Running of Neutrino Mass Parameters

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mostly based on based on work
in collaboration with S. Antusch, J. Kersten, M. Lindner, M. Ratz [JHEP 0503:024]
and part of my diploma thesis [hep-ph/0703...]

Why do we need the renormalization group?

Measurements by
low–energy experiments

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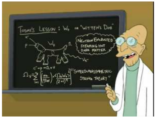
String theory

Extra Dimensions

Predictions from
high–energy theories

SU(5)

SO(10)



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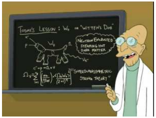

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Renormalization Group Evolution

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Measurements by
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Outline

1. Premlinaries

Outline

Outline

Outline

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Seesaw model

neutrino mass matrix

$$\begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix}^T \begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix}$$

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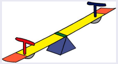
RH neutrinos decouple and give mass $(m_\nu)_{ij} = -\frac{v^2}{2}\frac{(Y_\nu)_{ki}(Y_\nu)_{kj}}{M_k}$ to light neutrinos by the seesaw mechanism [Minkowski;Yanagida;Glashow;Gell-Mann,Ramond,Slansky;Mohapatra,Senjanovic]



Seesaw model

neutrino mass matrix

$$\begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix}^T \begin{pmatrix} m_L & m_D^T \\ m_D & M_R \end{pmatrix} \begin{pmatrix} \nu \\ \overline{\nu} \end{pmatrix} \Rightarrow m_\nu = m_L - \frac{v^2}{2} Y_\nu^T M^{-1} Y_\nu$$

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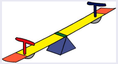
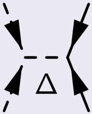


Higgs triplet $\Delta \sim (\mathbf{3}, \mathbf{1})$:



$$\Rightarrow \quad m_L = \frac{v^2}{2}\frac{\Lambda_6}{M_\Delta^2} Y_\Delta$$

[Magg, Wetterich;Lazaridis, Shafi, Wetterich; Mohapatra, Senjanovic]

General structure of β–function

1 loop β-function:

$P = P(Y_e,\ Y_\nu,\ Y_\Delta)$

$\alpha = \alpha(g_1,\ g_2,\ \mathrm{Tr}\, Y^\dagger Y,\ \Lambda_i)$

$$16\pi^2 \mu \frac{\mathrm{d}m_\nu}{\mathrm{d}\mu} = 16\pi^2 \beta_{m_\nu} = m_\nu P + P^T m_\nu + \alpha m_\nu$$

P.H. Chankowski, Z. Pluciennik (1993)

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$$P = C_e Y_e^\dagger Y_e + C_\nu Y_\nu^\dagger Y_\nu + C_\Delta Y_\Delta^\dagger Y_\Delta$$

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	SM			MSSM		
m_ν	C_e	C_ν	C_Δ	C_e	C_ν	C_Δ
$-\frac{v^2}{4}\kappa$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	1	1	$\frac{3}{2}$
$-\frac{v^2}{2} Y_\nu^T M^{-1} Y_\nu$	$-\frac{3}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	1	1	$\frac{3}{2}$
$\frac{v^2}{2}\frac{\Lambda_6}{M_\Delta^2} Y_\Delta$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{3}{2}$	1	1	$\frac{3}{2}$

Mixing parameters

MNS mixing matrix

$$U = \mathrm{diag}(e^{\mathrm{i}\delta_e}, e^{\mathrm{i}\delta_\mu}, e^{\mathrm{i}\delta_\tau}) \cdot V \cdot \mathrm{diag}(e^{-\mathrm{i}\varphi_1/2}, e^{-\mathrm{i}\varphi_2/2}, 1)$$

where ($s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$)

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,e^{-\mathrm{i}\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}\,e^{\mathrm{i}\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}\,e^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}\,e^{\mathrm{i}\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}\,e^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix}$$

Experimental data

T. Schwetz [hep-ph/0606060]

Parameter	Best-fit	Allowed range (3σ)
$\sin^2\theta_{12}$ [°]	0.30	0.24 . . . 0.40
$\sin^2\theta_{23}$ [°]	0.50	0.34 . . . 0.68
$\sin^2\theta_{13}$ [°]	0.000	$\leq$ 0.041
Δm^2_{21} [$10^{-5}\,\mathrm{eV}^2$]	7.9	7.1 . . . 8.9
$\lvert\Delta m^2_{31}\rvert$ [$10^{-3}\,\mathrm{eV}^2$]	2.5	1.9 . . . 3.2

