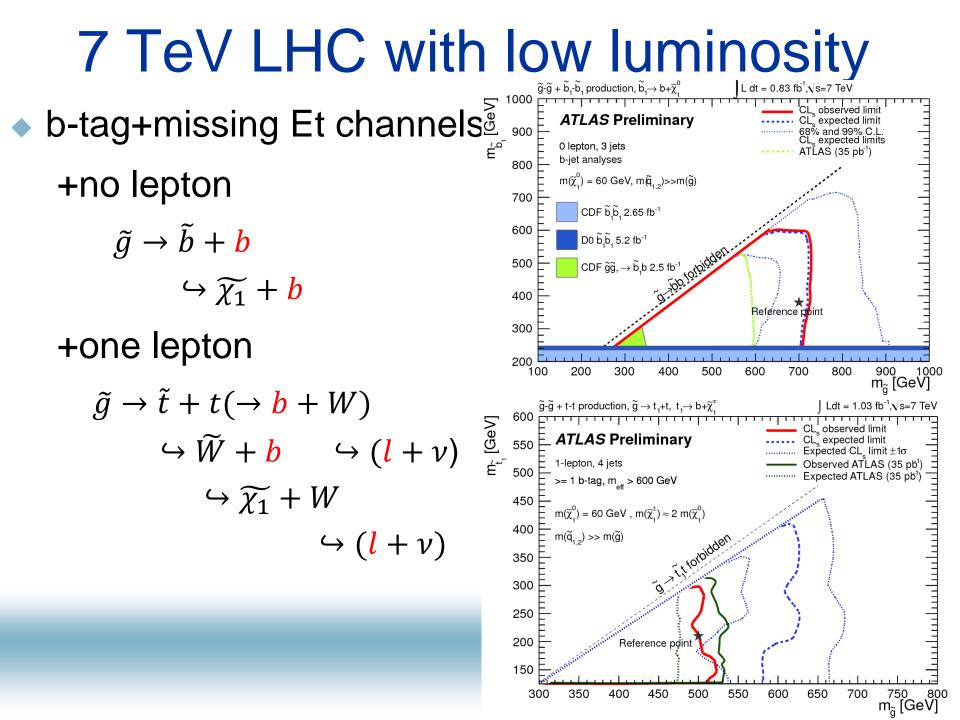
Quark jets becomes softer.

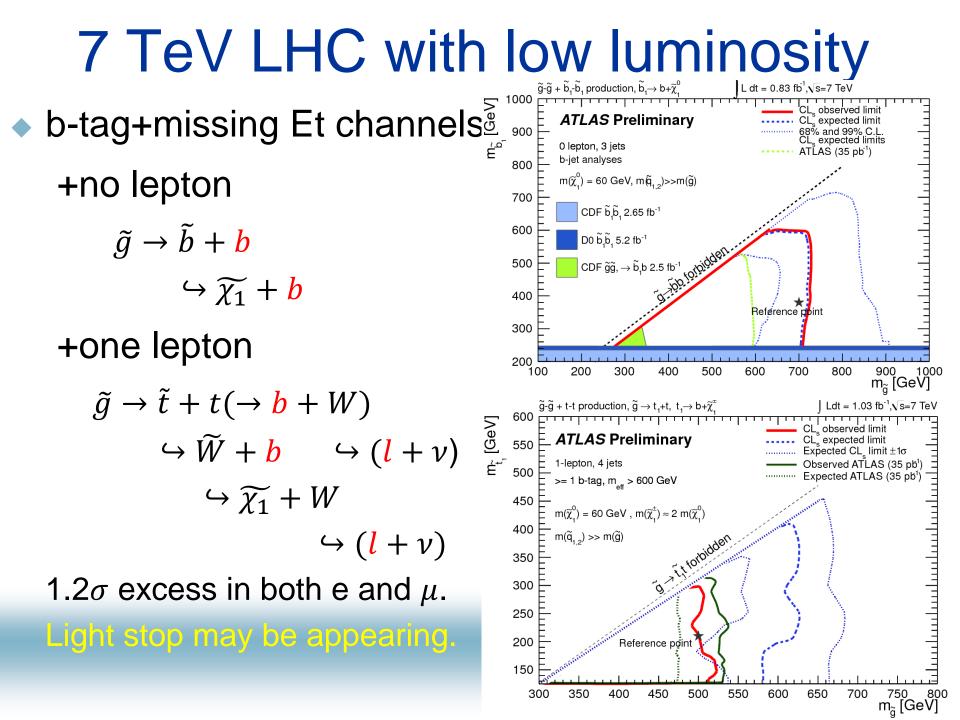
$$\begin{split} \widetilde{g} &\to \widetilde{t} + t (\to b + W) & \widetilde{g} \to \widetilde{b} + b \\ & \hookrightarrow \widetilde{W} + b & \hookrightarrow \widetilde{\chi_{1(2)}} + b \\ & \hookrightarrow \widetilde{\chi_{1}} + W & (\hookrightarrow \widetilde{\chi_{1}} + Z) \end{split}$$





