# The Potential of Minimal Flavour Violation

Belén Gavela Universidad Autónoma de Madrid (UAM) and IFT Today I will discuss flavour:

\* for quarks\* for leptons

and then focus on Minimal Flavour Violation (MFV) for both sectors

#### Beyond Standard Model because

1) Experimental evidence for new particle physics:

- **\*\*\* Neutrino masses**
- \*\*\* Dark matter
- **\*\* Matter-antimatter asymmetry**

#### 2) Uneasiness with SM fine-tunings



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The (Tevatron->) LHC allow us to explore it

# The happiness in the air of the LHC era

#### ... as we are almost "touching" the Higgs

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The Higgs excitation has the quantum numbers of the EW vacuum

#### **BSM because**

1) Experimental evidence for new particle physics:

- **\*\*\* Neutrino masses**
- \*\*\* Dark matter
- **\*\* Matter-antimatter asymmetry**

2) Uneasiness with SM fine-tunings, i.e. electroweak:

\*\*\* Hierarchy problem \*\*\* Flavour puzzle

#### **BSM electroweak**

#### \* HIERARCHY PROBLEM

fine-tuning issue: if BSM physics, why Higgs so light

Interesting mechanisms to solve it from SUSY; strong-int. Higgs, extra-dim....

In practice, none without further fine-tunings

#### \* FLAVOUR PUZZLE

# \* All quark flavour data are ~consistent with SM

### Kaon sector, B-factories, accelerators....

There are some  $\sim$ 2-3 sigma anomalies around, though:

- --  $sin 2\beta$  in CKM fit (Lunghi, Soni, Buras, Guadagnoli, UTfit, CKMfitter)
- -- anomalous like-sign dimuon charge asymmetry in B<sub>S</sub> decays (D0)

-- 
$$B \longrightarrow \tau v$$
 (UTfit)

# yet....we do NOT understand flavour

#### **The Flavour Puzzle**



#### Why 2 replicas of the first family?

when we only need one to account for the visible universe

#### **The Flavour Puzzle**



Why so diferent masses and mixing angles?

#### **The Flavour Puzzle**



Why has nature chosen the number and properties of families so as to allow for CP violation... and explain nothing? (i.e. not enough for matter-antimatter asymmetry)

#### **BSM electroweak**

#### \* HIERARCHY PROBLEM

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\* FLAVOUR PUZZLE: no progress

Understanding stalled since 30 years. Only new B physics data AND neutrino masses and mixings  $\bigwedge_{f} \sim 100$ 's TeV ???

**BSMs tend to worsen the favour puzzle** 

#### The FLAVOUR WALL for BSM

i) Typically, BSMs have **electric dipole moments** at one loop i.e susy MSSM:



< 1 loop in SM ---> Best (precision) window of new physics

#### ii) **FCNC**

i.e susy MSSM:

$$K^{0} - \overline{K}^{0} \operatorname{mixing} \begin{array}{c} \bar{s} \\ \tilde{g} \\ \underline{\tilde{g}} \\ \underline{\tilde{g}} \\ \underline{\tilde{d}}_{R_{\times}} \\ \tilde{s}_{R} \\ \tilde{s}_{R_{\times}} \\$$

competing with SM at one-loop

#### The FLAVOUR WALL for BSM

\* The **QCD** vaccuum : Strong CP problem,  $\theta_{QCD} < 10^{-10}$ 

#### **BSM in general induce** $\theta_{QCD} > 10^{-10}$



\* The **matter-antimatter asymmetry** : CP-violation from quarks in SM fails by ~10 orders of magnitude (+ too heavy Higgs)

#### **Neutrino light on flavour ?**

# \* Neutrino masses indicate BSM.... yet consistent with 3 standard families

- -- in spite of some 2-3 sigma anomalies:
  - \* Minos, 2 sigma, neutrinos differ from antineutrinos
  - \* Hints of steriles: LSND and MiniBoone in antineutrinos, new deficit in Double-Chooz nu\_efluxes, Gallex deficit in antinu\_e .....

# The Higgs mechanism can accomodate masses in SM... but neutrinos (?)



# The Higgs mechanism can accomodate masses in SM... but neutrinos (?)



#### **Maybe because of Majorana neutrinos?**

#### Lepton mixing in charged currents



$$V_{\rm CKM} = = \begin{pmatrix} c_{13}c_{12} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - s_{23}s_{13}c_{12}e^{-i\delta} & c_{12}c_{23} - s_{23}s_{13}s_{12}e^{-i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{-i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}$$

Quarks

#### Lepton mixing in charged currents

Leptons





#### **More wood for the Flavour Puzzle**



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**Maybe because of Majorana neutrinos?** 

### **Dirac o Majorana ?**

•The only thing we have really understood in particle physics is the gauge principle

•SU(3)xSU(2)xU(1) allow Majorana masses....

Lepton number was only an accidental symmetry of the SM

Anyway, it is for experiment to decide

# What are the main physics goals in v physics?

- To determine the absolute scale of masses
- To determine whether they are Majorana
- \* To discover Leptonic CP-violation

# The art of the possible:

#### We should at least measure the 3 active $\nu$ mass matrix



## Absolute mass scale



(Karlsruhe Katrin web page)

#### Neutrino mass hierarchy from cosmology



## Majorananess and v mass scale





#### **CP violation**



### $\theta_{13} \neq 0$ is the key to CP violation

# Entering the era of precision neutrino oscillation physics

~ % level

 $v_{\mu} < ---> v_e$  golden channel...

# $\theta_{13}$ future sensitivities






Plenty of possibilities to reach the  $sin^2 2\theta_{13} \sim 10^{-4}$  realm ...

# What are the main physics goals in $\nu$ physics?

• To determine the absolute scale of masses

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#### They would not "prove" leptogenesis, but they would give a strong argument for it

# What are the main physics goals in v physics?

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- \* To discover Leptonic CP-violation

#### Go for those discoveries !

## Can we foresee how to go beyond?

## Experimentally? i.e. beyond 3... \* Miniboone-like... \* Or rather cosmo?:

#### CMB+X bounds on $N_{ m eff}$



 Precise numbers depend on cosmological model and data sets used

Recent analysis:  $N_{\rm eff}$  = 4.47 <sup>+1.8</sup> CMB + SDSS-DR7-BAO + HST ACDM + neutrino mass +  $N_{\rm eff}$ [JH, Hannestad, Lesgourgues, Rampf, Wong (2010)]

today: arXiv:1009.0866 including ACT small scale CMB data

 $N_{\rm eff}$  = 4.56 ± 1.5 (95%)

Haman NOW2010

## Experimentally? i.e. beyond 3... \* Miniboone-like... \* Or rather cosmo?:





Haman NOW2010

## Assume only 3:

• To determine the absolute scale of masses

- To determine whether they are Majorana
- \* To discover Leptonic CP-violation

## Can we foresee how to go beyond?

Neutrino masses indicate new physics beyond the SM

Maybe new flavour physics could appear also in neutrino couplings ?

Neutrino masses indicate new physics beyond the SM ? Maybe new flavour physics could appear also in neutrino couplings ?

## How to advance in a modelindependent way?

- In quark flavour puzzle
- In lepton flavour puzzle

How to go about it model-independent ?....

Effective field theory

# Mimic travel from Fermi's beta decay to SM

$$\int_{U(1)em}^{Fermi} + \frac{O}{M^2} + \dots$$







If new physics scale M > v

$$\mathcal{L} = \mathcal{L}_{SU(3)\times SU(2)\times U(1)} + \frac{O^{d=5}}{M} + \frac{O^{d=6}}{M^2} + \dots$$

#### v masses beyond the SM

### The Weinberg operator



It's unique  $\rightarrow$  very special role of v masses: lowest-order effect of higher energy physics

#### v masses beyond the SM

### The Weinberg operator

Dimension 5 operator: H  $\lambda/M (L L H H) \rightarrow \lambda \sqrt{2}M (\nu \nu)$  $\int_{0}^{d=5}$ 

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This mass term violates lepton number (B-L) → Majorana neutrinos

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 $\mathbf{O}^{d=5}$  is common to all models of Majorana  $\mathbf{V}$ s







#### The Seesaw models

 Three types of models yield the Weinberg operator at tree level



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 Three types of models yield the Weinberg operator at tree level



Type I

#### Type II

#### Type III

Heavy fermion singlet  $N_R$  Heavy scalar triplet  $\Delta$  Heavy fermion triplet  $\Sigma_R$ Minkowski, Gell-Mann, Ramond,

Slansky, Yanagida, Glashow, Mohapatra, Senjanovic

Magg, Wetterich, Lazarides, Shafi, Mohapatra, Senjanovic, Schecter, Valle

Ma, Roy, Senjanovic, Hambye et al.,

$$\mathcal{L} = \mathcal{L}_{SU(3)\times SU(2)\times U(1)} + \frac{O^{d=5}}{M} + \frac{O^{d=6}}{M^2} + \dots$$

O<sup>d=6</sup> : conserve B, L... and lead new flavour effects for quarks and leptons





 $SU(2) \times U(1)_{em}$  gauge invariant

#### A humble ansatz:

## Minimal Flavour Violation

(Chivukula. Georgi) (Ambrosio, Giudice, Isidori, Strumia)

### A humble ansatz:

## Minimal Flavour Violation

....taking laboratory data at face value

(Chivukula. Georgi) (Ambrosio, Giudice, Isidori, Strumia)

## \* All quark flavour data are consistent with SM Kaon sector, B-factories, accelerators....

There are some  $\sim$ 2-3 sigma anomalies around, though: i.e. Fermilab's anomalous like-sign dimuon charge asymmetry in B<sub>S</sub> decays

#### = consistent with CKM

### = consistent with all flavour effects due to Yukawas







$$Y_{U} \quad Y_{U} = \mathcal{V}_{CKM}^{\dagger} \begin{pmatrix} y_{u} & 0 & 0 \\ 0 & y_{c} & 0 \\ 0 & 0 & y_{t} \end{pmatrix}$$

$$Q_{L} \quad U_{R}$$

•Flavour data (i.e. B physics) consistent with all flavour physics coming from Yukawa

 $\begin{array}{l} \mathsf{MFV} \ \mathsf{Hypothesis} \equiv \mathsf{The} \ \mathsf{Yukawas} \ \mathsf{are} \ \mathsf{the} \ \mathsf{only} \ \mathsf{sources} \ (\textit{irreducible}) \\ \mathsf{of} \ \mathsf{flavour} \ \mathsf{violation.} \ \mathsf{in} \ \mathsf{the} \ \mathsf{SM} \ \underline{\mathsf{and}} \ \underline{\mathsf{BSM}} \\ \mathbb{R}. \ \mathsf{S. \ Chivukula} \ \mathsf{and} \ \mathsf{H. \ Georgi,} \ \mathtt{Phys.} \ \mathsf{Lett.} \ \mathsf{B} \ \mathtt{188}, \ \mathtt{99} \ \mathtt{(1987)}. \end{array}$ 

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The global Flavour symmetry of the SM with massless fermions:

 $\begin{aligned} G_{\mathrm{f}} &= SU(3)_{Q_{L}} \times SU(3)_{u} \times SU(3)_{d} \times SU(3)_{L} \times SU(3)_{e} \\ Q_{\mathrm{L}} &\rightarrow \mathrm{L} \ Q_{\mathrm{L}} \quad \mathrm{D}_{\mathrm{R}} \rightarrow R_{d'} \ \mathrm{D}_{\mathrm{R}} \ \ldots \end{aligned}$ 

 $D_R = (d_{R, s_R, b_R}) \sim (1, 1, 3)$ 

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MFV Hypothesis  $\equiv$  The Yukawas are the only sources (*irreducible*) of flavour violation. in the SM and BSM R. S. Chivukula and H. Georgi, Phys. Lett. B **188**, 99 (1987).

