The Standard Model (I – IV)

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Goal of precourses:

astro T + E   cosmo T + E   particle T + E

get particle course   get cosmo course

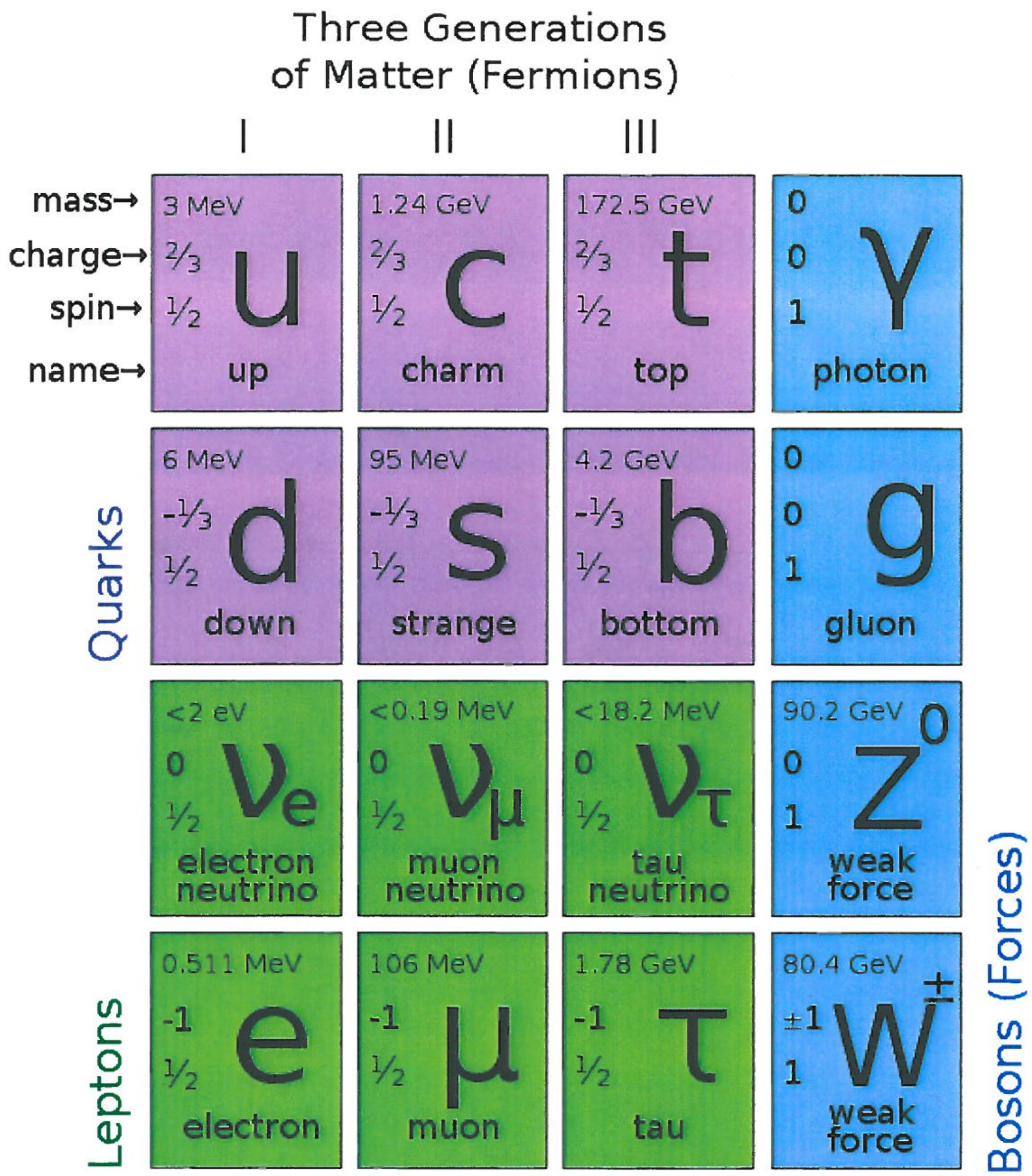
at the end: same level

⇒ impossible task!
- some theorists (may) know better than me
- too theoretical for some experimentalists (focus on theory)

⇒ give overview on basics of particle physics (theory), current status, problems, etc.

~~no~~ few technical details (i.e. what I think is technical...)

14/10/10



→ • Warum I, II, III, (?) ?   • L, B erhalten. Warum ?
- Was ist Masse ?
- warum γ, g masselos? Warum $W^\pm, Z$ nicht masselos?
- Spin 1/2 $\Leftrightarrow$ chirale Felder
- mischen die Generationen? Wie? warum? „Flavorphysik"
- Gibt es Kandidaten für DM?
- Kann ich Y_B erklären?
- Sind Neutrinos masselos?
- Anzahl der Parameter ? ⟶ b.w.
- ...

ABER: Formalismus beschreibt WW korrekt!

Basic basics

- natural units $\quad c = \hbar = 1 \quad (k_B = 1)$

  $\Rightarrow$ everything expressed in <u>energy</u> or mass

  $$[M] = [L]^{-1} = [T]^{-1}$$

- Dirac-equation

  $$(i\not{\partial} - m)\psi = (\not{p} - m)\psi = 0 \; ; \; \not{\partial} = \partial_\mu \gamma^\mu$$

  $$\psi = \begin{pmatrix} x \\ x \\ x \\ x \end{pmatrix} \text{ spinor}$$

  $$\partial_\mu = \frac{\partial}{\partial x^\mu} = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right)$$

  $$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad \text{with } \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \; ; \; \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

  adjoint spinor: $\bar{\psi} = \psi^+ \gamma^0 \quad (= (x, x, -x, -x))$

  $\bar{\psi}\gamma^\mu\psi$ is Lorentz-vector =: "current"

  $$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{with } \{\gamma_\mu, \gamma_5\} = 0 \; ; \; \gamma_5^2 = \mathbb{1}$$

  ~~$\gamma_5^2 = \mathbb{1}$~~ chirality-operator ; not to be confused with helicity-operator $\vec{\Sigma}\hat{p} = \begin{pmatrix} \vec{\sigma}\hat{p} & 0 \\ 0 & \vec{\sigma}\hat{p} \end{pmatrix}$; $\hat{p} = \frac{\vec{p}}{|\vec{p}|}$

  projection $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$

* same for $m = 0$;
* $m \neq 0$: helicity $\neq$ chirality

  helicity: not Lorentz-invariant

  chirality: not conserved

  $\Rightarrow$ Nature doesn't care, use γ_5 heavily...

$\Rightarrow (i\cancel{D} - m)\psi = 0 \Rightarrow (i\cancel{\partial} - q\cancel{A} - m)\psi = 0$

Or : use in <u>Lagrangian</u> $\mathcal{L} = \bar{\psi}(i\cancel{\partial} - m)\psi$

$\downarrow$

$\mathcal{L} = \bar{\psi}(i\cancel{\partial} - m)\psi$ → free particle

$-q\bar{\psi}\gamma_\mu\psi A^\mu$ → IA with <u>photon</u>

$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ → field strength of photon

<u>Note:</u> $\bar{\psi}\Gamma\psi X$ current can be any things:

$\Gamma = \mathbb{1}$	γ^μ	γ_5	$\gamma_\mu\gamma_5$	$\sigma_{\mu\nu}$	$\sigma_{\mu\nu}\gamma_5$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
S	V	P	A	T	AT

I) Basics

A) What do we deal with?

Fermions, Scalars, Bosons : fundamental

$\downarrow$ not yet verified

$$(\not\partial - m)\psi = 0 \; ; \; (\partial_\mu \partial^\mu + m^2)\phi = 0 \; ; \; (\partial_\mu \partial^\mu + M^2) A^\nu = 0$$

Dirac — Klein-Gordon — Proca

Lagrange-density $\mathcal{L}$; $S = \int d^4x \; \mathcal{L}(\phi, \partial_\mu \phi)$ „action"
$[S] = [\hbar]$

e.o.m. by minimizing Action

principle of least action : $\delta S = 0$

$$\Rightarrow \boxed{\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \frac{\partial \mathcal{L}}{\partial \phi}}$$

Euler-Lagrange equation

E.g.: $\mathcal{L} = \bar\psi (\not\partial - m)\psi$ $\quad (*)$

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu \phi)(\partial^\mu \phi) - m^2 \phi^2\right]$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j^\mu A_\mu \longrightarrow \partial_\mu F^{\mu\nu} = j^\nu$$

Field strength: $\partial_\mu A_\nu - \partial_\nu A_\mu$

$e\bar\psi \gamma^\mu \psi$ electrom. current

$\Rightarrow \partial_\nu j^\nu = 0$

Maxwell-eqs!

Gauge-invariance (*)

(*) is invariant under $\psi \to e^{i\alpha}\psi \simeq (1+i\alpha)\psi$

<u>global</u> transformation

$\alpha = \text{const}$ $\qquad \psi' = (1+i\alpha)\psi$

$\Rightarrow \delta \mathcal{L} = 0$ i.e. $\quad 0 = \frac{\partial \mathcal{L}}{\partial \psi}\,\delta\psi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\,\delta(\partial_\mu \psi)$

$$= \frac{\partial \mathcal{L}}{\partial \psi}(\psi' - \psi) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\left[(\partial_\mu \psi') - (\partial_\mu \psi)\right]$$

$$= \frac{\partial \mathcal{L}}{\partial \psi}\, i\alpha\psi + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}(i\alpha\,\partial_\mu \psi)$$

$$= i\alpha \underbrace{\left[\frac{\partial \mathcal{L}}{\partial \psi} - \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\right)\right]}_{=0}\psi + i\alpha\,\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\psi\right)$$

$$\Rightarrow \quad i\alpha\,\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)}\psi\right) = 0$$

with $\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi)} = \frac{\partial}{\partial(\partial_\mu \psi)}\left[\bar{\psi}(-i\gamma^\mu \partial_\mu - m)\psi\right] = -i\bar{\psi}\gamma^\mu\psi$

current!

$\Rightarrow \boxed{\partial_\mu j^\mu = 0} \Rightarrow$ conserved current due to invariance under <u>global</u> trafo („U(1)" or „Abelian gauge transformation")

$j^\mu = -e\bar{\psi}\gamma^\mu\psi$

this is of course the Noether-theorem

$$\frac{d}{dt} Q \equiv \frac{d}{dt} \int d^3x \; j^0 = \int d^3x \; \frac{d}{dt} j^0 = - \int d^3x \; \vec{\nabla} \vec{j}$$

$$\overset{\text{Gauss}}{=} - \oint d\vec{S} \, \vec{j} \; \overset{!}{=} 0$$

fields appearing in surface term vanish at infinity; $\lim_{x \to \pm\infty} \psi(x) = 0$

Gauss: any charge leaving V must be accounted by the $\vec{j}$ leaving volume

Volume d^3x covered by a surface S. Charge constant if no charge leaves volume

---

now we generalize $\alpha \to \alpha(x)$ in $\psi \to e^{i\alpha(x)} \psi$

"local" gauge trafo on free $\mathcal{L}$

problem: $\bar{\psi}(i\gamma_\mu \partial^\mu - m)\psi \to \bar{\psi} e^{-i\alpha(x)} \left(i\gamma_\mu \partial^\mu (\psi e^{i\alpha(x)}) - m\psi e^{i\alpha} \right)$

$$= -\bar{\psi} m \psi + \bar{\psi} e^{-i\alpha(x)} i\gamma^\mu \left((\partial_\mu \psi) e^{i\alpha(x)} + \cancel{i\alpha(x)(\partial_\mu\psi) e^{i\alpha(x)}} \; i(\partial_\mu \alpha) \psi e^{i\alpha(x)} \right)$$

$$= \bar{\psi}(i\partial\!\!\!/ - m)\psi - \bar{\psi}\gamma^\mu \psi \,(\partial_\mu \alpha(x))$$

Solution: $\bar{\psi}(iD\!\!\!/ - m)\psi \to \bar{\psi} e^{-i\alpha(x)} (iD\!\!\!/' - m) e^{i\alpha} \psi$

such that $D'_\mu = e^{i\alpha(x)} D_\mu$

$$\hookrightarrow \boxed{\begin{aligned} \partial_\mu &\to D_\mu = \partial_\mu - ieA_\mu \\ A_\mu &\to A_\mu + \frac{1}{e} \partial_\mu \alpha \end{aligned}}$$

proof: $\Rightarrow \quad D'_\mu \psi' = \left[\partial_\mu - ieA_\mu - i(\partial_\mu \alpha) \right] e^{i\alpha} \psi$

$$= \cancel{i(\partial_\mu \alpha) e^{i\alpha} \psi} - ieA_\mu e^{i\alpha}\psi - \cancel{i(\partial_\mu \alpha) e^{i\alpha} \psi} + e^{i\alpha}(\partial_\mu \psi)$$

$$= e^{i\alpha}(\partial_\mu - ieA_\mu)\psi = e^{i\alpha} D_\mu \psi$$

$$\Rightarrow \quad \mathcal{L} = \bar{\psi}(\not{p} - m)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

justification for "minimal substitution"

kinetic term, only gauge- and Lorentz-invariant function of A_μ

$$\equiv \frac{i}{e}[D_\mu, D_\nu]$$

$m^2 A_\mu A^\mu$ forbidden $\Rightarrow m_\gamma = 0$

$\Downarrow$

local Symmetry (gauge invariance)

needs existence of

gauge fields $\longrightarrow$ massless !

$\rightarrow$ lesson: "smallness" of a parameter not unnatural if a symmetry behind it

$\downarrow$

interactions

$\hookrightarrow$ generalize this, apply to all interactions

What does paticle physics want?

interactions: → weak: $n \to p\, e^-\, \bar{\nu}_e$ $\quad G_F m_N^2 \sim 10^{-5}$

$W^\pm, Z^0$ $\quad r_0 \sim$ fm

→ strong:

$r > 0.2$ fm $\quad$ Yukawa potential

$r_0 = \frac{1}{m_\pi} \sim$ fm $\quad \frac{g^2}{4\pi} = O(10)$

$r < 0.2$ fm $\quad$ QCD, gluons

$\frac{g^2}{4\pi} \lesssim 0.3$

→ electromagn: $\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$ $\quad r_0 = \infty$

~~... fast processes~~ (QED)

QED e.g. magn. moment of e^- calculated to precision of 10^{-12}

⇒ construct theories according to the principles which were successfull for QED

(Electro)weak Interaction and $SU(N)$ Non-Abelian Gauge-trafos

Fermi-theory: • $G_F \approx 10^{-5}\ \mathrm{GeV}^{-2}$ dimension...

