Discrete flavor symmetries

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Observations

- Masses of the charged fermions are strongly hierarchical

$$m_u : m_c : m_t \approx \lambda^8 : \lambda^4 : 1 \; , \quad m_d : m_s : m_b \approx \lambda^4 : \lambda^2 : 1 \; ,$$

$$m_e : m_\mu : m_\tau \approx \lambda^5 : \lambda^2 : 1 \quad \text{where } \lambda \approx \theta_C \approx 0.22$$

- Mass hierarchy in the ν sector is milder, ordering till now unknown.
- Mixing parameters:
  small mixings for quarks, large mixings for leptons.
- for lepton mixing special structures are allowed:
  - "tri-bimaximal" (TB): $(1\,\sigma)$
    $\sin^2(\theta_{12}^{TB}) = \frac{1}{3} \; , \quad \sin^2(\theta_{23}^{TB}) = \frac{1}{2} \; , \quad \sin^2(\theta_{13}^{TB}) = 0.$
  - "μ-τ" symmetric (MTS):
    $\sin^2(\theta_{23}^{MTS}) = \frac{1}{2} \; , \quad \sin^2(\theta_{13}^{MTS}) = 0.$

$\Rightarrow$ All these issues need a theoretical description: Flavor symmetry G_F!

Choice of Flavor Symmetry

- explain existence of three generations or at least unify two of them
  $\rightarrow$ needs *non-abelian* symmetry group
- avoid Goldstone/ gauge bosons coming from SSB of G_F
  $\rightarrow$ needs *discrete* symmetry group

possible symmetries

- permutation symmetries: S_N and A_N with $N \in \mathbb{N}$
- dihedral symmetries: D_n with $n \in \mathbb{N}$
- double-valued dihedral symmetries: D'_n with $n \in \mathbb{N}$
- further double-valued groups: T', O', I', ...
- subgroups of $SU(3)$: $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$ with $n \in \mathbb{N}$, ...

$\rightarrow$ several models using these symmetries ...

Examples in the Literature

- S_3: Pakvasa et al. (1978), Derman (1979), ..., Ma (2000), Kubo et al. (2003), Chen et al. (2004), Grimus et al. (2005), Dermisek et al. (2005), Mohapatra et al. (2006), ... and many more ...
- S_4: Pakvasa et al. (1979), Derman et al. (1979), Lee et al. (1994), Mohapatra et al. (2004), Ma (2006), CH et al. (2006), Caravaglios et al. (2006), ...
- A_4: Wyler (1979), Ma et al. (2001), Babu et al. (2003), Altarelli et al. (2005,2006), He et al. (2006) ... and many more ...
- D_4: Seidl (2003), Grimus et al. (2003,2004), Kobayashi et al. (2005), ...
- D_5: Ma (2004), CH et al. (2006).
- $D_{n>5}$: Chen et al. (2005), Kajiyama et al. (2007), ...
- D'_n: Frampton et al. (1995,1996,2000), Frigerio et al. (2005), Babu et al. (2005), Kubo (2005), ...
- T': Frampton et al. (1994,2007), Aranda et al. (1999,2000), Feruglio et al. (2007).
- $\Delta(3\,n^2)$ and $\Delta(6\,n^2)$: Kaplan et al. (1994), Chou et al. (1997), de Medeiros Varzielas et al. (2005), ...

Group Theory of A_4 and T'

- The group A_4 is the symmetry group of a regular tetrahedron, group of even permutations of four objects
- Order of the group: 12
- Four irreducible representations: 1, $1'$, $1''$ and 3
- Generator relations for generators S and T:

$$S^2 = \mathbb{1}\ , \quad T^3 = \mathbb{1}\ , \quad (ST)^3 = \mathbb{1}\ .$$

rep.	S	T
1	1	1
$1'$	1	ω
$1''$	1	ω^2
3	$\frac{1}{3}\begin{pmatrix} -1 & 2\omega & 2\omega^2 \\ 2\omega^2 & -1 & 2\omega \\ 2\omega & 2\omega^2 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$