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 $Q_L \rightarrow L \ Q_L \qquad D_R \rightarrow R_{d'} D_R \ ...$  $D_R = (d_{R, S_R, b_R}) \sim (1, 1, 3)$ 

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The global Flavour symmetry of the SM: Yukawas break it

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## $G_f = SU(3)_{Q_L} \times SU(3)_{U_R} \times SU(3)_{D_R}$

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The global Flavour symmetry of the SM: Yukawas break it unless  $G_{f} = SU(3)_{Q_{L}} \times SU(3)_{u} \times SU(3)_{d} \times SU(3)_{L} \times SU(3)_{e}$   $Q_{L} \rightarrow L \ Q_{L} \quad D_{R} \rightarrow R_{d'} D_{R} \quad Y_{d} \rightarrow L_{u} Y_{u} R_{d}^{\dagger}, \dots$   $D_{R} = (d_{R}, s_{R}, b_{R}) \sim (1, 1, 3)$   $\overline{Q}_{L} Y_{D} D_{R} H \qquad Y_{D} \sim (3, 1, \overline{3})$ 

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It is very predictive for quarks:

 $O^{d=6} \sim \overline{Q}_{\alpha} Q_{\beta} \overline{Q}_{\gamma} Q_{\delta}$ 

$$\mathcal{L} = \mathcal{L}_{SM} + \mathbf{c}^{d=6} \mathbf{O}^{d=6} + \dots$$

i.e. 
$$C^{d=6} \sim \frac{Y_{\alpha\beta}^+ Y_{\gamma\delta}}{\Lambda_{flavour}^2} \qquad O^{d=6} \sim \overline{Q}_{\alpha} Q_{\beta}$$






\* MFV can reconcile  $\Lambda_f$  and  $\Lambda_{electroweak}$ :

 $\Lambda_{f} \sim \Lambda_{electroweak} \sim TeV$ 

... and induce observable flavour changing effects

## WHY MFV?



Hierarchy Problem points to Λ~TeV

$\mathcal{O}_{d=6}^{i}$	$\Lambda_f$	$C_{d=6}$	= 1
$(\bar{s}_L \gamma^\mu d_L)^2$	$9.8 \times 10^2$	$1.6 \times$	$10^{4}$
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	$1.8\times 10^4$	$3.2 \times$	$10^{5}$
$(\bar{c}_L \gamma^\mu u_L)^2$	$1.2\times 10^3$	$2.9 \times$	$10^{3}$
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	$6.2\times 10^3$	$1.5 \times$	$10^{4}$
$(\bar{b}_L \gamma^\mu d_L)^2$	$5.1  imes 10^2$	$9.3 \times$	$10^{2}$
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	$1.9\times10^3$	$3.6 \times$	$10^{3}$
$(\bar{b}_L \gamma^\mu s_L)^2$		$1.1  imes 10^2$	
$(\bar{b}_R  s_L)(\bar{b}_L s_R)$		$3.7  imes 10^2$	

$\mathcal{O}_{d=6}^{i}$	$\Lambda_f$	
$H^{\dagger}\left(\overline{D}_{R}Y^{d\dagger}Y^{u}Y^{u\dagger}\mathcal{J}_{\mu\nu}Q_{L}\right)\left(eF_{\mu\nu}\right)$	$6.1~{\rm TeV}$	
$\frac{1}{2} (\overline{Q}_L Y^u Y^u \gamma_\mu Q_L)^2$	$5.9~{\rm TeV}$	
$H_D^{\dagger}\left(\overline{D}_RY^{d\dagger}Y^uY^u{}^{\dagger}\sigma_{\mu\nu}T^aQ_L\right)\left(g_sG^a_{\mu\nu}\right)$	$3.4~{\rm TeV}$	
$\left(\overline{Q}_L Y^u Y^u^\dagger \gamma_\mu Q_L\right) \left(\overline{E}_R \gamma_\mu E_R\right)$	$2.7~{\rm TeV}$	
$i\left(\overline{Q}_L Y^u Y^u^{\dagger} \gamma_{\mu} Q_L\right) H_U^{\dagger} D_{\mu} H_U$	$2.3~{\rm TeV}$	
$\left(\overline{Q}_L Y^u Y^u^\dagger \gamma_\mu Q_L\right) \left(\overline{L}_L \gamma_\mu L_L\right)$	$1.7~{\rm TeV}$	
$\left(\overline{Q}_L Y^u Y^u^\dagger \gamma_\mu Q_L\right) \left(e D_\mu F_{\mu\nu}\right)$	$1.5 { m ~TeV}$	

 $Z_{d-6} \equiv C_{d-6}(Y_u, Y_d)$ 

WITHOUT MFV:  $\Lambda_f \sim 10^2$  TeV

WITH MFV:  $\Lambda_f \sim \text{TeV}$ 

G. Isidori, Y. Nir, G. Perez, 1002.09





Gonzalez-Alonso

#### **Minimal Flavour violation (MFV)**

•Unitarity of CKM first row:

 $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9999 \pm 0.0006$ 

•\*Restrict to flavour blind ops.-> 4 operators

•Correction is only multiplicative to  $\beta$  and  $\mu$  decay rate

The direct experimental limit puts strong constraints on all 4 operators, at the level of the colliders constraints or better.

A rationale for the MFV ansatz?

- Flavour data (i.e. B physics) consistent with all flavour physics coming from Yukawa
- Inspired in "condensate" flavour physics a la Froggat-Nielsen (Yukawas ~  $\langle \Psi \Psi \rangle^{n} / \Lambda_{f}^{n}$ , rather than in susy-like options



# MFV suggests that Y<sub>U</sub> & Y<sub>D</sub> have a dynamical origin at high energies ......

-

$$Y \sim \langle \phi \rangle$$
 or  $\langle \phi \rangle \rangle$  or  $\langle \phi \rangle$  or  $\langle \phi \rangle$ ...



(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

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$$Y \sim \langle \phi \rangle$$
 or  $\langle \phi \chi \rangle$  or  $\langle ()^n \rangle$ ...

Spontaneous breaking of flavour symmetry dangerous

--> i.e. gauge it (Grinstein, Redi, Villadoro, 2010)

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That scalar field or aggregate of fields may have a potential (Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

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#### What is the potential of Minimal Flavour Violation ?

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#### What is the potential of Minimal Flavour Violation ?

# Can its minimum correspond to the observed masses and mixings?

(Alonso, Gavela, Merlo, Rigolin, arXiv 1103.2915)

We analyzed the scalar potential for both 2 and 3 families, two cases:

1) Y -- > one single field 
$$\Sigma \sim (3, 1, \overline{3})$$
  
2) Y -- > two fields  $\chi \chi^{+} \sim (3, 1, \overline{3})$ 



Y --> one single field  $\Sigma$ 

#### **Dimension 5 Yukawa operator**

 $\boldsymbol{\Sigma}$  are bifundamentals of  $G_f$  :



$$\overline{Q}_{L} \frac{\Sigma_{d}}{\Lambda} D_{R} H \qquad \Sigma_{d} \sim (3, 1, \overline{3})$$

$$\uparrow_{Y_{d}}$$

 $; V(\Sigma_u \Sigma_u H)?$ 

Dimension 5 Yukawa Operator

The potential will also respect the flavour symmetry, so it will be built with the  $G_f$  invariants:

$$\begin{aligned} & \textit{Tr}\left(\Sigma_{u}\Sigma_{u}^{\dagger}\right), \quad \det\left(\Sigma_{u}\right), \quad \textit{Tr}\left(\Sigma_{u}\Sigma_{u}^{\dagger}\Sigma_{d}\Sigma_{d}^{\dagger}\right), \\ & \textit{Tr}\left(\Sigma_{d}\Sigma_{d}^{\dagger}\right), \quad \det\left(\Sigma_{d}\right), \quad \cdots \end{aligned}$$

These invariants can be expressed in terms of masses and mixing angles (2 generations):

$$\langle \Sigma_u \rangle = \Lambda \cdot V^{\dagger} Diag\{y_{u_i}\}, \qquad \langle \Sigma_d \rangle = \Lambda \cdot Diag\{y_{d_i}\};$$

$$Y_D = \begin{pmatrix} y_d & 0\\ 0 & y_s \end{pmatrix}, \qquad Y_U = \mathcal{V}_C^{\dagger} \begin{pmatrix} y_u & 0\\ 0 & y_c \end{pmatrix}$$

$$\mathbf{V} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

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$$\operatorname{Tr}\left(\Sigma_{u}\Sigma_{u}^{\dagger}\right) = \Lambda_{f}^{2}\left(y_{u}^{2} + y_{c}^{2}\right) \qquad \det\left(\Sigma_{u}\right) = \Lambda_{f}^{2}y_{u}y_{c}$$

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These invariants can be expressed in terms of masses and mixing angles (2 generations):

$$\langle \Sigma_u \rangle = \Lambda \cdot V^{\dagger} Diag\{y_{u_i}\}, \qquad \langle \Sigma_d \rangle = \Lambda \cdot Diag\{y_{d_i}\};$$
  
$$Tr\left(\left\langle \Sigma_u \Sigma_u^{\dagger} \Sigma_d \Sigma_d^{\dagger} \right\rangle\right) = \frac{1}{2} \Lambda^4 \left[\left(y_c^2 - y_u^2\right) \left(y_s^2 - y_d^2\right) \cos 2\theta_c + \cdots\right]$$

Dimension 5 Yukawa Operator

The potential will also respect the flavour symmetry, so it will be built with the  $G_f$  invariants:

$$\begin{aligned} & \textit{Tr}\left(\Sigma_{u}\Sigma_{u}^{\dagger}\right), \quad \det\left(\Sigma_{u}\right), \quad \textit{Tr}\left(\Sigma_{u}\Sigma_{u}^{\dagger}\Sigma_{d}\Sigma_{d}^{\dagger}\right), \\ & \textit{Tr}\left(\Sigma_{d}\Sigma_{d}^{\dagger}\right), \quad \det\left(\Sigma_{d}\right), \quad \cdots \end{aligned}$$