It shows that the first generation squark is much heavier than gluino,





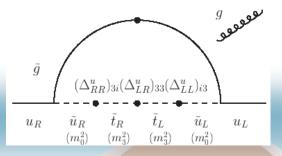
Spontaneous CP violation

Old and new type SUSY CP problems can be solved and several bonuses

SUSY CP Problem • EDM constraints => Real SUSY parameters. $\phi_{M_{1/2}}, \phi_{\mu}, \phi_{B}, \phi_{A} < 10^{-(2-3)}$ μ problem is solved by anomalous U(1) Complex Yukawa couplings => CEDM New type Hisano-Shimizu04 $\operatorname{Im}((\delta^{u}_{LL})_{13}(\delta^{u}_{RR})_{31})) < 3 \times 10^{-7}$ Griffith et.al.09 $(\delta^u_{LL})_{13}(\delta^u_{RR})_{31} \sim \lambda^6 \sim 10^{-4}$ in E6 GUT with SU(2) $\tilde{m}_{10}^{2} = \begin{pmatrix} m^{2} \\ m^{2} \\ m_{3}^{2} \end{pmatrix} \qquad \delta_{LL}^{u} \equiv V_{10} \begin{pmatrix} \tilde{m}_{10}^{2} \\ m^{2} \end{pmatrix} V_{10}^{\dagger} \sim \begin{pmatrix} 1 & \lambda^{5} & \lambda^{3} \\ \lambda^{5} & 1 & \lambda^{2} \\ \lambda^{3} & \lambda^{2} & O(1) \end{pmatrix} \\ \lambda \sim 0.22$ Complex Yukawa => $V_{10} \sim V_{CKM}$ is complex generically Additional (discrete) symmetry solves both problems

Decoupling features of SUSY CP problem EDM constraints from 1 loop $\mu = |\mu|e^{i\delta_{\mu}}, A = |A|e^{i\delta_{A}}$ $\delta_{\mu,A} < 10^{-(2-3)} \left(\frac{M_{SUSY}}{100 \,{\rm GeV}}\right)^2$ • CEDM from Hg(neutron) even if $\delta_{\mu,A} = 0$ Hisano-Shimizu '04 Im $(\delta_{d_L})_{13}(\delta_{d_R})_{31} < 8(24) \times 10^{-6}$ $\lambda^6 \sim 10^{-4}$ Im $(\delta_{u_L})_{13}(\delta_{u_R})_{31} < 3(9) \times 10^{-7}$

Contributions through stop loop are not decoupled. Complex Yukawa couplings induce them generically.



