Three-flavour and matter effects in neutrino oscillations

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3 neutrino flavours exist: $\nu_e\,,\ \nu_\mu\,,\ \nu_\tau$

But: 2f description – a good 1st approximation in many cases.

Reasons:

- Hierarchy of Δm^2: $\Delta m^2_{\rm sol} \ll \Delta m^2_{\rm atm}$
- Smalness of $|U_{e3}|$.

Exceptions: $P(\nu_\mu \leftrightarrow \nu_\tau)$, $P(\nu_\mu \to \nu_\mu)$ and $P(\nu_\tau \to \nu_\tau)$ when oscillations due to the solar frequency ($\sim \Delta m^2_{\rm sol}$) are not frozen.

In any case, coorections due to 3-flavorness can reach $\sim 10\%$ – cannot be ignored at present

Also: a number of pure 3f effects exist $\Rightarrow$

$\diamondsuit$ 3f analyses are a must !

Two types of 3f effects

I. “Trivial” effects

- Existence of new physical channels – in addition to $\nu_e \leftrightarrow \nu_\mu$ there are $\nu_e \leftrightarrow \nu_\tau$ and $\nu_\mu \leftrightarrow \nu_\tau$; mutual influence of channels through unitarity (conservation of probability).

- New “parameter channels” for the same physical channel. E.g.: $\nu_e \leftrightarrow \nu_\mu$ oscill. can be governed by $(\Delta m_{21}^2, \theta_{12})$ and $(\Delta m_{31}^2, \theta_{13})$

Two types of 3f effects – contd.

II. Nontrivial effects

- Fundamental $\not{C}\not{P}$ and $\not{T}$
- Matter-induced $\not{T}$
- Interference of different "parameter channels" in $\nu_e \leftrightarrow \nu_{\mu,\tau}$ – specific contributions to oscillation probabilities
- Matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations

Nontrivial 3f effects (except the last one): disappear if at least one mixing angle is 0 or $90°$, or at least one $\Delta m_{ij}^2 = 0$

3f neutrino mixing

Relates flavour eigenstates $\nu_f = (\nu_e,\, \nu_\mu,\, \nu_\tau)$ with mass eigenstates $\nu_m = (\nu_1,\, \nu_2,\, \nu_3)$:

$$\nu_f \;=\; \tilde{U} \nu_m$$

$$\tilde{U} = U K\,, \qquad\qquad K = \mathrm{diag}(1\,, e^{i\sigma_1}\,, e^{i\sigma_2})$$

Majorana-type phases $\sigma_{1,2}$ do not affect neutrino oscillations.

The relevant part of the mixing matrix:

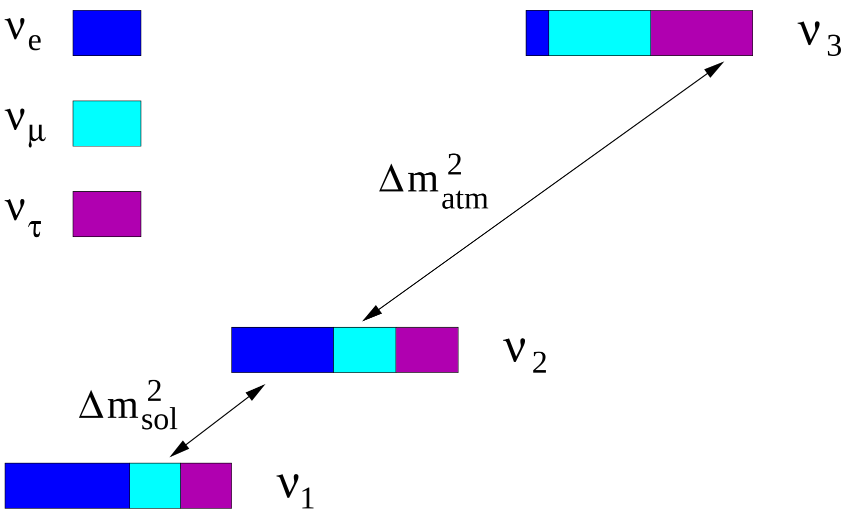
$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{\rm CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{\rm CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= O_{23}\, (\Gamma_\delta\, O_{13}\, \Gamma_\delta^\dagger)\; O_{12}\,, \qquad\qquad \Gamma_\delta \equiv \mathrm{diag}(1\,,\; 1\,,\; e^{i\delta_{\rm CP}})$$

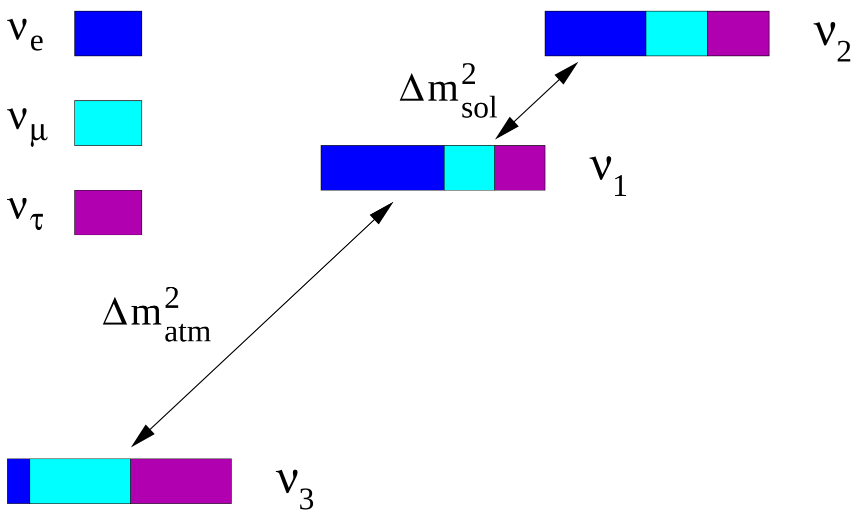
Leptonic mixing – contd.

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\rm CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{\rm CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{\rm CP}} & c_{13}c_{23} \end{pmatrix}$$

Normal hierarchy:



Inverted hierarchy:



Genuine 3f effects

$\not{C}\not{P}$ and $\not{T}$ in ν oscillations in vacuum

- $\not{C}\not{P}$: $P(\nu_a \to \nu_b) \neq P(\bar{\nu}_a \to \bar{\nu}_b)$
- $\not{T}$: $P(\nu_a \to \nu_b) \neq P(\nu_b \to \nu_a)$

CPT invariance: $\diamond$ $P(\nu_a \to \nu_b) \;=\; P(\bar{\nu}_b \to \bar{\nu}_a)$

$\not{C}\not{P} \;\Leftrightarrow\; \not{T}$ – *consequence of CPT*

Measures of $\not{C}\not{P}$ and $\not{T}$ – probability differences:

$$\Delta P_{ab}^{\rm CP} \equiv P(\nu_a \to \nu_b) - P(\bar{\nu}_a \to \bar{\nu}_b)$$

$$\Delta P_{ab}^{\rm T} \equiv P(\nu_a \to \nu_b) - P(\nu_b \to \nu_a)$$

From CPT:

$\diamond$ $\Delta P_{ab}^{\rm CP} = \Delta P_{ab}^{\rm T}\,;$ $\qquad \Delta P_{aa}^{\rm CP} = 0$

$\not{C}\not{P}$ and $\not{T}$ in ν oscillations in matter

Normal matter [(# of particles) $\neq$ (# of anti-particles)]:

The very presence of matter violates C, CP and CPT

$\Rightarrow$ Fake (extrinsic) $\not{C}\not{P}$. Exists even in 2f case. May complicate study of fundamental (intrinsic) $\not{C}\not{P}$

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Matter with density profile symmetric w.r.t. midpoint of neutrino trajectory does not induce any fake $\not{T}$. Asymmetric profiles do, but only for $N \geq 3$ flavors – an interesting 3f effect.

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Induced $\not{T}$: absent when either $U_{e3} = 0$ or $\Delta m^2_{\rm sol} = 0$ (2f limits)

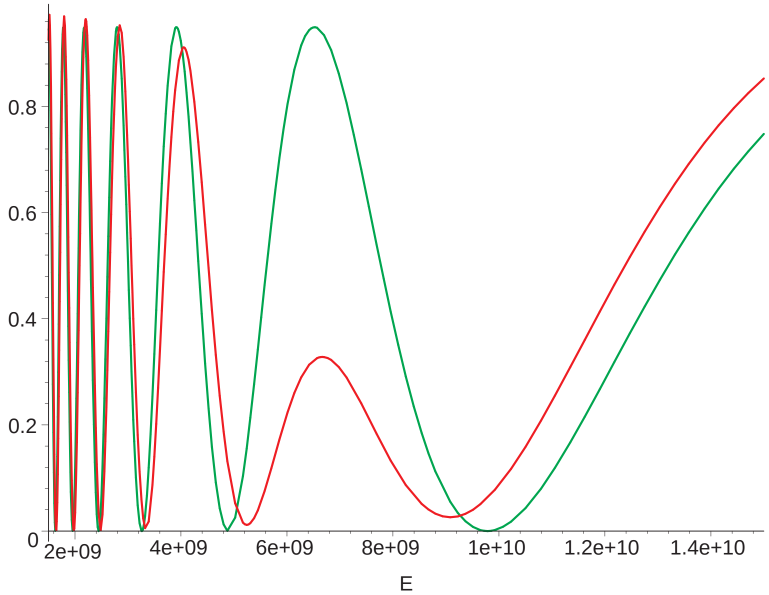
$\Rightarrow$ Doubly suppressed by both these small parameters

– effects in terrestrial experiments are small

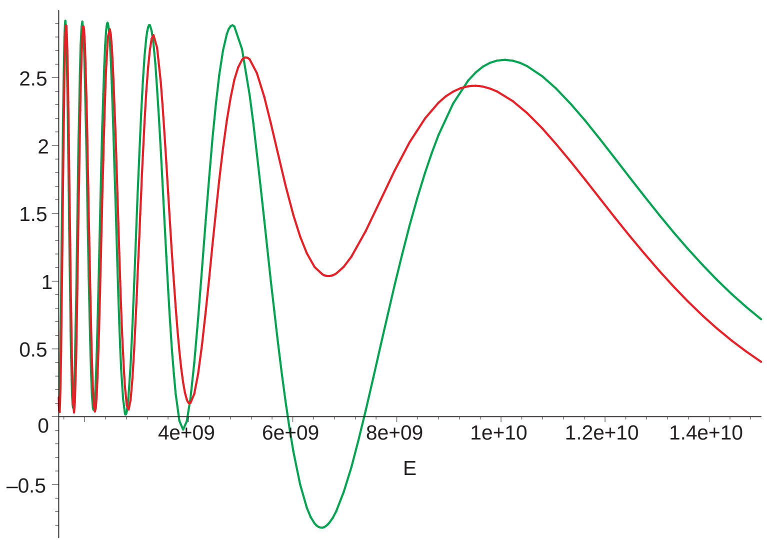
Matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations

In 2f approximation: no matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations [$V(\nu_\mu) = V(\nu_\tau)$ modulo tiny rad. corrections].

Not true in the full 3f framework! (E.A., 2002; Gandhi et al., 2004)



$P_{\mu\tau}$



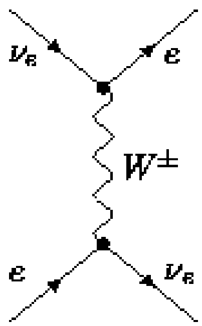
Oscillated flux of atm. ν_μ

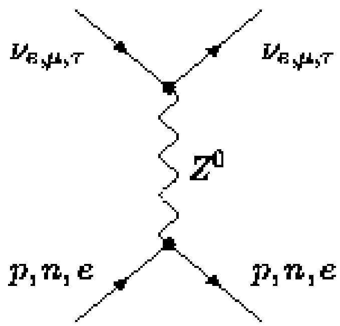
$\Delta m_{31}^2 = 2.5 \times 10^{-3}$ eV^2, $\sin^2\theta_{13} = 0.026$, $\theta_{23} = \pi/4$, $\Delta m_{21}^2 = 0$, $L = 9400$ km

Red curves – w/ matter effects, green curves – w/o matter effects on $P_{\mu\tau}$

Neutrino oscillations in matter

Coherent forward scattering on the particles in matter





$$V_e^{\rm CC} \equiv V = \sqrt{2}\, G_F\, N_e$$

2f neutrino evolution equation:

$\diamondsuit$

$$i\frac{d}{dt}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + V & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix}$$

Mixing in matter

◇ $$\sin^2 2\theta_m = \frac{\sin^2 2\theta \cdot (\frac{\Delta m^2}{2E})^2}{[\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e]^2 + (\frac{\Delta m^2}{2E})^2 \sin^2 2\theta}$$

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$|\nu_{1m}\rangle$, $|\nu_{2m}\rangle$ – eigenstates of H in matter (matter eigenstates)

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$$N_e \gg (N_e)_{\rm res}: \quad \theta_m \approx 90^\circ$$

$$N_e = (N_e)_{\rm res}: \quad \theta_m = 45^\circ$$

$$N_e \ll (N_e)_{\rm res}: \quad \theta_m \approx \theta$$

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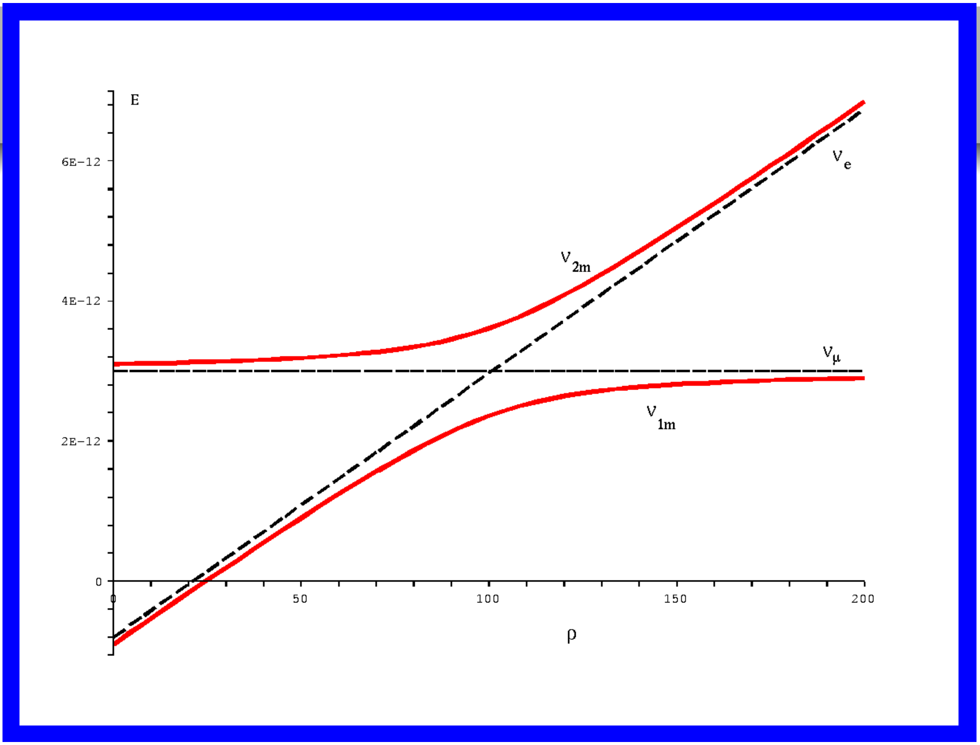
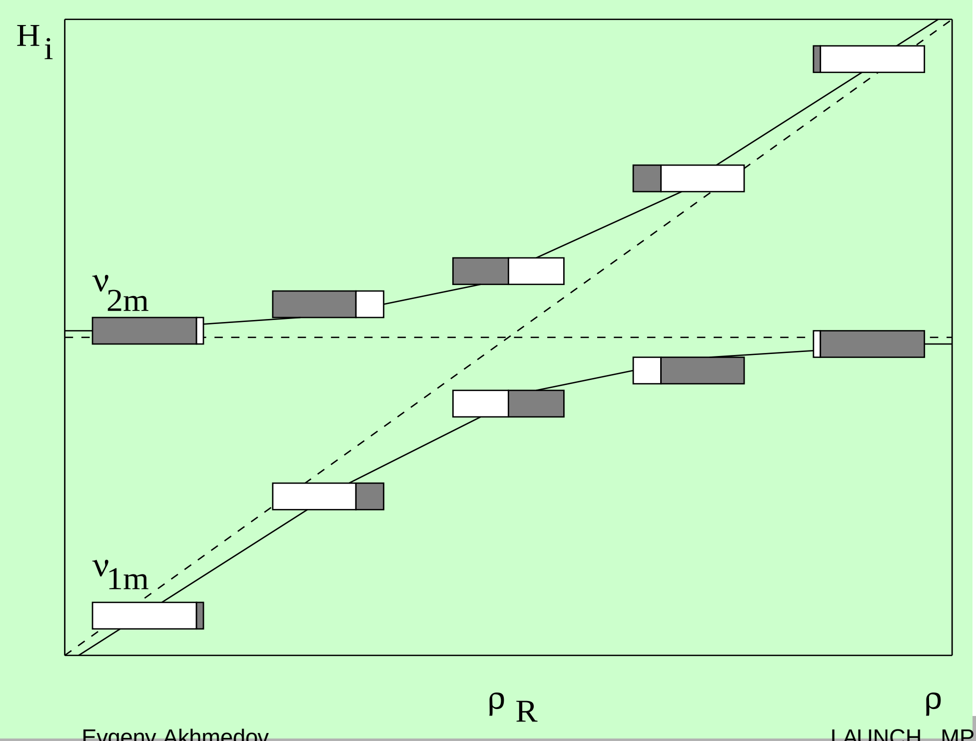




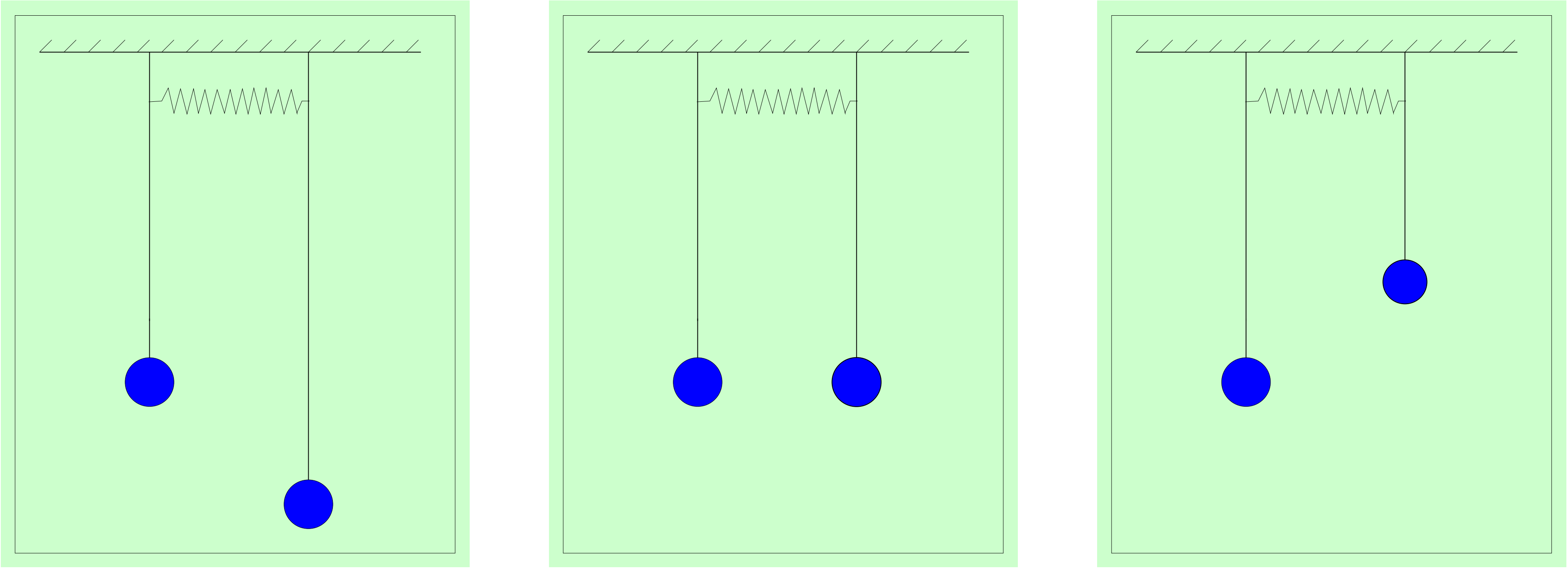
Adiabaticity: slow density change along the neutrino path

$$\frac{\sin^2 2\theta}{\cos 2\theta}\,\frac{\Delta m^2}{2E}\,L_\rho \gg 1$$

L_ρ – electron density scale hight:

$$L_\rho = \left|\frac{1}{N_e}\frac{dN_e}{dx}\right|^{-1}$$

Analogy: Two coupled pendula



Mechanical model of the MSW effect

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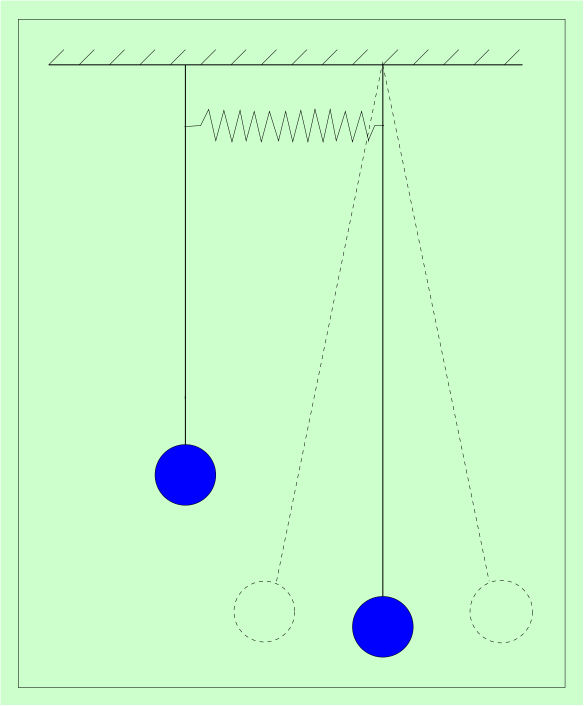

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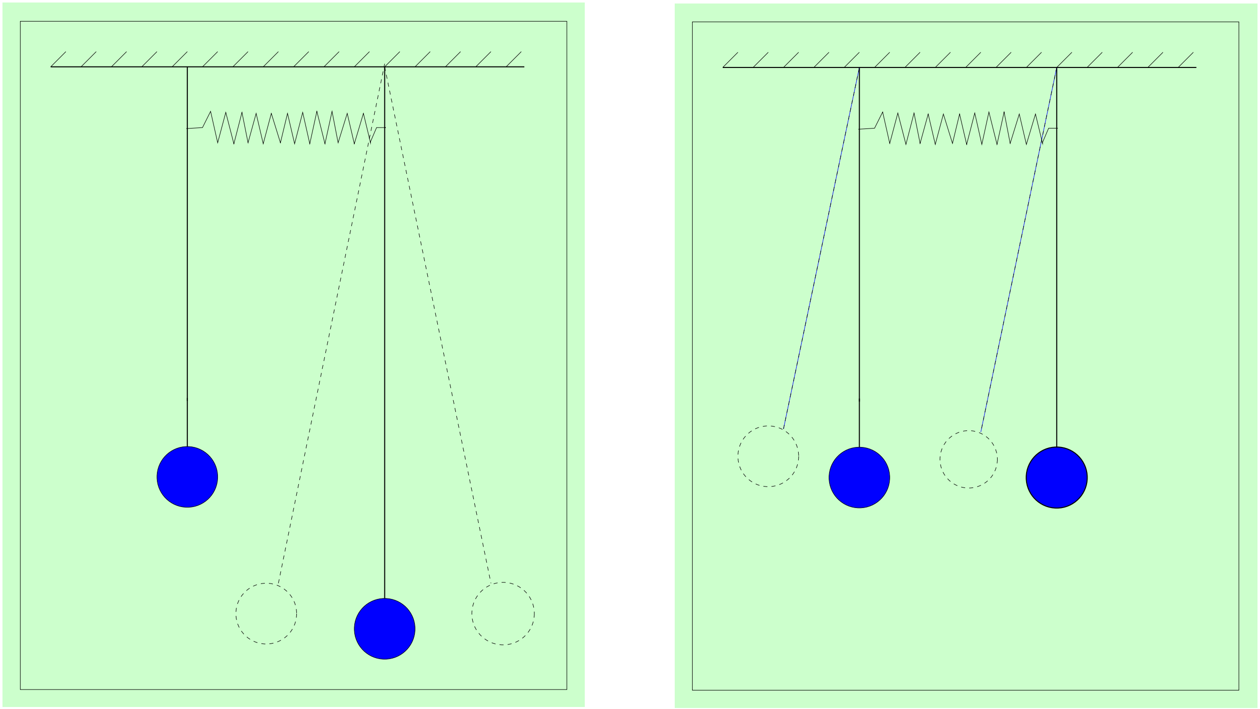

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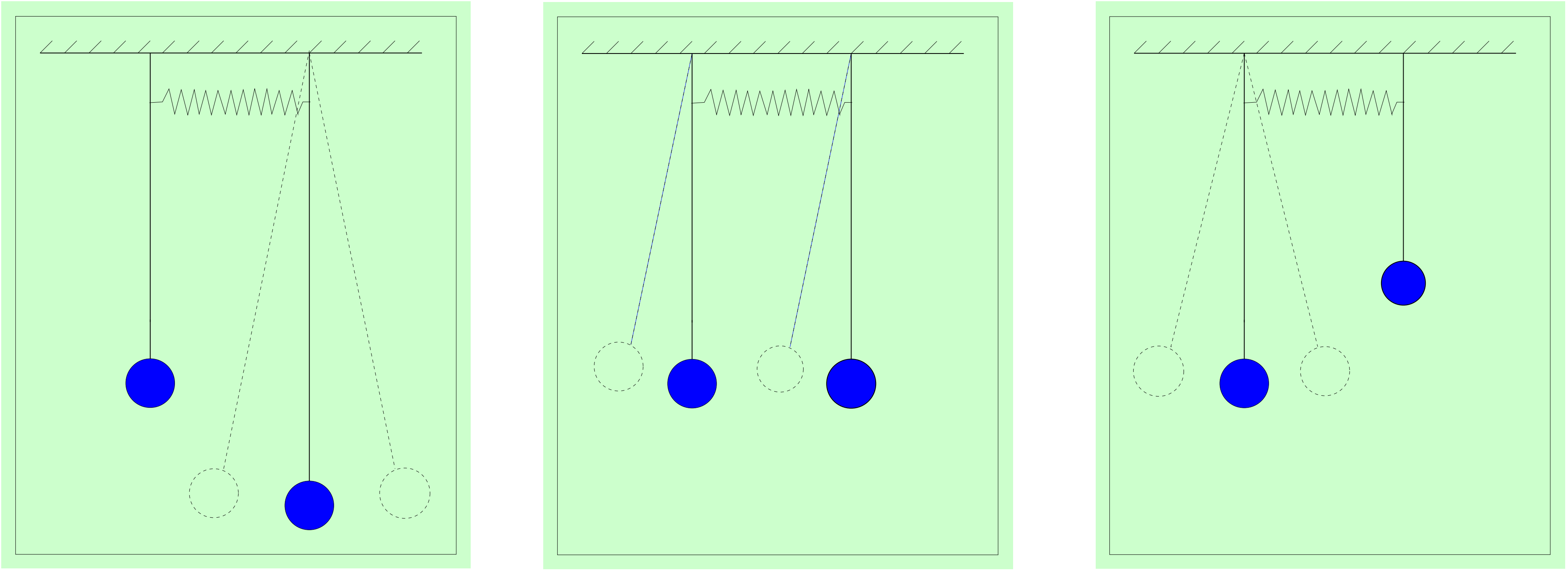
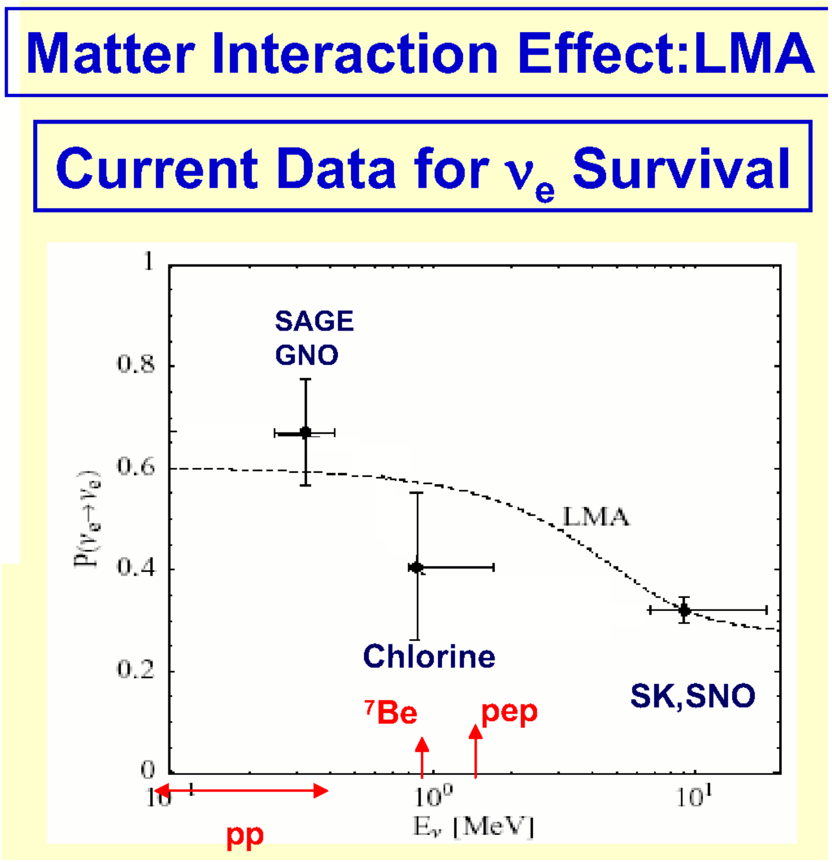
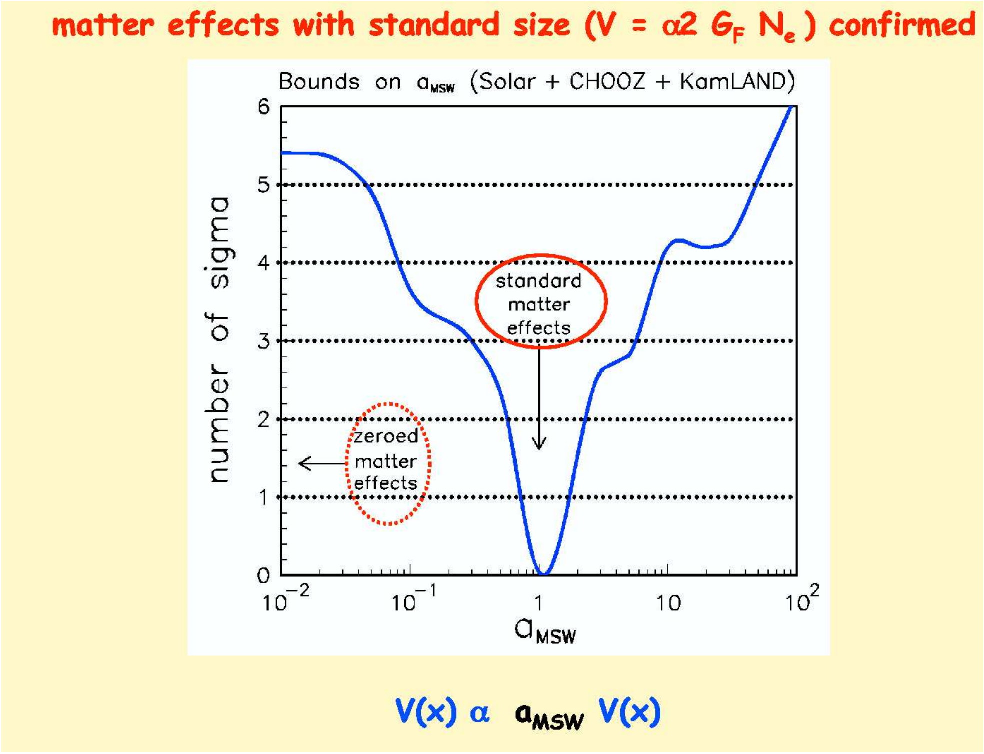


Mechanical model of the MSW effect

Evidence for the MSW effect





$V(x) \Rightarrow a_{\rm MSW} V(x)$; $a_{\rm MSW} = 1$ strongly favoured

(Fogli et al. 2003, 2004; Fogli & Lisi 2004)

Another possible matter effect

Parametric resonance in neutrino oscillations

Parametric resonance in oscillating systems with varying parameters: occurs when the rate of the parameter change is correlated in a certain way with the values of the parameters themselves

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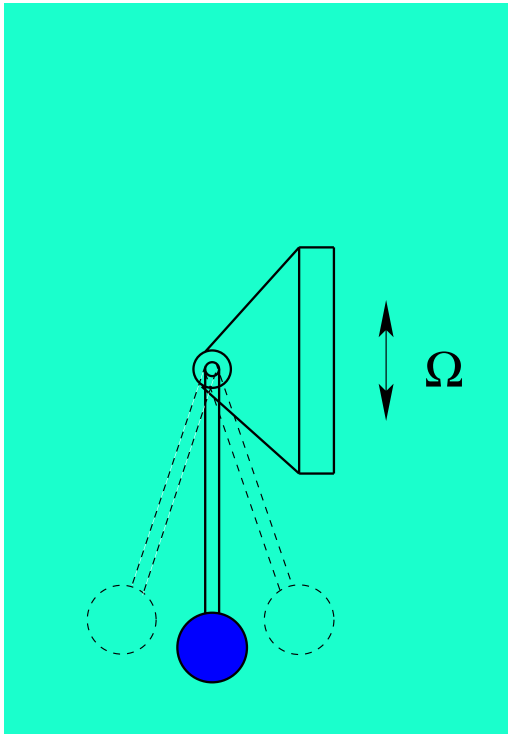

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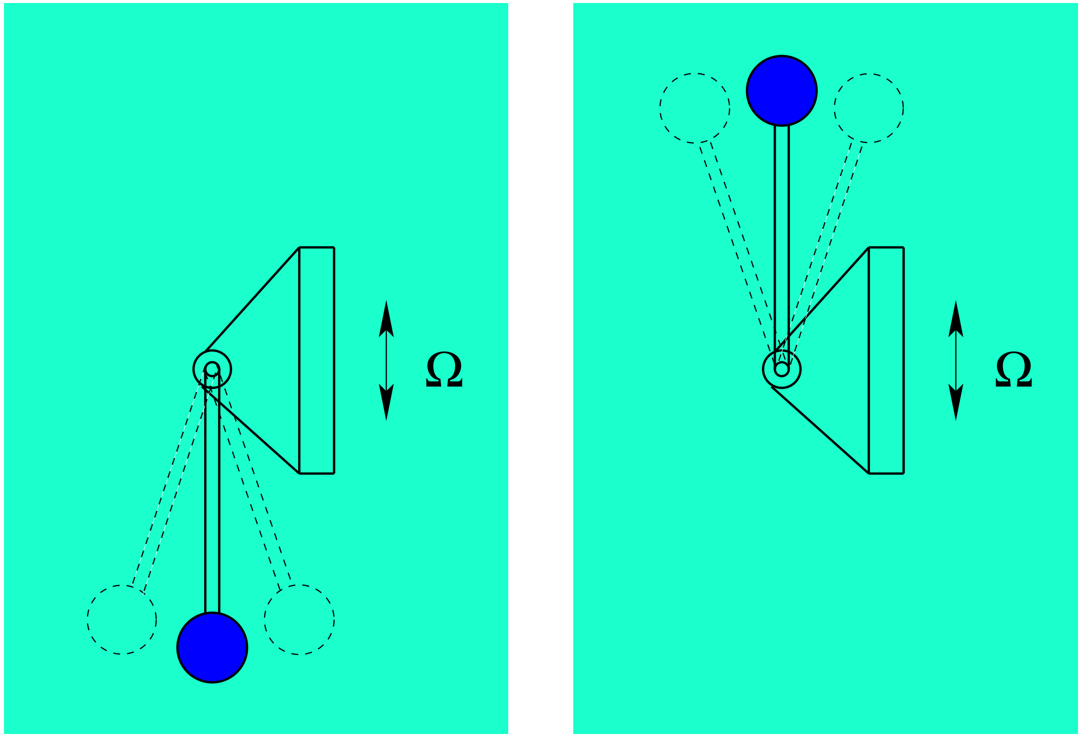

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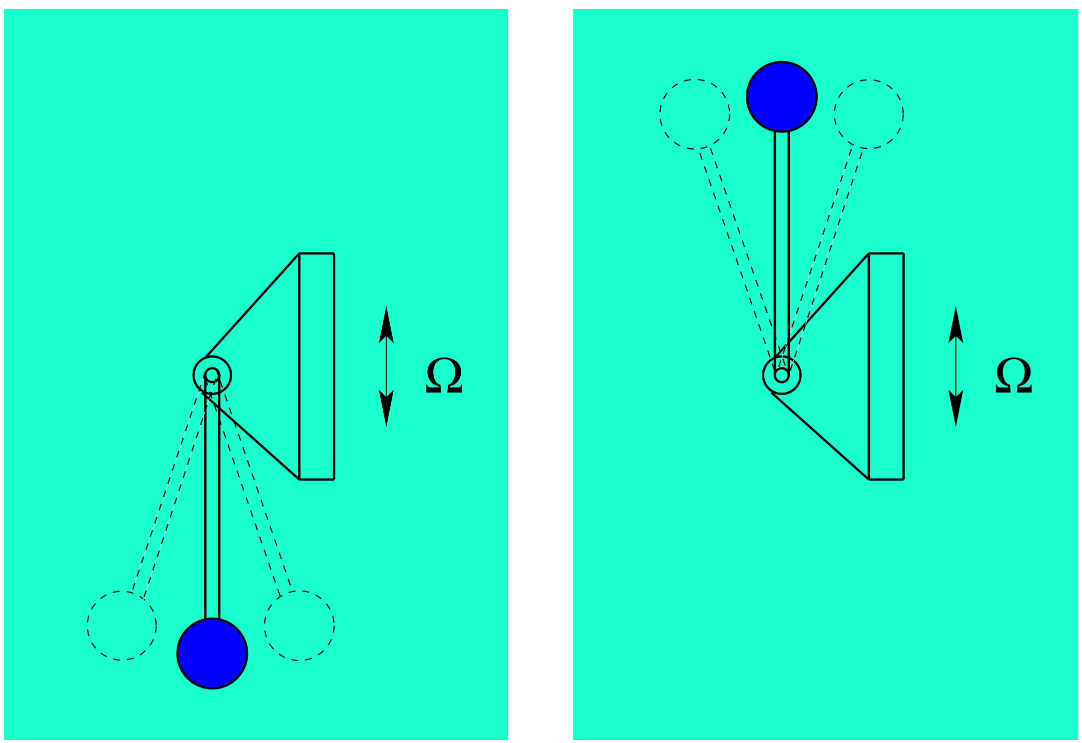


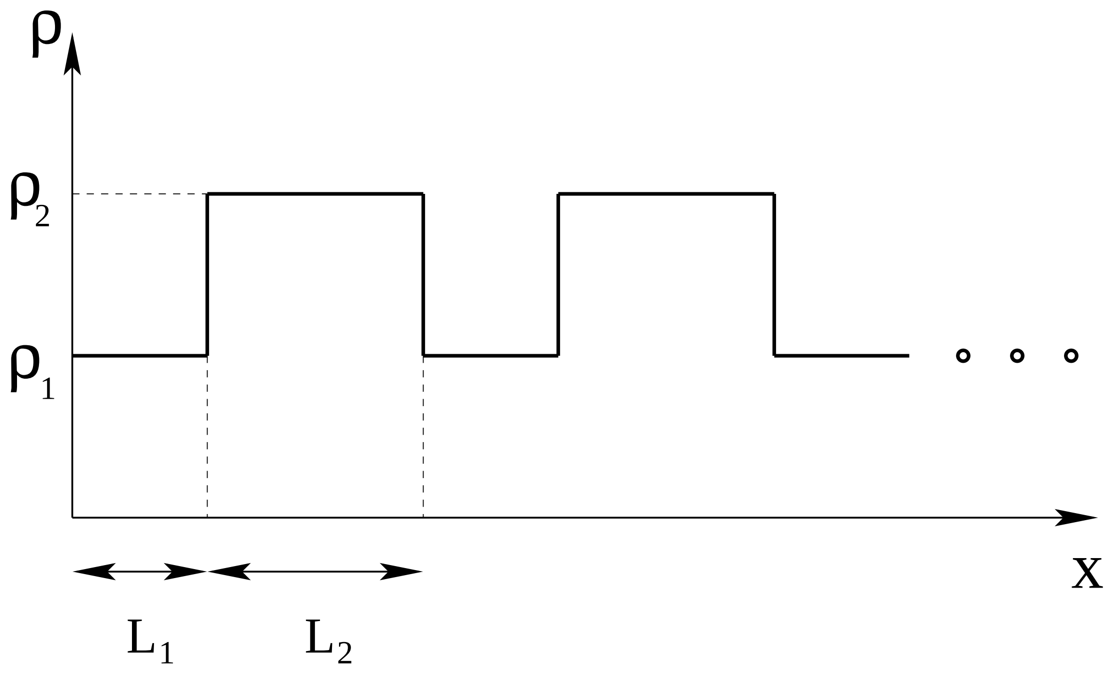
For small-ampl. osc.:

$$\Omega_{\rm res} = \frac{2\omega}{n}$$

$n = 1, 2, 3...$

Different from MSW eff. – no level crossing !

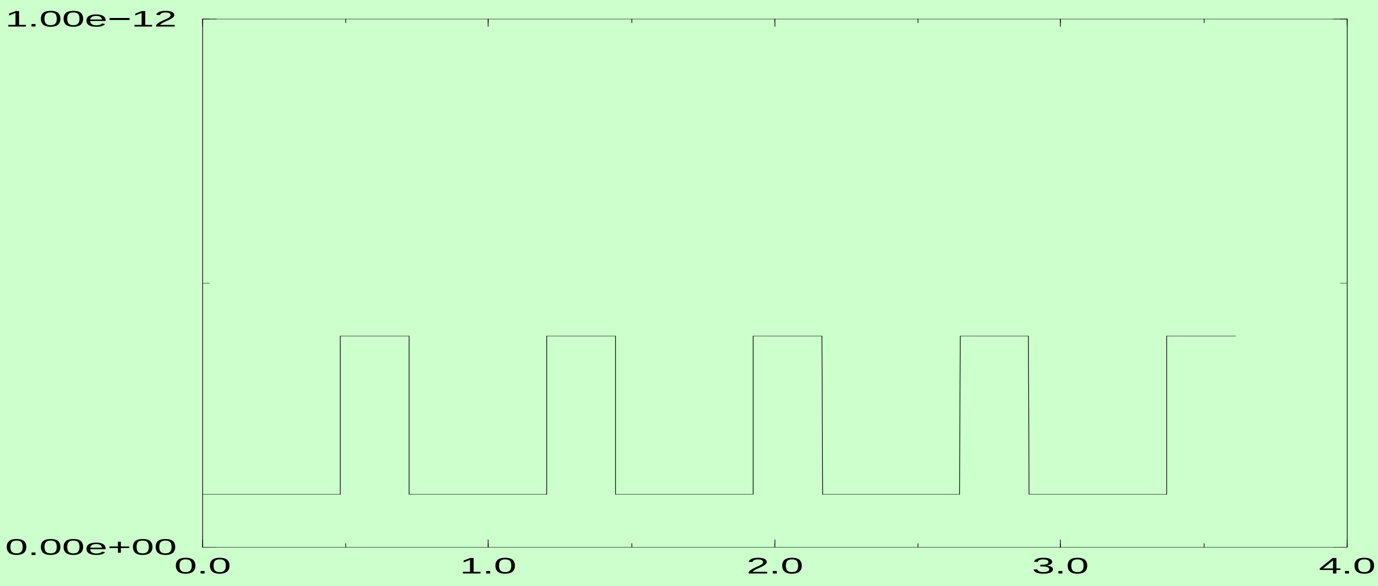
An example admitting an exact analytic solution – “castle wall” density profile (E.A., 1987, 1998):

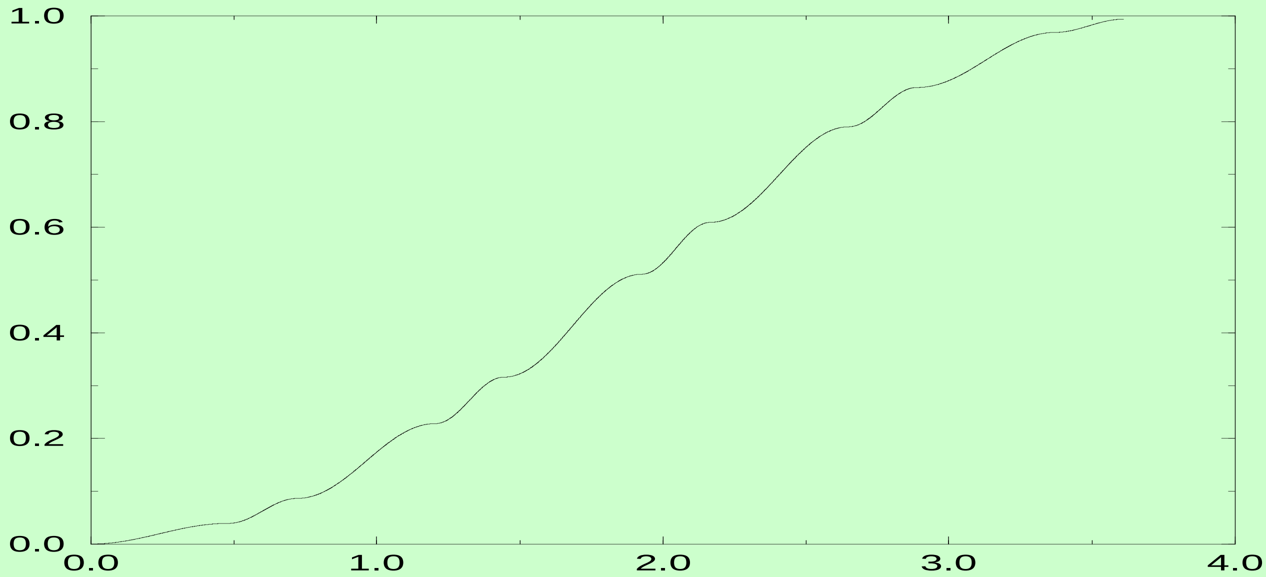


Resonance condition:

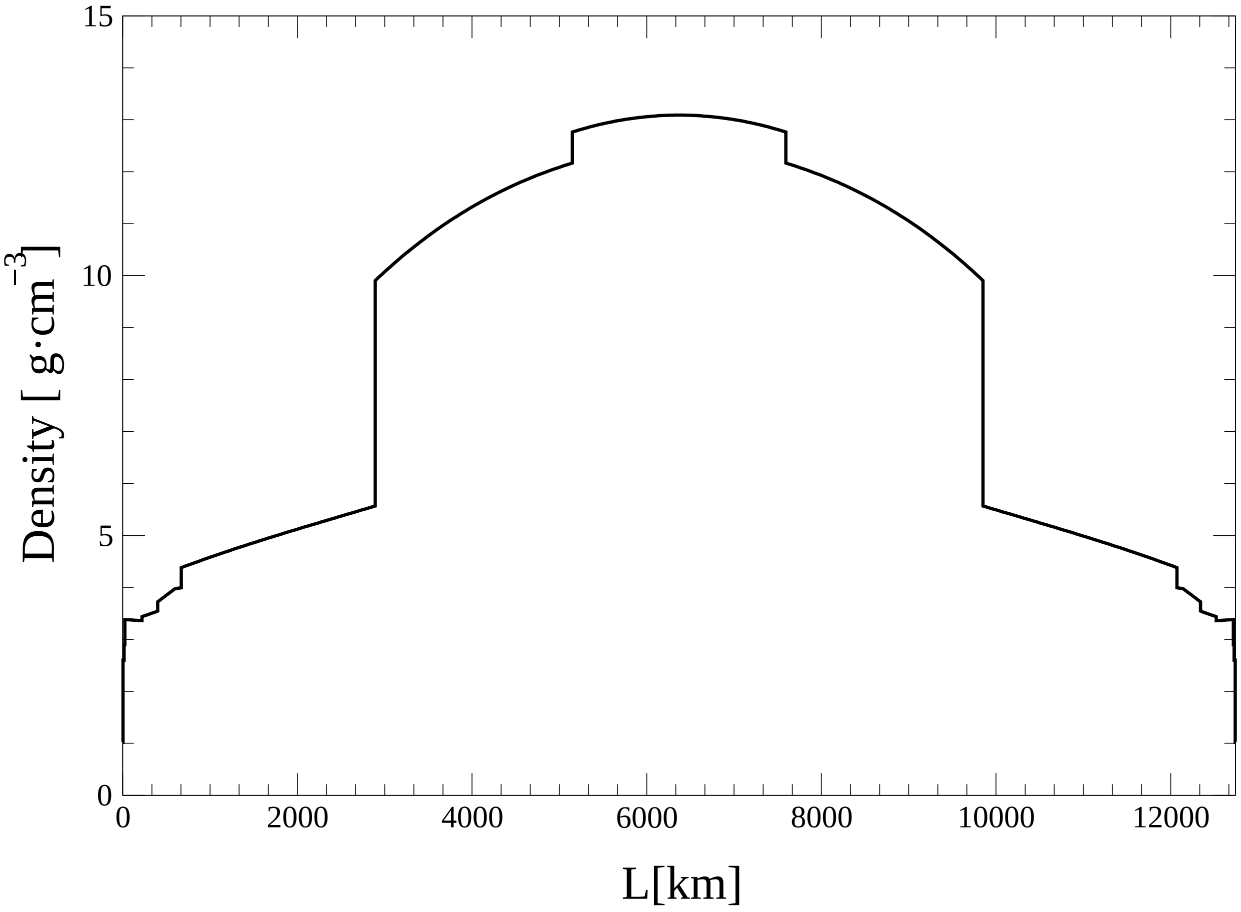
$$X_3 \equiv -(\sin\phi_1 \cos\phi_2 \cos 2\theta_{1m} + \cos\phi_1 \sin\phi_2 \cos 2\theta_{2m}) = 0$$

$\phi_{1,2}$ – oscillation phases acquired in layers 1, 2


1.00e−12
0.00e+00
0.0
1.0
2.0
3.0
4.0


1.0
0.8
0.6
0.4
0.2
0.0
0.0
1.0
2.0
3.0
4.0

Earth's density profile (PREM model) :



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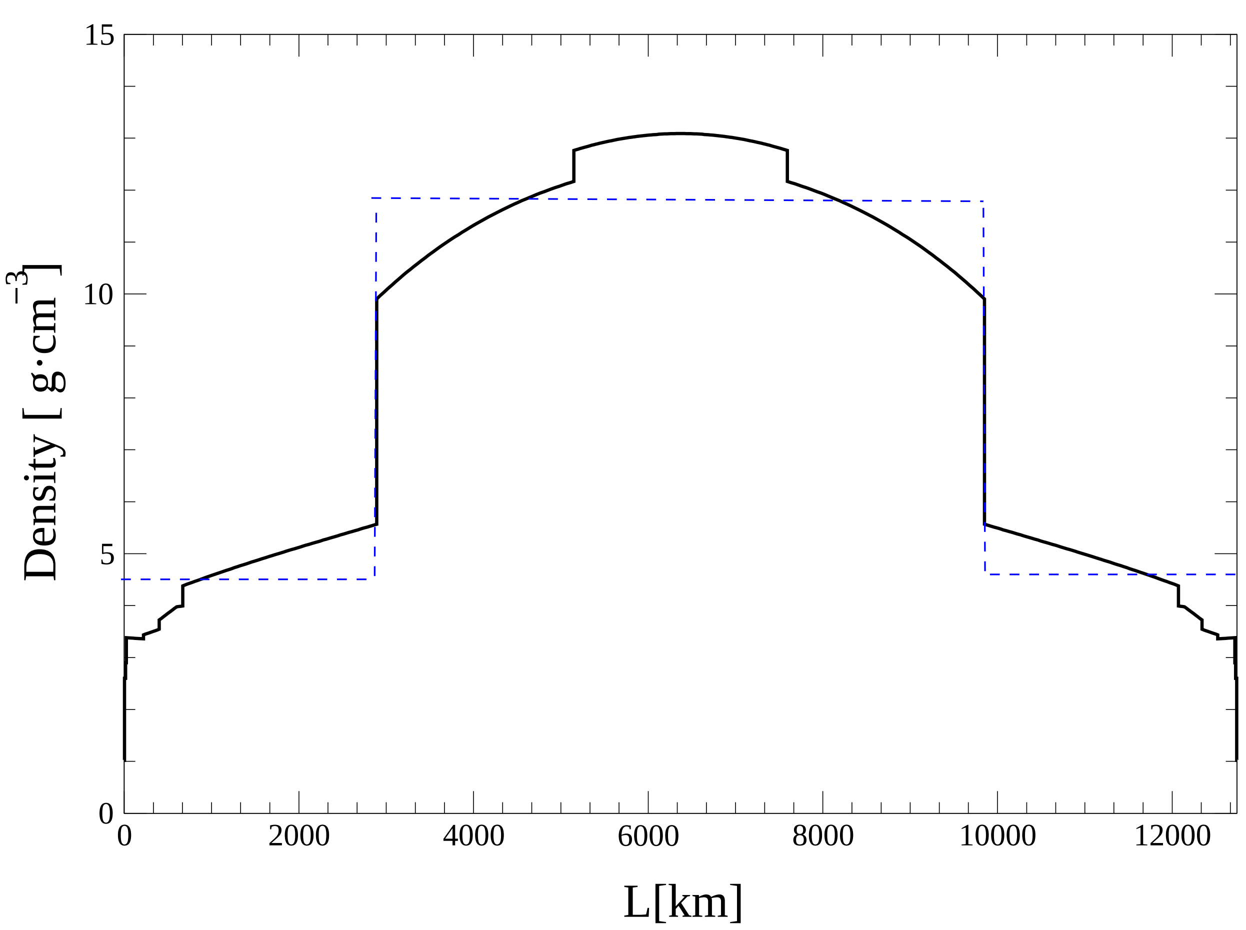
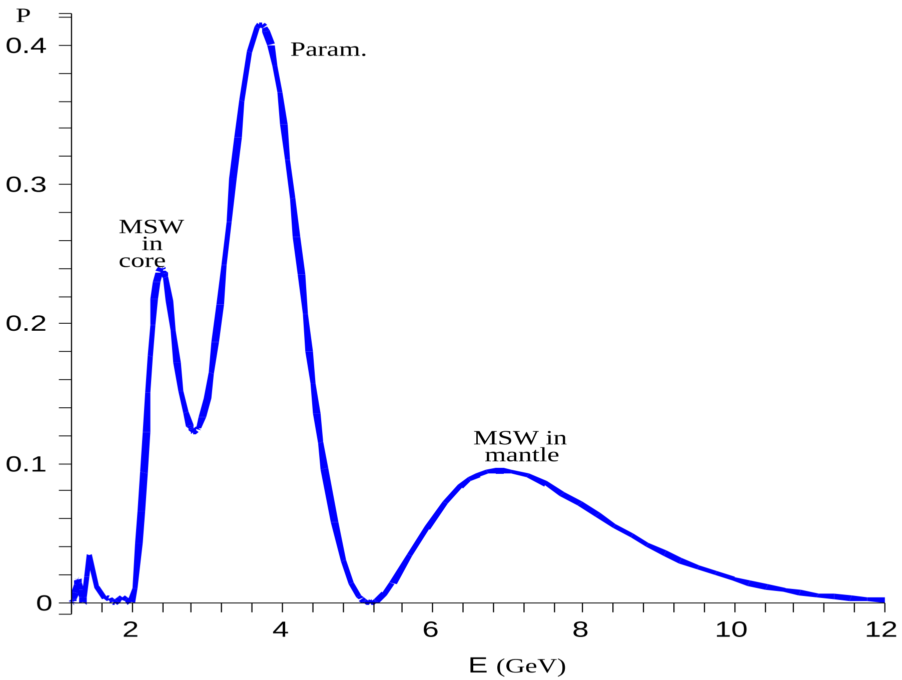
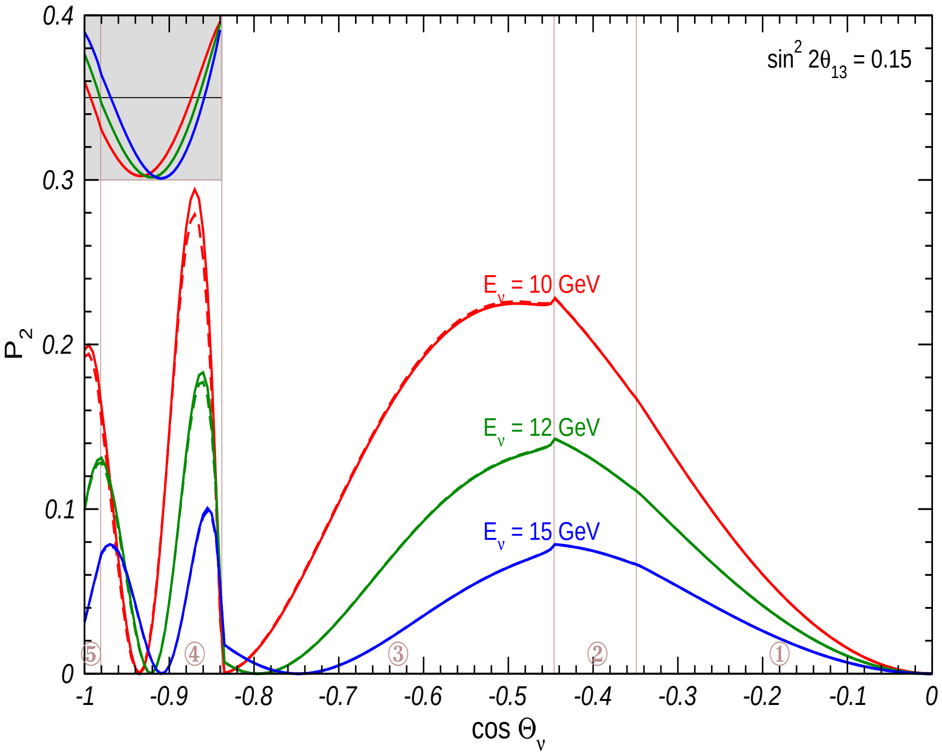


Param. res. condition: $(l_{osc})_{\rm matt} \simeq l_{\rm density\ mod.}$

Fulfilled for $\nu_e \leftrightarrow \nu_{\mu.\tau}$ oscillations of core-crossing ν's in the Earth for a wide range of energies and zenith angles !





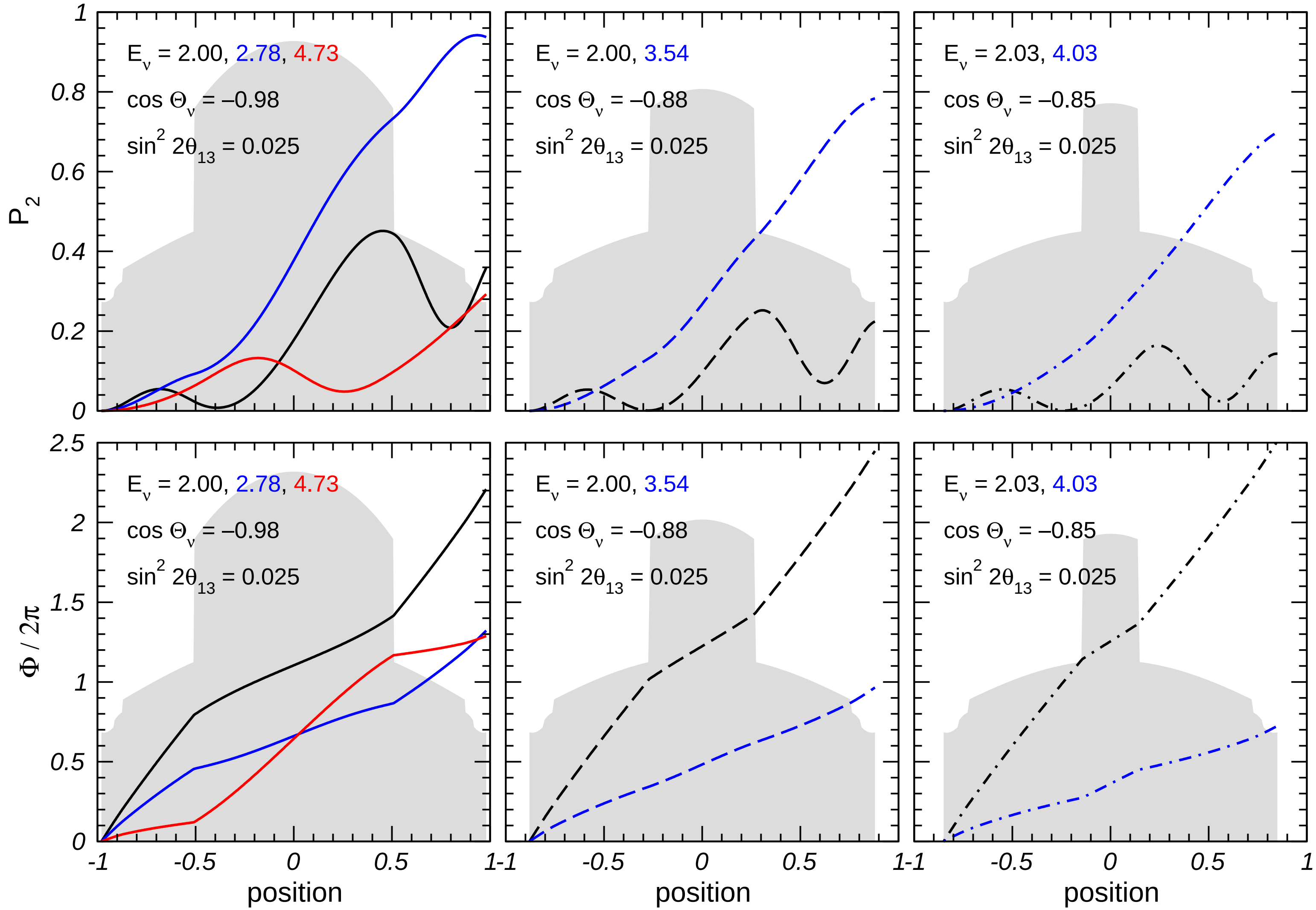
Intermed. energies

$\cos\Theta = -0.89 \qquad \sin^2 2\theta_{13} = 0.01$

(Liu, Smirnov, 1998; Petcov, 1998; E.A. 1998)

High energies, $\cos\Theta$ - dependence

(E.A., Maltoni & Smirnov, 2005)



♢ Parametric resonance of ν oscillations in the Earth: can be observed in future atmospheric or accelerator experiments if θ_{13} is not much below its current upper limit

Neutrino oscillations in the Earth

A coherent description in terms of different realizations of just 2 conditions – amplitude and phase conditions

Matter with $N_e = const$:

- amplitude condition = MSW resonance condition
- phase condition: $\phi \;=\; \pi/2 + \pi n$

3 layers of constant densities (or the "castle wall" density profile):

- amplitude condition = parametric resonance condition ($X_3 = 0$)
- phase condition: $\Phi \;\equiv\; \arccos Y = \pi/2 + \pi n$

Evolution matrix: $\nu(t) \;=\; U(t,\, t_0)\,\nu(0)$. For 2 layers:

$$U^{(2)}(t,\, t_0) \;=\; \begin{pmatrix} Y - iX_3 & -i(X_1 - iX_2) \\ -i(X_1 + iX_2) & Y + iX_3 \end{pmatrix}, \qquad Y^2 + \mathbf{X}^2 \;=\; 1$$

The meaning of the amplitude condition

Alignment of the transitions amplitudes in different layers.

Evolution matrices for individual layers:

$$U_i(t,\,t_0) \;=\; \begin{pmatrix} \alpha_i & \beta_i \\ -\beta_i^* & \alpha_i^* \end{pmatrix}, \qquad |\alpha_i|^2 + |\beta_i|^2 \;=\; 1\,, \qquad i = 1, 2, 3$$

For 2 layers: $U^{(2)} = U_2 U_1$,

$$\beta^{(2)} \;=\; \alpha_2\,\beta_1 \;+\; \beta_2\,\alpha_1^*$$

Alignment (collinearity) condition:

$$\arg(\alpha_2\,\beta_1) \;=\; \arg(\beta_2\,\alpha_1^*) \qquad \mathrm{mod}\ (\pi)$$

– potentially leads to maximal trans. probability.

For 2 layers of const. densities: align. cond. $\Leftrightarrow$ $s_1 s_2 X_3 \;=\; 0$

How about 3 layers?

$U^{(3)} = U_3 U_2 U_1$. For the Earth, $U^{(3)} = U_1^T U_2 U_1$.

Transition amplitude:

$$\beta^{(3)} = \alpha_1 \alpha_2 \beta_1 - \alpha_1^* \alpha_2^* \beta_1^* + |\alpha_1|^2 \beta_2 + |\beta_1|^2 \beta_2^*$$

$\Rightarrow$ If the 2-layer align. cond. is satisfied, so is the 3-layer one !

A consequence of

- The symmetry of the core density profile
- The symmetry of the overall density profile of the Earth (3rd layer's profile is the reverse of the 1st layer's one)

$\Rightarrow$ The generalized amplitude condition is the alignment condition in the case of non-constant density layers

Generalized phase condition

For constant density matter: $\phi \;=\; \pi/2 + \pi n \quad \Leftrightarrow \quad \mathrm{Im}\, \alpha^{(1)}\beta^{(1)*} = 0.$

$\Rightarrow$ Generalize to an arbitrary density profile:

$$\mathrm{Im}\, \alpha\beta^* \;=\; 0 \qquad \Leftrightarrow \qquad \frac{dP_{\mathrm{tr}}}{dL} \;=\; 0$$

The whole complex oscillation pattern:

- MSW resonances
- parametric resonances
- saddle points
- local maxima and minima
- absolute maxima and minima

can be understood in terms of the generalized amplitude and phase conditions ! (E.A., Maltoni & Smirnov, 2007)

Neutrino oscillograms of the Earth

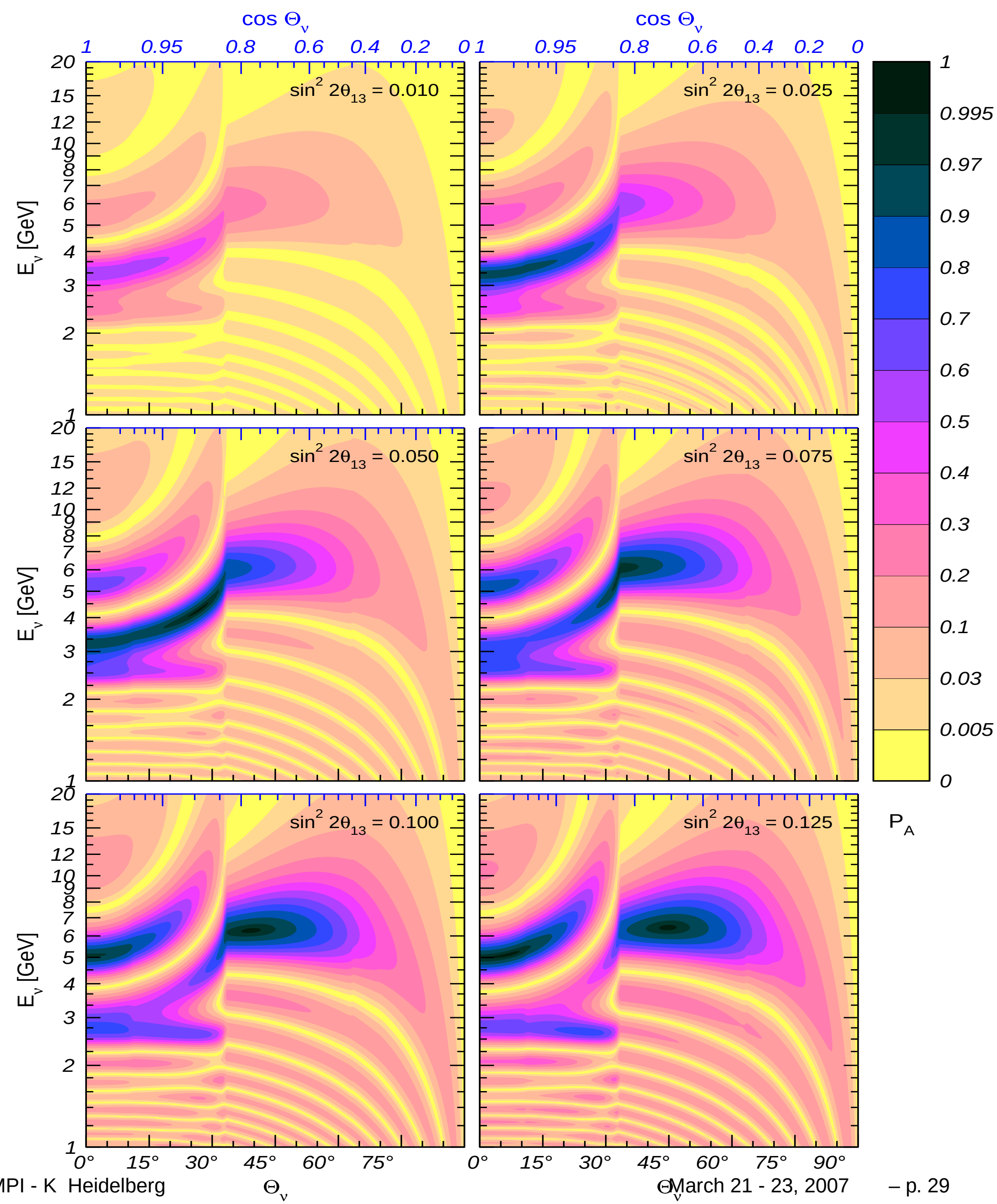
Contours of equal osc. probabilities in (Θ_ν, E_ν) plane

Θ_{13} - dependense of P_A $\Rightarrow$

P_A – effective 2f transition probability $(\Delta m^2_{\rm sol} \to 0)$

$$P_{e\mu} = s^2_{23} P_A$$

$$P_{e\tau} = c^2_{23} P_A$$

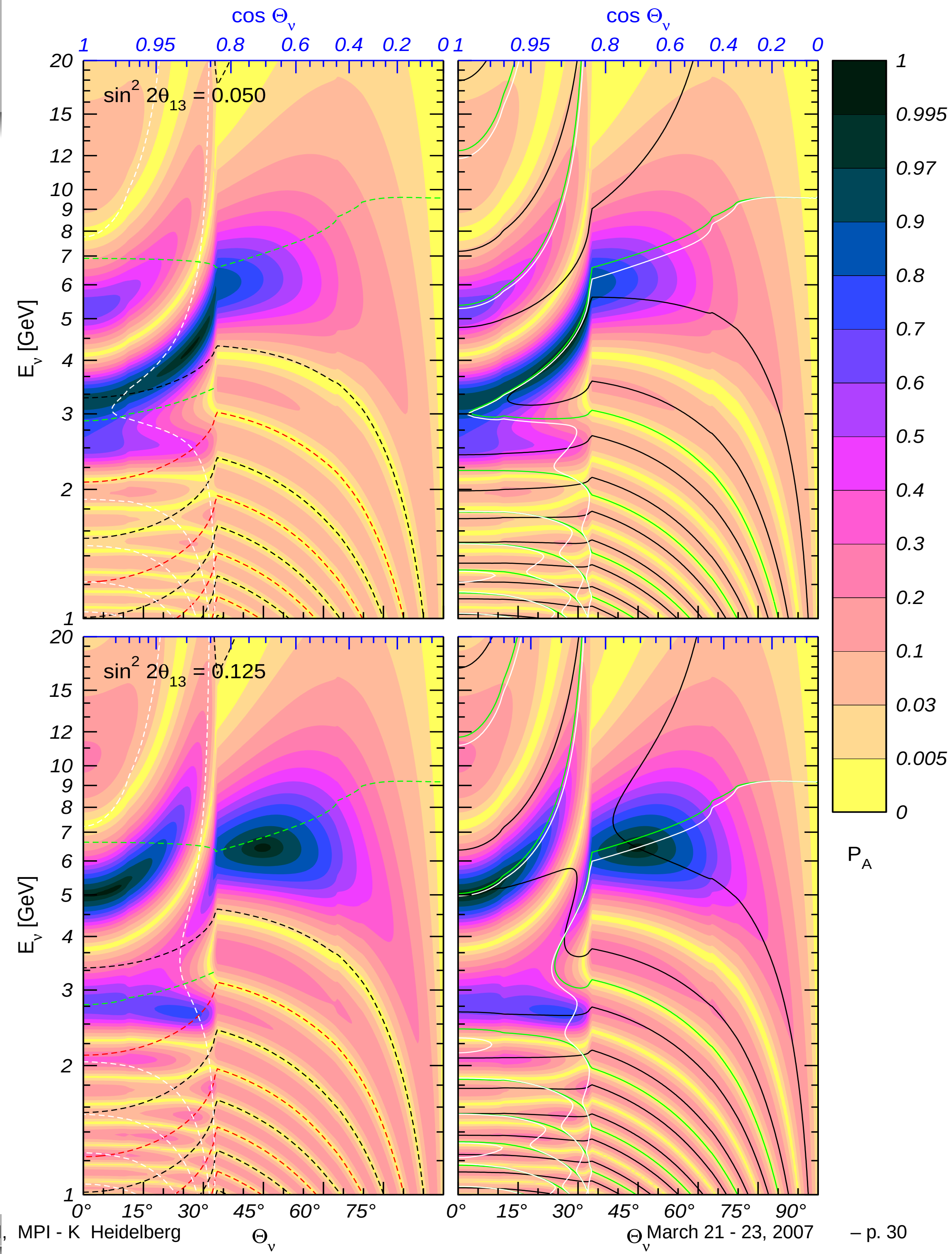


In the right panels:

– alignment (collin.) cond.

– generalized res. cond. ($\mathrm{Im}\,\alpha^{(2)} = 0$)

– generalized phase cond.



Including the effects of $\Delta m^2_{\rm sol}$

Fundamental $\not{C}\not{P}$ and $\not{T}$; dependence of P_{ab} on $\delta_{\rm CP}$ (also in CP - and T - even terms) $\Rightarrow$ parameter correlations and degeneracies (e.g. θ_{13} and $\delta_{\rm CP}$)

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"Magic" baseline: $L \simeq 7300$ km – dependence on $\delta_{\rm CP}$, $\Delta m^2_{\rm sol}$ disappears (Barger *et al.*, 2001; Huber & Winter, 2003; Huber *et al.*, 2006)

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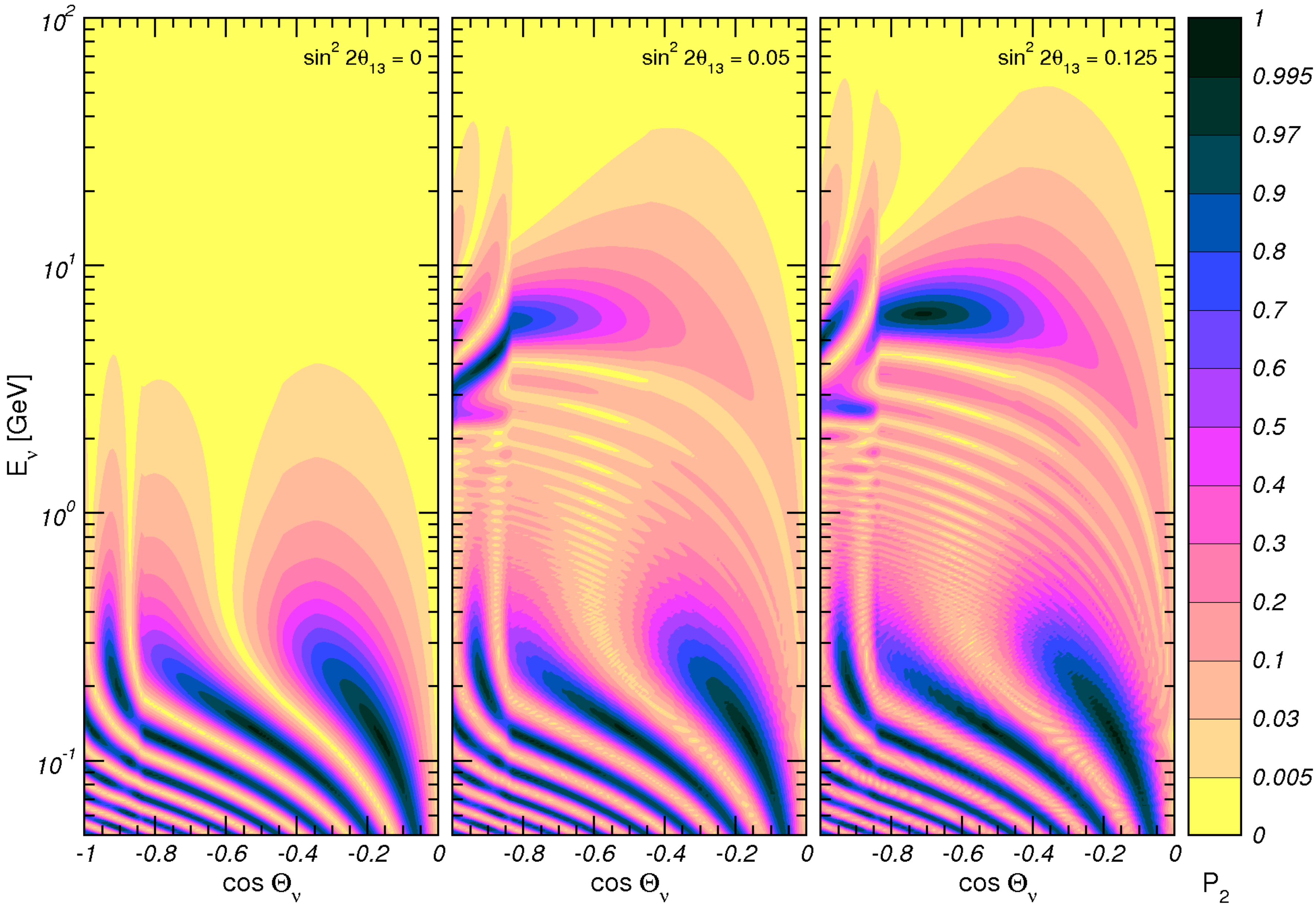
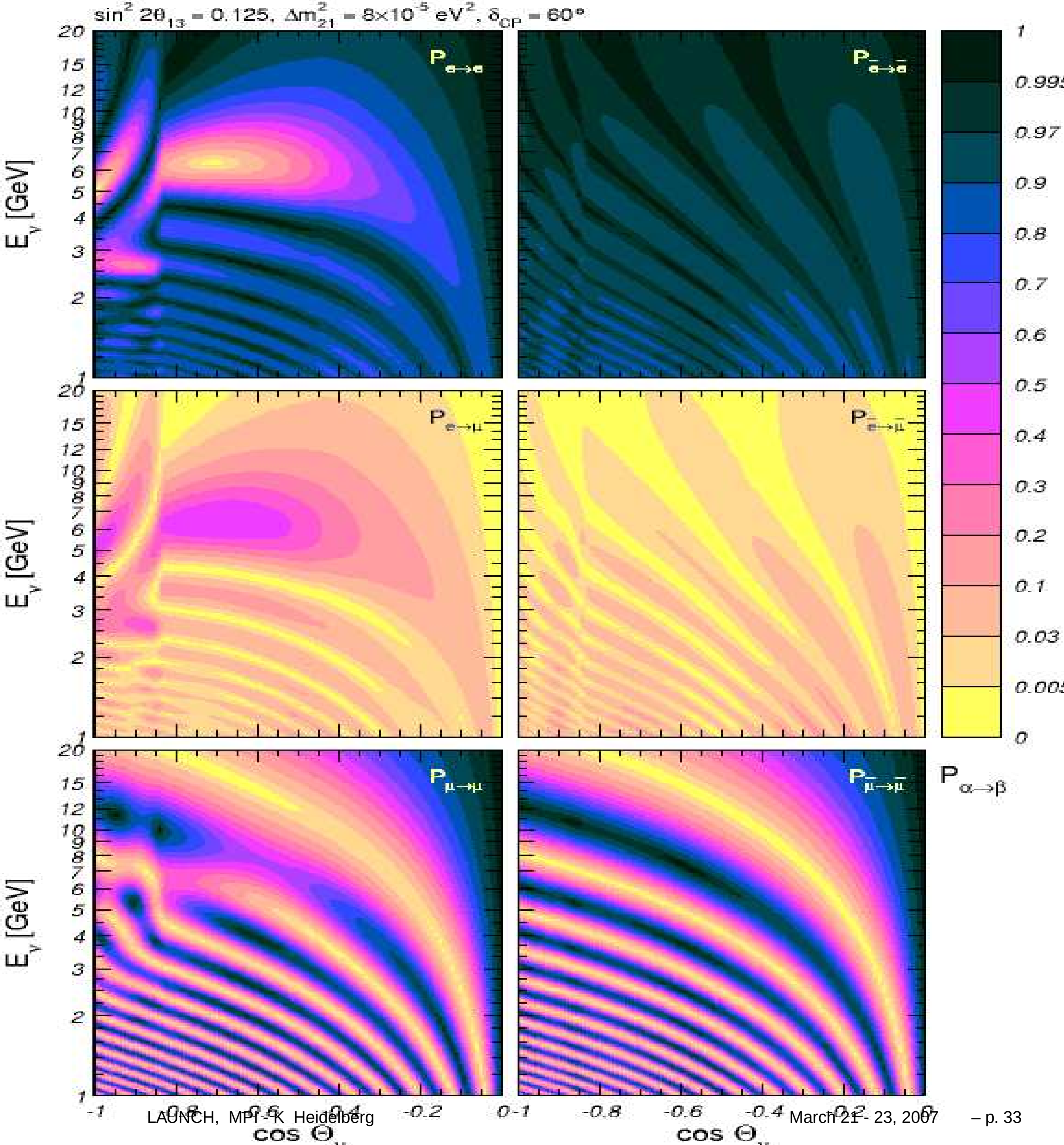
At high energies: $\boxed{VL = 2\pi n} \quad \Leftrightarrow \quad L = l_{\rm refr.} n$

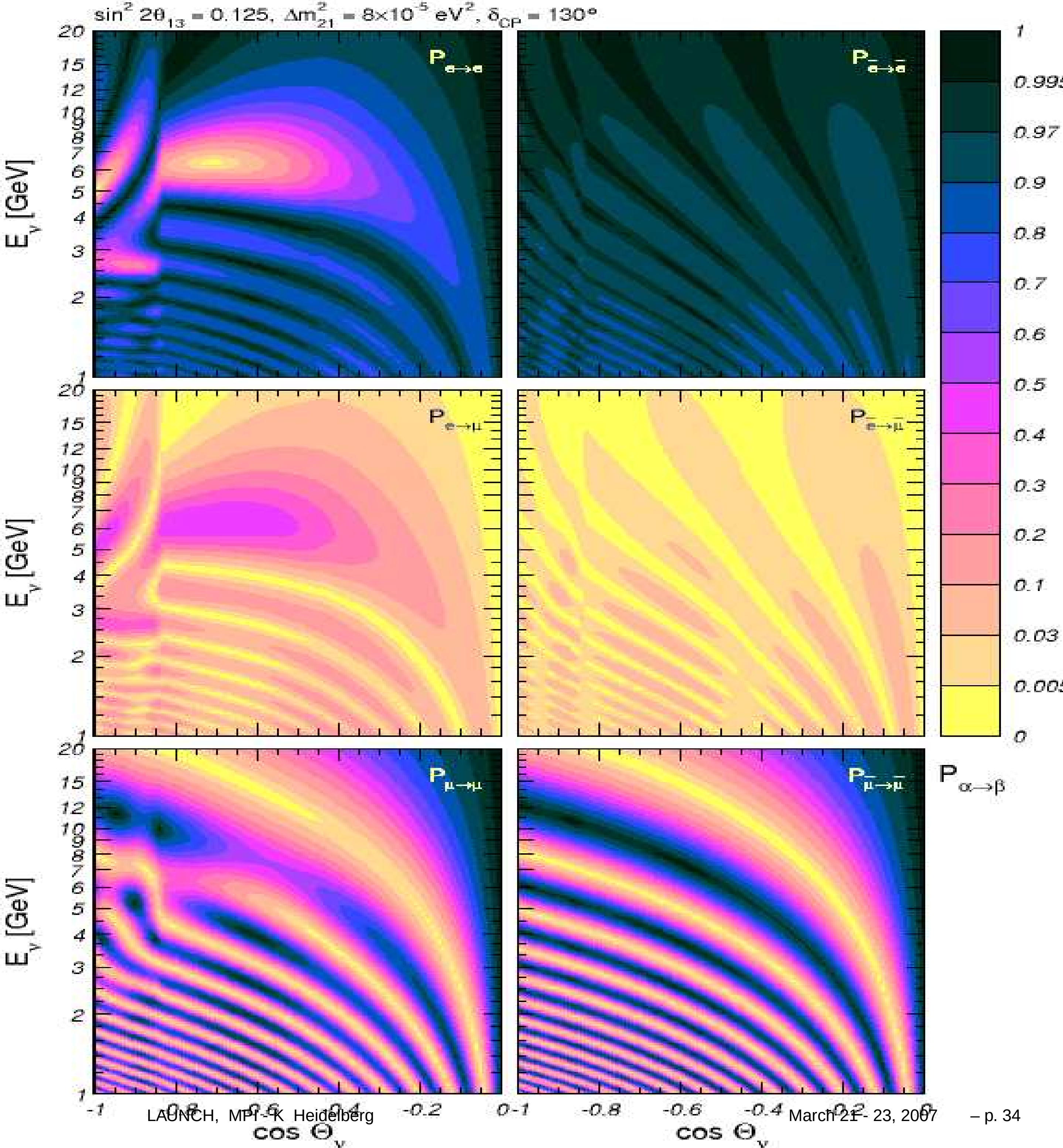
Including the effects of $\Delta m^2_{\rm sol}$: $(1 - P_{ee})$



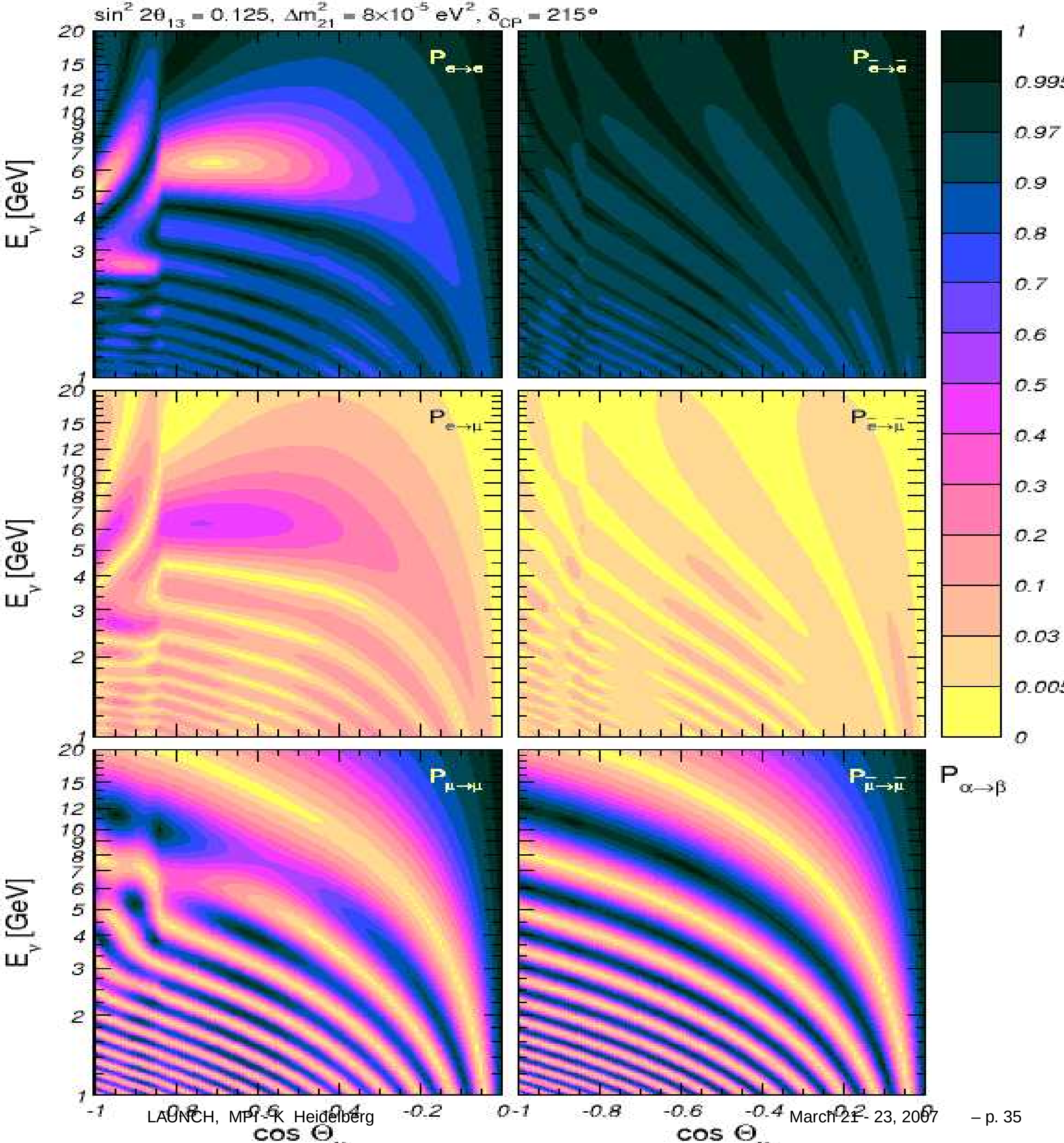
Dependence on $\delta_{\rm CP}$



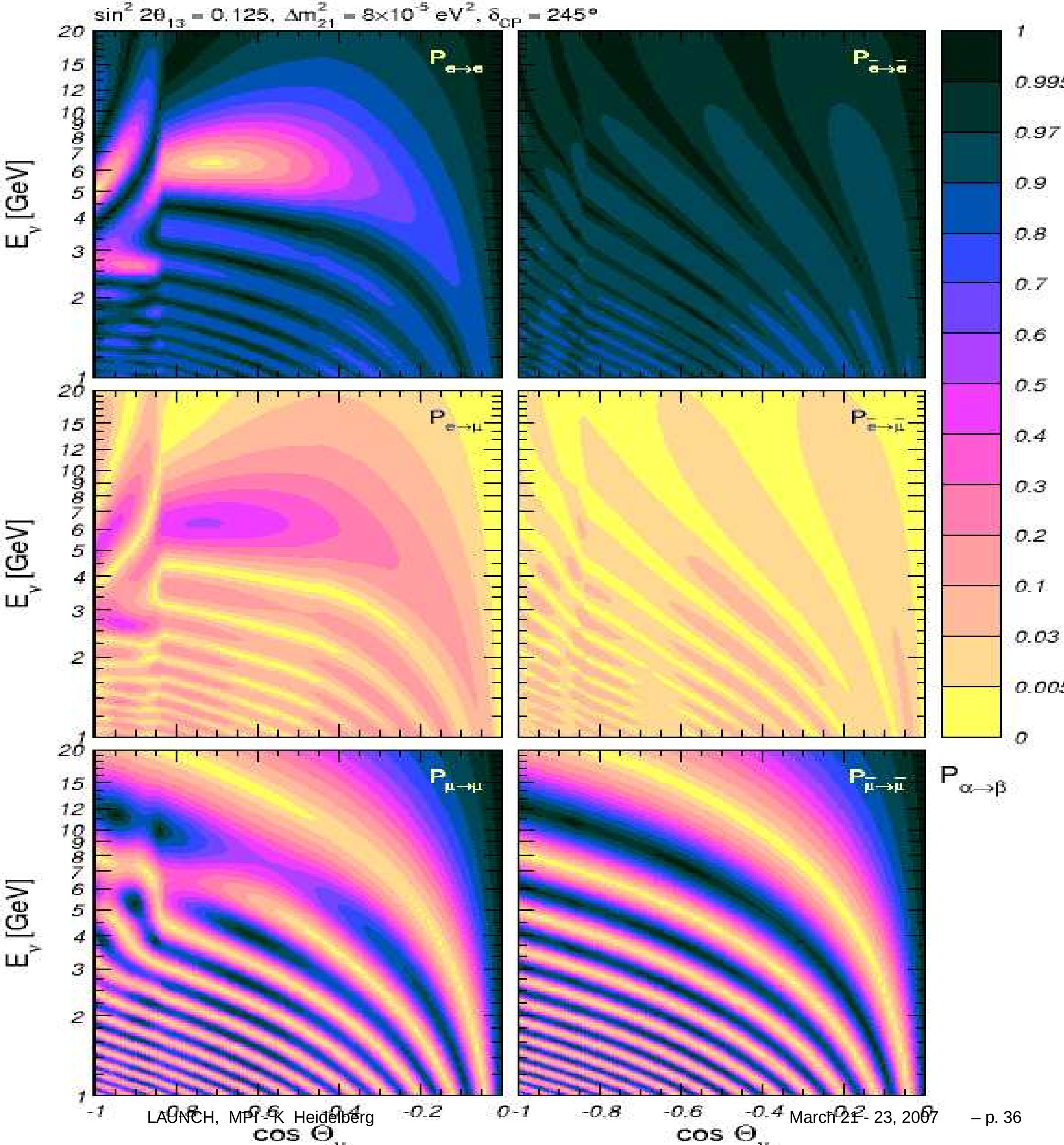
Dependence on δ_{CP}



Dependence on $\delta_{\rm CP}$



Dependence on $\delta_{\rm CP}$



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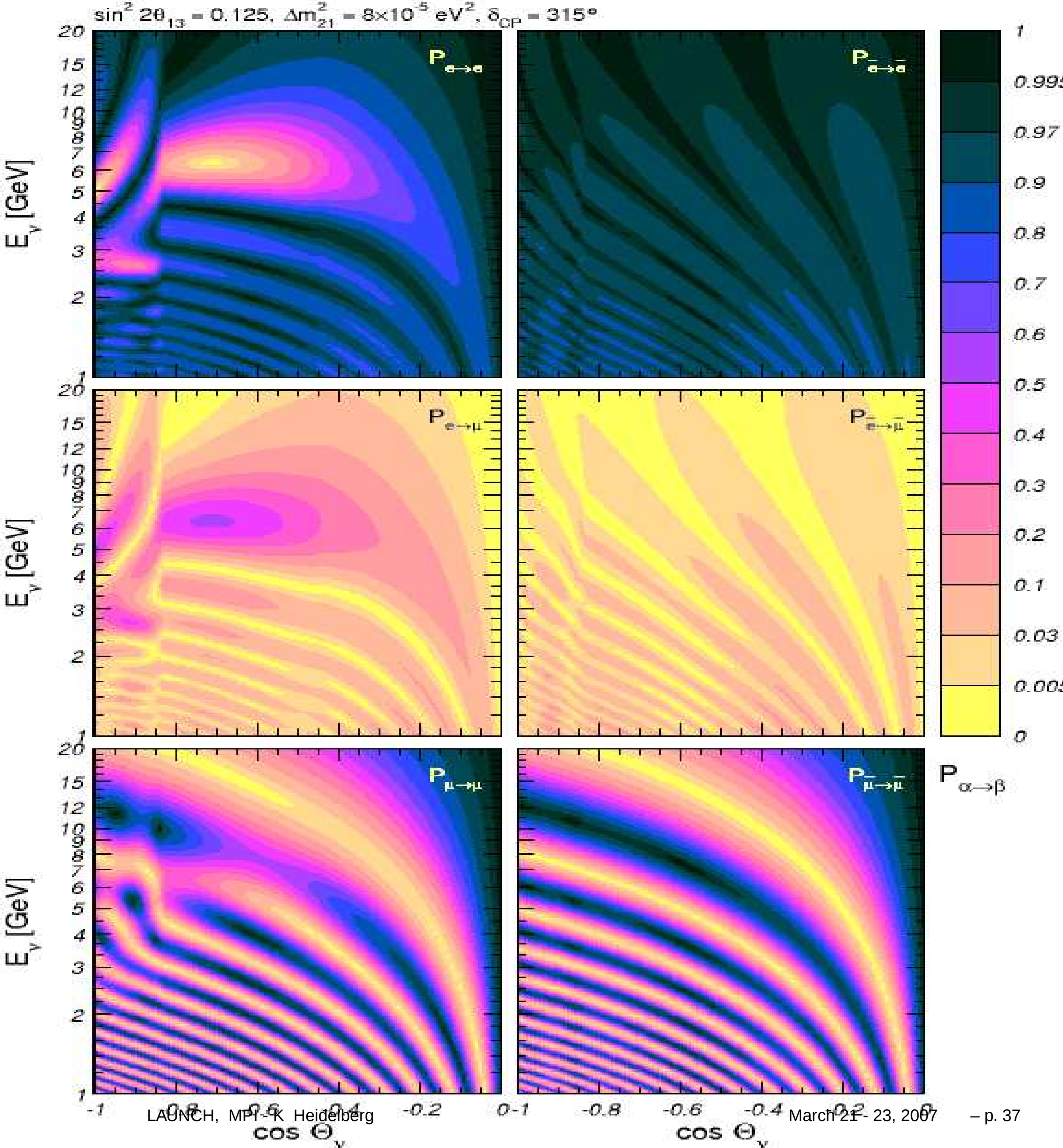
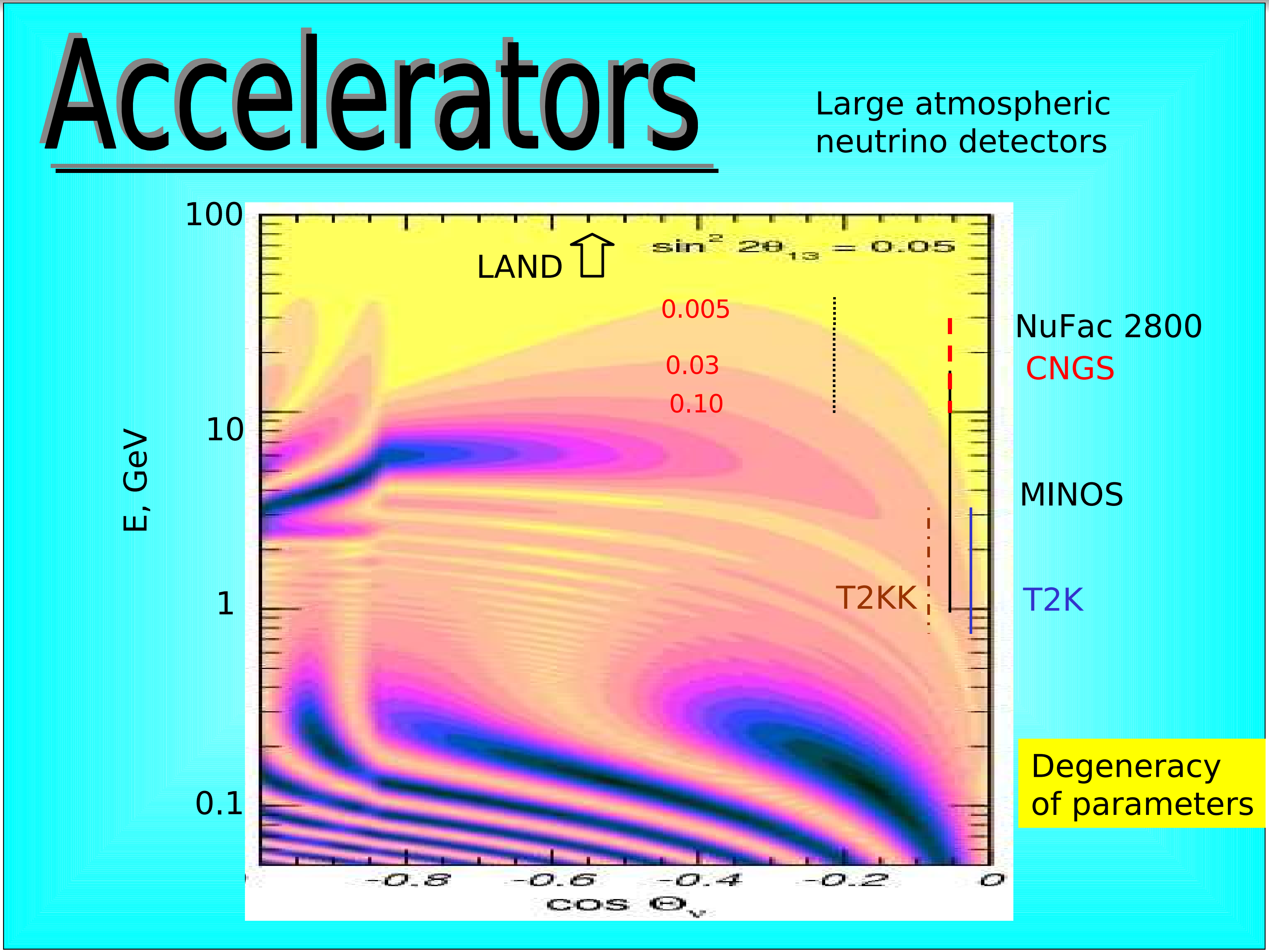


Producing the oscillograms



A. Smirnov, UCLA seminar

Producing the oscillograms – contd.

Huge atmospheric neutrino detectors may be necessary!

Would require :

- Very good energy and angle resolution
- Low threshold ($E_{\rm thr.} \sim 3$ GeV)
- Charge discrimination (?)
- High statistics

Very ambitious, but the gain may be overwhelming $\Rightarrow$

It is worth studying the oscillograms with Huge Atmospheric Neutrino Detectors !

Conclusions

- 3f corrections to 2f oscillation probabilities can reach $\sim 10\%$ – at the level of current experimental sensitivity. Depend on $|U_{e3}| = \sin\theta_{13}$
- A number of interesting pure 3f effects exist – fundamental CP and T-violation, matter - induced T violation, matter effects on $\nu_\mu \leftrightarrow \nu_\tau$ oscillations
- Matter can strongly affect ν oscillations inside the Earth through the MSW and parametric resonance effects
- Neutrino oscillograms of the Earth carry a wealth of information both on neutrinos and the Earth:

Conclusions

They :

- Depend strongly on the neutrino mass hierarchy and the value of θ_{13}
- Depend sensitively on the $\not{C}\not{P}$ phase $\delta_{\rm CP}$ and on the Earth density profile
- Their specific structures (MSW resonances, parametric ridges, local and global extrema, saddle points) and their dependence on ν parameters can by fully described in terms of the amplitude and phase conditions
- This can be used for looking for best strategies for future ν oscillations experiments