These invariants can be expressed in terms of masses and mixing angles (2 generations):

$$\langle \Sigma_u \rangle = \Lambda \cdot V^{\dagger} Diag\{y_{u_i}\}, \qquad \langle \Sigma_d \rangle = \Lambda \cdot Diag\{y_{d_i}\};$$
  
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The most general Potential is:

$$V = \sum_{i=u,d} \left( -\mu_i^2 \operatorname{Tr} \left( \Sigma_i \Sigma_i^{\dagger} \right) - \tilde{\mu}_i^2 \det (\Sigma_i) \right) + \sum_{i,j=u,d} \left( \lambda_{ij} \operatorname{Tr} \left( \Sigma_i \Sigma_i^{\dagger} \right) \operatorname{Tr} \left( \Sigma_j \Sigma_j^{\dagger} \right) + \tilde{\lambda}_{ij} \det (\Sigma_i) \det (\Sigma_j) + \cdots \right) \cdots$$

Dimension 5 Yukawa Operator

The potential will also respect the flavour symmetry, so it will be built with the  $G_f$  invariants:

$$\begin{aligned} & \textit{Tr}\left(\Sigma_{u}\Sigma_{u}^{\dagger}\right), \quad \det\left(\Sigma_{u}\right), \quad \textit{Tr}\left(\Sigma_{u}\Sigma_{u}^{\dagger}\Sigma_{d}\Sigma_{d}^{\dagger}\right), \\ & \textit{Tr}\left(\Sigma_{d}\Sigma_{d}^{\dagger}\right), \quad \det\left(\Sigma_{d}\right), \quad \cdots \end{aligned}$$

These invariants can be expressed in terms of masses and mixing angles (2 generations):

$$\langle \Sigma_u \rangle = \Lambda \cdot V^{\dagger} Diag\{y_{u_i}\}, \qquad \langle \Sigma_d \rangle = \Lambda \cdot Diag\{y_{d_i}\};$$
  
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The most general Potential is:

$$V = \sum_{i=u,d} \left( -\mu_i^2 \operatorname{Tr} \left( \Sigma_i \Sigma_i^{\dagger} \right) - \tilde{\mu}_i^2 \det(\Sigma_i) \right) + \sum_{i=u,d} \left( \lambda_{ij} \operatorname{Tr} \left( \Sigma_i \Sigma_i^{\dagger} \right) \operatorname{Tr} \left( \Sigma_j \Sigma_j^{\dagger} \right) + \tilde{\lambda}_{ij} \det(\Sigma_i) \det(\Sigma_j) + \cdots \right) \cdots$$

Y --> one single field  $\Sigma$ 

## Minimum of the Potential

Dimension 5 Yukawa Operator

The minimum of the Potential is given by:

$$\frac{\partial V}{\partial y_i} = 0 \qquad \frac{\partial V}{\partial \theta_i} = 0$$

Take the angle for example:

$$rac{\partial V}{\partial heta_c} \propto \left(y_c^2 - y_u^2
ight) \left(y_s^2 - y_d^2
ight) \sin 2 heta_c = 0$$



Non-degenerate masses  $\longrightarrow \sin 2\theta_c = 0$  No mixing !

Notice also that 
$$\frac{\partial V^{(4)}}{\partial \theta} \sim \sqrt{J}$$
 (Jarlskog determinant)

## Minimum of the Potential

Dimension 5 Yukawa Operator

The minimum of the Potential is given by:

$$\frac{\partial V}{\partial y_i} = \mathbf{0} \qquad \frac{\partial V}{\partial \theta_i} = \mathbf{0}$$

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ight) \sin 2 heta_c = 0$$



Non-degenerate masses  $\longrightarrow \sin 2\theta_c = 0$  No mixing !

Can the actual masses and mixings fit naturally in the minimum of the Potential? e.g. adding non-renormalizable terms... NO

- Not only we have to input the hierarchy of masses in the potential parameters, wich was to expect but ···
- To accommodate the mixing we must introduce wild fine tunnings of O(10<sup>-10</sup>) and nonrenormalizable terms of dimension 8

#### Y --> one single field $\Sigma$

#### three families

\* at renormalizable level: 7 invariants instead of the 5 for two families

$$\begin{aligned} \operatorname{Tr} \left( \Sigma_{u} \Sigma_{u}^{\dagger} \right) &\stackrel{vev}{=} \Lambda_{f}^{2} \left( y_{t}^{2} + y_{c}^{2} + y_{u}^{2} \right) , & Det \left( \Sigma_{u} \right) \stackrel{vev}{=} \Lambda_{f}^{3} y_{u} y_{c} y_{t} , \\ \operatorname{Tr} \left( \Sigma_{d} \Sigma_{d}^{\dagger} \right) \stackrel{vev}{=} \Lambda_{f}^{2} \left( y_{b}^{2} + y_{s}^{2} + y_{d}^{2} \right) , & Det \left( \Sigma_{d} \right) \stackrel{vev}{=} \Lambda_{f}^{3} y_{d} y_{s} y_{b} , \\ &= \operatorname{Tr} \left( \Sigma_{u} \Sigma_{u}^{\dagger} \Sigma_{u} \Sigma_{u}^{\dagger} \right) \stackrel{vev}{=} \Lambda_{f}^{4} \left( y_{t}^{4} + y_{c}^{4} + y_{u}^{4} \right) , \\ &= \operatorname{Tr} \left( \Sigma_{d} \Sigma_{d}^{\dagger} \Sigma_{d} \Sigma_{d}^{\dagger} \right) \stackrel{vev}{=} \Lambda_{f}^{4} \left( y_{b}^{4} + y_{s}^{4} + y_{d}^{4} \right) , \\ &= \operatorname{Tr} \left( \Sigma_{u} \Sigma_{u}^{\dagger} \Sigma_{d} \Sigma_{d}^{\dagger} \right) \stackrel{vev}{=} \Lambda_{f}^{4} \left( P_{0} + P_{int} \right) , \\ \\ \mathbf{Interesting angular dependence:} \quad P_{0} \equiv -\sum_{i < j} \left( y_{u_{i}}^{2} - y_{u_{j}}^{2} \right) \left( y_{d_{i}}^{2} - y_{d_{j}}^{2} \right) \sin^{2} \theta_{ik} \sin^{2} \theta_{jk} + \\ &- \left( y_{d}^{2} - y_{s}^{2} \right) \left( y_{c}^{2} - y_{t}^{2} \right) \sin^{2} \theta_{13} \sin^{2} \theta_{23} + \\ &+ \frac{1}{2} \left( y_{d}^{2} - y_{s}^{2} \right) \left( y_{c}^{2} - y_{t}^{2} \right) \cos \delta \sin 2\theta_{12} \sin 2\theta_{23} \sin \theta_{13} , \end{aligned}$$

Sad conclusions as for 2 families:

need non-renormalizable + super fine-tuning



Automatic strong mass hierarchy and one mixing angle ! Holds for 2 and 3 families !

#### **Dimension 6 Yukawa operator**

 $\chi$  are fundamentals of G<sub>f</sub> : vectors under, similar to quarks and leptons

i.e. 
$$Y_{D} \sim \chi^{L} d (\chi^{R} d)^{+} \sim (3, 1, 1) (1, 1, \overline{3}) \sim (3, 1, \overline{3})$$
  
 $\Lambda_{f}^{2}$ 

 $\chi^{L}_{u}, \chi^{L}_{d} \sim (3, 1, 1); \quad \chi^{R}_{u} \sim (1, 3, 1); \quad \chi^{R}_{d} \sim (1, 1, 3)$ 

It is very simple:

- a square matrix built out of 2 vectors

$$\begin{pmatrix} d \\ e \\ f \\ \vdots \end{pmatrix} (a, b, c \dots)$$

has only one non-vanishing eigenvalue



strong mass hierarchy at leading order: -- only 1 heavy "up" quark -- only 1 heavy "down" quark

## only $|\chi|$ 's relevant for scale

### Minimum of the Potential

Dimension 6 Yukawa Operator

The invariants are:

$$\begin{split} \chi_u^{L\dagger} \chi_u^L, & \chi_u^{R\dagger} \chi_u^R, & \chi_d^{L\dagger} \chi_d^L, \\ \chi_d^{R\dagger} \chi_d^R, & \chi_u^{L\dagger} \chi_d^L = \left| \chi_u^L \right| \left| \chi_d^L \right| \cos \theta_c \,. \end{split}$$





 $\theta_{c}$  is the angle between up and down L vectors

### Minimum of the Potential

Dimension 6 Yukawa Operator

The invariants are:

$$\begin{split} \chi_u^{L\dagger} \chi_u^L, & \chi_u^{R\dagger} \chi_u^R, & \chi_d^{L\dagger} \chi_d^L, \\ \chi_d^{R\dagger} \chi_d^R, & \chi_u^{L\dagger} \chi_d^L = \left| \chi_u^L \right| \left| \chi_d^L \right| \cos \theta_c \,. \end{split}$$



We can fit the angle and the masses in the Potential; as an example:

$$V' = \lambda_u \left( \chi_u^{L\dagger} \chi_u^L - \frac{\mu_u^2}{2\lambda_u} \right)^2 + \lambda_d \left( \chi_d^{L\dagger} \chi_d^L - \frac{\mu_d^2}{2\lambda_d} \right)^2 + \lambda_{ud} \left( \chi_u^{L\dagger} \chi_d^L - \frac{\mu_{ud}^2}{2\lambda_{ud}} \right)^2 + \cdots$$

Whose minimum sets (2 generations):

$$y_c^2 = \frac{\mu_u^2}{2\lambda_u \Lambda_f^2} \quad y_s^2 = \frac{\mu_d^2}{2\lambda_d \Lambda_f^2} \quad \cos\theta = \frac{\mu_{ud}^2 \sqrt{\lambda_u \lambda_d}}{\mu_u \mu_d \lambda_{ud}}$$

#### **Towards a realistic 3 family spectrum**

e.g. replicas of 
$$\chi^L$$
 ,  $\chi^R_u$  ,  $\chi^R_d$ 

???