Effective field theory

$$P = C_e Y_e^\dagger Y_e$$

$C_e^{\mathrm{SM}} = -\frac{3}{2}, \; C_e^{\mathrm{MSSM}} = 1$

S. Antusch, J. Kersten, M. Lindner, M. Ratz [hep-ph/0305273]

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General structure (also applicable for phases):

Renormalization scale

$$\mu \frac{\mathrm{d}\theta_{ij}}{\mathrm{d}\mu} \propto \frac{f(m_l, \delta, \varphi_1, \varphi_2)}{m_j^2 - m_i^2} \times F^{(ij)}(y_k, \theta_{lm})$$

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- The more degenerate the spectrum is, the larger is the running.
- Phases can damp the running.

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- $F^{(ij)} \propto$ Yukawa coupling $\Rightarrow \tan\beta$ dependence
- $F^{(ij)} = 0$ for zero mixing (fixed point)

S. Antusch, J. Kersten, M. Lindner, M. Ratz [hep-ph/0305273]

Outline

Full theory

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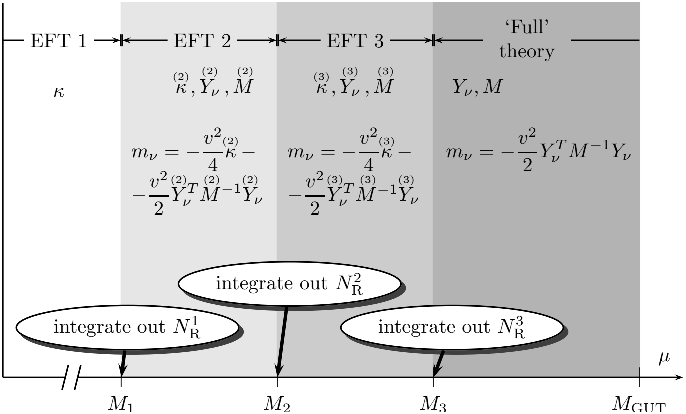
- off–diagonal terms $\rightarrow$ mixing can be generated
- GUT: Y_ν strongly hierarchical $\rightarrow$ P_{33} dominates

Thresholds


EFT 1
EFT 2
EFT 3
'Full' theory
κ
$\overset{(2)}{\kappa}, \overset{(2)}{Y_\nu}, \overset{(2)}{M}$
$\overset{(3)}{\kappa}, \overset{(3)}{Y_\nu}, \overset{(3)}{M}$
Y_ν, M
$m_\nu = -\frac{v^2}{4}\overset{(2)}{\kappa} - \frac{v^2}{2}\overset{(2)}{Y_\nu^T}\overset{(2)}{M}^{-1}\overset{(2)}{Y_\nu}$
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M_1
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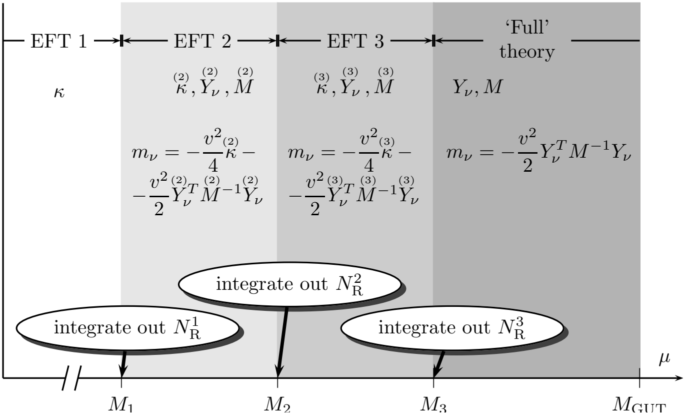
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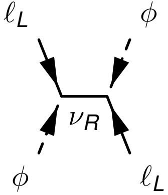
ℓ_L
ϕ
ν_R
ϕ
ℓ_L

