Comparison of Boltzmann Kinetics with Quantum Dynamics for Relativistic Quantum Fields

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Outline

1. Motivations
2. Boltzmann Kinetics
3. Quantum Dynamics
4. Comparison of Numerical Solutions
5. Conclusions

Motivations for going into the subject

Early Universe, High Energy Physics

- Baryogenesis, Leptogenesis
- Preheating after Inflation
- Heavy Ion Collisions, Quark-Gluon Plasma

Nonrelativistic Physics

- Bose-Einstein Condensation
- Transport of charged particles in (semi-)conductors

Fundamental Question

How does a closed many-body system which is initially arbitrarily far from equilibrium equilibrate?

Motivations (cont.)

The situation

- All of the above phenomena require the description of systems out of thermal equilibrium.
- In general, such nonequilibrium situations are treated by means of Boltzmann equations.
- However, Boltzmann equations are only a classical approximation to the quantum thermalization process described by Kadanoff-Baym equations.

An obvious question

How reliable are Boltzmann equations as compared to Kadanoff-Baym equations?

Boltzmann Equation

for a spatially homogeneous system in the framework of a real scalar Φ^4 quantum field theory:

$$\partial_t n(t,\mathbf{k}) = \frac{\lambda^2\pi}{48}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3q}{(2\pi)^3}\int d^3r\left[\frac{1}{E_k E_p E_q E_r}\right.$$
$$\times\,\delta(\mathbf{k}+\mathbf{p}-\mathbf{q}-\mathbf{r})\quad\delta(E_k+E_p-E_q-E_r)$$
$$\left.\times\left(\underbrace{(1+n_\mathbf{k})(1+n_\mathbf{p})\,n_\mathbf{q}n_\mathbf{r}}_{\text{gain term}}-\underbrace{n_\mathbf{k}n_\mathbf{p}(1+n_\mathbf{q})(1+n_\mathbf{r})}_{\text{loss term}}\right)\right]$$

Momentum conservation — Energy conservation

Isotropy: 9 dimensional integral $\Longrightarrow$ 2 dimensional integral.
Important for numerics! [Dolgov, Hansen, Semikoz (1997)]

Complete Schwinger-Keldysh Propagator

Definition

$$G(x,y) = \langle T_{\mathcal{C}} \{\Phi(x)\Phi(y)\}\rangle$$

The index $\mathcal{C}$ denotes time ordering along the closed Schwinger-Keldysh real-time contour.

Decomposition

$$G(x,y) = G_F(x,y) - \frac{i}{2}\mathrm{sign}_{\mathcal{C}}\left(x^0 - y^0\right) G_\varrho(x,y)$$

- Statistical propagator $\Longrightarrow$ effective particle number
- Spectral function $\Longrightarrow$ thermal mass, decay width

Effective Energy and Particle Number Densities

Free-field ansatz [Berges (2002)]

Effective kinetic energy density:

$$\omega^2(t,k) = \left(\frac{\partial_{x^0}\partial_{y^0} G_F\left(x^0, y^0, k\right)}{G_F\left(x^0, y^0, k\right)} \right)_{x^0=y^0=t}$$

Effective particle number density:

$$n(t,k) = \omega(t,k)\, G_F(t,t,k) - \frac{1}{2}$$

Advantages of these definitions

- They furnish a particle number density which thermalizes.
- They do not rely on any quasi-particle assumption.
- They comprise conserved charges, if present in the theory.

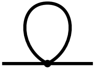
Kadanoff-Baym Equations

for a spatially homogeneous and isotropic system in the framework of a real scalar Φ^4 quantum field theory: [Berges (2002)]

$$\left[\partial_{x^0}^2 + k^2 + M^2\left(x^0\right)\right] G_F\left(x^0, y^0, k\right)$$

$$= \int_0^{y^0} dz^0 \, \Pi_F\left(x^0, z^0, k\right) G_\varrho\left(z^0, y^0, k\right)$$

$$- \int_0^{x^0} dz^0 \, \Pi_\varrho\left(x^0, z^0, k\right) G_F\left(z^0, y^0, k\right)$$

Effective Mass: $M^2\left(x^0\right) = m^2 +$ 

Nonlocal Self Energy: $\Pi\left(x^0, z^0, k\right) =$

Internal lines represent the complete Schwinger-Keldysh propagator!