• parity violation: pure $V-A$ $\bar{\psi}_L \gamma_\mu \psi_L$

• exchange of ~~...~~ heavy particles m_W, m_Z

$G_F \sim \frac{g^2}{m_W^2}$

$\Rightarrow m_W \sim 100$ GeV

@ leading order $\begin{pmatrix} u \\ d \end{pmatrix}$ $\begin{pmatrix} c \\ s \end{pmatrix}$

$u \to d$ $(n \to p)$
$c \to s$

„isospin"

• use now instead of $U(1)$ non-Abelian gauge group: $SU(N)$

det = 1 unitary $N \times N$ matrices

generators: $[\tau^a, \tau^b] = i f_{abc} \tau^c$ $a = 1, N^2-1$

anti-symmetric structure constants

$$\boxed{\psi_i \to U_{ij} \psi_j} \quad \text{with } U_{ij} = \exp\{-i\theta^a(x)\tau^a\}_{ij}$$

$$= (1 - i\theta^a \tau^a)_{ij}$$

$\begin{pmatrix} \nu \\ e \end{pmatrix}$ for $SU(2)$

$\begin{pmatrix} q_{rot} \\ q_{grün} \\ q_{blau} \end{pmatrix}$ for $SU(3)$

$\bar{\psi}(\not{p}-m)\psi$ invariant:

from $D'_\mu \mathcal{U}\psi \overset{!}{=} \mathcal{U} D_\mu \psi$
$\Rightarrow \mathcal{U}^\dagger D'_\mu \mathcal{U} = D_\mu$

$$\partial_\mu \to D_\mu \quad \text{such that} \quad \boxed{D'_\mu \psi' = \mathcal{U}\, D_\mu \psi \quad \text{or } \not{D}' = \mathcal{U}\not{D}\,\mathcal{U}^{-1}}$$

solution: $D_\mu = \partial_\mu - ig\, A_\mu^a \tau^a \equiv \partial_\mu - ig\, \vec{A}_\mu \vec{\tau} \equiv \partial_\mu - ig \hat{A}_\mu$

$$\cancel{\vec{A}_\mu \to \vec{A}_\mu - \frac{i}{g}(\partial_\mu \mathcal{U})\mathcal{U}^{-1} + \mathcal{U}\vec{A}_\mu \mathcal{U}^{-1}}$$

$$\tilde{A}_\mu \to \frac{-i}{g}(\partial_\mu \mathcal{U})\,\mathcal{U}^{-1} + \mathcal{U}\tilde{A}_\mu \mathcal{U}^{-1}$$

cf.: $A'_\mu = A_\mu + \frac{1}{e}\partial_\mu \alpha$

N^2-1 gauge fields...

one per generator

$$\Rightarrow A_\mu^a \to (1-i\theta^b\tau^b)\,A_\mu^a \tau^a\,(1+i\theta^b\tau^b)$$

$$-\frac{i}{g}\left[\partial_\mu(1-i\theta^a\tau^a)\right]\left[1+i\theta^b\tau^b\right]$$

$$= \dots = A_\mu^a - \frac{1}{g}\partial_\mu \theta^a + f_{abc}\,\theta^b A_\mu^c$$

$$\xrightarrow{\mathcal{U}(1)} A_\mu - \frac{1}{e}(\partial_\mu \alpha)$$

$$\psi_i \to (1-i\theta^a\tau^a)_{ij}\,\psi_j$$

Note: $\bar{\psi}_i A_\mu^a \tau^a \psi$ generate $SU(N)$ as well...
"current algebra"

the kinetic term comes from $F_{\mu\nu} = \frac{i}{g}[D_\mu, D_\nu]$

$$\Rightarrow F_{\mu\nu} = \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_\mu^b A_\nu^c\right)\tau^a$$

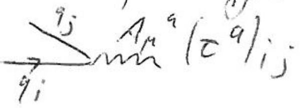
τ^a, τ^b do not commute

and with $D_\mu \to D'_\mu = U D_\mu U^{-1}$ it follows

$$F_{\mu\nu} \to F'_{\mu\nu} = U F_{\mu\nu} U^{-1}$$

$\Rightarrow \mathrm{Tr}\{F_{\mu\nu} F^{\mu\nu}\}$ is gauge-invariant

note:

- massive gauge bosons still forbidden...
- $F_{\mu\nu} F^{\mu\nu} \to A^3, A^4$ terms  
  - $\Rightarrow$ self-interaction of gauge fields
  - $\Rightarrow$ HERA 3 jets
  - $\Rightarrow$ characteristic for non-Abelian
- $g \bar\psi \gamma^\mu \tau^a \psi A_\mu^a$ terms 

- $N=2$: $SU(2)$ $\qquad N^2-1=3$

  $\tau^a = \frac{\sigma^a}{2} \qquad [\tau^a, \tau^b] = i\varepsilon_{abc}\tau^c$

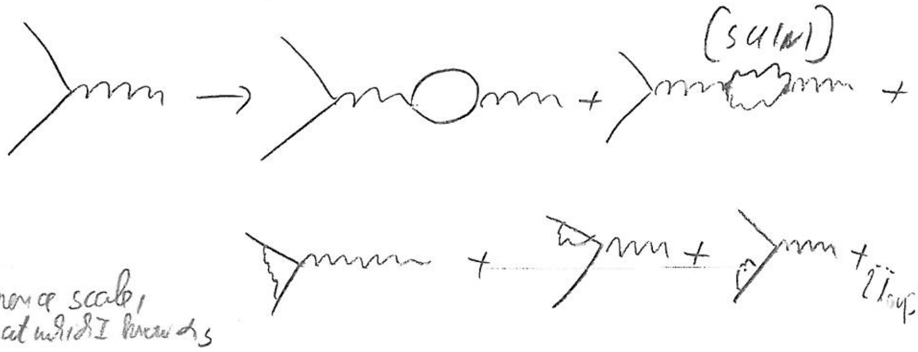
- $N=3$: $SU(3)$ $\qquad N^2-1=8$

  $\tau^a = \frac{\lambda_a}{2}$ „Gell-Mann matrices"

QCD, confinement und asymptotische Freiheit

„laufende Kopplung“ $g = g(Q^2)$

je näher ich rangehe, desto mehr Diagramme sehe ich



1 loop

Reference scale, at which I know α_s

$$\boxed{\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1+\frac{\alpha_s(\mu^2)}{4\pi}\beta_0 \log Q^2/\mu^2}}$$

$\beta_0 = \frac{11}{3}N - \frac{2}{3}N_f$ >0 für SM

für SU(N)

SU(3): $\alpha_s \searrow$ für $Q^2 \nearrow$ solange $N_f \le 16$ (ausserdem: $N_f = 6$)

(ASYMPTOTISCHE FREIHEIT)

→ wir können jetzt $\alpha_s(Q^2)$ vom exptl. bestimmten $\alpha_s(\mu^2)$ berechnen

ebenso: für kleine Q^2 wird α_s gross „confinement“ (nicht-perturbativ, kein formaler Beweis...)

$\alpha_s = \infty$ wenn $Q^2 = \mu^2 \exp\left\{\frac{-12\pi}{(33-2N_f)\,\alpha_s(\mu^2)}\right\} \equiv \Lambda^2$ ↔ Hadronisierungsskala der QCD

$\Lambda \approx (200..300)$ MeV

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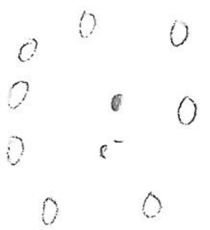
cf. mit: $\alpha_{QED}(Q^2) = \frac{\alpha_{QED}(\mu^2)}{1-\frac{\alpha_{QED}(\mu^2)}{3\pi}N_f \log Q^2/\mu^2}$ $\alpha_{QED} \nearrow$ für $Q^2 \nearrow$

Vakuum, $e^\pm$-Paaren



$Q^2 \nearrow$
Ladung $\nearrow$

Abschirmung bei grossen Abständen

Vakuum $q\bar{q}$-Paaren als auch gluon Paaren mit gegenteiliger Wirkung: falls $N_f \le 16$ Quanten-Beiträge dominieren und QCD antiges Verhalten.

The central value is determined as the weighted average of the individual measurements. For the error an overall, a-priori unknown, correlation coefficient is introduced and determined by requiring that the total χ^2 of the combination equals the number of degrees of freedom. The world average quoted in Ref. 172 is

$$\alpha_s(M_Z^2) = 0.1184 \pm 0.0007 \; ,$$

with an astonishing precision of 0.6%. It is worth noting that a cross check performed in Ref. 172, consisting in excluding each of the single measurements from the combination, resulted in variations of the central value well below the quoted uncertainty, and in a maximal increase of the combined error up to 0.0012. Most notably, excluding the most precise determination from lattice QCD gives only a marginally different average value. Nevertheless, there remains an apparent and long-standing systematic difference between the results from structure functions and other determinations of similar accuracy. This is evidenced in Fig. 9.2 (left), where the various inputs to this combination, evolved to the Z mass scale, are shown. Fig. 9.2 (right) provides strongest evidence for the correct prediction by QCD of the scale dependence of the strong coupling.

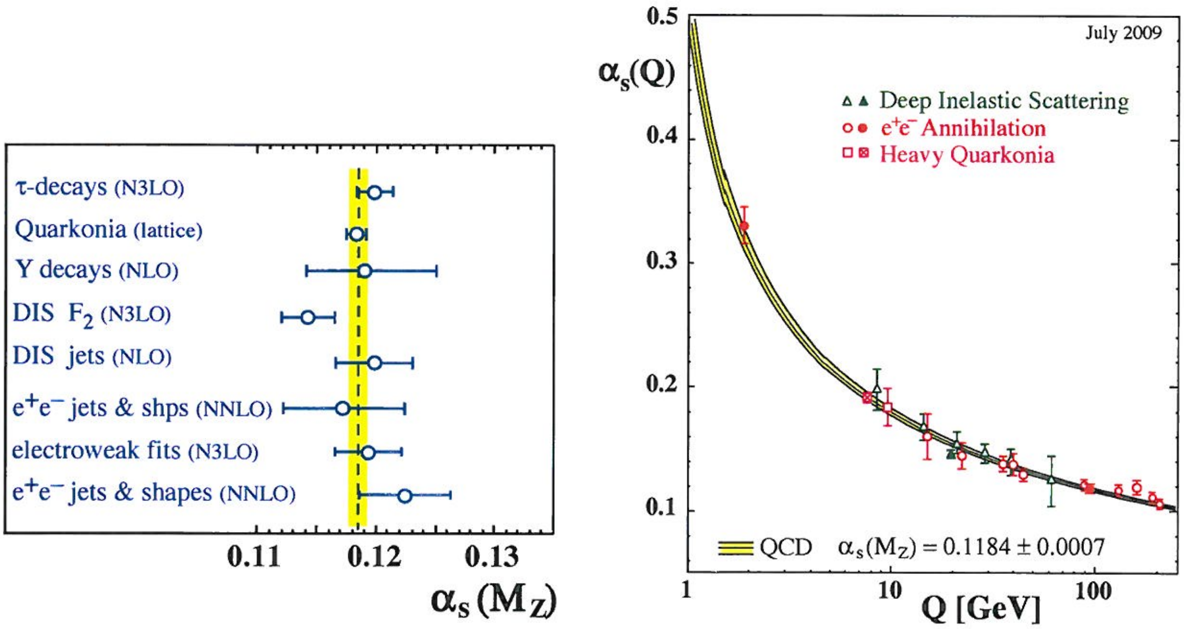
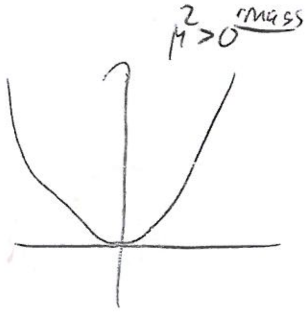


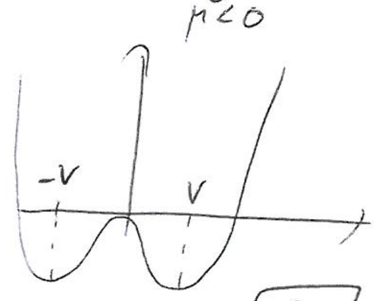
Figure 9.2: Left: Summary of measurements of $\alpha_s(M_Z^2)$, used as input for the world average value; Right: Summary of measurements of α_s as a function of the respective energy scale Q. Both plots are taken from Ref. 172.

Spontaneous Symmetry Breaking + Higgs-Mechanism

$\mathcal{L}$ posesses symmetry which the ground state does not obey

a) $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi)$; $V(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4}\phi^4$





$v = -\sqrt{\mu^2/\lambda}$

$\phi \to -\phi$ Symmetry „Z_2"

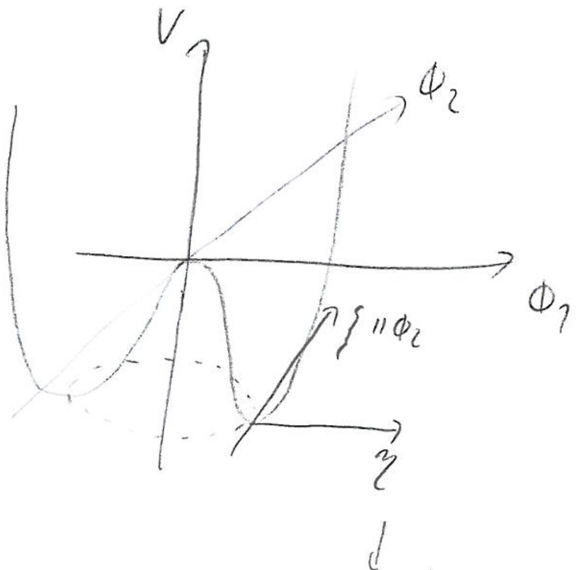
$\lambda>0$: energy bounded from below
otherwise: „vacuum instable"
$\to$ see later

choose $\langle\phi\rangle = +v$; do physics around minimum: $\phi = v+\eta(x)$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2 - \lambda v\eta^3 - \frac{1}{2}\lambda\eta^4$$

$\Rightarrow m_\eta^2 = 2\lambda v^2$ correct sign! Mass!

b) $\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi^*) - \mu^2\phi^*\phi - \lambda(\phi^*\phi)^2$



$\phi \to e^{i\alpha}\phi$ Symmetry global

$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$

choose $\langle\phi\rangle = +v = \sqrt{-\mu^2/\lambda}$

do physics around the minimum $\phi = (v+\eta(x)+i\xi(x))/\sqrt{2}$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 + \mu^2 \eta^2 + O(\eta^{3,4}, \xi^{3,4})$$

mass term note: sign

massive η

massless ξ „Goldstone - Boson"

flat direction tangent ξ has no resistance: moves along the circle...

~~U(1) ... of ...~~

\# is # of generators of the group which leave the vacuum <u>not</u> invariant
$\Downarrow$
1 GB for each broken generator (1),...

now gauged

c) $$\mathcal{L} = [(\partial^\mu \Phi - ieA^\mu)\Phi^*][(\partial_\mu + ieA_\mu)\Phi] - \mu^2 \Phi^*\Phi - \lambda(\Phi^*\Phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\Phi = \sqrt{\tfrac{1}{2}}(v + \eta + i\xi)$$

$$\Rightarrow \mathcal{L} = \frac{1}{2}(\partial_\mu \xi)^2 + \frac{1}{2}(\partial_\mu \eta)^2 - v^2\lambda\eta^2 + \underbrace{\frac{1}{2}e^2v^2 A_\mu A^\mu}_{!\ m_\gamma\ !} + \underbrace{evA_\mu \partial^\mu \xi}_{?}$$

m_η

\# degrees of freedom : before: $2+2$ (Φ + massless A)
after: $3+2$ (ξ, η + massive A)

$\Rightarrow$ note that ~~with~~ 2) $\Phi \simeq \sqrt{\frac{1}{2}}(v+\eta)\, e^{i\xi/v}$ looks like gauge trafo!