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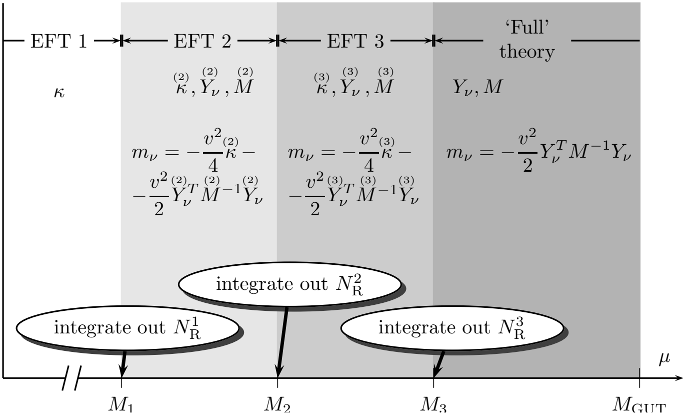
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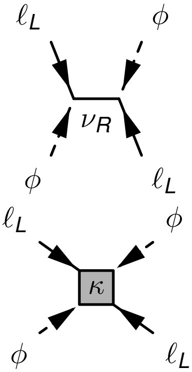
ℓ_L
ϕ
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κ
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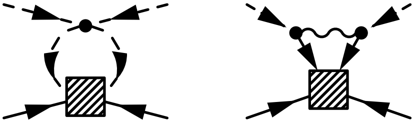

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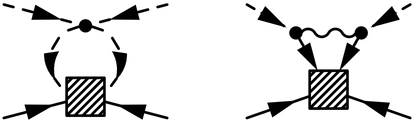


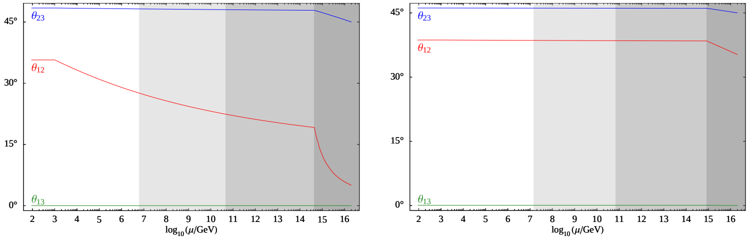
- rescaling of right–handed neutrino masses

M. Lindner, MS, A. Smirnov, hep-ph/0505162

$$m_\nu = Z_{\text{ext}}^T Y_\nu^T X M^{-1} Y_\nu Z_{\text{ext}}$$

additional vertex corrections

Example for RG evolution



(a) $\tan\beta = 20$, $m_1 = 0.05\,\text{eV}$ (b) $\tan\beta = 10$, $m_1 = 0.015\,\text{eV}$

$\delta = \varphi_1 = \varphi_2 = 0$

θ_{23}

Current	Beams	T2K+NuMI	JPARC-HK	NuFact-II
0.16	0.1	0.050	0.020	0.055

P. Huber, M. Lindner, M. Rolinec, T. Schwetz, W. Winter [hep-ph/0403068]

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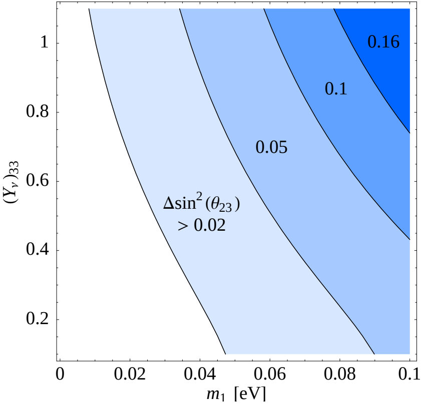

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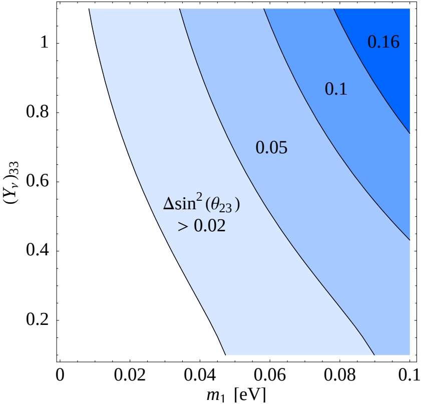

- $|0.5 - \sin^2\theta_{23}| \leq 0.16$

  T. Schwetz [hep-ph/0606060]

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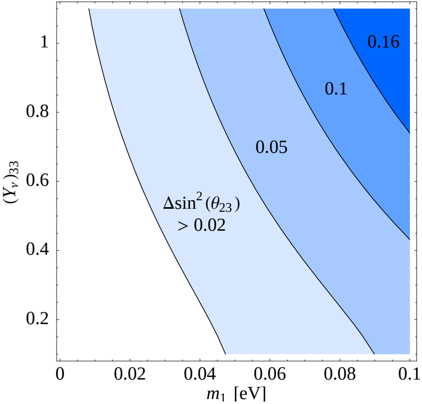