## Towards a realistic 3 family spectrum Combining fundamentals and bi-fundamentals

i.e. combining d=5 and d =6 Yukawa operators

$$\Sigma_u \sim (3,\overline{3},1) , \qquad \Sigma_d \sim (3,1,\overline{3}) , \qquad \Sigma_R \sim (1,3,\overline{3}) ,$$
$$\chi_u^L \in (3,1,1) , \qquad \chi_u^R \in (1,3,1) , \qquad \chi_d^L \in (3,1,1) , \qquad \chi_d^R \in (1,1,3) .$$

The Yukawa Lagrangian up to the second order in  $1/\Lambda_f$  is given by:

$$\mathscr{L}_{Y} = \overline{Q}_{L} \left[ \frac{\Sigma_{d}}{\Lambda_{f}} + \frac{\chi_{d}^{L} \chi_{d}^{R\dagger}}{\Lambda_{f}^{2}} \right] D_{R}H + \overline{Q}_{L} \left[ \frac{\Sigma_{u}}{\Lambda_{f}} + \frac{\chi_{u}^{L} \chi_{u}^{R\dagger}}{\Lambda_{f}^{2}} \right] U_{R}\tilde{H} + \text{h.c.} ,$$

\* At leading (renormalizable) order:

$$Y_{u} \equiv \frac{\langle \Sigma_{u} \rangle}{\Lambda_{f}} + \frac{\langle \chi_{u}^{L} \rangle \langle \chi_{u}^{R\dagger} \rangle}{\Lambda_{f}^{2}} = \begin{pmatrix} 0 & \sin \theta_{c} y_{c} & 0 \\ 0 & \cos \theta_{c} y_{c} & 0 \\ 0 & 0 & y_{t} \end{pmatrix},$$
$$Y_{d} \equiv \frac{\langle \Sigma_{d} \rangle}{\Lambda_{f}} + \frac{\langle \chi_{d}^{L} \rangle \langle \chi_{d}^{R\dagger} \rangle}{\Lambda_{f}^{2}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_{s} & 0 \\ 0 & 0 & y_{b} \end{pmatrix}.$$

#### without unnatural fine-tunings

\* The masses of the first family and the other angles from nonrenormalizable terms or replicas ?

#### **In summary**

\* We constructed the general scalar potential for MFV

\* The flavor symmetry imposes strong restrictions on the MFV potential: just a few invariants allowed at the renormalizable and non-renormalizable level. In general, to obtain realistic masses and mixing requires strong fine-tuning.

\* Flavons in the fundamental are tantalizing (Y ~  $<\chi^2 > /\Lambda_f^2$ ), providing naturally:

-strong mass hierarchy

- non-trivial mixing

## **Leptonic Minimal Flavour Violation**

T. Hambye, D. Hernández, P. Hernández, MBG

A rationale for the MFV ansatz?

- Flavour data (i.e. B physics) consistent with all flavour physics coming from Yukawa
- Inspired in "condensate" flavour physics a la Froggat-Nielsen (Yukawas ~  $\langle \Psi \Psi \rangle^n / \Lambda_f^n$ , rather than in susy-like options

•It makes you think on the relation between scales: electroweak vs. flavour vs lepton number scales
## What happens in the presence of neutrino masses?

Cirigliano, Isidori, Grinstein, Wise



## The Seesaw models

 Three types of models yield the Weinberg operator at tree level



# Seesaws are favorite lepton number theories

## **Are they MFV theories?**

## First condition for it:

#### \* $\Lambda_f$ must be ~O (TeV), to have observable effects

M~1 TeV is suggested by electroweak hierarchy problem

$$\begin{split} \int_{-\frac{\mu_{\Delta}^2}{2\pi^2}} & \delta m_H^2 = -3\frac{\lambda_3}{16\pi^2} \left[\Lambda^2 + M_{\Delta}^2 \left(\log\frac{M_{\Delta}^2}{\Lambda^2} - 1\right)\right] \\ & -\frac{\mu_{\Delta}^2}{2\pi^2} \log\left(\left|\frac{M_{\Delta}^2 - \Lambda^2}{M_{\Delta}^2}\right|\right) \end{split}$$



N

- -



 $\int m_{H}^{2} = -3 \frac{Y_{\Sigma}^{\dagger} Y_{\Sigma}}{16\pi^{2}} \left[ 2\Lambda^{2} + 2M_{\Sigma}^{2} \log \frac{M_{\Sigma}^{2}}{\Lambda^{2}} \right]$ 

(Abada, Biggio, Bonnet, Hambye, M.B.G.)

# First condition for it: to separate the effective lepton number scale $\Lambda_{LN}$ from the flavour scale $\Lambda_f$

\*  $\Lambda_f$  must be ~O (TeV), to have observable effects

\*  $\Lambda_{LN}$  effective ~O (Grand Unif.) for tiny  $m_V$ 

### First condition for it: to separate the effective lepton number scale $\Lambda_{LN}$ from the flavour scale $\Lambda_f$

\*  $\Lambda_f$  must be ~O (TeV)  $\leftarrow$  d=6 operators

\*  $\Lambda_{LN}$  effective ~O (Grand Unif.) for tiny m<sub>v</sub>  $\leftarrow$  d=5 op. (Weinberg)

# Could d=6 be stronger than d=5?

\* Two independent scales in d=5, d=6 may result from a <u>symmetry principle: lepton number</u>

Cirigliano et al; Kersten, Smirnov; Abada et al

- \* d=5 requires to violate lepton number
- \* d=6 does not violate any symmetry

$$\begin{split} & \Lambda_{\text{LN}} >> \Lambda_{\text{fl}} \sim \text{TeV} ? \\ & \mathcal{L} = \mathcal{L}_{SM} + \sum_{i} \frac{\alpha}{\Lambda_{LN}} O_i^{d=5} + \sum_{i} \frac{\beta_i}{\Lambda_{\text{flavour}}^2} O_i^{d=6} + \dots \\ & \text{Girigliano, et al} \end{split}$$

There is a sensible physics motivation:

Origin of lepton/quark flavour violation linked/close to the EW scale

>(Effective) Lepton number breaking scale higher and responsible for the gap between v and other fermion

What happens in the presence of neutrino masses? Cirigliano, Isidori, Grinstein, Wise

In the lepton sector



Delicate:

\* Majorana masses are model dependent :  $c^{d=5}(Y_e,?), c^{d=6}(Y_e,?)$ 

\* Requires to separate lepton number from flavour origin

# A successful model: Scalar-triplet Seesaw (type II)



 $\mathscr{L}_{\Delta} = \cdots + (D_{\mu}\Delta)^{\dagger}(D^{\mu}\Delta) - \mathsf{M}_{\Lambda}^{2}\Delta^{\dagger}\Delta + +$  $+ Y^{\alpha\beta}_{\Lambda}\overline{\widetilde{L}}(\tau \cdot \Delta) L + \mu_{\Delta}\widetilde{\phi}^{\dagger}(\tau \cdot \Delta)^{\dagger}\phi + \dots$ 

# A successful model: Scalar-triplet Seesaw (type II)



 $\mathscr{L}_{\Delta} = \dots + (D_{\mu}\Delta)^{\dagger}(D^{\mu}\Delta) - \mathsf{M}_{\Delta}^{2}\Delta^{\dagger}\Delta + +$  $+ \Upsilon_{\Delta}^{\alpha\beta}\overline{\widetilde{\mathsf{L}}}(\tau \cdot \Delta)\mathsf{L} + \mu_{\Delta}\widetilde{\phi}^{\dagger}(\tau \cdot \Delta)^{\dagger}\phi + \dots$  Correlations among weak processes, i.e.

 $\mu \longrightarrow e\gamma / \tau \longrightarrow e\gamma / \tau \longrightarrow \mu\gamma$ 

- \* Neutrino masses OK
- \* Measurable flavour OK
- \* Predictivity OK



V. Cirigliano, B. Grinstein, G. Isidori, M. Wise, hep-ph/050700 M. B. Gavela, T. Hambye, P. Hernández, D.H., 0906.1

## An unsuccessful model: simplest type I

Standard Seesaw (Type I) doesn't work

 $\mathscr{L} = \cdots - \mathbf{Y}_{N} \bar{N} \phi^{\dagger} L_{L} - \Lambda_{LN} \bar{N}^{c} N \ldots$ 



• Neutrino masses: Ok.  $m_v \propto Y_N^T \frac{1}{\Lambda_{LN}} Y_N$ 

• Measurable flavour: NOT OK!.  $\Lambda_{fl} \equiv \Lambda_{LN}$ 

• Predictivity: More or less Ok.  $c_{d=5} \propto c_{d=6}$  if no CP Hambye, Hernandez<sup>2</sup>, Gavela

One more mediator, one more scale.... i.e. Inverse seesaws

Instead of 
$$\mathcal{L}_m = \begin{pmatrix} 0 & \mathbf{Y}_N^{\mathsf{T}} \mathbf{v} \\ \mathbf{Y}_N & \mathbf{W}_N \end{pmatrix}$$

One more mediator, one more scale.... i.e. Inverse seesaws

$$\mathcal{L}_{i} \qquad \bar{\mathcal{N}}^{c} \qquad \bar{\mathcal{N}}^{\prime c} \\ \mathcal{L}_{i} \qquad \bar{\mathcal{N}}^{c} \qquad \bar{\mathcal{N}}^{\prime c} \\ \begin{pmatrix} 0 & \boldsymbol{Y}_{N}^{T} \boldsymbol{v} & \boldsymbol{0} \\ \boldsymbol{Y}_{N} \boldsymbol{v} & \boldsymbol{0} & \boldsymbol{\Lambda}^{T} \\ \boldsymbol{0} & \boldsymbol{\Lambda} & \boldsymbol{0} \end{pmatrix}$$

One more mediator, one more scale.... i.e. Inverse seesaws

$$\mathcal{L}_{M_{\nu}} = \begin{pmatrix} \bar{L}_{i} & \bar{N^{c}} & \bar{N^{\prime c}} \\ 0 & \boldsymbol{Y}_{N}^{T} \boldsymbol{v} & \boldsymbol{0} \\ \boldsymbol{Y}_{N} \boldsymbol{v} & \boldsymbol{0} & \boldsymbol{\Lambda}^{T} \\ 0 & \boldsymbol{\Lambda} & \boldsymbol{0} \end{pmatrix}$$



One more mediator, one more scale.... i.e. Inverse seesaws

$$\mathcal{L}_{M_{\nu}} = \begin{pmatrix} \bar{L}_{i} & \bar{N^{c}} & \bar{N^{\prime c}} \\ 0 & \boldsymbol{Y}_{N}^{T} \boldsymbol{v} & 0 \\ \boldsymbol{Y}_{N} \boldsymbol{v} & 0 & \boldsymbol{\Lambda}^{T} \\ 0 & \boldsymbol{\Lambda} & 0 \end{pmatrix}$$

Lepton number conserved.