$(\omega = e^{\frac{2\pi i}{3}})$

Group Theory of A_4 and T' (II)

- The group T' is the double covering of the group A_4. (Compare to $SU(2)$ and $SO(3)$)
- Order of the group: 24
- Irred. reps: 1, $1'$, $1''$, 3 and 2, $2'$, $2''$
- Generator relations for generators S and T:

$$S^2 = \mathbb{R}\,, \quad T^3 = \mathbb{1}\,, \quad (ST)^3 = \mathbb{1}\,, \quad \mathbb{R}^2 = \mathbb{1}.$$

rep.	S	T
2	A_1	ωA_2
$2'$	A_1	$\omega^2 A_2$
$2''$	A_1	A_2

$$A_1 = -\frac{1}{\sqrt{3}} \begin{pmatrix} i & \sqrt{2}e^{i\pi/12} \\ -\sqrt{2}e^{-i\pi/12} & -i \end{pmatrix},$$

$$A_2 = \begin{pmatrix} \omega & 0 \\ 0 & 1 \end{pmatrix}$$

T' Model

(Feruglio et al.(2007)

- T' broken at high energies
- low energy effective theory: MSSM
- Fermion generations transform according to A_4/ T' reps.:

$$l_i \sim 3\ , \quad e^c \sim 1\ , \quad \mu^c \sim 1''\ , \quad \tau^c \sim 1'\ , \qquad (A_4 \text{ is enough})$$

$$D_q, D_u^c, D_d^c \sim 2''\ , \quad q_3, t^c, b^c \sim 1\ .$$

$$(D_q = (q_1, q_2)^t,\ D_u^c = (u^c, c^c)^t,\ D_d^c = (d^c, s^c)^t)$$

  and Higgs fields $h_{u,d} \sim 1$
- further fields: gauge singlet Higgs fields (flavons) with heavy masses which transform non-trivially under T'
- results :
  - TB mixing in the lepton sector at leading order
  - $m_{u,d,s,c} \ll m_{b,t}$, all data can be accommodated at subleading order in the quark sector

Particle Content of the T' Model

	LEPTONS				*QUARKS*						*FLAVONS*				
Field	l	e^c	μ^c	τ^c	D_q	D_u^c	D_d^c	q_3	t^c	b^c	φ_T	φ_S	$\xi, \tilde{\xi}$	η	ξ''
T'	3	1	$1''$	$1'$	$2''$	$2''$	$2''$	1	1	1	3	3	1	$2'$	$1''$
Z_3	ω	ω^2	ω^2	ω^2	ω	ω^2	ω^2	ω	ω^2	ω^2	1	ω	ω	1	1
$U(1)_{FN}$	0	$2n$	n	0	0	n	0	0	0	0	0	0	0	0	0

additionally needed:

- Z_3 symmetry to separate charged fermion and neutrino sector:

$$\{\varphi_T, \eta, \xi''\} \;\rightarrow\; m_l, m_u, m_d \quad \text{and} \quad \left\{\varphi_S, \xi, \tilde{\xi}\right\} \;\rightarrow\; m_\nu$$

- $U(1)_{FN}$ for hierarchy $m_s \ll m_c$, $m_e \ll m_\mu \ll m_\tau$ (field θ: $Q(\theta) = -1$)