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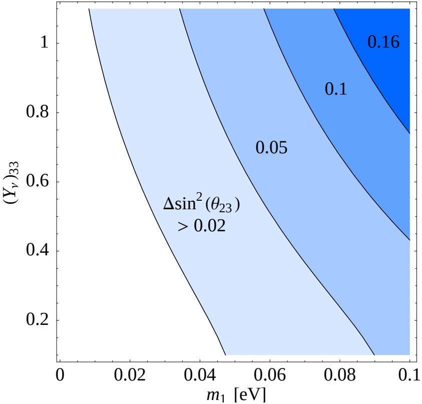

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  T. Schwetz [hep-ph/0606060]
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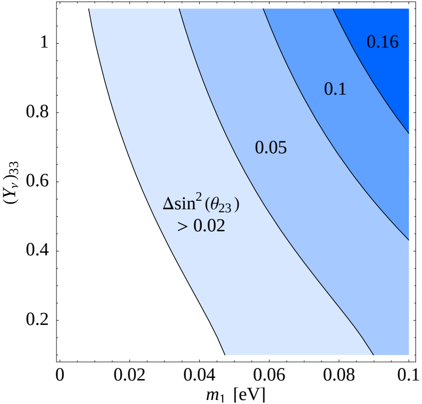

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- suppression by phases possible

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Only Higgs triplet

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convenient basis: $Y_\Delta = \mathrm{diag}(y_1, y_2, y_3)$ diagonal

$$16\pi^2 \dot{y}_i = 2C_\Delta y_i^3 + \alpha y_i$$

$$16\pi^2 \left(\dot{Y}_e\right)_{ij} = (Y_e)_{ij} D_\Delta y_j^2 + \alpha_e (Y_e)_{ij}$$

Chao,Zhang [hep-ph/0611323]

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$$16\pi^2 \left(\dot{Y}_e\right)_{ij} = (Y_e)_{ij} D_\Delta y_j^2 + \alpha_e (Y_e)_{ij}$$

Chao,Zhang [hep-ph/0611323]

- Neutrino mass matrix stays diagonal

Only Higgs triplet

$$P = \qquad C_\Delta Y_\Delta^\dagger Y_\Delta$$

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Chao,Zhang [hep-ph/0611323]

- Neutrino mass matrix stays diagonal
- Running of angles and phases described by charged leptons

General structure of RG equations I

$$16\pi^2\dot{\theta}_{12} = -\frac{D_\Delta}{2} y_{21}^2 \sin 2\theta_{12}$$

$$16\pi^2\dot{\theta}_{13} = -\frac{D_\Delta}{4}\left[y_{31}^2 + y_{32}^2 + y_{21}^2 \cos 2\theta_{12}\right] \sin 2\theta_{13}$$

$$16\pi^2\dot{\theta}_{23} = -\frac{D_\Delta}{2}\left[\left(y_{32}^2 c_{12}^2 + y_{31}^2 s_{12}^2\right) \sin 2\theta_{23} + y_{21}^2 \cos\delta \sin 2\theta_{12}\, c_{23}^2\, \theta_{13}\right] + \mathcal{O}(\theta_{13}^2)$$

$y_{ji}^2 = y_j^2 - y_i^2$, $y_i \epsilon \{y_1, y_2, y_3, y_e, y_\mu, y_\tau\}$

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General structure of RG equations II

$$16\pi^2\dot{\theta}_{12} = -\frac{1}{2}\left[D_\Delta y_{21}^2 + C_e y_\tau^2 \frac{y_2+y_1}{y_2-y_1}\sin\theta_{23}\right]\sin 2\theta_{12} + \mathcal{O}(\theta_{13})$$

$$16\pi^2\dot{\theta}_{13} = -\frac{C_e}{2}\frac{(y_2-y_1)y_3}{(y_3-y_1)(y_3-y_2)}y_\tau^2\cos\delta\sin 2\theta_{12}\sin 2\theta_{23} + \mathcal{O}(\theta_{13})$$

$$16\pi^2\dot{\theta}_{23} = -\frac{1}{2}\Bigg[D_\Delta\left(y_3^2 - y_1^2\sin^2\theta_{12} - y_2^2\cos^2\theta_{12}\right) + C_e\frac{y_3^2 - y_1 y_2 + (y_2-y_1)\cos 2\theta_{12}}{(y_3-y_2)(y_3-y_1)}y_\tau^2\Bigg]\sin 2\theta_{23} + \mathcal{O}(\theta_{13})$$