 $U(1)_{LN}$ 

$$\begin{array}{|c|c|c|c|c|} \Lambda_{\mathsf{fl}} = \Lambda & & & & & & \\ \Lambda_{\mathsf{LN}} = \infty & & & & & & & \\ \Lambda_{\mathsf{LN}}^{\mathsf{d}=6} & & & & & & & & \\ \end{array}$$

One more mediator, one more scale.... i.e. Inverse seesaws

$$\mathscr{L}_{M_{\nu}} = \begin{pmatrix} \bar{L}_{i} & \bar{N^{c}} & \bar{N^{\prime c}} \\ 0 & \boldsymbol{Y}_{N}^{T}\boldsymbol{v} & \boldsymbol{Y}_{N}^{\prime T}\boldsymbol{v} \\ \boldsymbol{Y}_{N}\boldsymbol{v} & \boldsymbol{\mu}^{\prime} & \boldsymbol{\Lambda}^{T} \\ \boldsymbol{Y}_{N}^{\prime}\boldsymbol{v} & \boldsymbol{\Lambda} & \boldsymbol{\mu} \end{pmatrix}$$

Lepton number violated. by any of those 3 entries

#### $\Lambda$ may be ~ TeV and Ys ~1, and be ok with m<sub>v</sub>

Wyler, Wolfenstein; Mohapatra, Valle, Branco, Grimus, Lavoura, Malinsky, Romao...

Small parameters ( $\mu$ ,  $\mu'$ , Y') unpleasant?

•They are technically natural

• There exist UV completions with only high scales (Bonnet, Hernandez(D), Ota, Winter 09)

Three light active families + one  $N_R$  + one  $N_R$ 

$$\mathscr{L}_{M_{\nu}} = \begin{pmatrix} \bar{L}_{i} & \bar{N^{c}} & \bar{N^{\prime c}} \\ 0 & \boldsymbol{Y}_{N}^{T}\boldsymbol{v} & \boldsymbol{Y}_{N}^{\prime T}\boldsymbol{v} \\ \boldsymbol{Y}_{N}\boldsymbol{v} & \boldsymbol{\mu}' & \boldsymbol{\Lambda}^{T} \\ \boldsymbol{Y}_{N}'\boldsymbol{v} & \boldsymbol{\Lambda} & \boldsymbol{\mu} \end{pmatrix}$$

 $\mu$  is irrelevant (at tree-level)

- -- one massless neutrino
- -- just one low-energy Majorana phase

arguably the símplest model of neutríno mass

Three light active families + one  $N_R$  + one  $N_R$ 

$$\mathscr{L}_{M_{\nu}} = \begin{pmatrix} \bar{L}_{i} & \bar{N^{c}} & \bar{N^{\prime c}} \\ 0 & \boldsymbol{Y}_{N}^{T}\boldsymbol{v} & \boldsymbol{Y}_{N}^{\prime T}\boldsymbol{v} \\ \boldsymbol{Y}_{N}\boldsymbol{v} & \boldsymbol{\mu}' & \boldsymbol{\Lambda}^{T} \\ \boldsymbol{Y}_{N}'\boldsymbol{v} & \boldsymbol{\Lambda} & \boldsymbol{\mu} \end{pmatrix}$$

 $\mu$  is irrelevant (at tree-level)

$$\mathbf{m}_{\mathbf{v}} \sim c_{\alpha\beta}^{d=5} = \left( Y_N^{\prime T} \frac{1}{\Lambda^T} Y_N + Y_N^T \frac{1}{\Lambda} Y_N^{\prime} \right)_{\alpha\beta} - \left( Y_N^T \frac{1}{\Lambda} \mu \frac{1}{\Lambda^T} Y_N \right)_{\alpha\beta}$$
  
ur effects ~  $c_{\alpha\beta}^{d=6} = \left( Y_N^{\dagger} \frac{1}{\Lambda^T} Y_N \right)_{\alpha\beta} + \cdots$ 

**flavour effects** ~ 
$$c_{\alpha\beta}^{d=6} \equiv \left(Y_N^{\dagger} \frac{1}{\Lambda^{\dagger}\Lambda} Y_N\right)_{\alpha\beta} + \cdots$$

Three light active families + one  $N_R$  + one  $N_R$ 

$$\mathscr{L}_{M_{\nu}} = egin{pmatrix} ar{L}_i & ar{N^c} & ar{N^{\prime c}} \ 0 & oldsymbol{Y}_N^T oldsymbol{v} & oldsymbol{Y}_N^{\prime T} oldsymbol{v} \ oldsymbol{Y}_N oldsymbol{v} & \mu^\prime & oldsymbol{\Lambda}^T \ oldsymbol{Y}_N oldsymbol{v} & \mu^\prime & oldsymbol{\Lambda}^T \ oldsymbol{Y}_N^\prime oldsymbol{v} & oldsymbol{\Lambda} & \mu \end{pmatrix}$$

 $\mu$  is irrelevant (at tree-level)

#### FUNDAMENTAL

	moduli	phases	
$Y_N$	3	3	
$Y'_N$	3	3	
٨	1	1	

#### LOW ENERGY

- 3 angles and 2 phases in the U<sub>PMNS</sub>
- 2 masses and 0 phases in M<sub>v</sub>
- 2 overall factors and 5 phases absorbed.

#### \*Yukawas are completely determined from $U_{PMNS}+m_{v}$ , except for a

VS

normalization + a degeneracy in the Majorana phase

Three light active families + one  $N_R$  + one  $N_R$ 

$$\mathscr{L}_{M_{\nu}} = egin{pmatrix} ar{L}_i & ar{N^c} & ar{N^{\prime c}} \ 0 & oldsymbol{Y}_N^T oldsymbol{v} & oldsymbol{Y}_N^T oldsymbol{v} \ oldsymbol{Y}_N oldsymbol{v} & \mu' & oldsymbol{\Lambda}^T \ oldsymbol{Y}_N oldsymbol{v} & \mu' & oldsymbol{\Lambda}^T \ oldsymbol{Y}_N' oldsymbol{v} & oldsymbol{\Lambda} & \mu \end{pmatrix}$$

 $\mu^{\prime}$  is irrelevant (at tree-level)

$$\begin{split} \text{i.e.} & Y_N^T \simeq y \begin{pmatrix} e^{i\delta}s_{13} + e^{-i\alpha}s_{12}r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha}r^{1/4}c_{12}c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha}r^{1/4}c_{12}s_{23} \end{pmatrix} \qquad r = \frac{|\Delta m_{12}^2|}{|\Delta m_{13}^2|} \\ \end{split}$$
 Normal hierarchy

\*Yukawas are completely determined from  $U_{PMNS}+m_V$ , except for a normalization + a degeneracy in the Majorana phase (+ 1 phase if  $\mu$  present)

## Normal hierarchy:

Up to terms of  $\mathcal{O}(\sqrt{r}, s_{13})$ , we find

$$\begin{split} \sqrt{r}, s_{13}), \text{ we find} & r = \frac{|\Delta m_{12}^2|}{|\Delta m_{13}^2|} \\ Y_N^T &\simeq y \begin{pmatrix} e^{i\delta}s_{13} + e^{-i\alpha}s_{12}r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha}r^{1/4}c_{12}c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha}r^{1/4}c_{12}s_{23} \end{pmatrix} . \end{split}$$

## **Inverted hierarchy:**

$$Y_N^T \simeq \frac{y}{\sqrt{2}} \left( \begin{array}{c} c_{12} e^{i\alpha} + s_{12} e^{-i\alpha} \\ c_{12} \left( c_{23} e^{-i\alpha} - s_{23} s_{13} e^{i(\alpha-\delta)} \right) - s_{12} \left( c_{23} e^{i\alpha} + s_{23} s_{13} e^{-i(\alpha+\delta)} \right) \\ -c_{12} \left( s_{23} e^{-i\alpha} + c_{23} s_{13} e^{i(\alpha-\delta)} \right) + s_{12} \left( s_{23} e^{i\alpha} - c_{23} s_{13} e^{-i(\alpha+\delta)} \right) \end{array} \right)$$



#### Degeneracy in the Majorana phase $\alpha$



Figure 3: Left: Ratio  $B_{e\mu}/B_{e\tau}$  for the normal hierarchy (solid) and the inverse hierarchy (dashed) as a function of  $\alpha$  for  $(\delta, s_{13}) = (0, 0.2)$ . Right: the same for the ratio  $B_{e\mu}/B_{\mu\tau}$ .



Figure 5:  $m_{ee}$  as a function of  $\alpha$  for the normal (solid) and inverted (dashed) hierarchies, for  $(\delta, s_{13}) = (0, 0.2)$ .



(Alonso, Gavela, Hernandez, Li ... ongoing)

#### \* It is a fermionic seesaw: bounds on non-unitarity apply

At present, best bound on Y<sup>2</sup> v<sup>2</sup>/M<sup>2</sup> < 10<sup>-4</sup> is from  $\mu$  --> e  $\gamma$ 

$$B_{\mu
ightarrow e\gamma} \propto |Y_{N_e}Y_{N_\mu}|^2$$

i.e. 
$$Y_N^T \simeq y \begin{pmatrix} e^{i\delta}s_{13} + e^{-i\alpha}s_{12}r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha}r^{1/4}c_{12}c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha}r^{1/4}c_{12}s_{23} \end{pmatrix} \qquad r = \frac{|\Delta m_{12}^2|}{|\Delta m_{13}^2|}$$
Normal hierarchy

 $\mu$ -e conversion being computed now

#### But in some regions the mu-tau sector can be stronger





Main Injector Non Standard InteractionS

Minsis is a project for a short baseline experiment in Fermilab



That would look for  $\nu_{\mu}$  dissappearance

$$v_{\mu} \rightarrow v_{\tau}$$

## Normal hierarchy: MINSIS and MFV

If we explore a wider range of parameters



We find that there are regions where an experiment as MINSIS would improve the present bounds on our Model

(Alonso, Gavela, Hernandez, Li ... ongoing)

#### **Ongoing:**

#### \* Bounds on non-unitarity apply

μ --> **e** γ

(Alonso, Gavela, Hernandez, Li ... ongoing)

- \* Impact on  $\nu_{\mu} \longrightarrow \nu_{\tau}$  can be very important
- \* Leptogenesis possible (Blanchet, Hambye, Josse-Michaux 09)

The Yukawas themselves are flavour vectors here!!

## Normal hierarchy:

Up to terms of  $\mathcal{O}(\sqrt{r}, s_{13})$ , we find

$$\begin{split} \sqrt{r}, s_{13}), \text{ we find} & r = \frac{|\Delta m_{12}^2|}{|\Delta m_{13}^2|} \\ Y_N^T &\simeq y \begin{pmatrix} e^{i\delta}s_{13} + e^{-i\alpha}s_{12}r^{1/4} \\ s_{23}\left(1 - \frac{\sqrt{r}}{2}\right) + e^{-i\alpha}r^{1/4}c_{12}c_{23} \\ c_{23}\left(1 - \frac{\sqrt{r}}{2}\right) - e^{-i\alpha}r^{1/4}c_{12}s_{23} \end{pmatrix} . \end{split}$$

## **Inverted hierarchy:**

$$Y_N^T \simeq \frac{y}{\sqrt{2}} \left( \begin{array}{c} c_{12} e^{i\alpha} + s_{12} e^{-i\alpha} \\ c_{12} \left( c_{23} e^{-i\alpha} - s_{23} s_{13} e^{i(\alpha-\delta)} \right) - s_{12} \left( c_{23} e^{i\alpha} + s_{23} s_{13} e^{-i(\alpha+\delta)} \right) \\ -c_{12} \left( s_{23} e^{-i\alpha} + c_{23} s_{13} e^{i(\alpha-\delta)} \right) + s_{12} \left( s_{23} e^{i\alpha} - c_{23} s_{13} e^{-i(\alpha+\delta)} \right) \end{array} \right)$$

The Yukawas themselves are flavour vectors here!!

