

The simplest Extension of the Standard Model



Alexander Lenz (TU München)

in collaboration with Markus Bobrowski, Otto Eberhardt (KIT Karlsruhe), Johann Riedl, Jürgen Rohrwild (RWTH Aachen)

MPIK Heidelberg, June 20, 2011

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- Flavor constraints
- Electro-weak constraints
- Results

Introduction

- What is the fourth family
- Dichtung und Wahrheit
 - ▶ History
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 - ▶ Arguments in favour of the fourth family
- Quark mixing
- Fitting the SM4 parameters

What is the new family?

Another sequential generation of fermions

Leptons: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}, \begin{pmatrix} \nu_4 \\ \ell_4 \end{pmatrix}$

Quarks: $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix}, \begin{pmatrix} t' \\ b' \end{pmatrix}$

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- **Resurrection:** Kribs et al 2007; Vysotsky, Okun, et al. 2000-...
Electro-weak Observables do not exclude a fourth family!

$$m_{t'} - m_{b'} = \left(1 - \frac{1}{5} \ln \frac{m_H}{115 \text{ GeV}} \right) \cdot 50 \text{ GeV}$$

Dichtung und Wahrheit

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- **Precision analyses:**

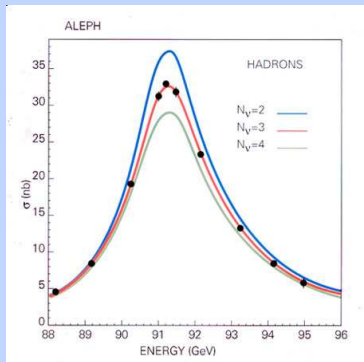
- ▶ Scans over the CKM4 parameter regions
0902.4883: Bobrowski, A.L., Riedl, Rohrwild - very large effects
0904.3570: Chanowitz - S, T, U shrinks the allowed region
1002.0595: Soni, ... - more Observables also 1011.6091
1002.2126: Buras et al. also 1004.4565 Charm; 1006.5356 Leptons
1005.3505: Eberhardt, A.L., Rohrwild - full CKM-dependence of S, T, U
- ▶ Complete Fit: with H. Lacker (CKMfitter) and U. Nierste
Kick-off Workshop: 13.10.2010 TU Dortmund
see also: 1011.2634 : Alok, Dighe, London

Main arguments against the fourth family

Number of light neutrinos from LEP

$$N_\nu = 2.9840 \pm 0.0082$$

[LEP '06]



Main arguments against the fourth family

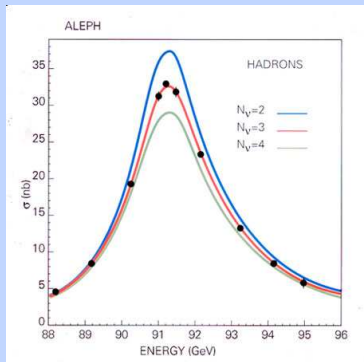
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But neutrinos do have a mass.

[Super-Kamiokande '98]



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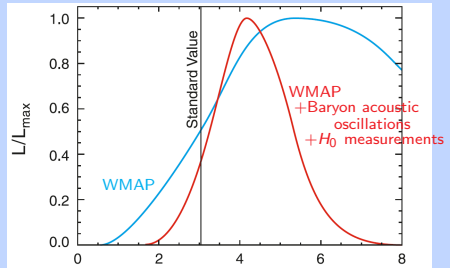
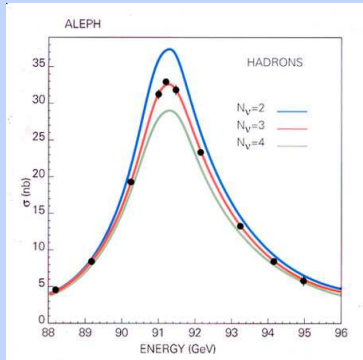
[LEP '06]

But neutrinos do have a mass.

[Super-Kamiokande '98]

$$\text{Astronomy: } N_{\nu}^{\text{eff}} = 4.34^{+0.86}_{-0.88}$$

[7y WMAP '10]



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- 2002: “an extra generation of ordinary fermions is excluded **at the 99.8% CL on the basis of the S parameter alone**. [...] This result assumes [...] that any new families are **degenerate**. This restriction can be relaxed [...] to 95%.”

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- 2002: “an extra generation of ordinary fermions is excluded **at the 99.8% CL on the basis of the S parameter alone**. [...] This result assumes [...] that any new families are **degenerate**. This restriction can be relaxed [...] to 95%.”
- 2010: “an extra generation of ordinary fermions is excluded **at the 6σ level** on the S parameter alone. This result assumes [...] that any new families are degenerate. [...] a fourth family is **disfavored but not excluded** by current data.”

[Erler/Langacker]

Some arguments in favour of the fourth family

- Softening of the Higgs mass bounds
[Novikov et al. '02,'09, Frere et al. '04, Kribs et al. '07]

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- Unification of the gauge couplings
[Hung '97]

Quark mixing

The CKM matrix

In the electroweak Lagrangian, the unitary transformation

$$q'_L = U_L^q q_L \quad \text{and} \quad q'_R = U_R^q q_R$$

from electroweak to mass eigenstates vanishes in all terms, except for:

$$\frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_L^i \gamma^\mu d_L^i + \text{h.c.}$$

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$$\frac{g}{\sqrt{2}} W_\mu^+ \bar{u}_L^i \gamma^\mu \underbrace{\left((U_L^u)^\dagger U_L^d \right)}_{V_{\text{CKM}}}{}^{ij} d_L^j + \text{h.c.}$$

Quark mixing

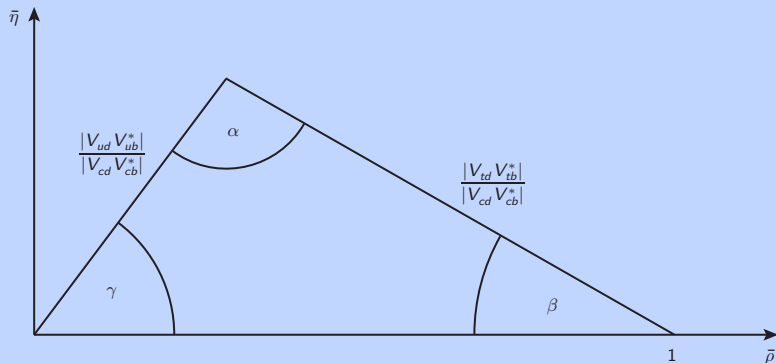
The CKM matrix

$$\begin{aligned} V^{\text{CKM3}} &\equiv \begin{pmatrix} V_{ud}^{\text{CKM3}} & V_{us}^{\text{CKM3}} & V_{ub}^{\text{CKM3}} \\ V_{cd}^{\text{CKM3}} & V_{cs}^{\text{CKM3}} & V_{cb}^{\text{CKM3}} \\ V_{td}^{\text{CKM3}} & V_{ts}^{\text{CKM3}} & V_{tb}^{\text{CKM3}} \end{pmatrix} & c_{ij} \equiv \cos \theta_{ij} \\ & & s_{ij} \equiv \sin \theta_{ij} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \end{aligned}$$