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General structure of RG equations II

$$16\pi^2\dot{\theta}_{12} = -\frac{1}{2}\left[D_\Delta y_{21}^2 + C_e y_\tau^2 \frac{y_2+y_1}{y_2-y_1}\sin\theta_{23}\right]\sin 2\theta_{12} + \mathcal{O}(\theta_{13})$$

$$16\pi^2\dot{\theta}_{13} = -\frac{C_e}{2}\frac{(y_2-y_1)y_3}{(y_3-y_1)(y_3-y_2)}y_\tau^2\cos\delta\sin 2\theta_{12}\sin 2\theta_{23} + \mathcal{O}(\theta_{13})$$

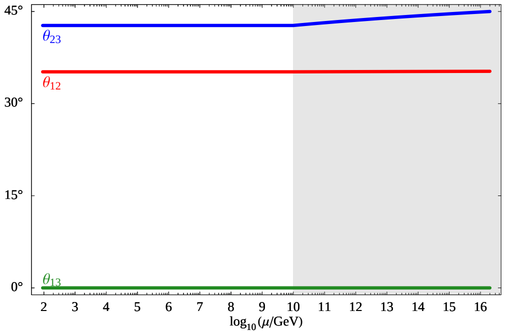
$$16\pi^2\dot{\theta}_{23} = -\frac{1}{2}\left[D_\Delta\left(y_3^2 - y_1^2\sin^2\theta_{12} - y_2^2\cos^2\theta_{12}\right)\right.$$
$$\left. + C_e\frac{y_3^2 - y_1 y_2 + (y_2-y_1)\cos 2\theta_{12}}{(y_3-y_2)(y_3-y_1)}y_\tau^2\right]\sin 2\theta_{23} + \mathcal{O}(\theta_{13})$$

$$16\pi^2\dot{\delta} = \frac{C_e}{2}\frac{(y_2-y_1)y_3}{(y_3-y_1)(y_3-y_2)}y_\tau^2\sin\delta\sin 2\theta_{12}\sin 2\theta_{23}\theta_{13}^{-1} + \mathcal{O}(\theta_{13})$$

$y_{ji}^2 = y_j^2 - y_i^2$, $y_i \epsilon \{y_1, y_2, y_3, y_e, y_\mu, y_\tau\}$

RG evolution of θ_{23}

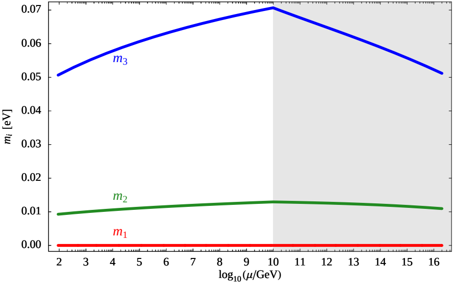
$$16\pi^2 \dot{\theta}_{23} \approx -\frac{D_\Delta}{2}\left(y_3^2 - y_1^2 \sin^2\theta_{12} - y_2^2 \cos^2\theta_{12}\right)\sin 2\theta_{23}$$



n.h., $m_1 = 0\,\mathrm{eV}$, $\langle\Delta\rangle \sim 0.1\,\mathrm{eV}$, $Y_\Delta \sim \mathcal{O}(0.1-1)$, $D_\Delta = -\frac{3}{2}$

Masses

$$16\pi^2 \dot{y}_i = 2C_\Delta y_i^3 + \alpha y_i$$



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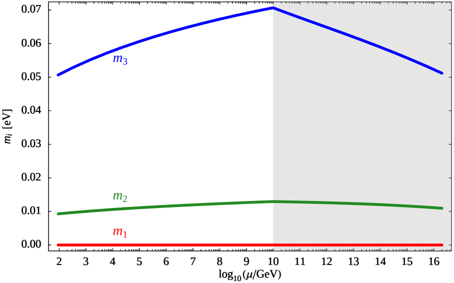

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REAP/MixingParameterTools: `http://www.ph.tum.de/~rge`

Outline

Summary

Standard seesaw

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- RG effect proportional to mass squared difference
- Sizable RG effect for θ_{23}