1) $A_\mu \partial^\mu \xi$ ~~bad~~

$\Rightarrow$ try to write $\Phi \to \sqrt{\frac{1}{2}}(v + h(x))\, e^{i\theta(x)/v}$
$A_\mu \to A_\mu + \frac{1}{ev}\partial_\mu \theta(x)$ particular gauge choice

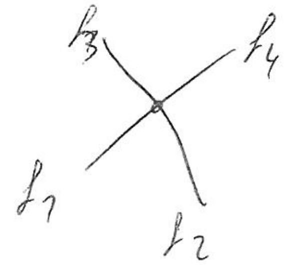
indeed: $$\mathcal{L} = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \lambda v^2 h^2 + \frac{1}{2}e^2v^2 A_\mu A^\mu + \ldots$$

$\boxed{\text{coupling of GB with gauge: } A^\mu \partial_\mu \xi}$

no massless Goldstone, „eaten" to become third polarization dof of photon

⇒ Electroweak Theory and massive gauge bosons

Fermi-interaction



$$\mathcal{L} = \frac{G_F}{\sqrt{2}} J_\mu J^{\mu +}$$

$$J_\mu = \bar{\nu}_e \gamma_\mu (1-\gamma_5) e + \bar{u} \gamma_\mu (1-\gamma_5) d + \dots$$

- V−A
- $[G_F] = -2 \Rightarrow$
  - not renormalizable
  - violates unitarity
- CC + NC
- unitarity: $\sigma(\nu_\mu e^- \to \mu^- \nu_e) = \frac{G_F^2 s}{\pi}$

  should have $\sigma \propto 1/s$

  amplitude → const

propagator $\frac{g^2 (g^{\mu\nu} - \frac{k^\mu k^\nu}{M_W^2})}{k^2 - m_W^2}$

$$\xrightarrow{\sqrt{s} \nearrow \infty} \frac{g^2}{m_W^2} \to \text{const}$$

- $\begin{pmatrix} \nu_e \\ e \end{pmatrix}$ $\begin{pmatrix} u \\ d \end{pmatrix}$ transitions
- still want QED, $U(1)$

$\Rightarrow$ $\boxed{\text{try } SU(2)_L \times U(1)_Y}$

define $\psi_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L$ or $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ $\sim 2_L$; Y_1

$$\left(\begin{array}{l} \text{where } \psi_{\substack{L \\ R}} = \frac{1}{2}(1 \mp \gamma_5)\psi = P_{L,R}\,\psi \\ P_L P_R = 0 \; ; \; P_{L,R}^2 = 1 \; ; \; P_L + P_R = 1 \; ; \; \bar{\psi}_L = \bar{\psi} P_R \end{array} \right)$$

$\psi_2 = u_R$ or ν_R $\sim 1_L$ $\quad Y_2$

$\psi_3 = d_R$ or e_R $\sim 1_L$ $\quad Y_3$

→ $u, e, \dots$ LH and RH part treated differently
⇒ parity violation! maximal!

typical for model building:
- choose group S
- choose how particles (new field) transform under S
- chiral why?

$$\psi_1 \to \psi_1' = e^{\frac{1}{2} i Y_1 \beta(x)} U_L \psi_1$$

$$\psi_2 \to \psi_2' = e^{\frac{1}{2} i Y_2 \beta(x)} \psi_2$$

$$\psi_3 \to \psi_3' = e^{\frac{1}{2} i Y_3 \beta(x)} \psi_3$$

* $U_L = \exp\{ i \, \sigma_i/2 \, \alpha_i(x) \}$

* 4 gauge-bosons

* choose Higgs-Doublet $\Phi = \begin{pmatrix} \phi_a \\ \phi_b \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \sim 2_L$
$Y = 1$

Φ has potential as usual $\mu \Phi\Phi^\dagger + \lambda (\Phi\Phi^\dagger)^2$

can be written as $\Phi(x) = e^{i \vec{\sigma}/2 \, \vec{\Theta}(x)} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} / \sqrt{2}$

choose gauge to get rid of $e^{i\dots}$

$\partial_\mu \to D_\mu = \partial_\mu + \frac{1}{2}\left(i g W_\mu^i \sigma^i + i g' B_\mu\right)$

and consider kinetic term $(D_\mu \Phi)^\dagger (D^\mu \Phi)$; define

$W_\mu^\pm = \sqrt{\tfrac{1}{2}} \left(W_\mu^1 \mp i W_\mu^2\right)$

$(W_\mu^-)^\dagger = W_\mu^+$

$$\mathcal{L} = \frac{1}{2} v^2 g^2 W_\mu^+ W_\mu^- + \underbrace{\frac{v^2}{8} (W_\mu^3 , B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}}_{\text{diagonalize to get physical names}}$$

$m_W^2 = \frac{1}{4} v^2 g^2$

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}} \qquad M_A = 0$$

$$Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}} \qquad M_Z^2 = \frac{v^2}{4} (g^2 + g'^2)$$

$\tan \Theta_W = g'/g$ Weinberg angle

$$\left.\begin{aligned} A_\mu &= c_W B_\mu + s_W W_\mu^3 \\ Z_\mu &= c_W W_\mu^3 - s_W B_\mu \end{aligned}\right\} \text{“physical fields”}$$

$\Rightarrow \frac{M_W}{M_Z} = \cos^2 \Theta_W \qquad s_W^2 = 0.2312$

need to rewrite covariant derivatives in physical fields

$$\partial_\mu \to D_\mu + \frac{i}{2} g W_\mu^i \sigma^i + \frac{i}{2} g' \frac{Y_1}{2} B_\mu \quad \text{in } \overline{\Psi}_1 \gamma_\mu D^\mu \Psi_1$$

contains terms Z_μ (later) (use: $\overline{\Psi}_1 \frac{\sigma_3}{2} \gamma^\mu \Psi_1 = \overline{u}_L \frac{1}{2} \gamma^\mu u_L - \overline{d}_L \frac{1}{2} \gamma^\mu d_L$
$\equiv I_3^u \overline{u}_L \gamma^\mu u_L + I_3^{d_L} \overline{d}_L \gamma^\mu d_L$

for all Ψ_i:
$$+ A_\mu \left[g I_3^{u_L} s_W \overline{u}_L \gamma^\mu u_L + g' \frac{Y^{u_L}}{2} c_W \overline{u}_L \gamma^\mu u_L \right]$$

cf. with electromagnetic IA: $e Q^{u_L} (\overline{u}_L \gamma^\mu u_L) + \cancel{\overline{u}_R \gamma^\mu u_R)}$
$= e Q^{u_L} \overline{u}_L \gamma^\mu u_L = e Q^{u} \overline{u}_R \gamma^\mu u_R$

see this also by writing:
$-i g J^3_\mu W^{3\mu} - i g' \frac{J^Y}{2} B^\mu$
$= -i g A^\mu (g s_W J^3_\mu + g' c_W J^Y/2)$
with
$J^3_\mu = \overline{\Psi} \gamma_\mu \frac{\tau_3}{2} \Psi$
$\frac{J^Y}{2} = \frac{1}{2} \overline{\Psi} \gamma_\mu \Psi$
MUST BE $e J_\mu^{em}$

$$\Rightarrow \boxed{I_3 + \frac{Y}{2} = Q}$$

$$\boxed{g s_W = c_W g' = e}$$

$$\tan \Theta_W = g'/g$$

e.g.: $Y_{u_L} = 1/3$ $\qquad Y_\phi = 1$

$Y_{d_R} = -2/3$ $\qquad Y_{u_R} = 4/3$

$Y_{\nu_L} = -1$

$Y_{\nu_R} = 0$ total singlet...

the now massive gauge fields couple to the currents which do not leave the vacuum invariant:
$W_\mu^+ J^{\mu+} + W_\mu^- J^{\mu-}$
$+ Z_\mu (J^\mu - s_W^2 J^\mu_Q)$
$+ A_\mu J^{Q\mu}$ leaves vacuum invariant

plus: $\begin{matrix} NC \\ CC \end{matrix}$ - terms $\mathcal{L}_{NC} = -\frac{e}{2 c_W s_W} Z_\mu \, \overline{f} \gamma^\mu (v_f - a_f \gamma_5) f$

~~also: CC + NC - terms~~

$\cancel{\mathcal{L}_{NC} = - Z_\mu [a_W \overline{f} \gamma^\mu (1-\gamma_5) f]}$ from $\overline{\Psi}_i D_\mu \Psi_i$

	u	d	ν	e	
$2v_f$	$1 - 8/3\, s_W^2$	$-1 + 4/3\, s_W^2$	1	$-1 + 4 s_W^2$	$= 2(I_3^f - 2 Q_f s_W^2)$
$2a_f$	1	-1	1	-1	$= 2 I_3^f$

„Ausintegration des W"

~~mal angenommen es gibt einen solchen Term...~~

$$\mathcal{L}_{fund} = -\frac{g}{2\sqrt{2}}\left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu + \bar{\nu}_e \gamma_\mu (1-\gamma_5) e\right] W^{\mu +} + \frac{1}{2} m_W^2 W_\mu W + \mathcal{L}_{kin}$$

Niedrigenergie $(E \ll m_W)$ $\partial_\mu W^\mu = 0$

$\Rightarrow$ kinetischer Term irrelevant

$$\Rightarrow \frac{\partial \mathcal{L}}{\partial W_\mu^+} = 0 \Rightarrow W_\mu^{\mu +} = \frac{g}{\sqrt{2}\,2}\left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu + \bar{\nu}_e \gamma_\mu (1-\gamma_5) e\right]\frac{1}{m_W^2}$$

$\hookrightarrow$ in $\mathcal{L}_{fund}$ einsetzen

$$\mathcal{L}_{eff} = -\frac{g^2}{8 m_W^2}\left[\bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu\right]\left[\bar{e} \gamma^\mu (1-\gamma_5)\nu_e\right]$$

$$\boxed{\frac{G_F}{\sqrt{2}}}$$

$\Rightarrow$ 4 Fermion Punkt-WW

+ Unitarity argument from page 11

$$\Rightarrow G_F = \frac{\sqrt{2}\, g^2}{8 m_W^2} \Rightarrow \frac{g}{m_W} \simeq 0.0087\ \mathrm{GeV}^{-1} = (123.75\ \mathrm{GeV})^{-1}$$

$$g = \frac{e}{s_W} = \frac{\sqrt{4\pi\alpha}}{\sqrt{0.23}} \simeq 0.65$$

$$\Rightarrow m_W = 80.05\ \mathrm{GeV}$$

$\left.\begin{array}{l} e \simeq 0.3 \\ g = 0.65 \end{array}\right\}$ „schwache WW"

Fermion - Masses

$$\mathcal{L} = -g_d \overset{Y=1/3}{\overline{\Psi_1}} \overset{Y=1}{\phi} \overset{Y=-2/3}{d_R} - g_u \overset{-1/3}{\overline{\Psi_1}} \overset{-1}{\tilde{\phi}} \overset{+4/3}{u_R}$$

are singlets • $\sum Y_i = 0$ ✓

• $\Psi_1 \to U_L \Psi_1 \quad \phi \to U_L \phi \Rightarrow \overline{\Psi_1} \phi \to \overline{\Psi_1} \phi$

• $\tilde{\phi} = i \sigma_2 \phi^* = \begin{pmatrix} \phi_2^* \\ -\phi_1^* \end{pmatrix} \longrightarrow \begin{pmatrix} v + h(x) \\ 0 \end{pmatrix} / \sqrt{2}$

w.r.t.: $\underset{(1-i\theta^a\sigma^a)\sigma_2(1-i\theta^a\sigma^a)}{\overset{\shortparallel}{U_L^+ i \sigma_2 U_L^*}} = i\sigma_2 \Rightarrow$ invariant

$$\Rightarrow \mathcal{L} = \underbrace{-g_d \frac{v}{\sqrt{2}}}_{m_d} \overline{d_L} d_R \underbrace{-g_u \frac{v}{\sqrt{2}}}_{m_u} \overline{u_L} u_R \underbrace{+ g_d \overline{d_L} d_R h(x)/\sqrt{2}}_{\text{Higgs-Fermion-IA}} + \dots$$

$\alpha\, g_d = \frac{m_d}{v}$

Have 3 generations: $\quad L_1 = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad L_2 = \begin{pmatrix} c \\ s \end{pmatrix}_L \quad L_3 = \begin{pmatrix} t \\ b \end{pmatrix}_L$

$u_R, c_R, t_R = u_{i,R} \quad ; \quad d_R, s_R, b_R = d_{i,R}$

$$\mathcal{L} = \sum_{i,j} \overline{L'_i} \left[g^d_{ij} \phi d_{jR} + g^u_{ij} \tilde{\phi} u_{jR} \right]$$

$$= \overline{d'_L} M^d d'_R + \overline{u'_L} M^u u'_L \quad \text{with} \quad d'_L = \begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix} \; u'_L = \begin{pmatrix} u'_L \\ c'_L \\ t'_L \end{pmatrix}$$

Mass matrices!

$$U_d^\dagger M^d V_d = D^d = \mathrm{diag}(m_d, m_s, m_b) \qquad U_{d,u} U_{d,u}^\dagger = \mathbb{1}$$

$$U_u^\dagger M^u V_u = D^u = \mathrm{diag}(m_u, m_c, m_t) \qquad V_{d,u} V_{d,u}^\dagger = \mathbb{1}$$

$$\Rightarrow \mathcal{L} = \underbrace{\overline{d'_L}\, U_d}_{\overline{d_L}}\, \underbrace{U_d^\dagger M^d V_d}_{\text{diag}}\, \underbrace{V_d^\dagger d'_R}_{d_R} + \underbrace{\overline{u'_L}\, U_u}_{\overline{u_L}}\, \underbrace{U_u^\dagger M^u V_u}_{\text{diag}}\, \underbrace{V_u^\dagger u'_R}_{u_R}$$

in CC-term:

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W_\mu^+ \overline{u'_L} \gamma^\mu d'_L$$

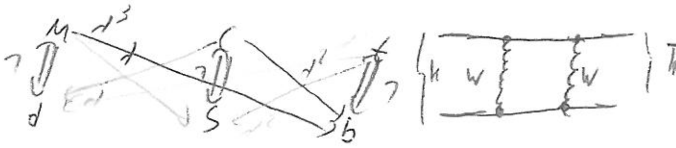
$$= -\frac{g}{\sqrt{2}} W_\mu^+ \underbrace{\overline{u'_L}\, U_u}_{\overline{u_L}} U_u^\dagger \gamma^\mu U_d \underbrace{U_d^\dagger d'_L}_{d_L}$$

$$\Rightarrow \boxed{V = U_u^\dagger U_d}$$

survives!