The unitarity triangle

The Wolfenstein parametrisation

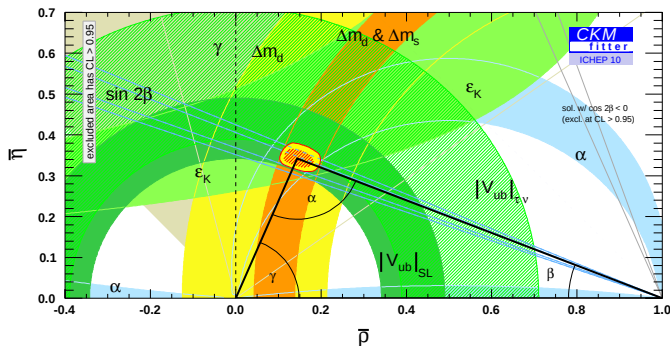
$$V^{\text{CKM3}} \approx \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



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Notation

The 4×4 CKM matrix

$$V^{\text{CKM}4} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & s_{24}e^{-i\delta_{24}} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24}e^{i\delta_{24}} & 0 & c_{24} \end{pmatrix} \cdot \begin{pmatrix} c_{14} & 0 & 0 & s_{14}e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix} \begin{pmatrix} & 0 \\ & 0 \\ & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} V^{\text{CKM}3}$$

$$c_{ij} \equiv \cos \theta_{ij}$$

$$s_{ij} \equiv \sin \theta_{ij}$$

Notation

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$$V^{\text{CKM}4} = \begin{pmatrix} c_{12}c_{13}c_{14} & c_{13}c_{14}s_{12} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ -c_{23}c_{24}s_{12} & c_{12}c_{23}c_{24} & c_{13}c_{24}s_{23} & c_{14}s_{24}e^{-i\delta_{24}} \\ -c_{12}c_{24}s_{13}s_{23}e^{i\delta_{13}} & -c_{24}s_{12}s_{13}s_{23}e^{i\delta_{13}} & -s_{13}s_{14}s_{24}e^{-i(\delta_{13}+\delta_{24}-\delta_{14})} & \\ -c_{12}c_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} & -c_{13}s_{12}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} & & \\ -c_{12}c_{23}c_{34}s_{13}e^{i\delta_{13}} & -c_{12}c_{34}s_{23} & c_{13}c_{23}c_{34} & c_{14}c_{24}s_{34} \\ +c_{34}s_{12}s_{23} & -c_{23}c_{34}s_{12}s_{13}e^{i\delta_{13}} & -c_{13}s_{23}s_{24}s_{34}e^{i\delta_{24}} & \\ -c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} & -c_{12}c_{23}s_{24}s_{34}e^{i\delta_{24}} & -c_{24}s_{13}s_{14}s_{34}e^{i(\delta_{14}-\delta_{13})} & \\ +c_{23}s_{12}s_{24}s_{34}e^{i\delta_{24}} & -c_{13}c_{24}s_{12}s_{14}s_{34}e^{i\delta_{14}} & & \\ +c_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} & +s_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} & & \\ -c_{12}c_{13}c_{24}c_{34}s_{14}e^{i\delta_{14}} & -c_{12}c_{23}c_{34}s_{24}e^{i\delta_{24}} & -c_{13}c_{23}s_{34} & c_{14}c_{24}c_{34} \\ +c_{12}c_{23}s_{13}s_{34}e^{i\delta_{13}} & +c_{12}s_{23}s_{34} & -c_{13}c_{34}s_{23}s_{24}e^{i\delta_{24}} & \\ +c_{23}c_{34}s_{12}s_{24}e^{i\delta_{24}} & -c_{13}c_{24}c_{34}s_{12}s_{14}e^{i\delta_{14}} & -c_{24}c_{34}s_{13}s_{14}e^{i(\delta_{14}-\delta_{13})} & \\ -s_{12}s_{23}s_{34} & +c_{23}s_{12}s_{13}s_{34}e^{i\delta_{13}} & & \\ +c_{12}c_{34}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} & +c_{34}s_{12}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} & & \end{pmatrix}$$

$$c_{ij} \equiv \cos \theta_{ij}$$

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Fitting the SM4 parameters

The “old” parameters

Quarks:	$m_u, m_d, m_s, m_c, m_b, m_t,$	$\theta_{12}, \theta_{13}, \theta_{23},$	δ_{13}
Leptons:	$m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau}, m_e, m_\mu, m_\tau,$	$\theta_{12}^\ell, \theta_{13}^\ell, \theta_{23}^\ell,$	δ_{13}^ℓ

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Quarks: $m_{b'}, m_{t'},$ $\theta_{14}, \theta_{24}, \theta_{34},$ δ_{14}, δ_{24}

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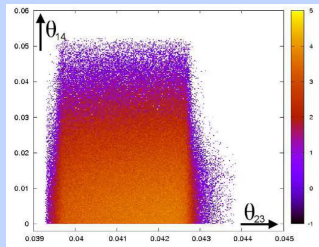
Leptons: $m_{\nu_4}, m_{\ell_4},$ $\theta_{14}^\ell, \theta_{24}^\ell, \theta_{34}^\ell,$ $\delta_{14}^\ell, \delta_{24}^\ell$

13 free parameters

Fitting the SM4 parameters

Our two approaches

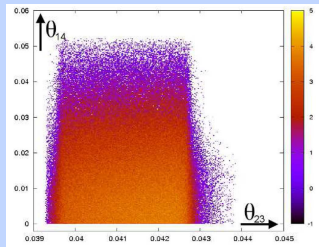
Scatter plots



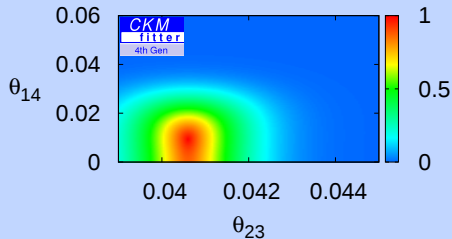
Fitting the SM4 parameters

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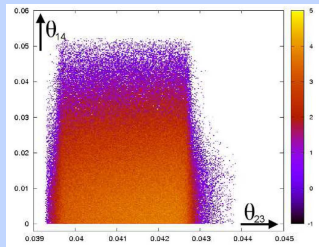
CKMfitter plots



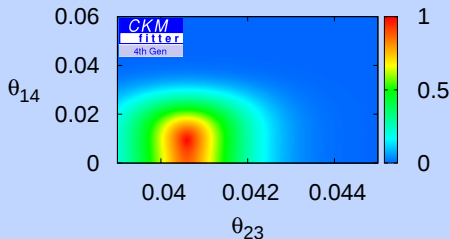
Fitting the SM4 parameters

Our two approaches

Scatter plots



CKMfitter plots



- Scatter: Create 10^{10} parameter points for the SM4: test if bounds from V_{CKM} , FCNC and electro-weak precision observables are fulfilled
- Fit: Full combined CKM-, (PMNS)- and electro-weak fit - [Work in progress](#)

Constraints on the SM4

Direct constraints

- Mass constraints (quarks, leptons, Higgs)
- Direct measurements of CKM elements
- Phase constraints on the CKM matrix