Is this model is automatically of the "fundamental" ( $\chi$ ) type....

... with  $2 \chi$  replicas ??
# What is the scalar potential of MFV including Majorana Vs?

- Work ongoing right now

- It should allow an answer to the question of whether leptonic mixing differs from quark mixing because of the different nature of mass

### Conclusions

#### 1) Scalar Potential for MFV

-- We constructed it for quarks and explored the minima

-- Quark masses and mixings difficult to accomodate Scalar fields in the fundamental induce naturally: strong quark mass hierarchy + mixing !

#### 2) MFV vs Seesaw

 Scalar mediated Seesaw ("type II") is automatically MFV
 Fermionic Seesaws (I and III) are NOT in minimal version but with approximate U(1)<sub>LN</sub> - e.g. inverse seesawsthey can be MFV
 We found maybe the simplest model of neutrino masses: add just 2 heavy right-handed neutrinos to SM: extremely predictiv, and it is MFV
 and Yukawas are in the fundamental of the flavour group
 We are exploring the leptonic MFV scalar potential

### Back-up slides

**Y** --> quadratic in fields  $\chi$ 

#### Fundamental Fields

Dimension 6 Yukawa Operator

It holds also for 3 families: one heavy "up", one heavy "down", one angle

$$Y_D = \frac{\langle \chi_d^L \chi_d^{R\dagger} \rangle}{\Lambda_f^2} \qquad Y_U = \frac{\langle \chi_u^L \chi_u^{R\dagger} \rangle}{\Lambda_f^2}$$

The Yukawas are composed of two 'vectors'. Such a structure has only one eigenvalue, one mass. This fact becomes evident when rotating the v.e.v.s of the fields to the form:

$$\begin{split} V_L^{\dagger} Y_D V_{D_R} &= \frac{|\chi_d^L| |\chi_d^R|}{\Lambda^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ V_L^{\dagger} Y_U V_{U_R} &= \frac{|\chi_u^L| |\chi_u^R|}{\Lambda_f^2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{split}$$

This means a hierarchy among the masses and an angle only by construction! already at renormalizable level

In fact, it usually assumes more, e.g. top dominance:

$$\begin{bmatrix} Y^{u}(Y^{u})^{\dagger} \end{bmatrix}_{i \neq j}^{n} \approx y_{t}^{2n}V_{ti}^{*}V_{tj}$$

$$\longrightarrow \qquad \mathcal{A}(d^{i} \to d^{j})_{\mathrm{MFV}} = (V_{ti}^{*}V_{tj}) \ \mathcal{A}_{\mathrm{SM}}^{(\Delta F=1)} \left[1 + a_{1}\frac{16\pi^{2}M_{W}^{2}}{\Lambda^{2}}\right]$$

$$\longrightarrow \qquad \mathrm{d-d} \sim \mathrm{s-d} \sim \mathrm{b-s} \quad \mathrm{transitions of} \sim \mathrm{equal strength}$$

while it may not be so...

for instance for SM+ 2 Higgses and Z<sub>3</sub>. light quarks dominate (Branco, Grimus, Lavoura)

All this underlies the importance of searching for  $v_{\mu} \leftrightarrow v_{\tau}$  transitions or NSI involving those flavours in general (i.e. at near detectors) "Why not" NSIs (Non-Seesaw NSIs)

## i.e., purely matter NSI?

Extra effects in matter propagation



$$\mathscr{L}_{\rm NSI} \propto -\epsilon^\ell_{lphaeta} (ar{
u}^lpha \gamma^
ho {\cal P}_L 
u_eta) (ar{\ell} \gamma_
ho \ell)$$

$$\mathcal{H}_{F} = U \begin{pmatrix} 0 & & \\ & \frac{\Delta m_{21}^{2}}{2E} & \\ & & \frac{\Delta m_{31}^{2}}{2E} \end{pmatrix} U^{\dagger} + V \begin{pmatrix} 1 + \epsilon_{ee}^{m} & \epsilon_{e\mu}^{m} & \epsilon_{e\tau}^{m} \\ \epsilon_{\mu e}^{m} & \epsilon_{\mu\mu}^{m} & \epsilon_{\mu\tau}^{m} \\ \epsilon_{\tau e}^{m} & \epsilon_{\tau\mu}^{m} & \epsilon_{\tau\tau}^{m} \end{pmatrix}$$
$$V \propto N_{e}$$

## i.e., purely matter NSI?

Extra effects in matter propagation



$$\mathscr{L}_{_{\mathrm{NSI}}} \propto -\epsilon^{\ell}_{lphaeta} (ar{
u}^{lpha} \gamma^{
ho} P_L 
u_{eta}) \, (ar{\mathrm{q}} \, \gamma_{\mu} \mathrm{q})$$

$$\mathscr{H}_{F} = U \begin{pmatrix} 0 & & \\ & \frac{\Delta m_{21}^{2}}{2E} & \\ & & \frac{\Delta m_{31}^{2}}{2E} \end{pmatrix} U^{\dagger} + V \begin{pmatrix} 1 + \epsilon_{ee}^{m} & \epsilon_{e\mu}^{m} & \epsilon_{e\tau}^{m} \\ \epsilon_{\mu e}^{m} & \epsilon_{\mu\mu}^{m} & \epsilon_{\mu\tau}^{m} \\ \epsilon_{\tau e}^{m} & \epsilon_{\tau\mu}^{m} & \epsilon_{\tau\tau}^{m} \end{pmatrix}$$
$$V \propto N_{e}$$

### BOUNDS

\*Absolute maxima:  

$$\begin{aligned} & \left| \varepsilon_{\alpha\beta}^{\oplus} \right| < \begin{pmatrix} 4.2 & 0.33 & 3.0 \\ 0.33 & 0.068 & 0.33 \\ 3.0 & 0.33 & 21 \end{pmatrix} & \text{from $v$ scattering} \\ & \text{in NuTev} \\ & \text{and in CHARM II} \end{aligned}$$

C. Biggio, M. Blennow, E. Fdez-Mtnez, 0907.0097

### BOUNDS



C. Biggio, M. Blennow, E. Fdez-Mtnez, 0907.0097

•Also from atmospheric data, unless cancellations among epsilons:

$$|\varepsilon_{\mu\tau}| < 5 \ 10^{-2}$$

Fornengo, Maltoni, Tomás-Bayo, Valle, hep-ph 0108043 Gonzalez-García, Maltoni, Phys. Ret. 460, 2008

#### **Potential Trouble:**

NSI <sup>SU(2) x U(1)</sup> gauge invariance Dangerous four charged lepton couplings

## • Gauge invariance $(SU(3) \times SU(2) \times U(1))$ $\frac{1}{\Lambda^2} (\bar{\nu}^e \gamma^\rho P_L \nu^\mu) (\bar{e}_L \gamma_\rho e_L) \rightarrow \frac{1}{\Lambda^2} (\bar{L}^e \gamma^\rho L^\mu) (\bar{L}_e \gamma_\rho L_e)$

Trouble, for instance  $\mu \rightarrow eee$ 



 $\mu \rightarrow eee$ 

Systematical analysis

#### Two possibilities

A) There could be NO lepton charged processes involved Ex: For d = 6

(Davidson, Kuypers) 
$$(\bar{L^c}i\tau^2 L)(\bar{L}i\tau^2 L^c) \rightarrow (\bar{\nu_{\tau}^c} e_L)(\bar{\nu_{\mu}}e_L^c)$$

Ex: For d = 8

$$O_{\rm NSI} = (\bar{L}H)\gamma^{\mu}(H^{\dagger}L)(\bar{E}\gamma_{\mu}E) \rightarrow v^{2}(\bar{\nu}_{U}\gamma^{\mu}\nu^{\mu})(\bar{e}_{L}\gamma_{\mu}e)$$

(Berezhiani, Rossi)

Systematical analysis

#### Two possibilities

A) There could be NO lepton charged processes involved Ex: For d = 6

(Davidson, Kuypers) 
$$(\bar{L}^c i \tau^2 L) (\bar{L} i \tau^2 L^c) \rightarrow (\bar{\nu}_{\tau}^c e_L) (\bar{\nu}_{\mu} e_L^c)$$
  
But it also produces  $\tau \rightarrow \mu v_e \bar{\nu}_e$  !  
Ex: For  $d = 8$  And  $\mu \rightarrow e v_{\tau} \bar{\nu}_e$   $\epsilon_{\mu\tau} < 3 \ 10^{-2}$   
 $O_{\rm NSI} = (\bar{L}H) \gamma^{\mu} (H^{\dagger}L) (\bar{E} \gamma_{\mu} E) \rightarrow v^2 (\bar{\nu}_{\tau} \gamma^{\mu} \nu^{\mu}) (\bar{e}_L \gamma_{\mu} e)$  Fdez-Martinez

(Berezhiani, Rossi)

Systematical analysis

### Two possibilities

B) In general, "fine tune" some of them to obtain desired suppression

Ex:

$$\mathcal{L}_{\rm eff} = \frac{\mathcal{C}^1}{\Lambda^2} (\bar{L}^e \gamma^\rho L^\mu) (\bar{L}^e \gamma_\rho L^\mu) + \frac{\mathcal{C}^3}{\Lambda^2} (\bar{L}^e \gamma^\rho \vec{\tau} L^\mu) (\bar{L}^e \gamma_\rho \vec{\tau} L^\mu)$$

We can avoid charged lepton interactions  $(\bar{e}_L \gamma^{\mu} P_L \mu)(\bar{e} \gamma_{\mu} P_L e)$ if

$$\mathcal{C}^{1}+\mathcal{C}^{3}\simeq \mathbf{0}$$

## ALL cancellation conditions examined

(Antusch, Baunman, Fdez-Martinez; D. Hernandez, Ota, Winter + MBG)

Bottom line: d = 6 doesn't look promising

Maybe things get better at d = 8

Is it possible to generate d = 8 and NO d = 6 operators??

Several possibilities



Require at least 2 new fields ( and unrelated to seesaw)

(Antusch, Baunman, Fdez-Martinez;

D. Hernandez, Ota, Winter + MBG )

Bottom line: d = 6 doesn't look promising

Maybe things get better at d = 8

Is it possible to generate d = 8 and NO d = 6 operators??