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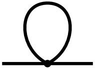
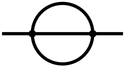
Effective Mass: $M^2\left(x^0\right) = m^2 +$ 

Nonlocal Self Energy: $\Pi\left(x^0, z^0, k\right) =$ 

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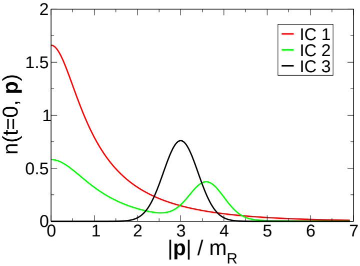
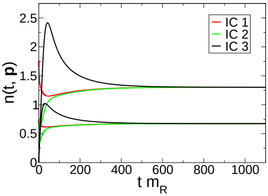
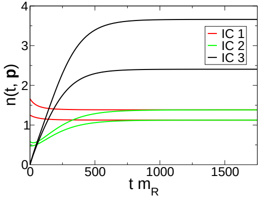
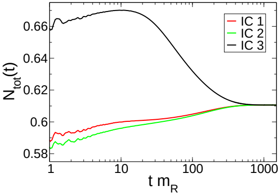
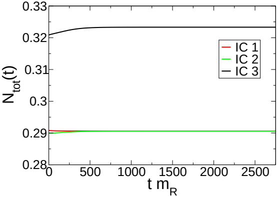
Initial Conditions



- All initial conditions correspond to the same (conserved) average energy density.
- The initial conditions IC1 and IC2 correspond to the same initial total particle number.

[Manfred Lindner, MMM (2006)]

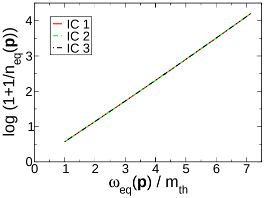
Universality

Kadanoff-Baym		Boltzmann
	Evolution of the particle number densities	
	Evolution of the total particle numbers	
Full Universality		Restricted Universality

[Manfred Lindner, MMM (2006)]

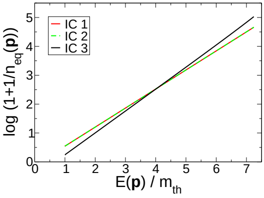
Chemical Equilibration

Kadanoff-Baym



Equilibrium particle number densities

Boltzmann



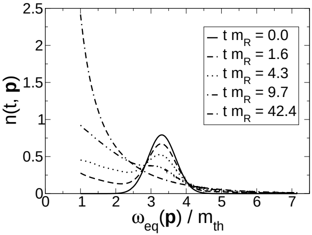
- Full Universality
- Chemical Equilibration

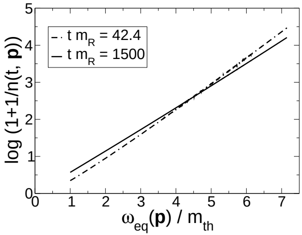
- Restricted Universality
- No Chemical Equilibration

[Manfred Lindner, MMM (2006)]

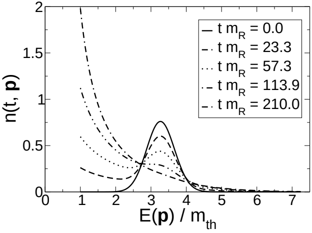
Separation of Time Scales

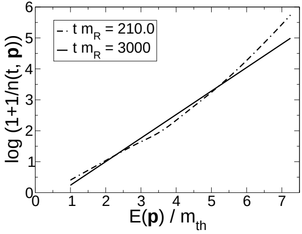
Kadanoff-Baym





Boltzmann





Generalization to fermionic theories

$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetric Yukawa model:

$$\lambda \left(\Phi_a \Phi_a\right)^2 + i\eta \bar{\Psi} \Phi_a \left(\sigma_a P_R - \sigma_a^\dagger P_L\right) \Psi$$

[Gell-Mann and Levy (1960), Deshpande et al. (1991)]

Kadanoff-Baym Equations

- Full universality, chemical equilibration [Berges et al. (2003)]
- Prethermalization [Berges et al. (2004)]

Boltzmann equations [MMM (2006)]

- Restricted universality
- classical, but no quantum chemical equilibration
- no separation of time scales

Conclusions

Quantum Dynamics (Kadanoff-Baym equations)

- take memory and off-shell effects into account.
- respect full universality.
- include chemical equilibration.
- separate time scales between kinetic and chemical equilibration.

Classical Kinetics (Standard Boltzmann equations)

- do not take memory and off-shell effects into account (molecular chaos for quasi-particles).
- comprise fake constants of motion.
- respect only a restricted universality.
- do not include quantum chemical equilibration, and therefore
- cannot separate time scales between kinetic and chemical equilibration.