Constraints on the SM4

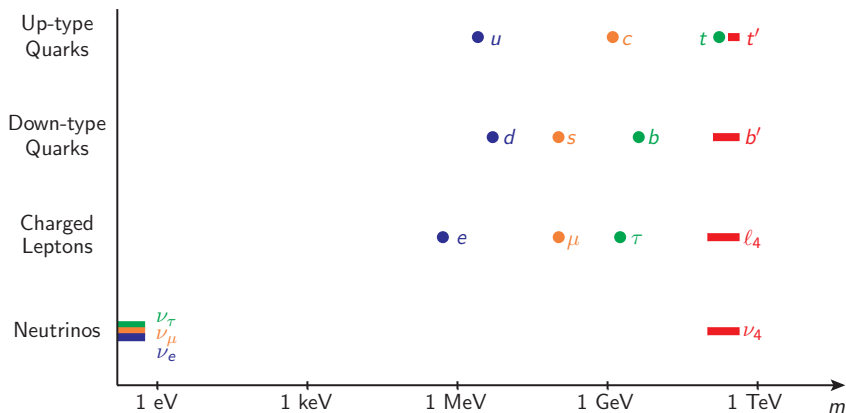
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Indirect constraints

- FCNC
- Lepton observables
- Electro-weak precision observables

Direct mass limits



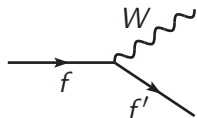
$m_{q'} > \mathcal{O}(300\text{GeV})$: D0, CDF, CMS

$m_{\nu_4} > \mathcal{O}(50\text{GeV})$: LEP

$m_{l_4} > \mathcal{O}(100\text{GeV})$: LEP

$m_H \notin [137\text{ GeV}, 207\text{ GeV}]$: D0, CDF, CMS

Tree level constraints



$$V_{CKM4} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}$$

Tree level constraints

$$\beta \text{ decays} \longrightarrow \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}$$

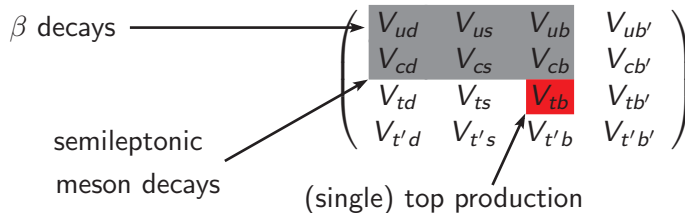
Tree level constraints

β decays $\longrightarrow$

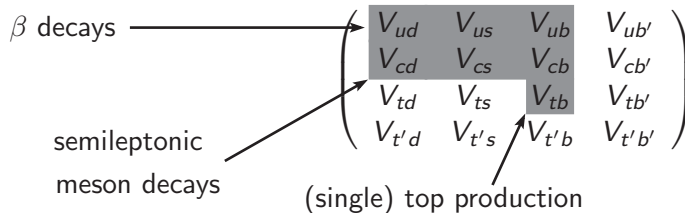
semileptonic meson decays $\longrightarrow$

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \\ V_{t'd} & V_{t's} & V_{t'b} & V_{t'b'} \end{pmatrix}$$

Tree level constraints

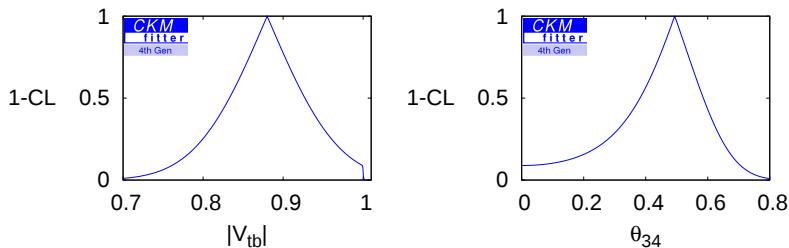


Tree level constraints



The latest PDG value for V_{tb} is 0.88 ± 0.07 .

Tree level constraints alone

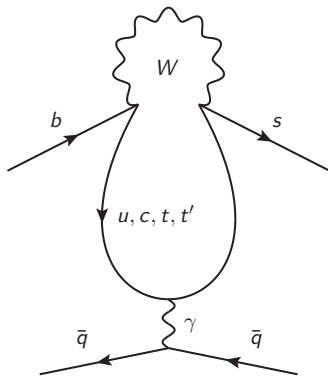
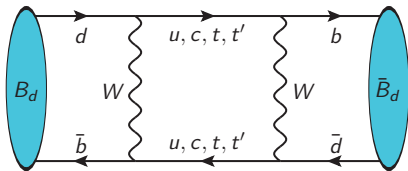


Still huge mixing with a fourth family possible!

⇒ include also loop observables!

Further constraints: FCNC

Flavour observables:



FCNC - in the SM

CKM picture works very - Corrections should be small

- Calculation of the box diagram gives transition rate between the flavor eigenstates B_s and $\bar{B}_s$: M_{12} and Γ_{12}
- The mass difference of the Mass eigenstates B_H and B_L is given by

$$\Delta M_s = m_{B_H} - m_{B_L} \approx 2|M_{12}|$$

- Performing the loop in the box diagram gives

$$M_{12} \propto (V_{tb} V_{ts}^*)^2 \cdot S_0 \left(x_t = \frac{m_t^2}{M_W^2} \right) \cdot f_{B_s}^2 B_{B_s}$$

- We have no direct information on V_{td} and V_{ts} . Indirect information is obtained from FCNCs, if we assume the unitarity of the 3x3 CKM matrix

FCNC in the SM4

Analysis: still huge corrections possible!

There are two effects that change the value of M_{12} in the SM4

- t' running in the loop
- The t loop is also changed, because now the CKM elements from the 3x3 fit can not be use anymore!

This was overseen many times!

Huge cancellations between these two effects are possible

0902.4883,1005.3505: Parameter sets with $\mathcal{O}(300\%)$ effects found

Bounds from FCNC

Observables used as bounds

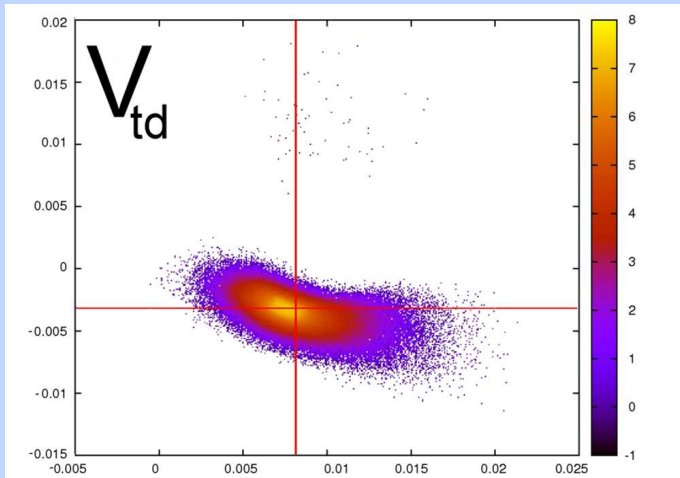
- $\Delta M_s, \Delta M_d, \Delta M_D$
- ϵ_K : CP violation in K decays
- $b \rightarrow s\gamma, B_s \rightarrow \mu\mu$
- $\sin 2\beta$ from $B_d \rightarrow \psi K_s$

Observables not yet used

- $B \rightarrow K^{(*)} \ell\ell$
- Rare K decays
- Semileptonic asymmetries, Dimuon asymmetry
- $\sin 2\beta_s$ from $B_s \rightarrow \psi\phi$