Spontaneous CP violatoin in SU(2) model

 Doublets under SU(2) horizontal symmetry $\begin{array}{ll} \mathbf{27}_{a}, F_{a}, \bar{F}^{a} & \langle \bar{F} \rangle \sim \begin{pmatrix} 0 \\ \bar{v} \end{pmatrix}, & \langle F \rangle \sim \begin{pmatrix} 0 \\ v e^{i\rho} \end{pmatrix} \\ \mathbf{27}_{1} \sim \epsilon^{ab} \mathbf{27}_{a} \langle F_{b} \rangle & \mathbf{27}_{2} \sim \mathbf{27}_{a} \langle \bar{F}^{a} \rangle & \mathbf{27}_{3} \end{array}$ $W = (Y_H)_{ij} 27_i 27_j 27_H + (Y_C)_{ij} 27_i 27_j 27_C$ $\epsilon^{ab} \mathbf{27}_a \mathbf{78}_A \mathbf{27}_b \mathbf{27}_H (\epsilon^{ab} \mathbf{27}_a \mathbf{27}_b \mathbf{27}_H) \quad \langle \mathbf{78}_A \rangle = \mathbf{Q}_{B-L} \mathbf{V}$ $27_H = 16_H + 10_H + \langle 1_H \rangle \quad E_6 \to SO(10)$ $\mathbf{27}_C = \langle \mathbf{16}_H \rangle + \mathbf{10}_C + \mathbf{1}_C \quad SO(10) \to SU(5)$ $E_6 \text{ Higgs sector} \Rightarrow \begin{cases} H_u \sim 10_H \rightarrow Y_u = Y_H \\ (H_d \sim 10_H + 16_C) \end{cases}$ Real μ and B require non-trivial discrete charge for F and 27_C .

A solution for μ problem

- Negative Higgs charges=> massless Higgs SUSY(holomorphic) zero mechanism
- SUSY breaking induces the non-vanishing VEV of superheavy positive charged singlet

 $W = SH_{u}H_{d} + \Lambda^{2}S + \Lambda SZ$ $V_{SB} = A_{SHH}SH_{u}H_{d} + A_{S}\Lambda^{2}S + \dots$ $\langle S \rangle \sim \frac{A_{S}\Lambda^{2}}{M_{S}^{2}} \sim A_{S} \qquad \mu \sim A_{S} \sim O(\tilde{m})$ $B\mu \sim A_{SHH}\langle S \rangle + F_{S} \sim O(\tilde{m}^{2})$

Additional discrete symmetry $W = SH_{\mu}H_{d} + \Lambda^{2}(1 + \overline{F}F)S + \Lambda SZ$ $V_{SB} = A_{SHH}SH_uH_d + A_S\Lambda^2S + \dots$ $\langle S \rangle \sim \frac{A_S \Lambda^2}{M_S^2} \sim A_S$ Complex $\mu \sim A_S \sim O(\tilde{m})$ $B\mu \sim A_{SHH} \langle S \rangle + F_S \sim O(\tilde{m}^2)$

Non trivial discrete charge for $\overline{F}F$ to forbid $S\overline{F}F$

What happens by the discrete symmetry? $W = (Y_H)_{ij} 27_i 27_j 27_H + (Y_C)_{ij} 27_i 27_j 27_C$ $Y_{H} = \begin{pmatrix} \lambda^{6} & \lambda^{5} & \lambda^{3} \\ \lambda^{5} & \lambda^{4} & \lambda^{2} \\ \lambda^{3} & \lambda^{2} & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & Q_{B-L}\lambda^{5} & 0 \\ Q_{B-L}\lambda^{5} & \lambda^{4} & \lambda^{2} \\ \lambda^{2} & \lambda^{2} & 1 \end{pmatrix} \text{ real}$ $\epsilon^{ab} 27_a 78_A 27_b 27_H \qquad \langle 78_A \rangle = \mathbf{Q}_{B-L} \mathbf{V}$ $Y_C = \begin{pmatrix} \lambda^0 & \lambda^3 & \lambda^3 \\ \lambda^5 & 0 & 0 \\ \lambda^3 & 0 & 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & \lambda^5 & \lambda^3 \\ \lambda^5 & 0 & 0 \\ \lambda^3 & 0 & 0 \end{pmatrix} \text{ complex}$ SUSY zero mechanism $\Rightarrow Y_u$ is real , Y_d is complex $\langle ar{F}
angle \sim \left(egin{array}{c} 0 \ \overline{v} \end{array}
ight), \quad \langle F
angle \sim \left(egin{array}{c} 0 \ v e^{i
ho} \end{array}
ight)$