„CKM-matrix"

$\propto g V_{ij}$

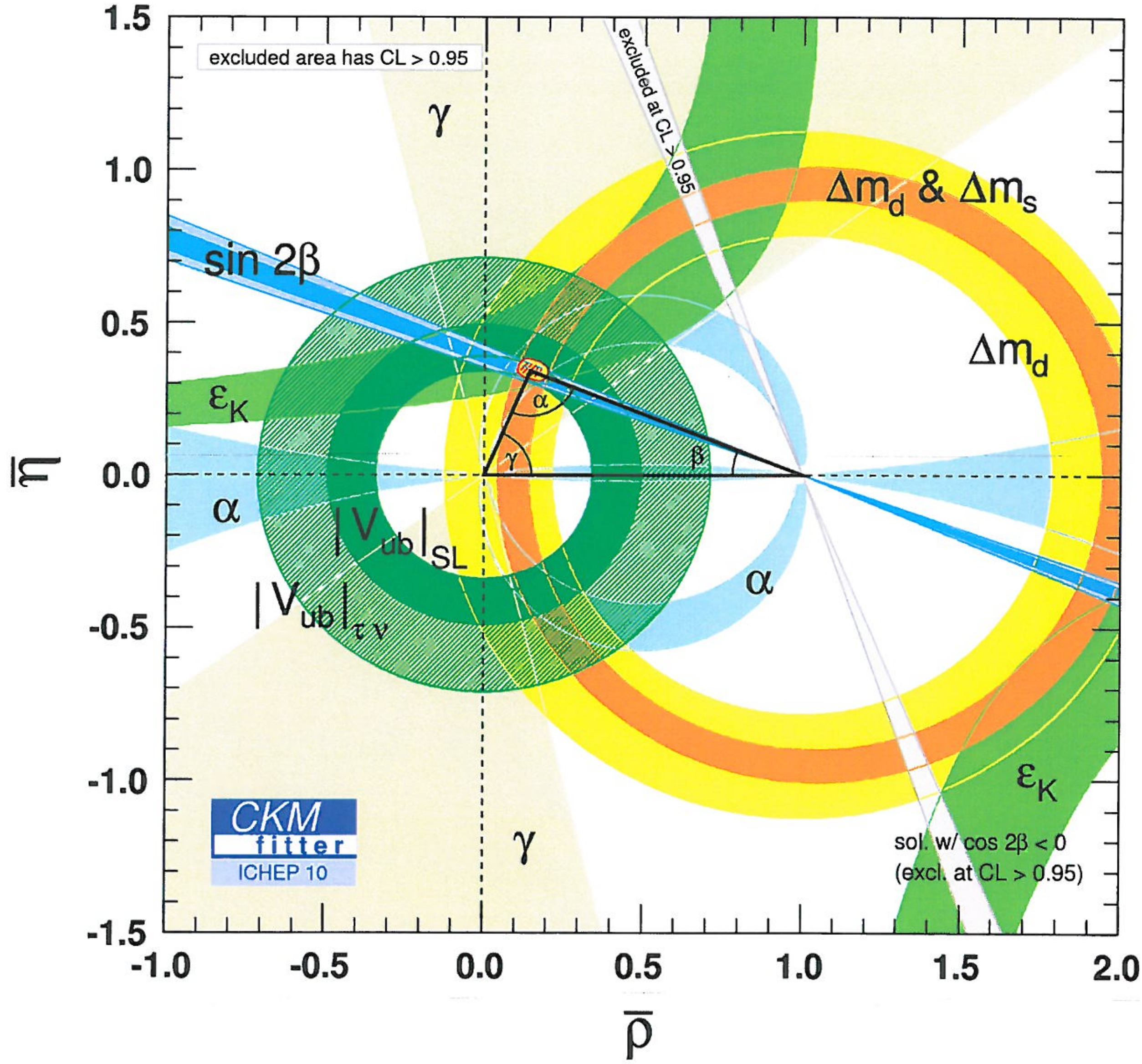


$$V \simeq \begin{array}{c|ccc} & d & s & b \\ \hline u & & & \\ c & & & \\ t & & & \end{array} \quad \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$\lambda \simeq 0.2257 \qquad A = 0.814$

$\bar\rho = \rho(1-\lambda^2/2) = 0.135 \qquad \bar\eta = 0.349$

4 Parameters (λ, A, ρ, η) or $(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$ or ...



$V_{ud} V_{ub}^* + V_{cd} V_{cd}^* + V_{td} V_{tb}^* = 0$ (all order λ^3)

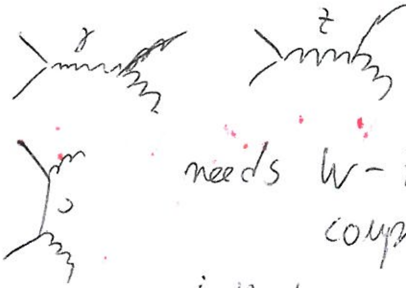
$\Rightarrow$ triangle: $(0,0)$ $(1,0)$ $(\bar\rho, \bar\eta)$

- should close
- different sets should give same results

0.5: CPV in K small, in B large ✓

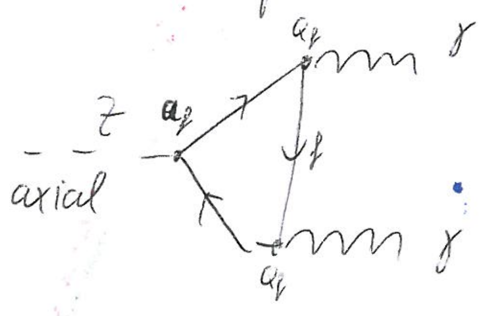
Other tests of the SM: what do we know (about Higgs)

- CKM fits doing okay; no FCNC @ tree level, 1 phase enough
  $\Rightarrow$ known particle content seems to do okay
  NO EXPLANATION FOR MIXING
- chiral couplings tested
- $e^+e^- \to W^+W^-$ @ LEP:



needs $W-Z-Z$ coupling!
both to reproduce data, as well as to restore unitarity of σ

- <u>miracle</u>: Anomalies: gauge symmetry violated @ loop level ($\mathcal{M}(\varepsilon^\mu \to k^\mu)$ should vanish)



$f = \nu, u, d, e$

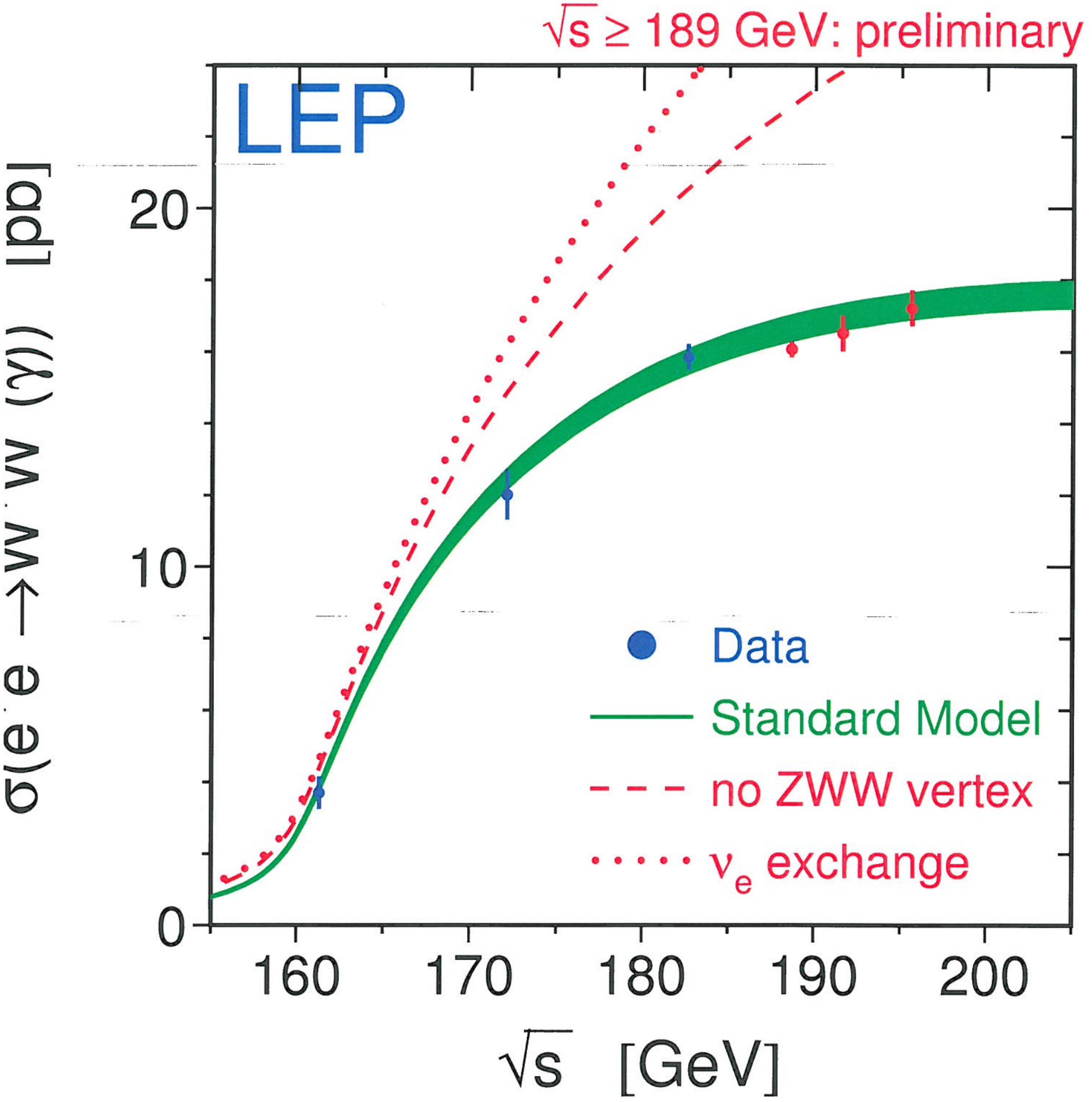
is proportional to $a_f Q_f^2 = \frac{1}{2} \cdot 0^2 + \frac{1}{2} N_c \left(\frac{2}{3}\right)^2$

$-\frac{1}{2} N_c \left(-\frac{1}{3}\right)^2 - \frac{1}{2}(-1)^2$

$= 0$

for $N_c = 3$

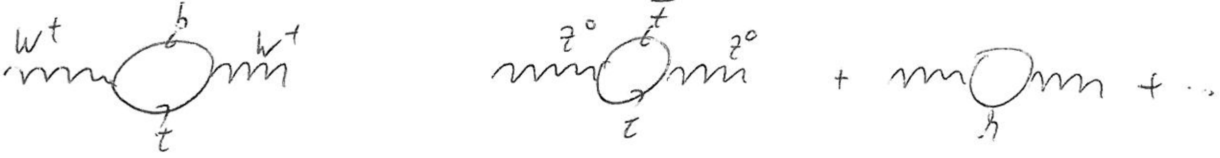
WTF?



$\sqrt{s} \geq 189$ GeV: preliminary
LEP
$\sigma(e^+e^- \to W^+W^-(\gamma))$ [pb]
20
10
0
160
170
180
190
200
$\sqrt{s}$ [GeV]
Data
Standard Model
no ZWW vertex
ν_e exchange

Loop-level tests „EWPT"

$$\rho = \frac{M_W^2}{M_Z^2 c_W^2} = 1$$ @ tree-level in SM



vacuum polarization: propagator modified „mass" is pole of propagator

$$\Delta\rho = \frac{G_F}{8\pi^2}\left(m_t^2 \cancel{+} m_b^2 + 2\frac{m_t^2 m_b^2}{m_t^2 - m_b^2}\ln \frac{m_t^2}{m_b^2} - \frac{3 m_W s_W^2}{c_W^2}\ln\frac{m_h^2}{m_W^2}\right)$$

vanishes for $m_t = m_b$, $g' = 0$ $\qquad + m_{W,Z}$

* top Quark mass „predicted" before its observation
* $\Delta\rho = 0$ for $g_1 = 0$, Yukawa $= 0$ „custodial SU(2)" — Higgs-potential has $O(4) = SU(2) \times SU(2)$ symmetry; $\rho = 1$; no large λ-corrections ⇒ „custodial"
* logarithmic Higgs-mass dependence
* t, b always leading ($y \sim \frac{m}{v}$; not suppressed (Heisenberg / Appelquist–Carazzone)
* sensitive to new physics
* also: $\overbrace{S, T}^{\text{dim } 6}, \overbrace{U}^{\text{dim } 8}$ parameters (Peskin, Takeuchi)

S: $\Pi_{VV} \sim \frac{m_X^2}{m_W^2} + \ln m_H^2$ ⇒ scalar particle preferred; NP contribution to NC at different energy scales

T: isospin-violation, $\Delta S \neq \Delta T$, # NC to CC

U: m_W, Γ_W small in most NP

$$S = 0.02 \pm 0.11$$
$$T = 0.05 \pm 0.12$$
$$U = 0.07 \pm 0.12$$
all are $= 0$ in SM

S, T, U „oblique parameters"

~~oblique~~
hidden, not to fermions

S, T: dim 6
U: dim 9

vacuum polarizations - if $\Lambda_{NP}^2 \gg m_Z^2$: 3 parameters are enough

defined to be 0 in SM (at some reference point)

S: ~~N~~NP to NC processes at different energies

T: diff between ~~diff~~ NP contribution to NC and CC $\propto \Delta\rho$
isospin violation

U: m_W, Γ_W small in most NP - models

1107.0975: (yesterday!)

$S = 0.04 \pm 0.10$
$T = 0.05 \pm 0.11$
$U = 0.08 \pm 0.11$

e.g.:  $\sim \frac{m_X^2}{m_W^2} + \ln m_H^2 \Rightarrow$ scalar particle better than „composite Higgs"

Custodial SU(2)

$$\Phi = \begin{pmatrix} \varphi_2 + i\varphi_3 \\ \varphi_0 + i\varphi_1 \end{pmatrix} \rightarrow \begin{pmatrix} \varphi_0 \\ \varphi_1 \\ \varphi_2 \\ \varphi_3 \end{pmatrix} \Rightarrow \Phi^+\Phi = \varphi^T\varphi$$

$$\Rightarrow \mathcal{L}(\Phi^+\Phi) = \mathcal{L}(\varphi^T\varphi) \Rightarrow O(4) \text{ Symmetry}$$

$$O(4) \simeq \underbrace{SU(2)}_{\text{geeichte } SU(2)_L} \times \underbrace{SU(2)}_{\substack{\text{global } SU(2)_R \\ \text{„custodial"}}}$$

„accidental", only in Higgs-sector: explicitely violated by Yukawas (t, b, rest $=0$) and gauge fields

Can also write: $\boldsymbol{\Phi} = (\Phi, \tilde{\Phi}) \longrightarrow \langle\boldsymbol{\Phi}\rangle = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} v/\sqrt{2}$

$$\boldsymbol{\Phi} \rightarrow \boldsymbol{\Phi}' = U_L \boldsymbol{\Phi} U_R \quad \Rightarrow \operatorname{Tr}\{\boldsymbol{\Phi}^+\boldsymbol{\Phi}\} \Rightarrow \operatorname{Tr}\{U_R^+\boldsymbol{\Phi}^+U_L^+U_L\boldsymbol{\Phi}U_R\}$$
$$= \operatorname{Tr}\{\boldsymbol{\Phi}^+\boldsymbol{\Phi}\} \text{ invariant}$$

and furthermore: $\operatorname{Tr}\{\boldsymbol{\Phi}^+\boldsymbol{\Phi}\} = 2\,\Phi^+\Phi \Rightarrow \mathcal{L}(\Phi^+\Phi) = \mathcal{L}(\operatorname{Tr}\{\boldsymbol{\Phi}^+\boldsymbol{\Phi}\})$

$SU(2)_R$ broken through Yukawas

$$\mathcal{L} = -g_t \bar{Q}\Phi t_R - g_b \bar{Q}\tilde{\Phi} b_R = -\bar{Q}\,\boldsymbol{\Phi}\begin{pmatrix} g_t & 0 \\ 0 & g_b \end{pmatrix} R$$

where $R = \begin{pmatrix} t_R \\ b_R \end{pmatrix}$

$$\Rightarrow \mathcal{L} = -\frac{g_t+g_b}{2}\, \bar{L}\, \Phi\, R \;-\; \frac{g_t-g_b}{2}\, \underbrace{\bar{L}\, \Phi\, \sigma_3\, R}$$

$U_L^+ U_L$ $\quad$ $U_R^+ U_R$

invariant

$\bar{L}\, U_L^+ U_L\, \Phi\, U_R\, U_R^+\, \sigma_3\, U_R^+ U_R R$

but $U_R^+ \sigma_3 \neq \sigma_3 U_R^+$

$\Rightarrow$ breaks $SU(2)_R$

$\propto g_t - g_b \simeq g_t$

same for $g' \neq 0$; with $g_t^2 \Rightarrow g'^2$: only g_t here

tree level : $g = 1$: $SU(2)_R$ transforms 3 (neutral + charged) GB into each other (they have same mass) ;

eaten $\Rightarrow$ enter m_W, m_Z $\Rightarrow$ relation between m_W, m_Z

$\Rightarrow$ $\Delta g \propto g_t^2, g'^2$, large m_H^2 corrections avoided due to custodial $SU(2)$

$\rightarrow \frac{1}{\Lambda^2}\left(H^+ D_\mu H\right)^2$ contributes to g in NP (dim 6)

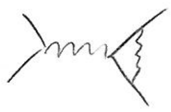
$\Rightarrow$ forbidden if $SU(2)_R$ custodial symmetry is present !

Fit parameters „precision observables"

Γ_Z ; both partial and full

Γ_W, m_W

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$ $e^+e^- \to \bar{b}\, b$



$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$ polarized e^-

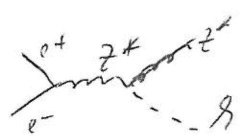
...

+ low energy observables ($\leq$ GeV)
  - Q_W (Cs) γ, Z interference
  - Møller-scattering γ, Z-interference
  - νN - scattering Θ_W
  - $\frac{\sigma_{\nu N} - \sigma_{\bar\nu N}}{+}$ Paschos-Wolfenstein

Plots...

indirect limit on $\boxed{m_H = 96^{+37}_{-24} \text{ GeV} \Rightarrow \leq 169 \text{ GeV}}$ @95%

LEP: $e^+e^- \to Z^* \to h Z^*$

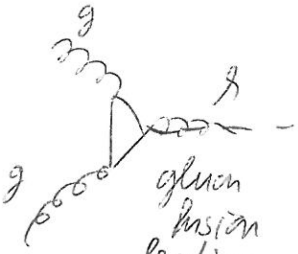


$m_H \geq 114$ GeV

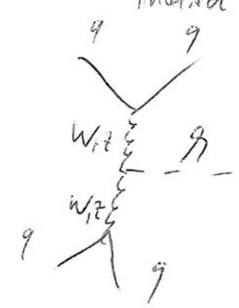
combine with indirect $\Rightarrow \boxed{m_H \leq 143 \text{ GeV}}$

TeVatron:

$h \to b\bar{b}$
$h \to WW$
$h \to \tau^+\tau^-$
$h \to \gamma\gamma$



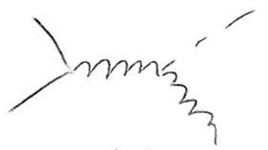
gluon fusion leading



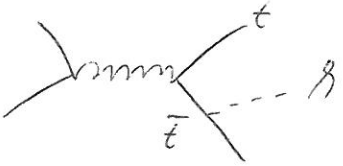
vector boson fusion

$158 < m_h < 175$ ausgeschlossen (August 2010)

$\Downarrow$



associated production

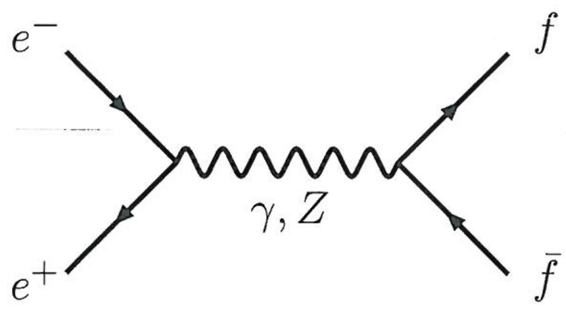


$\frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$ /cm

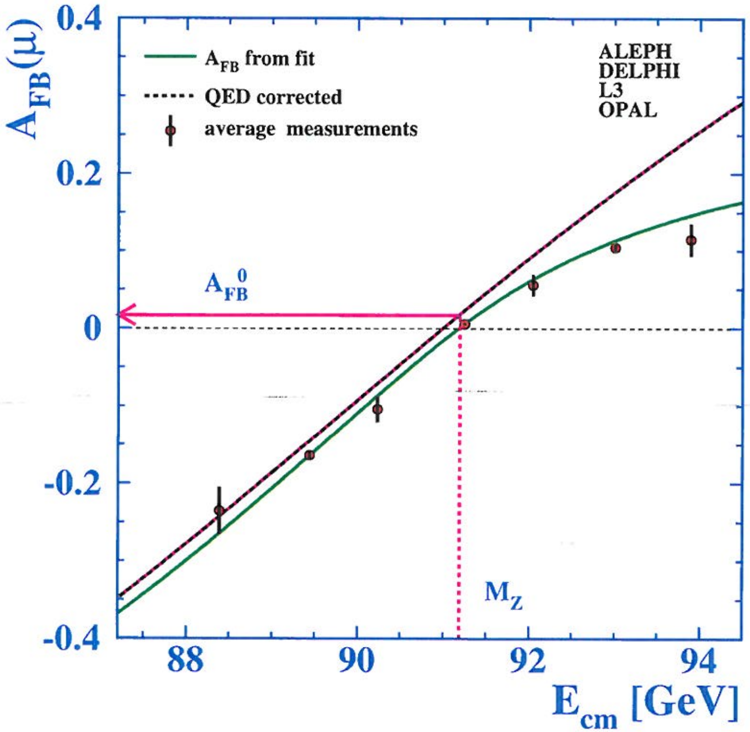
$e^+e^- \to \ell^+\ell^-$

Quantity	Value	Standard Model	Pull	Dev.
m_t [GeV]	$170.9 \pm 1.8 \pm 0.6$	171.1 ± 1.9	-0.1	-0.8
M_W [GeV]	80.428 ± 0.039	80.375 ± 0.015	1.4	1.7
	80.376 ± 0.033		0.0	0.5
M_Z [GeV]	91.1876 ± 0.0021	91.1874 ± 0.0021	0.1	-0.1
Γ_Z [GeV]	2.4952 ± 0.0023	2.4968 ± 0.0010	-0.7	-0.5
$\Gamma(\text{had})$ [GeV]	1.7444 ± 0.0020	1.7434 ± 0.0010	–	–
$\Gamma(\text{inv})$ [MeV]	499.0 ± 1.5	501.59 ± 0.08	–	–
$\Gamma(\ell^+\ell^-)$ [MeV]	83.984 ± 0.086	83.988 ± 0.016	–	–
σ_{had} [nb]	41.541 ± 0.037	41.466 ± 0.009	2.0	2.0
R_e	20.804 ± 0.050	20.758 ± 0.011	0.9	1.0
R_μ	20.785 ± 0.033	20.758 ± 0.011	0.8	0.9
R_τ	20.764 ± 0.045	20.803 ± 0.011	-0.9	-0.8
R_b	0.21629 ± 0.00066	0.21584 ± 0.00006	0.7	0.7
R_c	0.1721 ± 0.0030	0.17228 ± 0.00004	-0.1	-0.1
$A_{FB}^{(0,e)}$	0.0145 ± 0.0025	0.01627 ± 0.00023	-0.7	-0.6
$A_{FB}^{(0,\mu)}$	0.0169 ± 0.0013		0.5	0.7
$A_{FB}^{(0,\tau)}$	0.0188 ± 0.0017		1.5	1.6
$A_{FB}^{(0,b)}$	0.0992 ± 0.0016	0.1033 ± 0.0007	-2.5	-2.0
$A_{FB}^{(0,c)}$	0.0707 ± 0.0035	0.0738 ± 0.0006	-0.9	-0.7
$A_{FB}^{(0,s)}$	0.0976 ± 0.0114	0.1034 ± 0.0007	-0.5	-0.4
$\bar{s}_\ell^2(A_{FB}^{(0,q)})$	0.2324 ± 0.0012	0.23149 ± 0.00013	0.8	0.6
	0.2238 ± 0.0050		-1.5	-1.6
A_e	0.15138 ± 0.00216	0.1473 ± 0.0011	1.9	2.4
	0.1544 ± 0.0060		1.2	1.4
	0.1498 ± 0.0049		0.5	0.7
A_μ	0.142 ± 0.015		-0.4	-0.3
A_τ	0.136 ± 0.015		-0.8	-0.7
	0.1439 ± 0.0043		-0.8	-0.5
A_b	0.923 ± 0.020	0.9348 ± 0.0001	-0.6	-0.6
A_c	0.670 ± 0.027	0.6679 ± 0.0005	0.1	0.1
A_s	0.895 ± 0.091	0.9357 ± 0.0001	-0.4	-0.4
g_L^2	0.3010 ± 0.0015	0.30386 ± 0.00018	-1.9	-1.8
g_R^2	0.0308 ± 0.0011	0.03001 ± 0.00003	0.7	0.7
$g_V^{\nu e}$	-0.040 ± 0.015	-0.0397 ± 0.0003	0.0	0.0
$g_A^{\nu e}$	-0.507 ± 0.014	-0.5064 ± 0.0001	0.0	0.0
A_{PV}	$(-1.31 \pm 0.17)\cdot 10^{-7}$	$(-1.54 \pm 0.02)\cdot 10^{-7}$	1.3	1.2
$Q_W(\text{Cs})$	-72.62 ± 0.46	-73.16 ± 0.03	1.2	1.2
$Q_W(\text{Tl})$	-116.4 ± 3.6	-116.76 ± 0.04	0.1	0.1
$\frac{\Gamma(b\to s\gamma)}{\Gamma(b\to Xe\nu)}$	$\left(3.55^{+0.53}_{-0.46}\right)\cdot 10^{-3}$	$(3.19 \pm 0.08)\cdot 10^{-3}$	0.8	0.7
$\frac{1}{2}(g_\mu - 2 - \frac{\alpha}{\pi})$	$4511.07(74)\cdot 10^{-9}$	$4509.08(10)\cdot 10^{-9}$	2.7	2.7
τ_τ [fs]	290.93 ± 0.48	291.80 ± 1.76	-0.4	-0.4

Vorwärts-Rückwärts-Asymmetrie



$$\frac{d\sigma(s)}{d\cos\theta} = \sigma(s)\left[\frac{3}{8}(1+\cos^2\theta) + A_{FB}^f(s)\cos\theta\right]$$

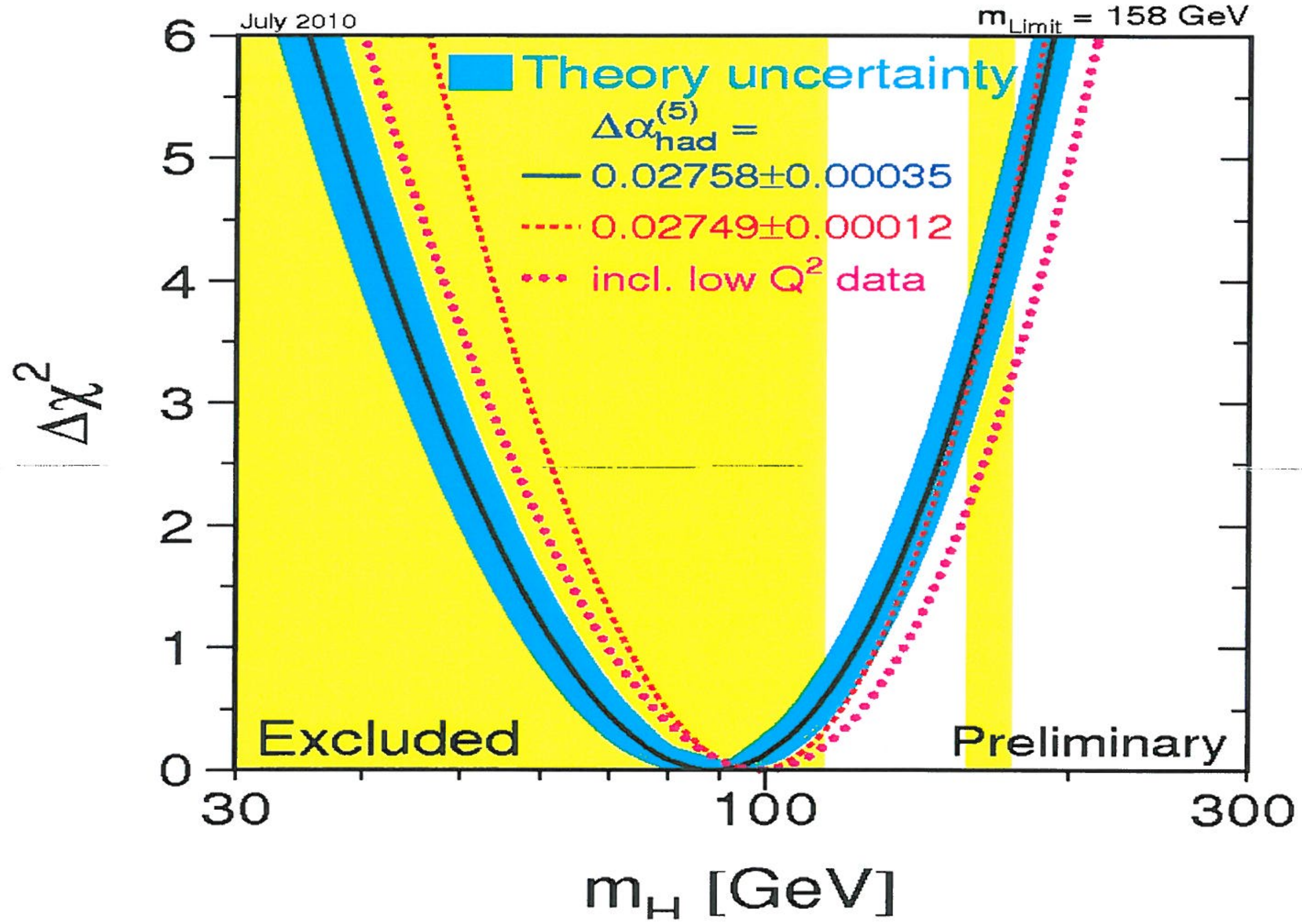


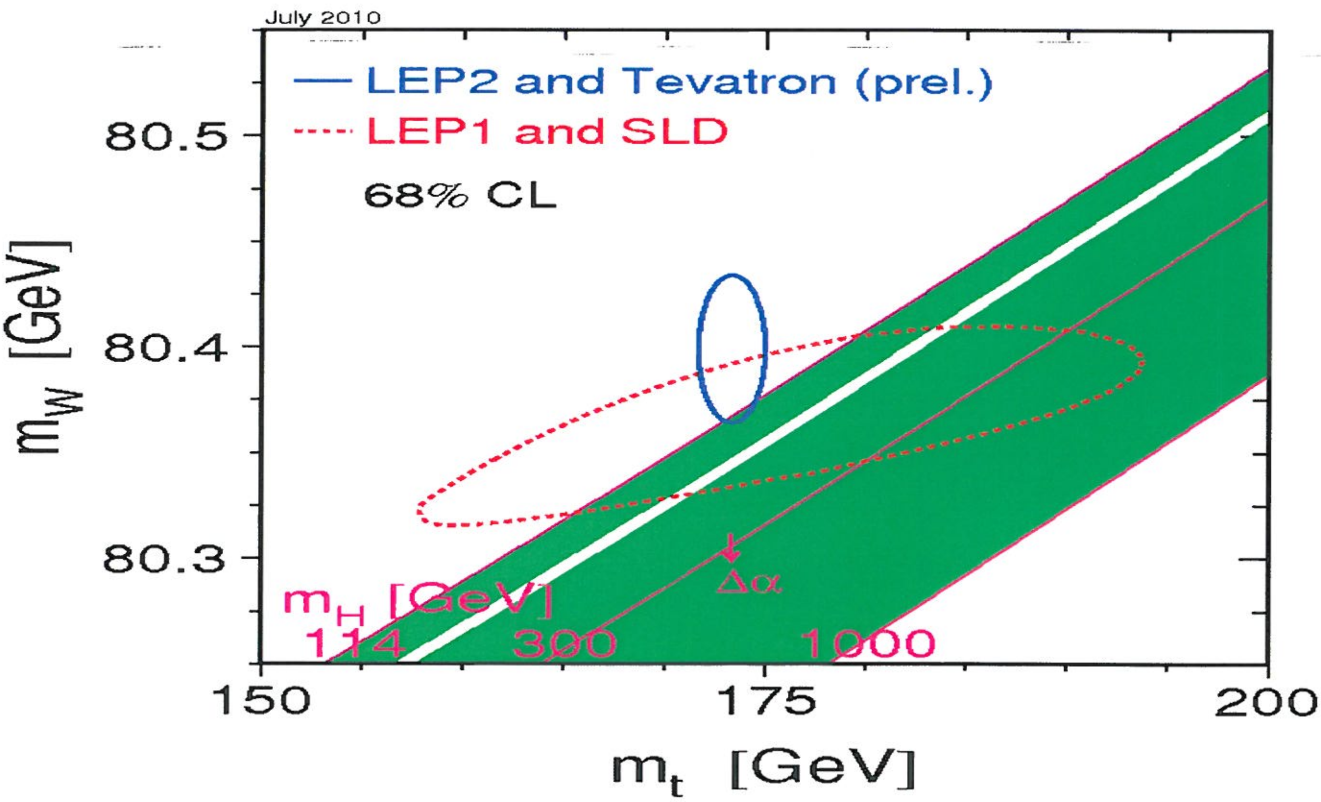
$$A_{FB}^f = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f\,,$$