Bounds on the CKM element V_{td}

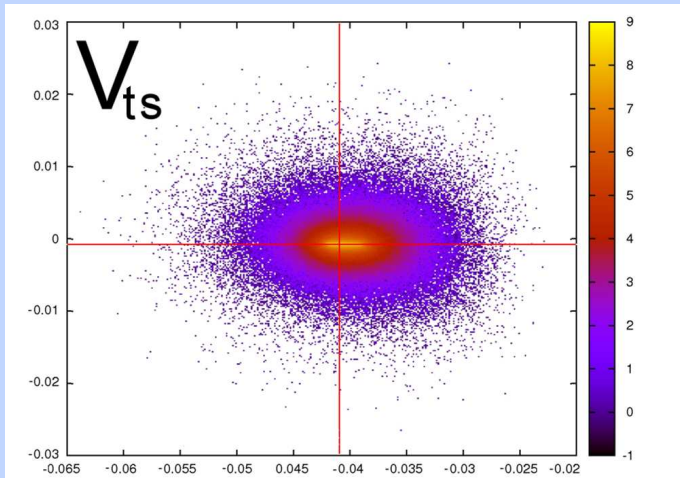
Im V_{td} vs. Re V_{td}



Eberhardt, A.L., Rohrwild: 1005.3505

Bounds on the CKM element V_{ts}

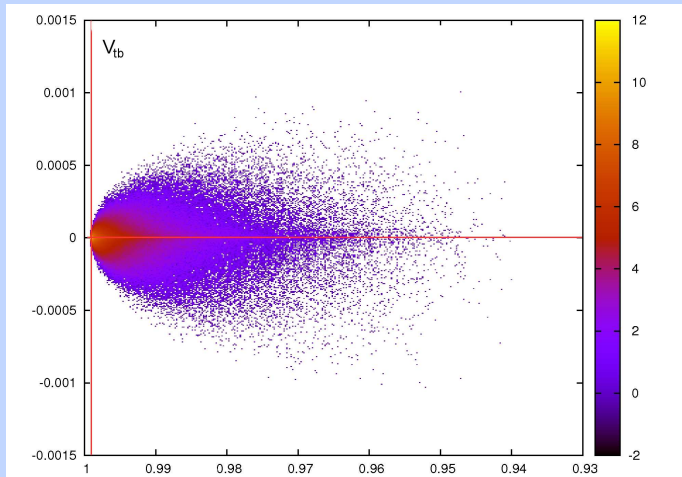
Im V_{ts} vs. Re V_{ts}



Eberhardt, A.L., Rohrwild: 1005.3505

Bounds on the CKM element V_{tb}

Im V_{tb} vs. Re V_{tb}



Eberhardt, A.L., Rohrwild: 1005.3505

The oblique parameters S , T and U

- What are S , T and U ?
- Explicit expressions
- The S - T ellipse
- Mass degeneracy
- The Higgs mass

What are S , T and U ?

- a.k.a. Peskin-Takeuchi parameters
- a.k.a. oblique parameters
- a.k.a. electroweak pseudo-observables

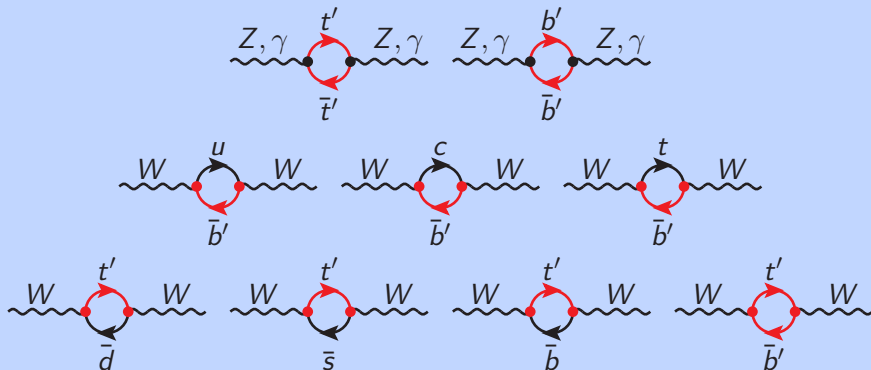
$$(S, T, U)_{\text{full}} = \underbrace{(S, T, U)_{\text{ferm}}^{\text{SM3}} + (S, T, U)_{\text{ferm}}^{\text{SM4}}}_{(S, T, U)_{\text{ferm}}} \leftarrow \text{fermion loop}$$

$$+ \underbrace{(S, T, U)_{\text{gauge}} + (S, T, U)_{\text{Higgs}}}_{(S, T, U)_{\text{bos}}} \leftarrow \text{boson loops}$$

$$(S, T, U) = (S, T, U)_{\text{ferm}}^{\text{SM4}} + (S, T, U)_{\text{Higgs}}^{\text{SM4}}$$

What are S , T and U ?

Self-energy contributions by the fourth family



What are S , T and U ?

The S parameter

$$S_{\text{ferm}} = \frac{16\pi s^2}{e^2} \sum_f N_c \cdot$$

$$\frac{\partial}{\partial p^2} \left[c^2 \left(\text{Z-loop}(f, \bar{f}) \right) + \frac{c}{s} (s^2 - c^2) \left(\text{Z}\gamma\text{-loop}(f, \bar{f}) \right) - c^2 \left(\text{}\gamma\text{-loop}(f, \bar{f}) \right) \right]_{p^2=0}$$

$$c \equiv \cos \theta_W$$

$$s \equiv \sin \theta_W$$

What are S , T and U ?

The T parameter

$$T_{\text{ferm}} = \frac{4\pi}{e^2 c^2 M_Z^2} \sum_f N_c \left[\sum_{f'} \left(\text{W loop with } f, \bar{f}' \right) - c^2 \left(\text{Z loop with } f, \bar{f} \right) - 2cs \left(\text{W loop with } f, \bar{f} \text{ and } \gamma \right) - s^2 \left(\text{Z loop with } f, \bar{f} \text{ and } \gamma \right) \right]_{p^2=0}$$

The diagrammatic terms in the brackets represent fermion loops with various gauge boson insertions:

- $\sum_{f'} \left(\text{W loop with } f, \bar{f}' \right)$: A fermion loop with incoming fermion f and outgoing antifermion $\bar{f}'$, connected by two W boson lines.
- $c^2 \left(\text{Z loop with } f, \bar{f} \right)$: A fermion loop with incoming fermion f and outgoing antifermion $\bar{f}$, connected by two Z boson lines.
- $2cs \left(\text{W loop with } f, \bar{f} \text{ and } \gamma \right)$: A fermion loop with incoming fermion f and outgoing antifermion $\bar{f}$, connected by a W boson line and a photon (γ) line.
- $s^2 \left(\text{Z loop with } f, \bar{f} \text{ and } \gamma \right)$: A fermion loop with incoming fermion f and outgoing antifermion $\bar{f}$, connected by a Z boson line and a photon (γ) line.

What are S , T and U ?

The U parameter

$$U_{\text{ferm}} = \frac{16\pi s^2}{e^2} \sum_f N_c \frac{\partial}{\partial p^2} \left[\sum_{f'} \left(\text{W loop with } f, \bar{f}' \right) - c^2 \left(\text{Z loop with } f, \bar{f} \right) - 2cs \left(\text{\gamma-Z loop with } f, \bar{f} \right) - s^2 \left(\text{\gamma-\gamma loop with } f, \bar{f} \right) \right]_{p^2=0}$$

The diagrammatic terms in the brackets represent fermion loops with various gauge boson insertions:

- $\left(\text{W loop with } f, \bar{f}' \right)$: A fermion loop with incoming fermion f and outgoing antifermion $\bar{f}'$, connected by two W boson lines.
- $\left(\text{Z loop with } f, \bar{f} \right)$: A fermion loop with incoming fermion f and outgoing antifermion $\bar{f}$, connected by two Z boson lines.
- $\left(\text{\gamma-Z loop with } f, \bar{f} \right)$: A fermion loop with incoming fermion f and outgoing antifermion $\bar{f}$, connected by one γ and one Z boson lines.
- $\left(\text{\gamma-\gamma loop with } f, \bar{f} \right)$: A fermion loop with incoming fermion f and outgoing antifermion $\bar{f}$, connected by two γ boson lines.