A model with a discrete symmetry Ishiduki-Kim-N.M.-Sakurai09 (Z_6) Bonus 1 • Real up-type Yukawa couplings \Rightarrow real $\delta_{u_L}, \delta_{u_R}$ CEDM constraints can be satisfied. Complex down-type Yukawa couplings KM phase can be induced. $\langle F_a \rangle \sim \left(\begin{array}{c} 0 \\ v e^{i\delta} \end{array} ight)$ The point $Y^H(\overline{F}, \overline{F})$: real, $H_u \sim 10_H$ $Y_{u} = Y^{H}$ $Y^C(F, \overline{F})$: complex $H_d \sim 10_H + 16_C$ $W = Y^H 27_i 27_j \langle 27_H \rangle + Y^C 27_i 27_j \langle 27_C \rangle$

A model with a discrete symmetry

• Bonus 2: small up quark mass is realized. Usually, to obtain the CKM matrix $V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$

$$Y_{u} = \begin{pmatrix} \lambda^{6} & \lambda^{5} & \lambda^{3} \\ \lambda^{5} & \lambda^{4} & \lambda^{2} \\ \lambda^{3} & \lambda^{2} & 1 \end{pmatrix} \xrightarrow{0} \begin{pmatrix} 0 & Q_{B-L}\lambda^{5} & 0 \\ \lambda^{4} & \lambda^{2} \\ 0 & \lambda^{2} & 1 \end{pmatrix}$$

$$\underbrace{\epsilon^{ab} \Psi_{a} \Psi_{b} H}_{\langle \epsilon^{ab} \Psi_{b} \land \Psi_{b} H}_{\langle \epsilon^{ab} F_{a} \rangle \Psi_{b} \sim \Psi_{1}} \langle A \rangle \propto Q_{B-L}$$

$$y_{u} \sim \lambda^{6} \qquad \Longrightarrow \qquad (\frac{1}{3})^{2} \lambda^{6}$$
Too large \rightarrow good value!

A model with a discrete symmetry

Bonus 3?: # of O(1) parameters =9-12

13 physical parameters

 $\implies m_u, m_d, m_e, V_{CKM}$

One of the relations

 $m_b = m_\tau (1 + O(\lambda))$

$$\left(egin{array}{l} m_s = O(1) m_\mu \ m_d = O(1) m_e \end{array}
ight)$$

Interesting result(perturbation in λ) • $V_{ub} \sim \lambda^4$ is obtained $V_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^4 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$ $V_{CKM} = \begin{pmatrix} 1 & \lambda & 4\lambda^3(\rho - i\eta) \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda & 1 \end{pmatrix} \sim \lambda^4$ $A \sim 0.8, \rho, \eta \sim 0.2 - 0.4$

This cancellation depends on the adjoint VEV. Kawase, N.M 10 (B factory measured the direction of GUT breaking?) $E_6 \rightarrow SO(10) \times U(1)_{V'}$ $E_6 \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{V'}$ $E_6 \rightarrow SU(4) \times SU(2)_L \times U(1)_R \times U(1)_{V'}$ $* |Y_bV_{cb}| = |Y_c| \rightarrow \tan \beta \sim 6$