Several possibilities



#### Complete list of d=8 operators and their mediators

#	Dim. eight operator	Clen	Clen	O <sub>NSI</sub> ?	Mediators
Combination LL					
1	$(\bar{L}\gamma^{\rho}L)(\bar{E}\gamma_{\rho}E)(H^{\dagger}H)$	1			18
2	$(L\gamma^{\mu}L)(EH^{\dagger})(\gamma_{\mu})(HE)$	1			$1_0^v + 2_{-3/2}^{L/R}$
3	$(\bar{L}\gamma^{\rho}L)(\bar{E}H^{T})(\gamma_{\rho})(H^{*}E)$	1			$1_0^v + 2_{-1/2}^{t/h}$
4	$(\bar{L}\gamma^{\rho}\vec{\tau}L)(\bar{E}\gamma_{\rho}E)(H^{\dagger}\vec{\tau}H)$		1		35 + 15
5	$(L\gamma^{\rho} \vec{\tau} L)(EH^{\dagger})(\gamma_{\rho} \vec{\tau})(HE)$		1		$35 + 2^{L/R}_{-3/2}$
6	$(\bar{L}\gamma^{\rho}\vec{\tau}L)(\bar{E}H^T)(\gamma_{\rho}\vec{\tau})(H^*E)$		1		$3_0^v + 2_{-1/2}^{L/R}$
Combination EL					
7	$(\overline{L}E)(\overline{E}L)(H^{\dagger}H)$	-1/2			2 <sup>*</sup> +1/2
8	$(\overline{L}E)(\vec{\tau})(\overline{E}L)(H^{\dagger}\vec{\tau}H)$		-1/9		2 <sup>#</sup> +1/2
9	$(LH)(H^{\dagger}E)(\bar{E}L)$	-1/4	-1/4	1	$2_{\pm 1/2}^s + 1_0^n + 2_{\pm 1/2}^n$
10	$(\overline{L}TH)(H^{\dagger}E)(d)(\overline{E}L)$	-3/4	1/4		$\frac{2!}{-1/2} + \frac{2!}{0} + \frac{2!}{-1/2}$
11	$(\bar{L}i\tau^{2}H^{*})(H^{T}E)(i\tau^{2})(\bar{E}L)$	1/4	-1/4		$2_{\pm 1/2}^{*} + 1_{-1}^{L/R} + 2_{-3/2}^{L/R}$
12	$(\overline{L}\vec{\tau}i\tau^2H^*)(H^TE)(i\tau^2\vec{\tau})(\overline{E}L)$	3/4	1/4		$2_{\pm 1/2}^s + 3_{\pm 1}^{L/R} + 2_{\pm 3/2}^{L/R}$
Combination $\overline{E^c}L$					
13	$(\overline{L}\gamma^{\rho}E^{c})(\overline{E^{c}}\gamma_{\rho}L)(H^{\dagger}H)$	-1			2°_3/2
14	$(\overline{L}\gamma^{\rho}E^{c})(\vec{\tau})(\overline{E^{c}}\gamma_{\rho}L)(H^{\dagger}\vec{\tau}H)$		$^{-1}$		2° 3/2
15	$(\overline{L}H)(\gamma^{\rho})(H^{\dagger}E^{c})(\overline{E^{c}}\gamma_{\rho}L)$	-1/2	-1/2	1	$2^{\circ}_{-3/2} + 1^{R}_{0} + 2^{L/R}_{+3/2}$
16	$(\overline{L}\overrightarrow{\tau}H)(\gamma^{\rho})(H^{\dagger}E^{c})(\overrightarrow{\tau})(E^{c}\gamma_{\rho}L)$	-3/2	1/2		$2_{-3/2}^{\nu} + 3_{0}^{L/R} + 2_{+3/2}^{L/R}$
17	$(\overline{L}i\tau^2 H^*)(\gamma^{\rho})(H^T E^c)(i\tau^2)(E^c\gamma_{\rho}L)$	-1/2	1/2		$2^{\nu}_{-3/2} + 1^{L/R}_{-1} + 2^{L/R}_{+1/2}$
18	$(L\vec{\tau}i\tau^2 H^*)(\gamma^{\rho})(H^T E^c)(i\tau^2\vec{\tau})(E^c\gamma_{\rho}L)$	-3/2	-1/2		$2^{u}_{-3/2} + 3^{L/R}_{-1} + 2^{L/R}_{+1/2}$
Combination $H^{\dagger}L$					
19	$(\overline{L}E)(\overline{E}H)(H^{\dagger}L)$	-1/4	-1/4	1	$2_{\pm 1/2}^{\prime} + 1_{0}^{R} + 2_{\pm 1/2}^{L/R}$
20	$(\overline{L}E)(\overrightarrow{\tau})(\overline{E}H)(H^{\dagger}\overrightarrow{\tau}L)$	-3/4	1/4		$2^{*}_{\pm 1/2} + 3^{L/R}_{0} + 2^{L/R}_{\pm 1/2}$
21	$(\bar{L}H)(\gamma^{\rho})(H^{\dagger}L)(\bar{E}\gamma_{\rho}E)$	1/2	1/2	1	$1_{0}^{v} + 1_{0}^{R}$
22	$(L\vec{\tau}H)(\gamma^{\rho})(H^{\dagger}\vec{\tau}L)(E\gamma_{\rho}E)$	3/2	-1/2		$1_0^v + 3_0^{t/H}$
23	$(\overline{L}\gamma^{\rho}E^{c})(\overline{E^{c}}H)(\gamma^{\rho})(H^{\dagger}L)$	-1/2	-1/2	1	$2^{v}_{-3/2} + 1^{R}_{0} + 2^{L/R}_{+3/2}$
24	$(\overline{L}\gamma^{\rho}E^{c})(\overline{E^{c}}H)(\gamma^{\rho})(H^{\dagger}L)$	-3/2	1/2		$2_{-3/2}^{\nu} + 3_{0}^{L/R} + 2_{+3/2}^{L/R}$
Combination HL					
25	$(\bar{L}E)(i\tau^{2})(\bar{E}H^{*})(H^{T}i\tau^{2}L)$	1/4	-1/4		$2_{\pm 1/2}^s \pm 1_{\pm 1}^{L/R} \pm 2_{\pm 3/2}^{L/R}$
26	$(LE)(\vec{\tau}i\tau^2)(\vec{E}H^*)(H^Ti\tau^2\vec{\tau}L)$	3/4	1/4		$2_{\pm 1/2}^s + 3_{-1}^{L/R} + 2_{-3/2}^{L/R}$
27	$(\overline{L}i\tau^2 H^*)(\gamma^{\rho})(H^Ti\tau^2 L)(\overline{E}\gamma_{\rho}E)$	-1/2	1/2		$1_0^v + 1_{-1}^{L/R}$
28	$(\tilde{L} \eta^{*} \tau^{2} H^{\bullet})(\gamma^{\rho})(H^{T} i \tau^{2} \eta^{*} L)(\tilde{E} \gamma_{\rho} E)$	-3/2	-1/2		$1_0^v + 3_{-1}^{L/R}$
29	$(\overline{L}\gamma^{\rho}E^{c})(ir^{2})(\overline{E^{c}}H^{*})(\gamma_{\rho})(H^{T}ir^{2}L)$	1/2	-1/2		$2_{-3/2}^{\nu} + 1_{-1}^{L/R} + 2_{+1/2}^{L/R}$
30	$(\bar{L}\gamma^{\rho}E^{c})(\vec{\tau}i\tau^{2})(E^{c}H^{*})(\gamma_{\rho})(H^{T}i\tau^{2}\vec{\tau}L)$	3/2	1/2		$2_{-3/2}^{\nu} + 3_{-1}^{L/R} + 2_{+1/2}^{L/R}$

Table 3: Complete list of  $LL\bar{E}E$ -type d = 8 interactions which involve two SM fields at any possible vertex of interaction (field bilinears within brackets). The columns show an ordinal for each operator, the d = 8 interaction, the corresponding combination of interactions in the BR basis, whether  $O_{\rm NSI}$  is satisfied and the necessary mediators, respectively. Those mediators leading as well to d = 6 operators in Table 2 are in boldface. The superscript L/R indicates massive vector fermions. The flavor structure is to be understood as  $\bar{L}^{\beta}L_{\alpha}\bar{E}^{\delta}E_{\gamma}$ .

D. Hernandez, Ota, Winter + MBG



#### **Finally, gauge invariance implies:**

•From d=6 ops.:  $\epsilon_{\mu\tau} < 3 \ 10^{-2}$ 

# Or avoid altogether d=6 ops. and finetune cancellations between d=8 ones -unbelievable- !

(check cancellantions in our table if you have the stomach for it)

#### TREE-LEVEL MEDIATOR DECOMPOSITION



(Antusch, Baunman, Fdez-Martinez;

D. Hernandez, Ota, Winter + MBG )

#### Mediator analysis of NSI Constraints are then stronger and odds even worse:



We can open the d = 6 vertex (remember Fermi, Weinberg?) Ex: For instance, take the  $(\overline{L}^c i \tau^2 L)(\overline{L} i \tau^2 L^c)$  Davidson+Kuypers







SEVERELY CONSTRAINED:

S. Antusch, J. P. Baumann, E. Fdez-Mtnez; 0807.1003 F. Cuypers, S. Davidson; hep-ph/ 9609487 Moreover...





•This S is disconnected from the seesaw mechanism... although connected to radiatively generated masses -Zee model-

•d=6 NSI are very very constrained.

#### TREE-LEVEL MEDIATOR DECOMPOSITION

Would give even stronger bounds... •  $\epsilon_{\mu\tau} < 10^{-3}$ 

(Antusch, Baunman, Fdez-Martinez;

D. Hernandez, Ota, Winter + MBG )

# MINOS: neutrinos versus antineutrino difference??





Mann et al. arXiv:1006.5720

 $\mathbf{e}_{\mu\tau}$ 





$$\frac{1}{\Lambda} \bar{v}_{\tau} v_{\mu} \bar{e} e$$

$$\frac{1}{\Lambda^2} \bar{v}_{\tau} v_{\mu_{\downarrow}} \bar{u} u$$

Mann et al. arXiv:1006.5720

8<sub>μτ</sub>



Mann et al. arXiv:1006.5720

8<sub>μτ</sub>



$$\frac{\text{They}}{\text{claim}} \varepsilon_{\mu\tau} = -(0.12 \pm 0.21), \ \Delta m_{32}^2 = 2.56^{+0.27}_{-0.24} \times 10^{-3} \text{ eV}^2$$
$$\sin^2 2\theta_{23} = 0.90 \pm 0.05.$$

#### Kopp, Machado, Parke arXiv:0076594 "Could it be $\varepsilon_{\mu\tau}$ matter NSI?"

\* Similar analysis, but simulating MINOS event spectrum:

$$\begin{array}{ll} {}^{\rm They} \, \epsilon^m_{\mu\tau} = -0.40 = 0.40 \, e^{1.0 i \pi} & \sin^2 \theta_{23} = & 0.38 \\ {}^{\rm claim} \, \epsilon^m_{\tau\tau} = -2.16 & \Delta m^2_{32} = +2.86 \times 10^{-3} \ {\rm eV}^2 \\ {}^{\rm (Signs \ can \ be \ changed, \ eightfold \ degeneracy)} \end{array}$$

 $\ast$  Discovery at NOvA in less than one nominal year

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\* Discovery at NOVA in less than one nominal year

Plausible? NO! : gauge invariance ->  $\varepsilon_{\mu\tau}$  < 3 10<sup>-2</sup> from d=6, or d=8 ops. with ad hoc cancellat.