Explicit expressions

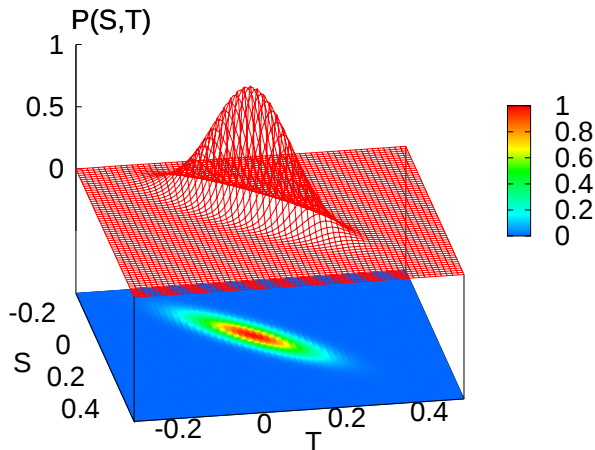
The exact formulae for S and T

$$S_{\text{ferm}} = \frac{N_c}{6\pi} \sum_{(U,D)} \left[1 - \frac{2}{3} \ln \left(\frac{m_U}{m_D} \right) \right] + \frac{1}{6\pi} \sum_{(\nu,l)} \left[1 + 2 \ln \left(\frac{m_\nu}{m_l} \right) \right]$$

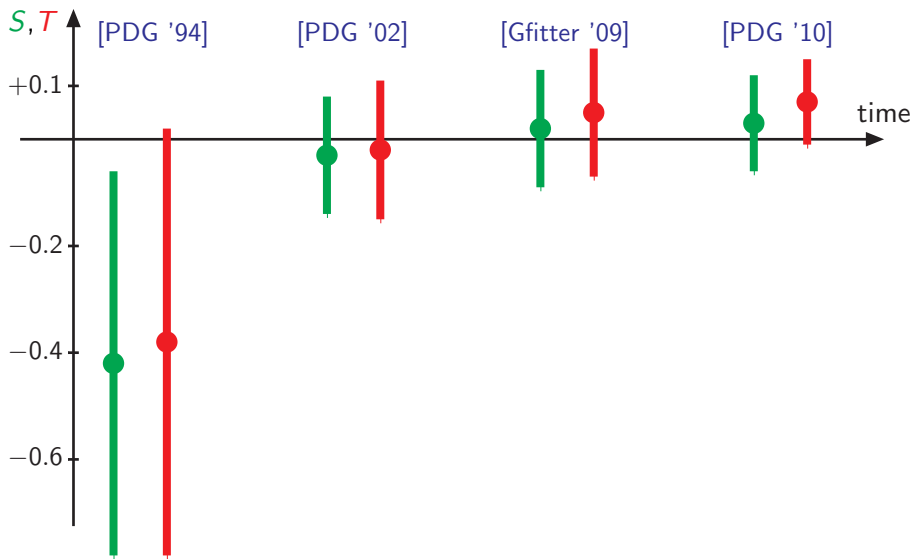
$$T_{\text{ferm}} = \frac{N_c}{16\pi s^2 c^2 M_Z^2} \left[\sum_{i=U,D} m_i^2 - 4 \sum_{U,D} |V_{UD}^{(\text{CKM})}|^2 \frac{m_U^2 m_D^2}{m_U^2 - m_D^2} \ln \left(\frac{m_U}{m_D} \right) \right] \\ + \frac{1}{16\pi s^2 c^2 M_Z^2} \left[\sum_{i=\nu,l} m_i^2 - 4 \sum_{\nu,l} |V_{\nu l}^{(\text{PMNS})}|^2 \frac{m_\nu^2 m_l^2}{m_\nu^2 - m_l^2} \ln \left(\frac{m_\nu}{m_l} \right) \right] \geq 0$$

The S - T ellipse

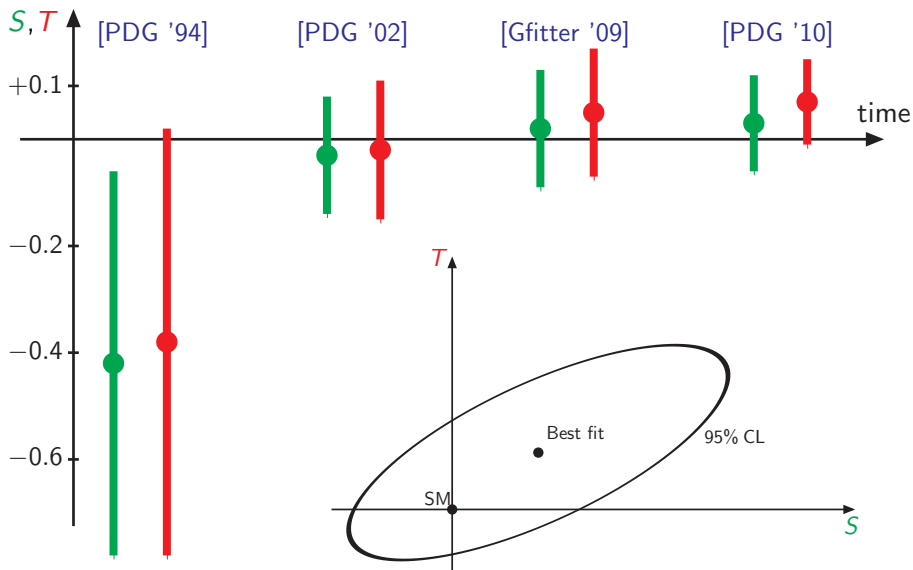
We assume Gaussian distributions for S and T :