Superpotential

$$w = w_l + w_q + w_d$$

for leptons :

$$w_l = y_e e^c(\varphi_T l)h_d/\Lambda\ \left(\frac{\theta}{\Lambda}\right)^{2n} + y_\mu \mu^c(\varphi_T l)' h_d/\Lambda\ \left(\frac{\theta}{\Lambda}\right)^{n} + y_\tau \tau^c(\varphi_T l)'' h_d/\Lambda$$
$$+(x_a\xi + \tilde{x}_a\tilde{\xi})(ll)h_u h_u/\Lambda^2 + x_b(\varphi_S ll)h_u h_u/\Lambda^2$$

for quarks :

$$w_q = y_t\,(t^c q_3)\,h_u + y_b\,(b^c q_3)\,h_d + y_1(\varphi_T D^c_u D_q)h_u/\Lambda\ \left(\frac{\theta}{\Lambda}\right)^{n} +$$
$$y_5(\varphi_T D^c_d D_q)h_d/\Lambda + y_2\ \xi''(D^c_u D_q)h_u/\Lambda\ \left(\frac{\theta}{\Lambda}\right)^{n} + y_6\ \xi''(D^c_d D_q)h_d/\Lambda$$
$$+\{y_3\ t^c(\eta D_q) + y_4\ (D^c_u \eta)q_3\ \left(\frac{\theta}{\Lambda}\right)^{n}\}h_u/\Lambda + \{y_7\ b^c(\eta D_q) + y_8\ (D^c_d \eta)q_3\}h_d/\Lambda$$

Higgs superpotential w_d: VEV structure:

$$G_S \quad : \quad \langle\varphi_S\rangle = (v_S, v_S, v_S)\ ,\quad \langle\xi\rangle = u\ ,\quad \langle\tilde{\xi}\rangle = 0\ ,$$
$$G_T \quad : \quad \langle\varphi_T\rangle = (v_T, 0, 0)\ ,\quad \langle\eta\rangle = (v_1, 0)\ ,\quad \langle\xi''\rangle = 0\ .$$

Leading order

- charged leptons: $m_l = \frac{v_T}{\sqrt{2}\Lambda} v_d$ diag$\left(y_e \left(\frac{\langle\theta\rangle}{\Lambda}\right)^{2n}, y_\mu \left(\frac{\langle\theta\rangle}{\Lambda}\right)^{n}, y_\tau\right)$

- neutrinos: $m_\nu = \frac{v_u^2}{\Lambda} \begin{pmatrix} a+2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a-b/3 \\ -b/3 & a-b/3 & 2b/3 \end{pmatrix} \Rightarrow$ **TB mixing!**

  with masses: $\frac{v_u^2}{\Lambda}$ diag$(a+b, a, -a+b)$ and $a = x_a \frac{u}{\Lambda}$, $b = x_b \frac{v_S}{\Lambda}$

- up-type and down-type quarks:

$$m_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_1 v_{\mathsf{T}}/\Lambda \left(\frac{\langle\theta\rangle}{\Lambda}\right)^n & y_4 v_1/\Lambda \left(\frac{\langle\theta\rangle}{\Lambda}\right)^n \\ 0 & y_3 v_1/\Lambda & y_t \end{pmatrix} v_u \;, \quad m_d = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_5 v_{\mathsf{T}}/\Lambda & y_8 v_1/\Lambda \\ 0 & y_7 v_1/\Lambda & y_b \end{pmatrix} v_d$$

$\Rightarrow m_b$ and m_t large, $\frac{m_c}{m_t} \sim \mathcal{O}(\lambda^4)$, $\frac{m_s}{m_b} \sim \mathcal{O}(\lambda^2)$, $|V_{cb}| \sim \mathcal{O}(\lambda^2)$

Higgs superpotential

$$\begin{aligned} w_d &= M\,(\varphi_T^0\,\varphi_T) + g\,(\varphi_T^0\,\varphi_T\,\varphi_T) + g_1\,(\varphi_S^0\,\varphi_S\,\varphi_S) + g_2\,\tilde{\xi}\,(\varphi_S^0\,\varphi_S) \\ &+ g_3\,\xi^0\,(\varphi_S\,\varphi_S) + g_4\,\xi^0\,\xi^2 + g_5\,\xi^0\,\xi\,\tilde{\xi} + g_6\,\xi^0\,\tilde{\xi}^2 \\ &+ M_\eta\,(\eta\,\eta^0) + M_\xi\,\xi''\,\xi'^0 + g_7\,\xi''\,(\varphi_T^0\,\varphi_T)' + g_8\,(\varphi_T^0\,\eta\,\eta) \\ &+ g_9\,(\varphi_T\,\eta\,\eta^0) + g_{10}\,\xi'^0\,(\varphi_T\,\varphi_T)'' \end{aligned}$$

- further fields needed: “driving fields” $\varphi_T^0 \sim (3,1)$, $\varphi_S^0 \sim (3,\omega)$, $\xi^0 \sim (1,\omega)$, $\eta^0 \sim (2'',1)$, $\xi'^0 \sim (1',1)$ under (T', Z_3)
- introduce $U(1)_R$ to construct Higgs potential:

$$Q(\text{matter}) = +1\,,\ \ Q(\text{Higgs}) = 0\,,\ \ Q(\text{driving field}) = +2$$

$\rightarrow w_d$ linear in driving fields
(Yukawa couplings are $U(1)_R$ invariant)

Subleading order

a.) take all terms up to $\mathcal{O}(\frac{1}{\Lambda^2})$ for charged fermions and all terms up to $\mathcal{O}(\frac{1}{\Lambda^3})$ for neutrinos

b.) take all terms up to $\mathcal{O}(\frac{1}{\Lambda})$ in Higgs sector

$\rightarrow$ induce VEV shifts: $\frac{\delta\text{VEV}}{\Lambda} \approx \left(\frac{\text{VEV}}{\Lambda}\right)^2 \approx \lambda^4$

<u>results</u>:

- important corrections to charged fermions are due to $\mathrm{b.)}$
- contributions from $\mathrm{a.)}$ and $\mathrm{b.)}$ correct TB mixing
- corrections should be $\lesssim \lambda^2$ for TB mixing and at the same time reproduce $\theta_C \approx \lambda$ in the quark sector
  $\rightarrow$ careful check needed $\quad \surd$
  (not possible in A_4 models considered before)

Subleading order (II)

- orders of mass matrix elements:

$$m_u = \begin{pmatrix} \lambda^8 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_u \; , \quad m_d = \begin{pmatrix} \lambda^6 & \lambda^3 & \lambda^4 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{pmatrix} v_d \; .$$

- two predictions:

$$\sqrt{\frac{m_d}{m_s}} = |V_{us}| + \mathcal{O}(\lambda^2)$$

and $\sqrt{\frac{m_d}{m_s}} = \left|\frac{V_{td}}{V_{ts}}\right| + \mathcal{O}(\lambda^2)$ (due to $|V_{ub}| \sim \mathcal{O}(\lambda^4)$)

- furthermore:

$$\frac{m_u}{m_c} \sim \mathcal{O}(\lambda^4) \; , \quad \frac{m_d}{m_s} \sim \mathcal{O}(\lambda^2) \; , \quad |V_{ud}| \approx |V_{cs}| \approx 1 + \mathcal{O}(\lambda^2) \; , \quad |V_{tb}| \approx 1 \; ,$$

$$|V_{us}| \approx |V_{cd}| \sim \mathcal{O}(\lambda) \; , \quad |V_{cb}| \approx |V_{ts}| \sim \mathcal{O}(\lambda^2) \; , \quad |V_{td}| \sim \mathcal{O}(\lambda^3) \; .$$

Outlook

- combine idea of flavor symmetry with GUT ?
  ... till now: no predictive and simple model
- origin of flavor symmetry ?
  ... till now: only postulated
- anomalies of flavor symmetries ?
  ... till now: not used as guideline for model building
- mechanism of VEV alignment
  ... till now: in most cases very complicated, simpler version ?
- breaking to non-trivial subgroup of flavor symmetry by VEV alignment
  ... till now: only used in A_4/ T' model, but maybe key to produce TB mixing, MTS and generally large and small mixings at same time
- group theory reason for success of A_4/ T' ... other symmetries ?
  ... till now: not explored

Thank you.