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\* Discovery at NOVA in less than one nominal year

\* They acknowledge that gauge invariance disfavours d=6 ops., and d=8 ops. unlikely:

breaking 4-fermion couplings can arise. However, dimension 8 operators of this type are typically accompanied by phenomenologically problematic dimension 6 operators unless the coefficients of different operators obey certain cancellation conditions [32]. Thus, if the MINOS results were indeed caused by NSI, this would point to a rather non-trivial model of new physics.

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One week ago:

Kopp, Machado, Parke "Could it be charged NSI + CP-viol.?"


#### But gauge invariance strikes again

\* Their bound  $|\epsilon_{\tau\mu}^{d}| \lesssim 0.20$  obtained from one loop contrib. to Kopp, Machado,Parke  $\tau \longrightarrow \mu^{\tau} \qquad \mu^{\pm} \rightarrow \mu^{\pm} \pi^{0}$ 

 $\mathcal{L}_{
m NSI} \supset -2\sqrt{2}G_F \epsilon^d_{\tau\mu} V_{ud} \left[ \bar{u} \gamma^{
ho} d \right] \left[ \bar{\mu} \gamma_{
ho} P_L \nu_{ au} \right] \cdot$ 

#### But gauge invariance strikes again

\* Their bound  $|\epsilon_{\tau\mu}^{d}| \lesssim 0.20$  obtained from one loop contrib. to Kopp, Machado,Parke  $\tau \longrightarrow \mu^{\nu_{\tau}} \qquad \mu^{\mu} \qquad \tau^{\pm} \rightarrow \mu^{\pm}\pi^{0}$ 

\* But a d=6 gauge inv. formulat. of the coupling

$$\mathcal{L}_{\text{NSI}} \supset -2\sqrt{2}G_F \epsilon_{\tau\mu}^d V_{ud} (\bar{Q} \gamma^{\rho} \tau Q) (\bar{L}_{\mu} \gamma_{\rho} \tau L_{\tau}) \cdot$$

$$\frac{\tau^{\pm}}{\tau^{\pm}} \rightarrow \mu^{\pm} \pi^0 \text{ and } \mu^{\pm} \rho \longrightarrow |\epsilon_{\tau\mu}^d| \lesssim 10^{-4}$$

$$\frac{\tau^{\pm}}{\tau^{\pm}} \rightarrow \mu^{\pm} \pi^0 \text{ and } \mu^{\pm} \rho \longrightarrow |\epsilon_{\tau\mu}^d| \lesssim 10^{-4}$$

$$\frac{\tau^{\pm}}{\tau^{\pm}} \rightarrow \mu^{\pm} \pi^0 \text{ and } \mu^{\pm} \rho \longrightarrow |\epsilon_{\tau\mu}^d| \lesssim 10^{-4}$$

My conclusion:



### More promising ? :

What about lighter states? ... ie light steriles?

Steriles lighter than  $M_W$  evade non-unitarity bounds -if light enough- and some of the pure matter NSI bounds

Ie. Ann Nelson and collab.; light steriles, gauged B-L



Engelhardt, Nelson and Walsh 2010 (Heeck, Rodejohan 2010 for gauging  $L_{\mu}$ - $L_{\tau}$  and light Z'?)

#### And light steriles for the new MiniBoone data?

•Interesting: Same L/E than LSND, but different L and E --> different backgrounds

CP in vacuum?:CP does not depend on L/E if matter effects negligible, but differs for neutrinos and antineutrinos

seems difficult (arXiv:0906.1997 and arXiv:0705.0107)

\* Combination of 3+1 and NSI ? Akhmedov+Schwetz 2010 A. Nelson and colab. 2010







# 2.6 $\sigma$ effect for the world cup ! (Marc Sher)



## It is an appearance experiment (Spain)



 $W^{-}$ U<sub>PMNS</sub> ν 11 Matter Standard scenarío

11 Matter Non-Unitary Mixing Matrix





These NSI are a generic signature of fermionic Seesaws









These NSI are a generic signature of fermionic Seesaws Fermion-triplet seesaws:

similar - although richer! - analysis



#### → For the Triplet-fermion Seesaws (type III):

$$(\mathrm{NN^{+}-1})_{\alpha\beta} = \frac{v^{2}}{2} |c^{d=6}|_{\alpha\beta} = \frac{v^{2}}{2} |Y_{\Sigma}^{\dagger} \frac{1}{M_{\Sigma}^{\dagger}} \frac{1}{M_{\Sigma}} Y_{\Sigma}|_{\alpha\beta} \lesssim \begin{pmatrix} 3 \cdot 10^{-3} & < 1.1 \cdot 10^{-6} & < 1.2 \cdot 10^{-3} \\ < 1.1 \cdot 10^{-6} & 4 \cdot 10^{-3} & < 1.2 \cdot 10^{-3} \\ < 1.2 \cdot 10^{-3} & < 1.2 \cdot 10^{-3} \end{pmatrix}$$

(Abada et al 07)

# Scalar triplet seesaw Bounds on c<sup>d=6</sup>

Process	Constraint on	Bound $\left(\times \left(\frac{M_{\Delta}}{1 \text{ TeV}}\right)^2\right)$	
$M_W$	$ Y_{\Delta \mu e} ^2$	$< 7.3  imes 10^{-2}$	
$\mu^- \to e^+ e^- e^-$	$ Y_{\Delta \mu e}  Y_{\Delta e e} $	$< 1.2  imes 10^{-5}$	
$\tau^- \to e^+ e^- e^-$	$ Y_{\Delta au e}  Y_{\Delta ee} $	$< 1.3  imes 10^{-2}$	
$\tau^- \to \mu^+ \mu^- \mu^-$	$ Y_{\Delta au\mu}  Y_{\Delta\mu\mu} $	$< 1.2 \times 10^{-2}$	
$ au^-  ightarrow \mu^+ e^- e^-$	$ Y_{\Delta au\mu}  Y_{\Delta ee} $	$< 9.3  imes 10^{-3}$	
$\tau^- \to e^+ \mu^- \mu^-$	$ Y_{\Delta au e}  Y_{\Delta\mu\mu} $	$< 1.0  imes 10^{-2}$	
$\tau^- \to \mu^+ \mu^- e^-$	$ Y_{\Delta au\mu}  Y_{\Delta\mu e} $	$< 1.8  imes 10^{-2}$	
$\tau^- \to e^+ e^- \mu^-$	$ Y_{\Delta au e}  Y_{\Delta\mu e} $	$< 1.7  imes 10^{-2}$	
$\mu  ightarrow e \gamma$	$\left  \Sigma_{l=e,\mu, au} Y_{\Delta l\mu}^{\dagger} Y_{\Delta el}  ight $	$<4.7 imes10^{-3}$	
$\tau \to e \gamma$	$\left \Sigma_{l=e,\mu, au}Y_{\Delta_{l au}}^{\dagger}Y_{\Delta_{el}} ight $	< 1.05	
$ au  ightarrow \mu \gamma$	$\left  \Sigma_{l=e,\mu, au} Y_{\Delta} ^{\dagger}_{l au} Y_{\Delta \mu l}  ight $	$< 8.4  imes 10^{-1}$	

## Scalar triplet seesaw

Combined bounds on c<sup>d=6</sup>

Combined bounds				
Process	Yukawa	Bound $\left( \times \left( \frac{M_{\Delta}}{1 \text{ TeV}} \right)^4 \right)$		
$\mu  ightarrow e \gamma$	$\left Y_{\Delta\mu\mu}^{\dagger}Y_{\Delta\mu e}+Y_{\Delta au\mu}^{\dagger}Y_{\Delta au e} ight $	$< 4.7  imes 10^{-3}$		
$\tau \rightarrow e \gamma$	$Y_{\Delta  au  au  au} Y_{\Delta  au  au} Y_{\Delta  au e}$	< 1.05		
$\tau \rightarrow \mu \gamma$	$Y_{\Delta  au  au  au}^{\dagger}Y_{\Delta  au \mu}$	$< 8.4  imes 10^{-1}$		

Obervable non-standard interactions from

$$\mathbf{Y}_{\Delta}^{+}\mathbf{Y}_{\Delta}/\mathbf{M}^{2} \ (\overline{\mathbf{L}}_{\alpha} \ \mathbf{L}_{\beta}) \ (\overline{\mathbf{L}}_{\gamma} \ \mathbf{L}_{\delta})$$

in scalar triplet seesaw ???

#### Barely so ! (Malisnky Ohlsson and Zhang 08):

- --- Require Yukawa couplings are almost diagonal--> degenerate neutríno spectrum
- --- Not excluded are

 $\mu^{-} -> e^{-} v_{e} v_{\mu} \dots$  Wrong sign muons at near detector

No v masses in the SM because the SM *accidentally* preserves B-L i.e. Adding singlet neutrino fields N<sub>R</sub>

• right-handed  $N_R \rightarrow Y_N \ \widetilde{H} \ \overline{L} \ N_R + h.c. \ \mathbb{R} \quad m_D \overline{v_L} \ N_R + h.c.$ 

Would require  $Y_N \sim 10^{-12}$  !!! Why  $v_s$  are so light??? Why  $N_R$  does not acquire large Majorana mass?  $\delta \mathcal{L} \sim M (N_T N_R)$ OK with gauge invariance



## Seesaw model

Which allows  $Y_N \sim 1 \rightarrow M \sim M_{Gut}$ 

## *N* elements from oscillations & decays

Μυν		.7589	.4565	<.20
without unitarity OSCILLATIONS +DECAYS	N  =	.1955	.4274	.5782
<b>3</b> σ		Antusch, Big López-Pavón	gio, <b>36</b> ern <b>75</b> de , M.B.G. 06	z-Martinez,
		.7988	.4761	< .20
with unitarity OSCILLATIONS	U  =	.1952	.4273	.5882
		.2053	.4474	.5681
with unitarity OSCILLATIONS	U  =	.7988 .1952 .2053 M. C. Gonzale	.4761 .4273 .4474 z Garcia hep-ph	<ul> <li>20</li> <li>.20</li> <li>.5882</li> <li>.5681</li> <li>.0410030</li> </ul>