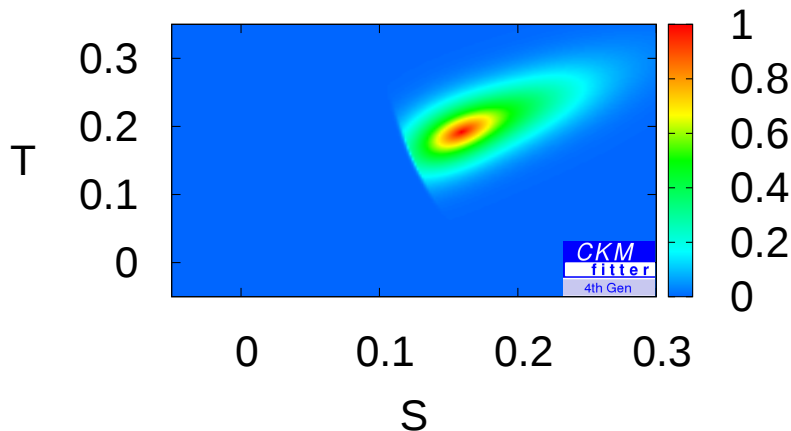
The S - T ellipse: Experimental values



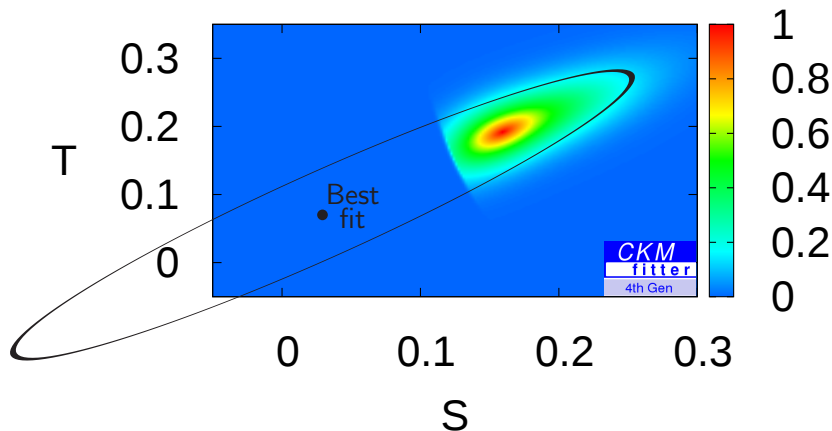
The S - T ellipse: Experimental values



The S - T ellipse: SM4-fit

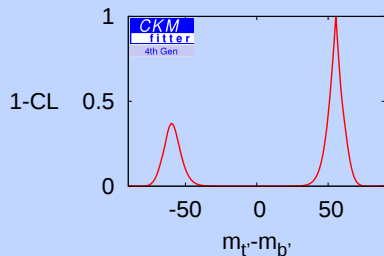


The S - T ellipse: SM4-fit



Fermion mass difference

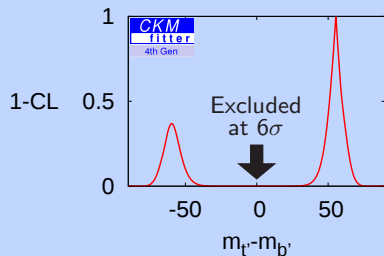
Neglecting the leptons



$$(V^{\text{CKM}} = \mathbb{1})$$

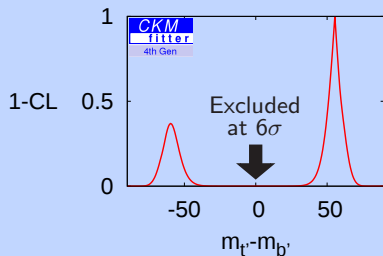
Fermion mass difference

Neglecting the leptons

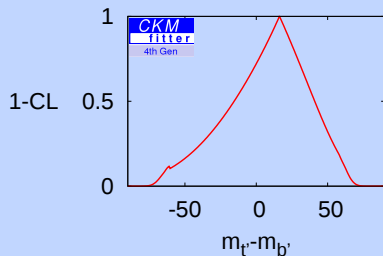


Fermion mass difference

Neglecting the leptons



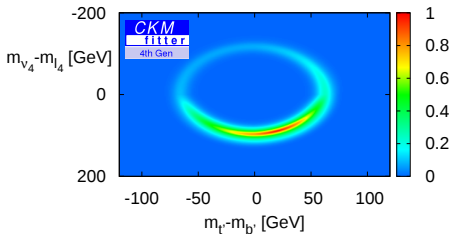
$$(V^{CKM} = 1)$$



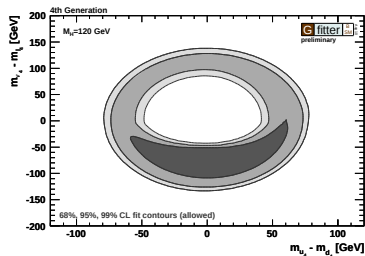
$$(V^{CKM} \neq 1)$$

Fermion mass difference

Taking also the leptons into account:

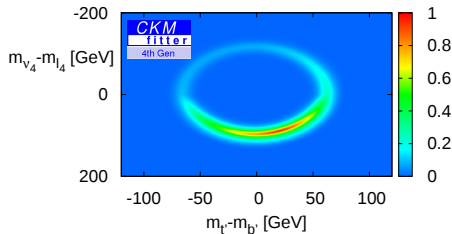


$$(V^{\text{CKM}} = \mathbb{1})$$

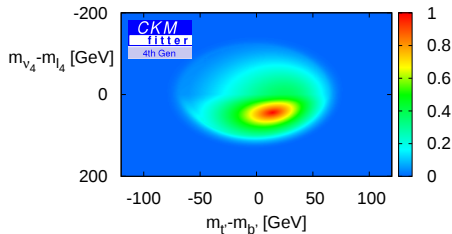


Fermion mass difference

Taking also the leptons into account:



$$(V^{\text{CKM}} = \mathbb{1})$$

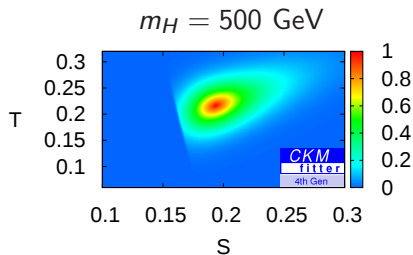
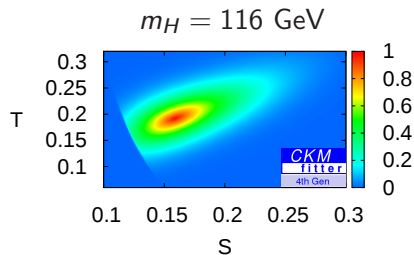


$$(V^{\text{CKM}} \neq \mathbb{1})$$

The Higgs mass

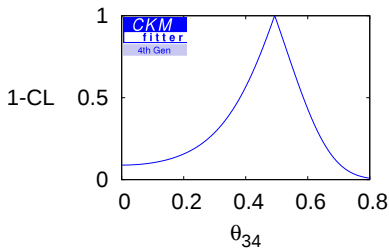
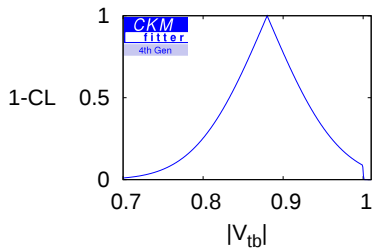
Until now, m_H was fixed to 116 GeV.

$m_H > 116$ GeV $\Rightarrow$ shift to the right:



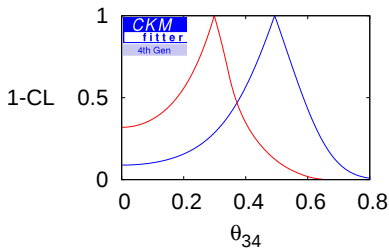
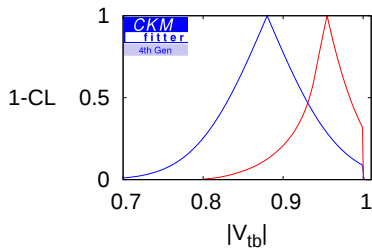
The impact of S and T