Numerical calculation

Kim-N.M.-Sakurai09 O(1) coefficients(10 parameters) $a = 0.6, b = -0.5, c = -0.7, d_5 = -0.9$ $d_q = 0.4, d_l = -0.5, f = 1.5, g = -0.9$ $\beta_H = 0.9, \delta = 1.4$ $Y_t = 6(5) \times 10^{-1}$ $Y_h = 2(3) \times 10^{-2}$ $Y_{\tau} = 3(4) \times 10^{-2}$ $Y_c = 3(1) \times 10^{-3}$ $Y_s = 5(6) \times 10^{-4}$ $Y_{\mu} = 1(3) \times 10^{-3}$ $Y_u = 4(3) \times 10^{-6}$ $Y_d = 8(3) \times 10^{-5}$ $Y_e = 3(1) \times 10^{-5}$ $|V_{CKM}| = \begin{pmatrix} 1 & 2(2) \times 10^{-1} & 2(4) \times 10^{-3} \\ 2(2) \times 10^{-1} & 1 & 10(4) \times 10^{-2} \\ 20(7) \times 10^{-3} & 10(4) \times 10^{-2} & 1 \end{pmatrix}$ $J_{CP} = 1(3) \times 10^{-5}$ Ref:Ross-Serna07 MSSM 2loop RGE

Heavy fields' contribution must be taken into account.

E6 Higgs sector

N.M.-Yamashita02 Ishiduki-Kim-N.M.-Sakurai 09

• E6 can be broken into the SM gauge group. 1, Natural doublet-triplet splitting 2, $H_u \sim 10_H$, $H_d \sim 10_H + 16_C$ is realized

3, consistent with the discrete symmetry

 $ar{F}^a$ H \overline{H} C \overline{C} C' $\overline{C'}$ A' Ψ_a Ψ_3 A F_{a} E_6 78 $\begin{bmatrix} 2 & 1 & 2 & \overline{2} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & \frac{3}{2} & -\frac{3}{2} & -\frac{5}{2} & -3 & 2 & -4 & 0 & 7 & 9 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 5 & 0 & 3 & 3 & 3 \end{bmatrix}$ $SU(2)_H$ 1 $U(1)_A$ 9 -1 4 3 Z_6

Neutrino sector

N.M.-Takayama 11

• $\overline{5}$ fields at the low energy are determined as 3 massless modes of this 3×6 matrix.

- For lepton doublets, $\overline{5}_1'$ of three massless modes, $(\overline{5}_1, \overline{5}_1', \overline{5}_2)$, has no mixing with $\overline{5}_3$, because $Q_{B-L} = 0$ for lepton doublets.
- Yukawa couplings of charged lepton in $\overline{5}_1'$ can be obtained through $Y_c 16_i 10_1 16_c$
- Dirac neutrino Yukawa couplings of $\overline{5}_1'$ through $Y_H 1_i 10_1 10_H$ is vanishing because $Q_{B-L} = 0$. $\rightarrow m_{\nu_{\mu}} = 0$ That's a problem.
- Fortunately, we have higher dimensional interactions

$$\epsilon^{ab}(\overline{27}_{\overline{H}}27_a)(27_b27_H27_H) \longrightarrow 1_210_110_H$$

We have many parameters on the right-handed neutrino masses.

Predictions on neutrino masses and mixings are the same as the usual E_6 GUT.

• $(\overline{27}_{\overline{H}}27_{H})$ can couple with many interactions, but the effects are only for the neutrino sector in the model.

Discussion & Summary

- KM theory is naturally realized by spontaneous CP violation in E6 GUT with horizontal symmetry
 - 1, Real μ and B parameters are realized by introducing a discrete symmetry.
 - 2, The symmetry solves CEDM problem. $(Y_u)_{ij}$:real $(Y_d)_{ij}$:complex
 - 3, Predicted EDM induced by RGE effect is sizable.
 - 4, $V_{ub} \sim \lambda^3 \rightarrow \lambda^4$, $|Y_c V_{bc}| = |Y_b| \rightarrow \tan \beta \sim 6$ 5, $Y_u \sim \lambda^6 \rightarrow \left(\frac{1}{3}\right)^2 \lambda^6$
 - E6 Higgs sector consistent with the above scenario with natural realization of D-T splitting