$$\mathcal{A}_f = 2\frac{g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2} = 2\frac{g_V^f/g_A^f}{1+(g_V^f/g_A^f)^2}$$

$$\frac{g_V^f}{g_A^f} = 1 - 4|Q_f|\sin^2\theta_{\text{eff}}^f$$

$\longrightarrow$ effektiver Mischungswinkel


July 2010
m_{Limit} = 158 GeV
Theory uncertainty
$\Delta\alpha^{(5)}_{had}$ =
0.02758±0.00035
0.02749±0.00012
incl. low Q^2 data
Excluded
Preliminary
$\Delta\chi^2$
m_H [GeV]
0
1
2
3
4
5
6
30
100
300


July 2010
LEP2 and Tevatron (prel.)
LEP1 and SLD
68% CL
m_W [GeV]
m_t [GeV]
80.3
80.4
80.5
150
175
200
$\Delta\alpha$
m_H [GeV]
114
300
1000

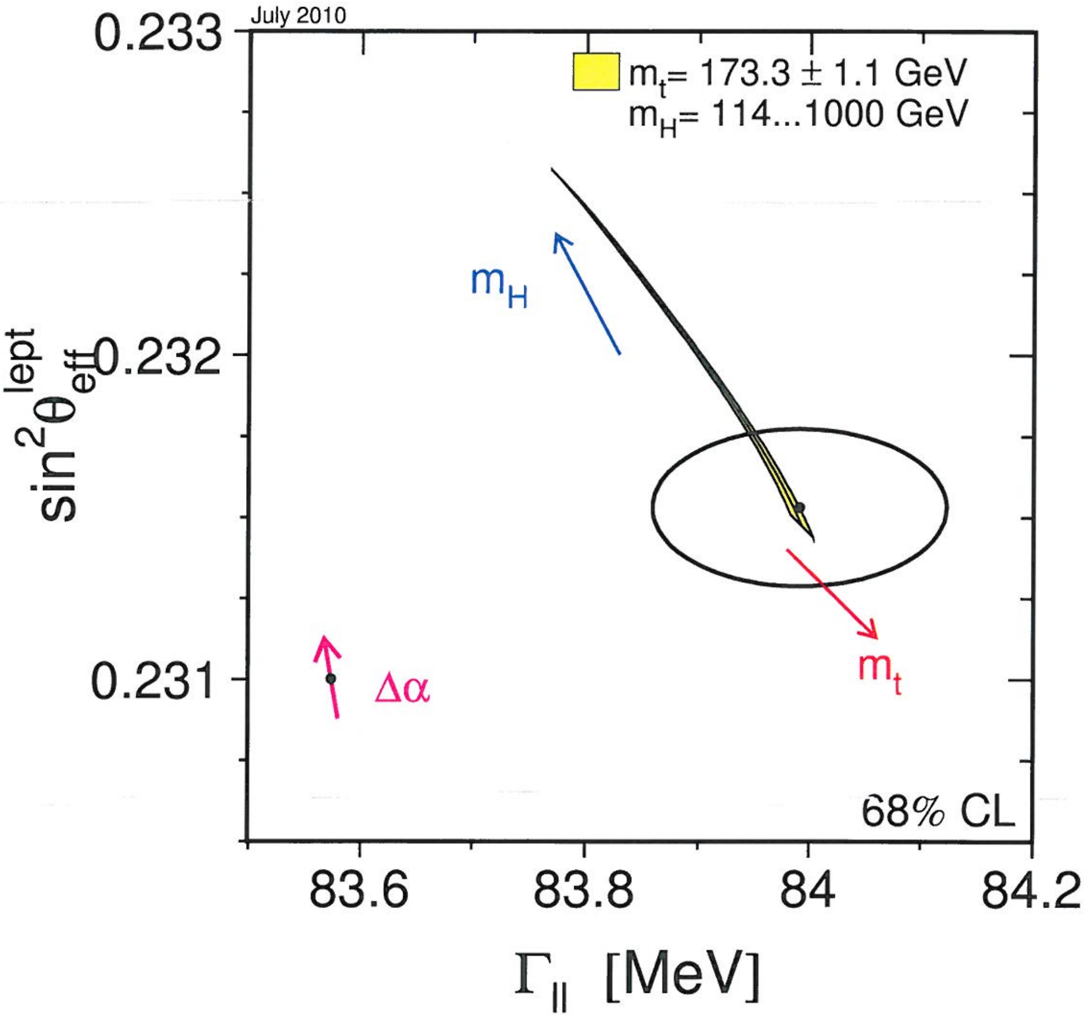


Figure 1: LEP-I+SLD measurements [1] of $\sin^2\theta_{\rm eff}^{\rm lept}$ and $\Gamma_{\ell\ell}$ and the SM prediction. The point with the arrow labelled $\Delta\alpha$ shows the prediction when only the photon vacuum polarisation is included in the electroweak radiative corrections. The associated arrow shows the variation in this prediction if $\alpha(m_Z^2)$ is changed by one standard deviation. This variation gives an additional uncertainty to the SM prediction shown in the figure.

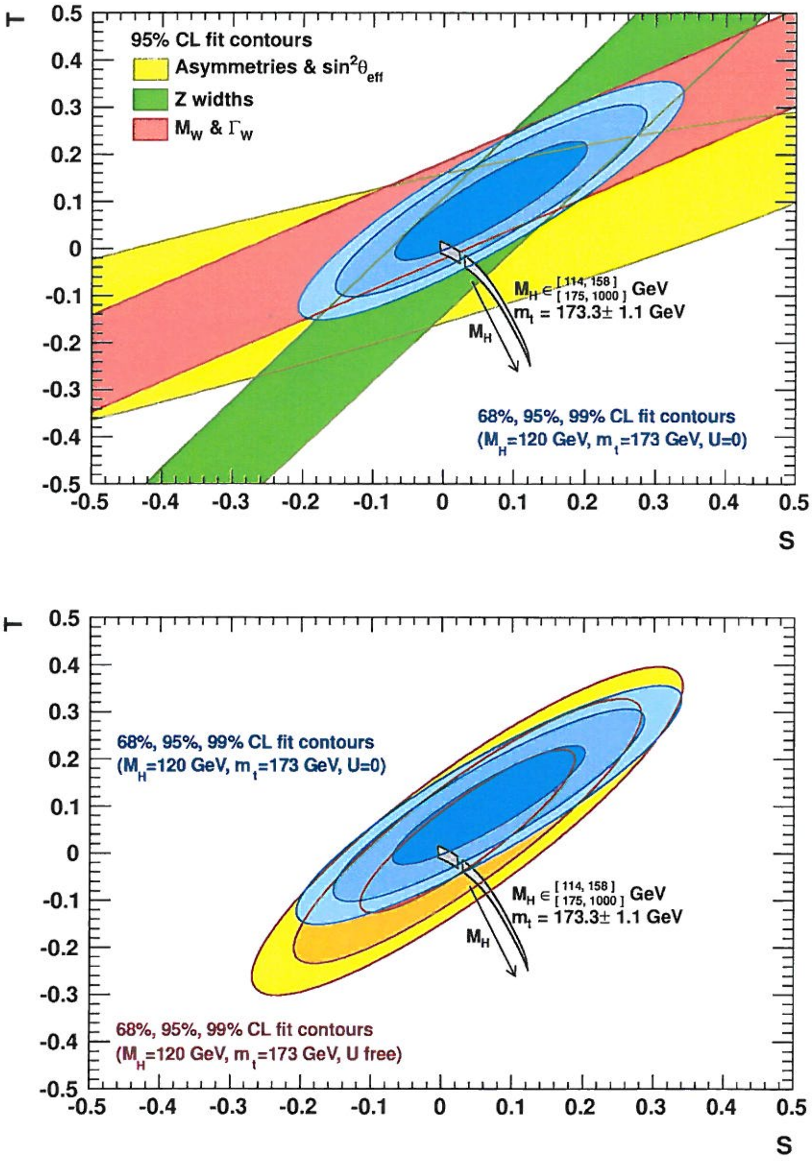


Figure 10: Experimental constraints on the S, T parameters with respect to the SM reference represented by $M_{H,\mathrm{ref}} = 120$ GeV, $m_{t,\mathrm{ref}} = 173$ GeV and the corresponding best fit values for the remaining SM parameters. Shown are the 68%, 95% and 99% CL allowed regions with the U parameter fixed to zero (blue ellipses on top and bottom panels) or let free to vary in the fit (orange ellipses on bottom panel). The top plot also shows for $U = 0$ the individual constraints from the asymmetry measurements (yellow), the Z partial and total widths (green), and the W mass and width (orange). The narrow dark grey bands illustrate the SM prediction for varying M_H and m_t values (see figures for the ranges used).

LHC: • gg is dominant production mechanism

$\sigma \sim 10^{4...5}$ fb (total pp-cross section $\sigma_{pp} \approx 0.1$ b)

• Ende 2011 : $1 fb^{-1}$ @ 7 TeV

• $H \to b\bar{b}$ dominant but huge QCD background

$\Rightarrow \tau\tau$

$\gamma\gamma$ (LHC does this mode, note small BR)

• $m_H \gtrsim 135$ GeV: $m_H \to \left.\begin{matrix} W^+W^- \\ ZZ \end{matrix}\right\}$ dominate

very clean "golden"
narrow width needs large mass

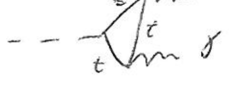
a) $H \to WW^*$: 2 high p_T leptons, opp. charge, small transverse opening angle

$\Rightarrow$ future exclusion with $1 fb^{-1}$; $\sqrt{s} = 7$ TeV $140 \leq m_H \leq 180$

masses around 165 GeV maybe observable

b) $H \to ZZ^*$ ~~not~~/go requires large m_H

c) $H \to \gamma\gamma$



promising in $110 \leq m_H \leq 140$
needs high luminosity, because BR small

LHC will run till end 2012 with 7 TeV

"will know by the end of 2012" ....
premature, optimistic, requires "low Higgs"

Higgs-properties

- $m_H^2 = \frac{1}{2} \lambda v^2$

v: only energy scale of SM

λ: only unknown parameter of SM
all couplings to $m_{W,Z}$, fermions fixed

$$v^2 = (246\ \text{GeV})^2 = \frac{4 m_W^2}{g^2} = \left(\frac{1}{\sqrt{2}\, G_F}\right)$$

~~Couplings:~~ $\cancel{g_{Hff} = \frac{m_f}{v}}$

Note: direct + indirect constraints on SM sensitive to new physics!

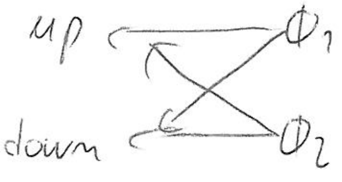
Example: Φ_2 instead of $\tilde{\Phi}$: 2HDM

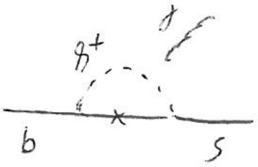
$$\rho = \frac{\sum v_i^2 \left[T_i(T_i+1) - Y_i^2/4\right]}{\sum \frac{1}{2} v_i^2 Y_i^2}$$ stays 1 @ tree level

(note: $\rho = 1$ for Higgs Sextuplett, 25 plet...)

→ 5 Higgs-Bosons! charged pair, 3 neutrals $\tan\beta = v_2 / v_{SM}$

→ "MSSM": one for up-type, one for down type → could be different





FCNC

avoid if parallel structure
Glashow-Weinberg-Paschos

$$\longrightarrow \quad \left.\begin{aligned} m_t &= g_t v_1 \\ m_b &= g_b v_2 \end{aligned}\right\} \quad v_2 < v_1 \Rightarrow g_b \text{ larger than in SM}$$

$\Rightarrow \Delta\rho$ modified

$H \to b\bar{b}$ larger

<u>other example:</u> • 4th generation of SM-fermions

$\Rightarrow gg \to H$ larger!

• S, T: need „sum-rule" ($m_{b'} \simeq m_{t'}$ 10%) why @ low energy?

<u>other example:</u> Higgs-triplet $\Delta \sim 3_L$ $\langle \Delta^0 \rangle = v_T$

$$\Rightarrow \rho = \frac{\frac{1}{2}v^2 + v_T^2}{\frac{1}{2}(v^2 + 4v_T^2)} \simeq 1 - 2\, v_T^2/v^2$$

$\Rightarrow v_T \lesssim$ few GeV

<u>other examples:</u> • $SU(2)_L \times U(1)$ singlet: no coupling to $W^\pm, Z$

$\Rightarrow$ okay

• new fermions with SSB scale $v' \gg v$

$\Rightarrow g_Y = \frac{m_f'}{v'}$ could be small...

<u>other example:</u> Vector-like quarks: L, R component opposite hypercharge, <u>same</u> isospin: $\bar{Q}Q$ singlet

$\Rightarrow$ mass arbitrary not $\frac{Y_b'}{v}$ ...

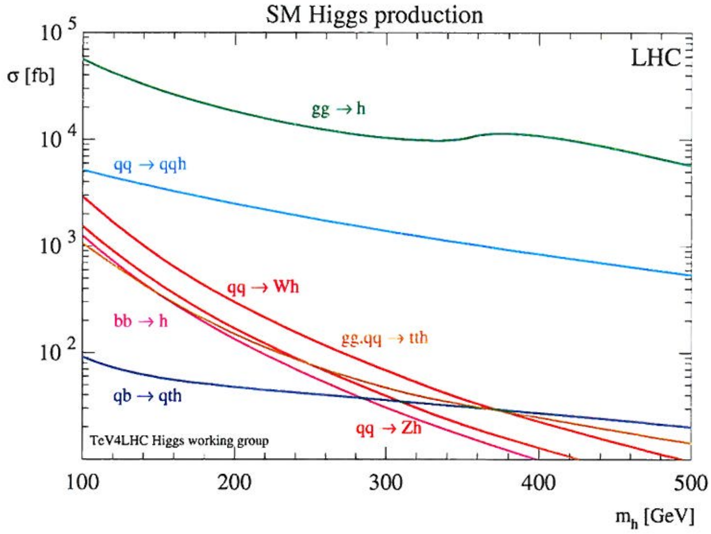


Figure 4. SM Higgs production cross-sections for $\sqrt{s} = 14$ TeV at the LHC (taken from ref. [26]).

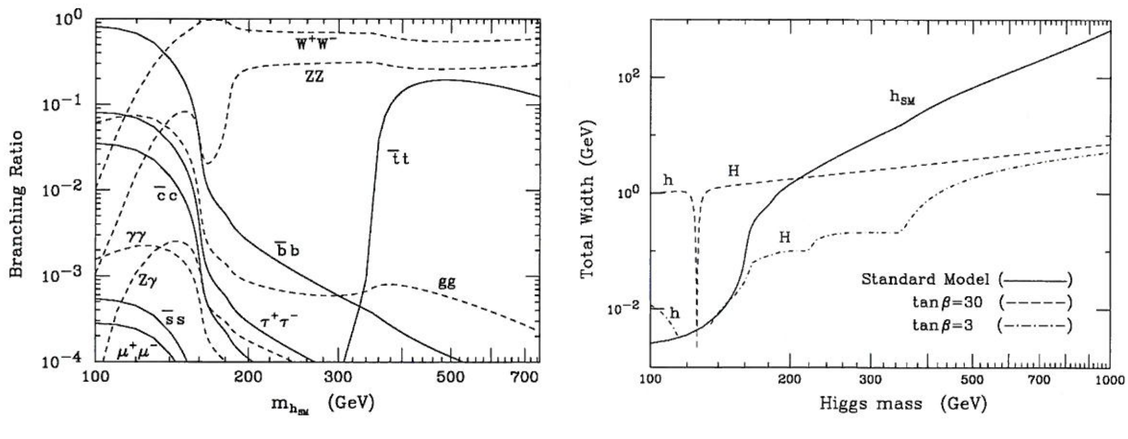
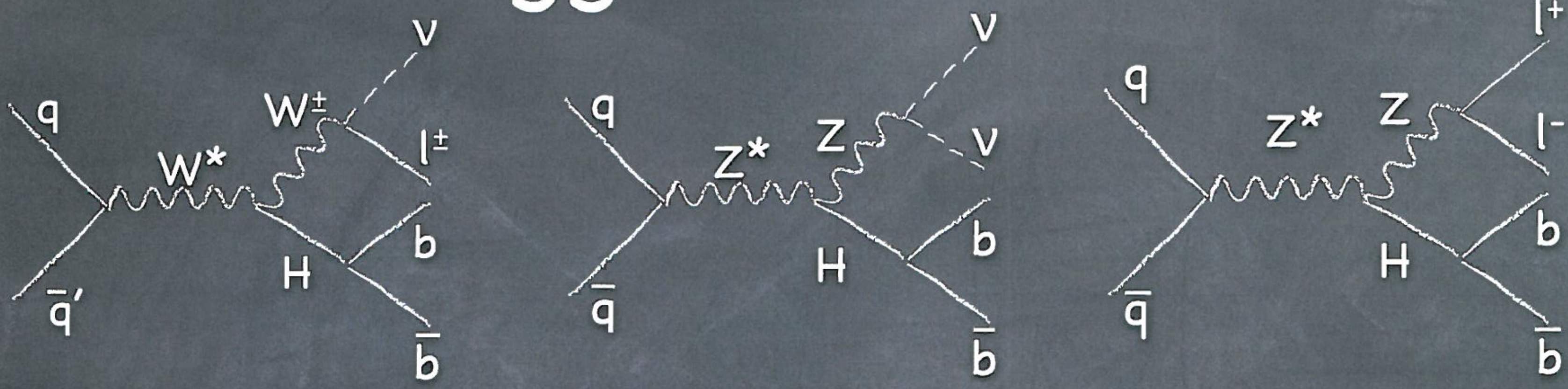


Figure 5. (a) In the left panel, the branching ratios of the SM Higgs boson are shown as a function of the Higgs mass. Two-boson [fermion-antifermion] final states are exhibited by solid [dashed] lines. (b) In the right panel, the total width of the Standard Model Higgs boson (denoted by $h_{\rm SM}$) is shown as a function of its mass. For comparison, the widths of the two CP-even scalars, h^0 and H^0 of the MSSM are exhibited for two different choices of MSSM parameters ($\tan\beta = 3$ and 30 in the maximal mixing scenario; the onset of the $H^0 \to h^0h^0$ and $H^0 \to t\bar{t}$ thresholds in the $\tan\beta = 3$ curve are clearly evident). Taken from ref. [10].

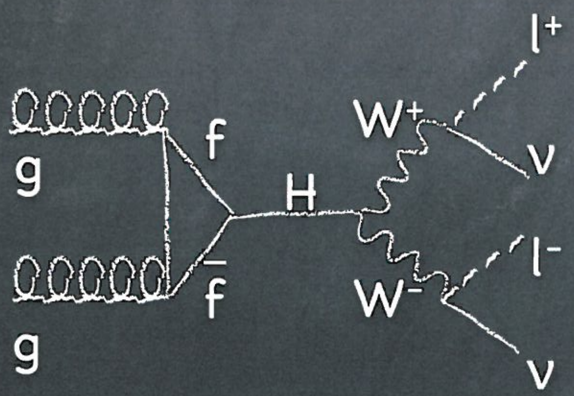
5.3. LHC prospects for SM Higgs discovery

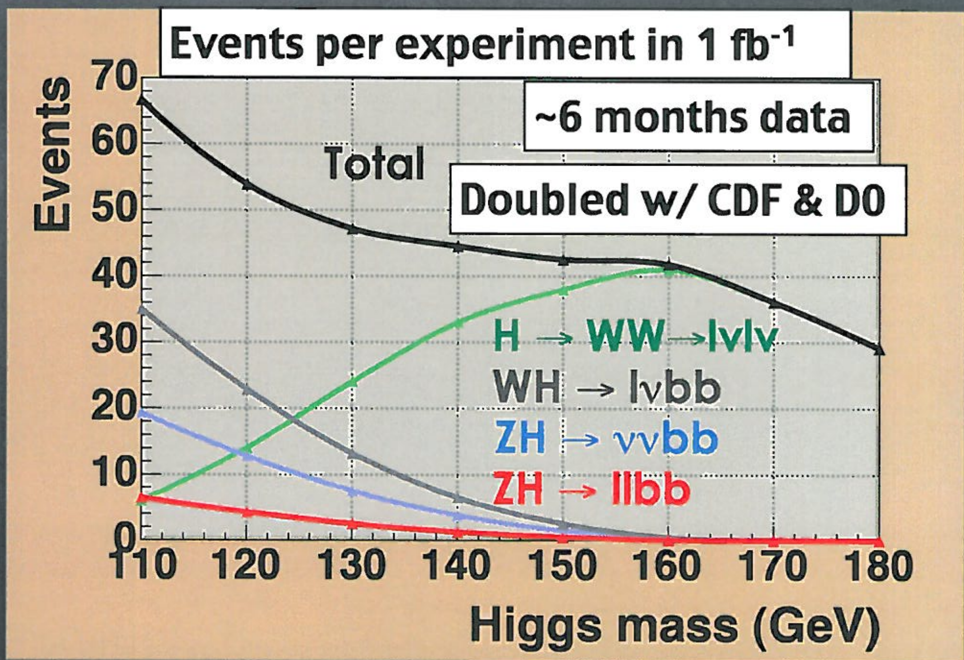
An examination of Fig. 5 indicates that for $m_h < 135$ GeV, the decay $h \to b\bar{b}$ is dominant, whereas for $m_h > 135$ GeV, the decay $h \to WW^{(*)}$ is dominant (where one of the W bosons must be virtual if $m_h < 2m_W$). These two Higgs mass regimes require different search strategies. For the lower mass Higgs scenario, gluon-gluon fusion to the Higgs boson followed by $h \to b\bar{b}$ cannot be detected as this signal is overwhelmed by QCD two-jet backgrounds. Instead, the Tevatron

SM Higgs at the Tevatron



Main decay modes





Higgs acceptance

Higgs rate small, we reconstruct additional topologies

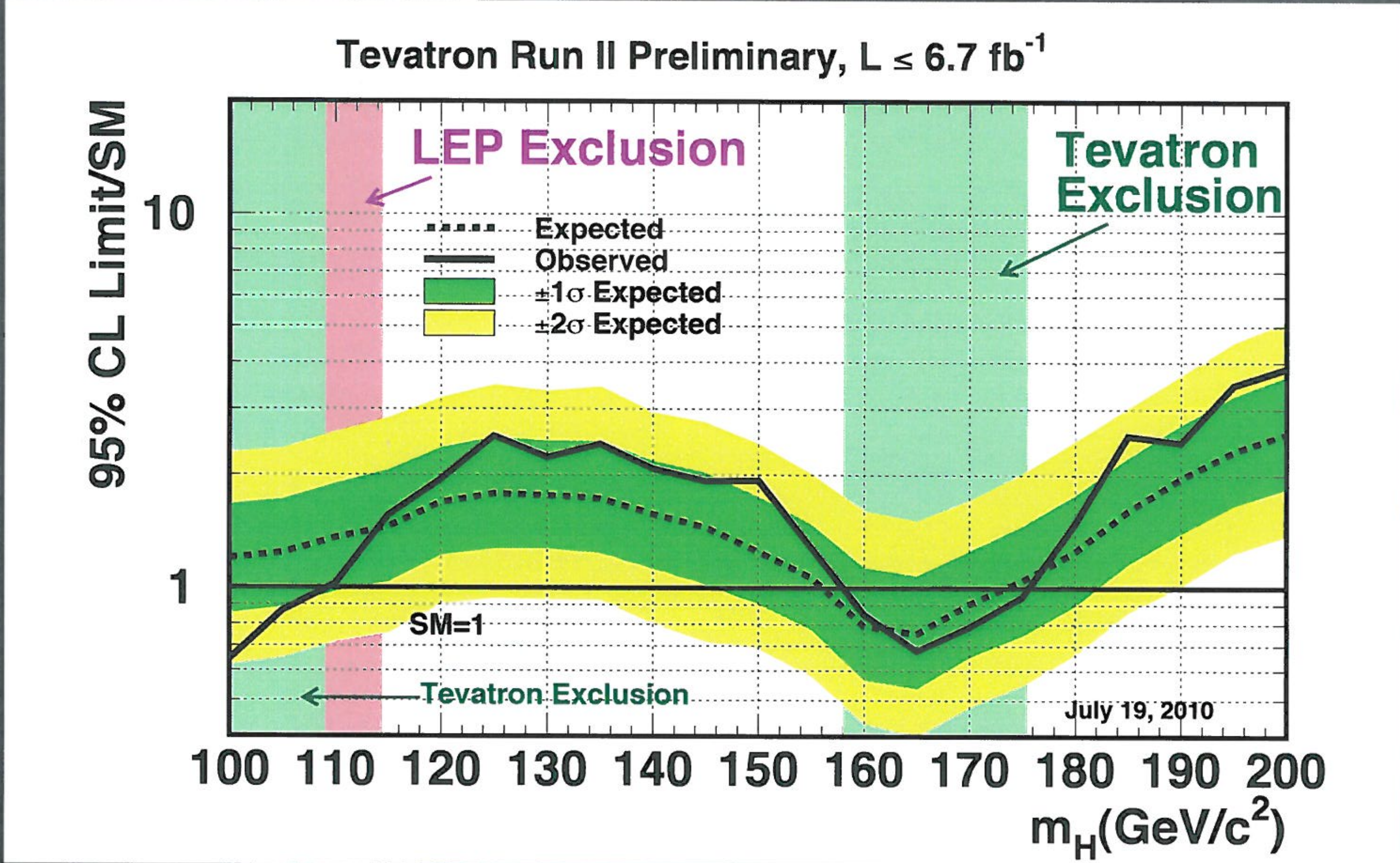
Production:	Decay:	W , Z decays :
$gg \rightarrow H$	$H \rightarrow WW$	$W \rightarrow l\nu$
$qq \rightarrow H + W$	$H \rightarrow bb$	$Z \rightarrow ll$
$qq \rightarrow H + Z$	$H \rightarrow \tau\tau$	$Z \rightarrow \nu\nu$
$qq \rightarrow H + qq$	$H \rightarrow \gamma\gamma$	$W \rightarrow \tau\nu$
		$W \rightarrow qq$

For example :
$qq \rightarrow HZ \rightarrow WWZ \rightarrow l\nu llqq$

Select : electrons,
muons, MET, jets

Tevatron combination

"Expected sensitivity"



- Low mass sensitivity approaching LEP exclusion :
  - Expected 1.45*SM @ 115 GeV
  - Expected 1.24*SM @ 105 GeV
- High mass 95% CL exclusion :
  - 158 < m_H < 175 GeV
    - 4 times previous (162 - 166 GeV)
    - Expected (156 < m_H < 175 GeV)

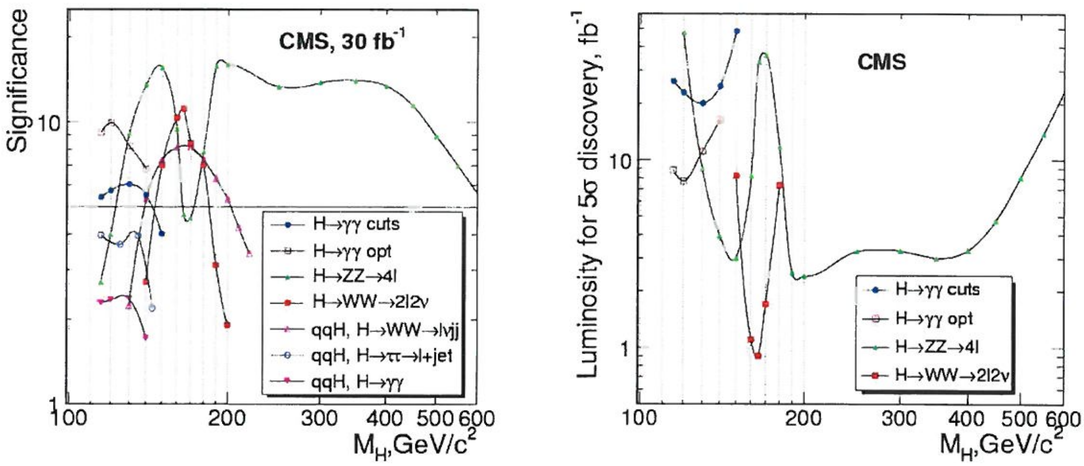


Figure 7. (a) The left panel depicts the signal significance as a function of the SM Higgs boson mass for 30 fb^{-1} at $\sqrt{s} = 14$ TeV, for the different Higgs boson production and decay channels. (b) The right panel depicts the integrated luminosity required for a 5σ discovery of the inclusive Higgs boson production, $pp \to h + X$, with the Higgs boson decay modes $h \to \gamma\gamma$, $h \to ZZ \to \ell^+\ell^-\ell^+\ell^-$ and $h \to W^+W^- \to \ell^+\nu_\ell\ell^-\bar{\nu}_\ell$. Taken from ref. [28].

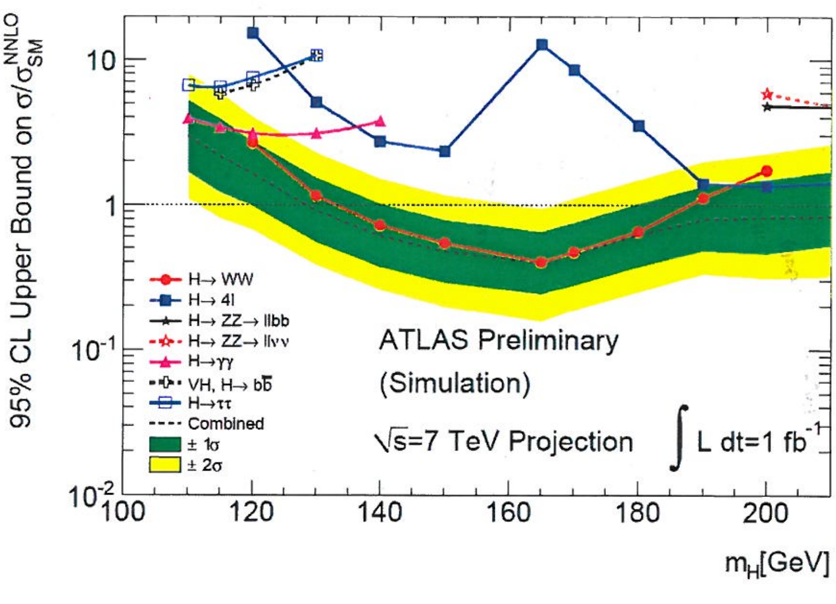


Figure 8. The multiple of the cross-section of a SM Higgs boson that can be excluded using 1 fb^{-1} of data at $\sqrt{s} = 7$ TeV. At each mass, every channel giving reporting on it is used. The green and yellow bands indicate the range in which the limit is expected to lie, depending on the data. Taken from ref. [29]

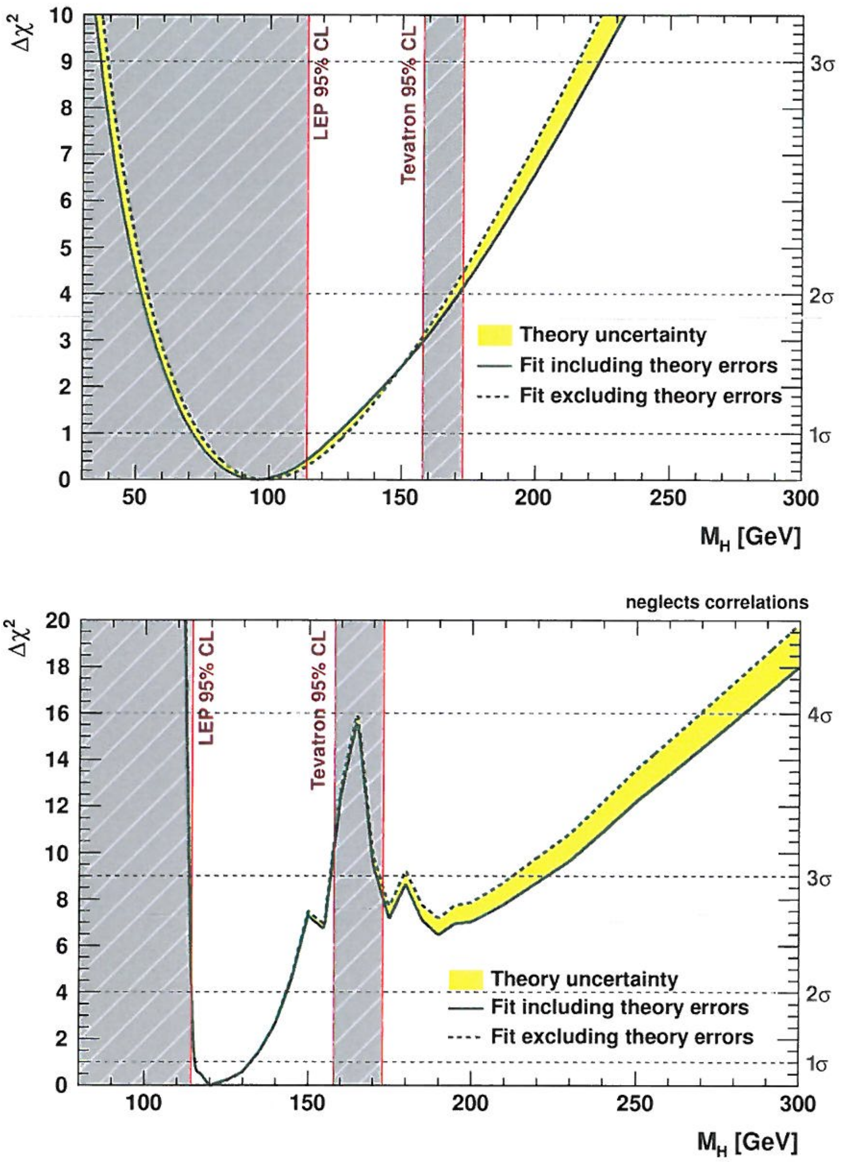


Figure 4: Indirect determination of the Higgs boson mass: $\Delta\chi^2$ as a function of M_H for the standard fit (top) and the complete fit (bottom). The solid (dashed) lines give the results when including (ignoring) theoretical errors. Note that we have modified the presentation of the theoretical uncertainties here with respect to our earlier results [1]. Before, the minimum $\chi^2_{\min}$ of the fit including theoretical errors was used for both curves to obtain the offset-corrected $\Delta\chi^2$. We now individually subtract each case so that both $\Delta\chi^2$ curves touch zero. In spite of the different appearance, the theoretical errors used in the fit are unchanged and the numerical results, which always include theoretical uncertainties, are unaffected.

Theoretical Higgs-Bounds

1) Unitarity

$$W_L W_L \to W_L W_L : \mathcal{A}_{J=0} = \begin{cases} \frac{G_F M_H^2}{4\pi\sqrt{2}} & \sqrt{s} \gg M_H \quad (*) \\ \frac{G_F s}{16\pi\sqrt{2}} & \sqrt{s} \ll M_H \quad (**) \end{cases}$$

$(*)$ $\left|\mathrm{Re}\{\mathcal{A}_{J=0}\}\right| < \frac{1}{2} \Rightarrow M_H \lesssim 900$ GeV from $(*)$

if heavier: non-perturbative

$(**)$ $\left|\mathrm{Re}\{\mathcal{A}_{J=0}\}\right| < 1/2 \Rightarrow \sqrt{s} \lesssim 1.8$ TeV

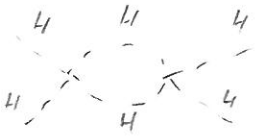
$\Rightarrow$ „there must be NP @ TeV scale"

2) Triviality

$$\lambda(Q) = \frac{\lambda(v)}{1 - \frac{3\lambda(v)}{4\pi^2}\log Q^2/v^2} \quad \text{or} \quad \lambda(v^2) = \frac{\lambda(Q^2)}{1 + \frac{3\lambda(Q)}{4\pi^2}\log Q^2/v^2}$$

$$\beta_\lambda = \frac{3\lambda^2}{4\pi^2} + \underset{m_{t,W,Z}}{\ldots}$$

$\lambda \nearrow$ for $Q \nearrow$



want to avoid „Landau-pole" : λ finite for $Q \to \infty$

$\Rightarrow$ $\lambda(v) = 0$ „trivial theory"

$\rightarrow$ only scalar theory which makes sense at all energies is ~~th~~ a trivial theory

but: $\Rightarrow$ enough if theory makes sense till Λ_{NP}

$\Rightarrow$ $\lambda(v) \simeq 0.3$ $\Rightarrow$ $m_h \lesssim 200$ GeV

$\nearrow$ $\lambda(\Lambda_{NP}) = 1$

$\Lambda_{NP} = M_{Pl}$

($\Lambda_{NP} \downarrow$: $m_h^{max} \nearrow$)

correct SM calculation, lattice ($\rightarrow$ λ range)

$m_H \lesssim 700$ GeV for $2m_H = \Lambda$

3) vacuum stability

shape of potential stable ; $\lambda \overset{!}{>} 0$ required to bound E from below

small λ : $\lambda(Q) = \lambda(Q_0) + \beta_\lambda \log\left(Q^2/Q_0^2\right)$

insert in potential $\Rightarrow$ $V(\phi) = \mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2 + \beta_\lambda(\phi^\dagger\phi)^2 \log Q^2/Q_0^2$

$$M_H^2 = \frac{1}{2}\frac{\partial^2 V}{\partial \phi^2}\Big|_{\phi = v/\sqrt{2}} \quad ; \text{ Minimum: } \frac{\partial V}{\partial \phi}\Big|_{\phi = v/\sqrt{2}} \overset{!}{=} 0$$

SSB occurs if $V(v) < V(0)$ $\rightarrow$ 

absolute stable vacuum requires

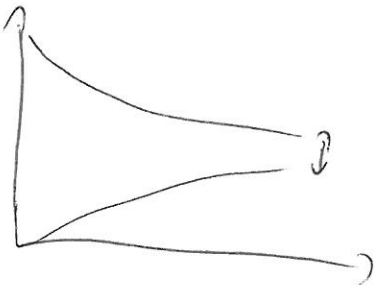
~~absolute~~ minimum of rad. corrected Higgs potential at v must be absolute minimum! ($V(v) < V(0)$ okay for $m_H > 80$ GeV

$$V(\phi = \Lambda) > V(\phi = v)$$

$$\Rightarrow m_H^2 \gtrsim 50 \text{ GeV} \quad (\Lambda = \text{TeV})$$

$$130 \text{ GeV} \quad (\Lambda = M_{Pl})$$

⇒ Linde-Weinberg
Coleman-Weinberg
not important)

⇒ 

could be stable up to M_{Pl} ...
$134 \leq m_H < 180$

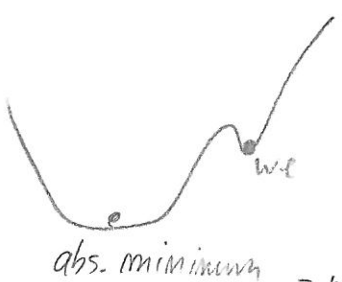
main message

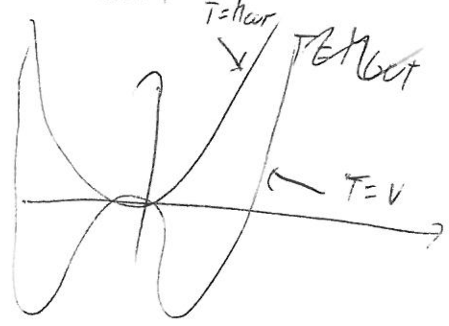
also: λ not < 0

4) Metastability

→ Minimum could be „false vacuum" metastable

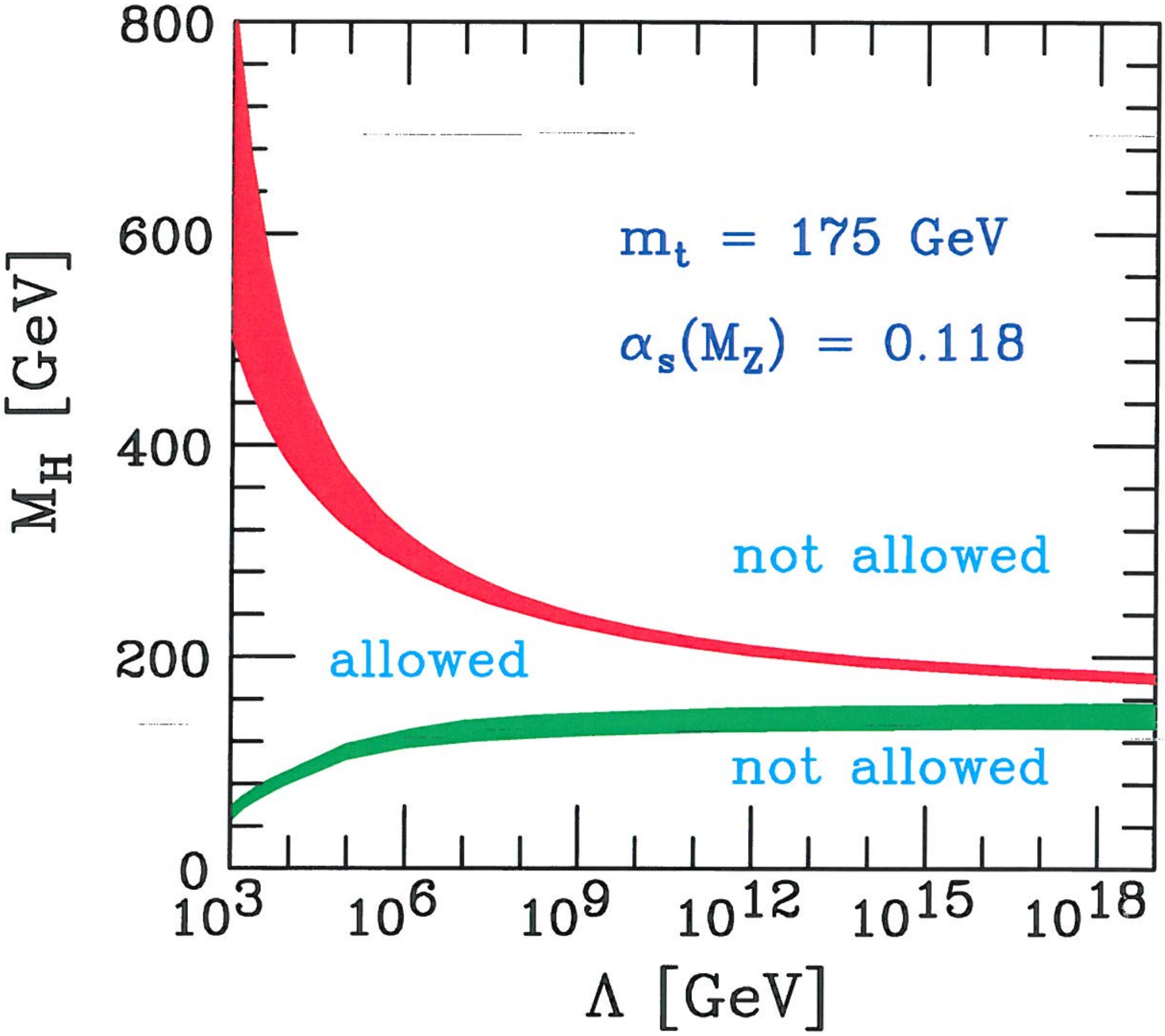
→ constraint: tunneling time to true vacuum $\geq \tau_u$



→ understanding: 

if going from ~~low~~ high to low: highly unlikely to ~~...~~ end up in false vacuum...

⇒ academic



800
600
400
200
0
M_H [GeV]
m_t = 175 GeV
$\alpha_s(M_Z)$ = 0.118
not allowed
allowed
not allowed
10^3
10^6
10^9
10^{12}
10^{15}
10^{18}
Λ [GeV]