Tree-level



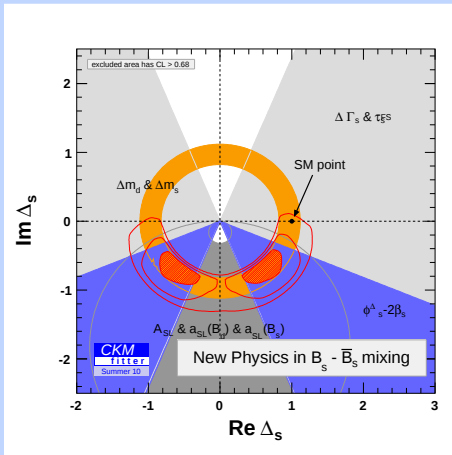
The impact of S and T

Tree-level
+ S and T



New physics in B_s mixing

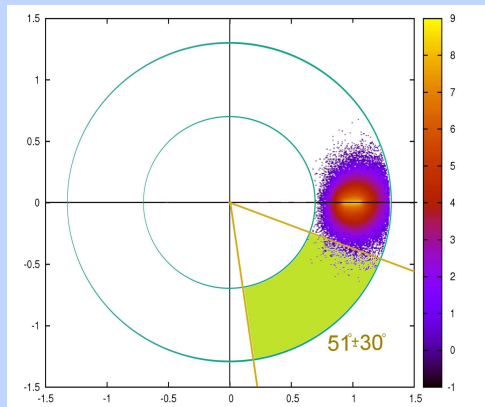
$$M_{12} = M_{12}^{\text{SM}} \cdot \Delta$$



A.L., Nierste, CKMfitter 1008.1593

New physics in B_s mixing due to SM4?

$$M_{12} = M_{12}^{\text{SM}} \cdot \Delta$$



Eberhardt, A.L., Rohrwild, 1005.3505

Outlook

Future project: Prepare a combined CKM and electro-weak fit

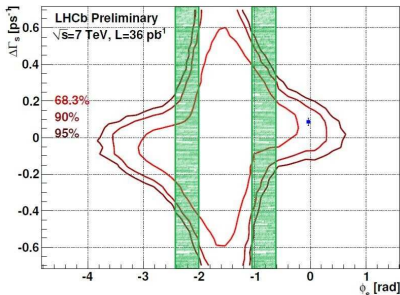
- Include further flavor observables to the fits
- Replace S , T , U by the explicit observables
Gonzalez, Rohrwild, Wiebusch, 1105.3434
- Include the whole lepton sector to the fits
Lacker, Menzel 1003.4532; A.L., Paes, Schalla 1104.2465
- Include new data to the fits - Summer conferences 2011
New physics in B_s mixing at LHCb?
- This kind of fit has also to be performed for other NP models

Outlook

SM4 is not yet excluded by experimental data

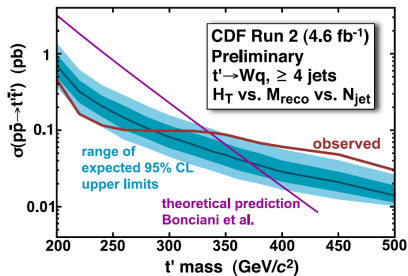
- Perform Fits *without* approximations
- Investigate interesting theoretical aspects
 - ▶ Lepton sector Aparici et al. '11, A.L., Paes, Schalla '11
 - ▶ DM candidates Soni et al '11
 - ▶ SM4 + other extensions of the SM, e.g.
SUSY Lebed '11, Garg '11
2HDM Soni '11
 - ▶ DSB Wise et al '11
 - ▶ Non-perturbative effects Jansen et al '11
 - ▶ Baryogenesis Murayama et al '11, Wise et al '11
- Exclude or discover the fourth family

Back-up slides



[WMAP '10]

mily:

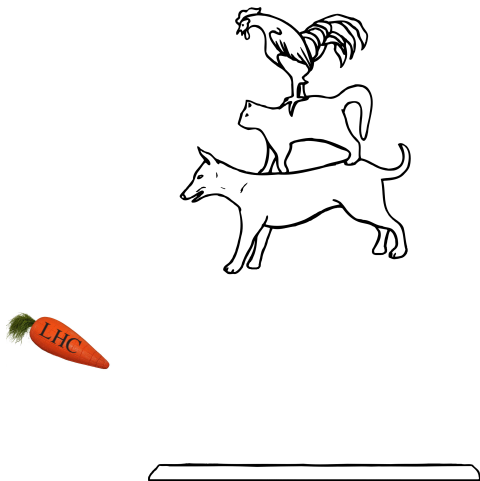


[CDF '10]

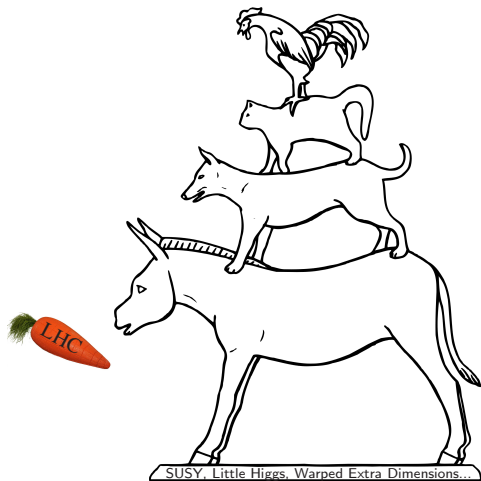
Results



Results



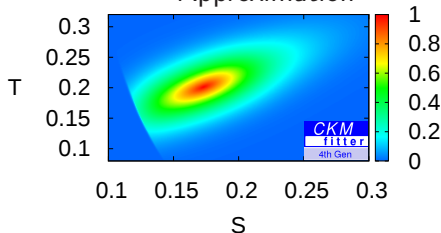
Results



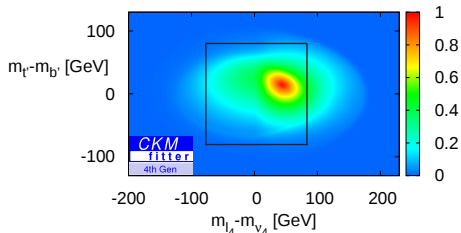
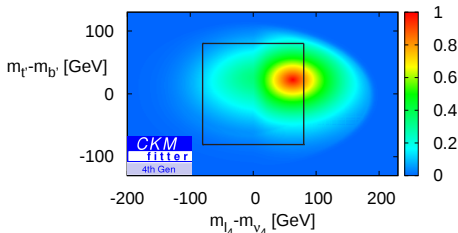
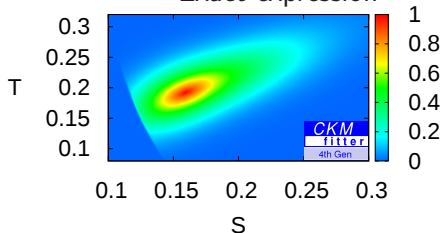
Approximative and exact Higgs contributions to S and T

For “free” Higgs masses:

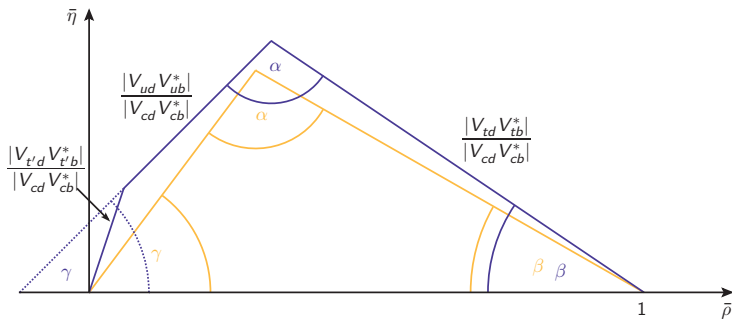
Approximation



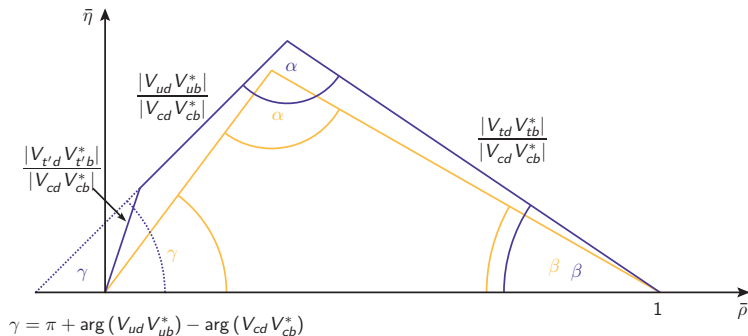
Exact expression



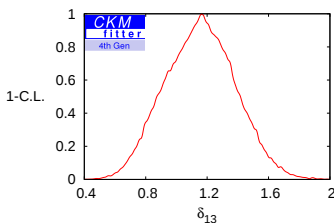
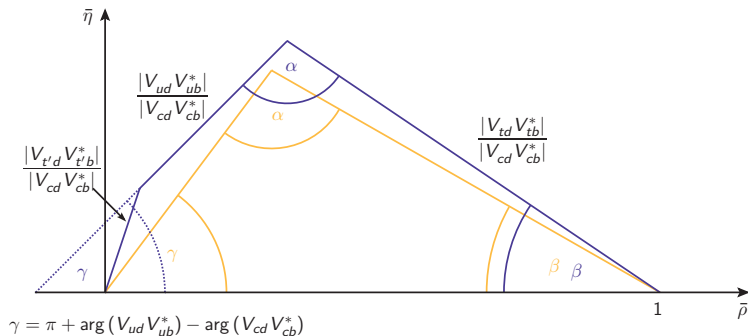
Back-up: The unitarity quadrangle



Back-up: The unitarity quadrangle



Back-up: The unitarity quadrangle



What is the effective number of neutrinos?

The total energy density of massive neutrinos

$$\begin{aligned}\rho_\nu(a) &= 2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{p}{T_\nu(a)}\right) + 1} \sum_i \sqrt{p^2 + m_{\nu_i}^2} \\ &= \frac{1}{a^4} \int \frac{q^2 dq}{\pi^2} \frac{1}{\exp\left(\frac{q}{T_{\nu 0}}\right) + 1} \sum_i \sqrt{q^2 + m_{\nu_i}^2 a^2}\end{aligned}$$

can – for relativistic neutrinos – be related to the photon energy density

$$\rho_\nu(a) \rightarrow \frac{7}{8} \left(\frac{4}{11}\right)^{\frac{4}{3}} N_\nu^{\text{eff}} \rho_\gamma(a)$$

taking the present-day neutrino temperature $T_{\nu 0} = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_{\text{cmb}}$.
(a is the cosmic scale and $q = pa$ the comoving momentum. It was assumed that $m_{\nu_i} = m_{\nu_j}$.)

Explicit expressions

The exact formula for U

$$\begin{aligned} U_{\text{ferm}} = & \frac{N_c}{3\pi} \sum_{U,D} |V_{UD}^{(\text{CKM})}|^2 \left[\frac{2m_U^2 m_D^2}{(m_U^2 - m_D^2)^2} + \left(\frac{m_U^2 + m_D^2}{m_U^2 - m_D^2} - \frac{2m_U^2 m_D^2 (m_U^2 + m_D^2)}{(m_U^2 - m_D^2)^3} \right) \ln \left(\frac{m_U}{m_D} \right) \right] \\ & + \frac{1}{3\pi} \sum_{\nu,l} |V_{\nu l}^{(\text{PMNS})}|^2 \left[\frac{2m_\nu^2 m_l^2}{(m_\nu^2 - m_l^2)^2} + \left(\frac{m_\nu^2 + m_l^2}{m_\nu^2 - m_l^2} - \frac{2m_\nu^2 m_l^2 (m_\nu^2 + m_l^2)}{(m_\nu^2 - m_l^2)^3} \right) \ln \left(\frac{m_\nu}{m_l} \right) \right] \\ & - \frac{5N_c}{36\pi} \sum_q 1 - \frac{5}{36\pi} \sum_{\nu,l} 1 \end{aligned}$$

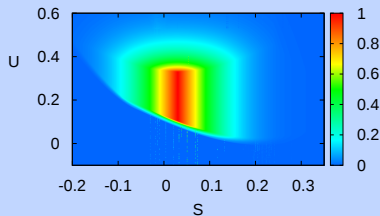
Explicit expressions

Fourth family contributions to S and T

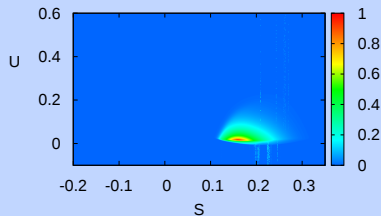
$$\begin{aligned} S_{\text{ferm}}^{\text{SM4}} &= \frac{1}{3\pi} \left[2 + \ln \left(\frac{m_{b'} m_{\nu_4}}{m_{t'} m_{\ell_4}} \right) \right] \\ T_{\text{ferm}}^{\text{SM4}} &\approx \frac{1}{16\pi s^2 c^2 M_Z^2} \left[3 (m_{t'}^2 + m_{b'}^2) + m_{\ell_4}^2 + m_{\nu_4}^2 \right. \\ &\quad - 12 \sum_{i=t,t'} \sum_{j=b,b'} |V_{ij}^{\text{CKM}}|^2 \frac{m_i^2 m_j^2}{m_i^2 - m_j^2} \ln \left(\frac{m_i}{m_j} \right) \\ &\quad - 4 \frac{m_{\nu_4}^2 m_{\ell_4}^2}{m_{\nu_4}^2 - m_{\ell_4}^2} \ln \left(\frac{m_{\nu_4}}{m_{\ell_4}} \right) \\ &\quad \left. + 12 \frac{m_t^2 m_b^2}{m_t^2 - m_b^2} \ln \left(\frac{m_t}{m_b} \right) \right] \end{aligned}$$

The S - T ellipse

The U parameter



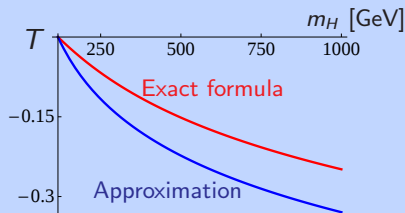
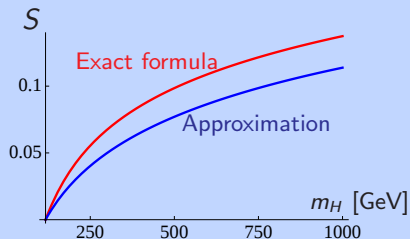
(without T)



(with T)

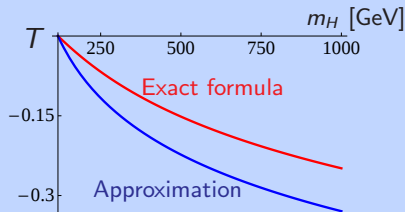
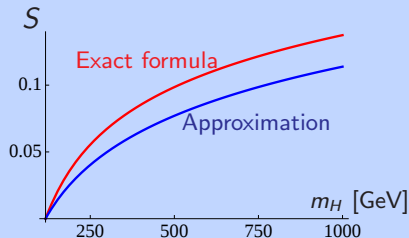
The Higgs mass

The exact Higgs contributions



The Higgs mass

The exact Higgs contributions



Relative errors:

