Introduction to Dark Energy

ISAPP Heidelberg

Luca Amendola, University of Heidelberg
l.amendola@thphys.uni-heidelberg.de

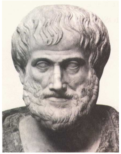
July 8, 2011

Outline

- Gravity, expansion and acceleration
- Cosmological observations
- Dark energy: linear and non-linear properties

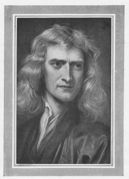
More details in: L. Amendola & S. Tsujikawa, *Dark Energy, Theory and Observations*, Cambridge University Press 2010

Historical perspective, circa 350 b.c.e.



- Gravity is always attractive: how to avoid that the **sky** falls on our head?
- **Aristotle**'s answer: quintessence

Historical perspective, circa 1700 c.e.



Sir Isaac Newton

- Gravity is always attractive: how to avoid that the **stars** fall on our head?
- **Newton**'s answer: God's initial conditions

Historical perspective, circa 1900 c.e.

- Gravity is always attractive: how to avoid that the **Universe** fall on our head?
- **Einstein**'s answer: to avoid collapse (to make the universe stable) it is necessary to introduce a form of repulsive gravity, by modifying the equations of General Relativity.

There is a simple Newtonian solution to the eq. $\nabla^2\Phi = const$,

$$\Phi = -G\frac{M}{r} + \lambda r^2$$

Newton himself noted the possibility of the r^2 repulsive behavior but did not comment further!

Original GR equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$geometry \qquad matter$$

$$T_\mu^\nu = \left\{ \begin{array}{cccc} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{array} \right\}$$

- These are the most general equations that are

1. covariant
2. covariantly conserved
3. second order in the metric
4. reducing to Newton at low energy

Box on $T_{\mu\nu}$

Basic hydrodynamic equations for a non-relativistic fluid at rest:

$$
\begin{aligned}
\dot{\rho} &= 0 \\
\nabla p &= 0
\end{aligned} \tag{1}
$$

where the energy density $\rho = nmc^2$ and the pressure is $p_i = nmv_i^2$. If we define the matrix

$$T_{\mu\nu} = diag(\rho, p, p, p)$$

then, more simply

$$\frac{\partial T^{\mu\nu}}{\partial x^\mu} \equiv T^{\mu\nu}_{,\mu} = 0$$

The relativistic version is the only tensor that depends on $\rho, p, u^\mu = dx^\mu / ds, g_{\mu\nu}$ and reduces to this limit in the Minkowski space

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu - p g^{\mu\nu} \tag{2}$$

Einstein's equations are complete only when a relation between p and ρ is given: the equation of state:

$$p = w\rho$$

Back to

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- For instance, neglecting the third condition (second order in the metric), we could add whatever rank-2 tensor covariantly conserved, e.g. any E_{μ}^{ν} such that

$$E_{\mu;\nu}^{\nu} = 0$$

- like e.g. a linear combination of

$$R R_{\mu\nu},\ R_{;\mu\nu},\ R^2 g_{\mu\nu}, \ldots$$

as for instance $[F = F(R)]$

$$E_{\mu\nu} \equiv F' R_{\mu\nu} - \frac{1}{2} F g_{\mu\nu} + g_{\mu\nu} \Box F' - F'_{;\mu;\nu}$$

obtaining a higher-order gravity.

- Neglecting instead the fourth condition, we can add a (*small*) term $\Lambda g_{\mu\nu}$ and rewrite the equations as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

- The new term is the **cosmological constant**.

- The big idea of recent years has been to move the new term from right to left

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu}$$

thereby introducing a new form of matter

$$T_{\mu\nu(\Lambda)} = \left(\frac{\Lambda}{8\pi}\right) g_{\mu\nu}$$

- This matter has a fundamental property. Writing

$$T^{\nu}_{\mu(\Lambda)} = \left(\frac{\Lambda}{8\pi}\right) \delta^{\nu}_{\mu}$$

or

$$\left\{\begin{array}{cccc} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{array}\right\} = \left\{\begin{array}{cccc} \frac{\Lambda}{8\pi} & 0 & 0 & 0 \\ 0 & \frac{\Lambda}{8\pi} & 0 & 0 \\ 0 & 0 & \frac{\Lambda}{8\pi} & 0 \\ 0 & 0 & 0 & \frac{\Lambda}{8\pi} \end{array}\right\}$$

one gets immediately

$$p_{\Lambda} = -\frac{\Lambda}{8\pi}, \quad \rho_{\Lambda} = \frac{\Lambda}{8\pi}$$

that is, the cosmological constant has **negative pressure** (if $\Lambda > 0$).

- Introducing the equation of state

$$p = w\rho$$

one has that the cosmological constant has a negative eq. of state

$$w = -1$$

As a comparison, the eq. of state of matter (dust or cold dark matter) is

$$p = mv^2 \approx 0 \rightarrow w = 0$$

while for radiation

$$p = \rho/3 \rightarrow w = 1/3$$

A repulsive gravity

- What a **negative pressure** has to do with a **repulsive gravity**?
- Homogeneous and isotropic Friedmann metric

$$ds^2 = dt^2 - a^2 \left(\frac{dr^2}{1 - kr^2} + r^2 \sin\theta d\phi^2 + r^2 d\theta^2 \right)$$

For a single perfect fluid, the ten Einstein equations reduce to two equations for the scale factor and the energy density (here we put for simplicity $k = 0$ and always assume $a_0 = 1$)

$$H^2 \equiv \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi}{3}\rho \tag{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}(\rho + 3p) = -\frac{4\pi}{3}\rho(1 + 3w) \tag{4}$$

From the second one it appears that if

$$w < -1/3$$

then we get accelerated expansion. Therefore the cosmological constant (or any fluid with $w < -1/3$) **accelerates** the expansion → “**repulsive gravity**”. We call this hypothetical fluid ***Dark Energy.***

- Consider now only the cosm. constant

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\rho_\Lambda = \frac{\Lambda}{3}$$

from which

$$a = a_0 e^{\sqrt{\frac{\Lambda}{3}}t}$$

This accelerated expansion is a prototype of **primordial inflation** **(de Sitter metric).**

- Generally speaking, there are at least three components (plus curvature) so that dynamics is more complicate:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\left(\rho_\gamma + \rho_M + \rho_\Lambda\right) - \frac{k}{a^2}$$

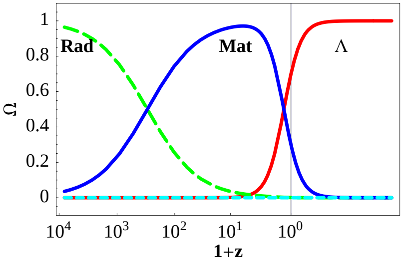
$$\dot{\rho}_i + 3H(\rho_i + p_i) = 0$$

Ordinary matter (baryons plus dark matter) conserves energy during expansion, so that we have **four** different behaviors

$$\begin{array}{rcl}
\rho_\gamma & \sim & a^{-4} \\
\rho_M & \sim & a^{-3} \\
\rho_k \equiv \dfrac{k}{a^2} & \sim & a^{-2} \\
\rho_\Lambda & \sim & a^0
\end{array}$$

- In general, therefore, we have

$$rad. \rightarrow matter \rightarrow curvature \rightarrow cosm.const.$$



$$\Omega_i \equiv \frac{\rho_i}{\rho_t}$$

Quantistic interpretation

- Think of a field, eg a scalar field, as a series of classical oscillators. Then, every oscillator contributes an energy due to the sum of its potential and kinetic energy.
- When at rest, every oscillator has only its potential energy of the lowest level, that we can always put to zero.
- Quantistically, however, the state of minimum is not at zero energy but rather

$$E_0 = \frac{1}{2}\hbar\omega$$

- Therefore, for a field, the total zero-point energy is

$$E_0 = \sum_i \frac{1}{2}\hbar\omega_i$$

  summing over all possible modes. Summing over $k_i = 2\pi/\lambda_i$ where $\lambda_i = L/n_i$ are all the wavelengths of the modes contained in a box of size L, we obtain $dn_i = dk_i L/2\pi$ modes in the range dk_i, so that

$$E_0 = \frac{1}{2}\hbar L^3 \int \frac{d^3k}{(2\pi)^3}\omega_k$$

- where the oscillation frequency is in relation to the particle's mass:

$$\omega^2 = k^2 + m^2/\hbar^2$$

The total energy density integrating up to a cut-off frequency k_{max} is then

$$\rho_{vacuum} = lim \frac{E}{L^3} = \hbar \frac{k_{max}^4}{16\pi^2}$$

- The energy diverges at the high frequencies (ultraviolet divergence). We must suppose then that there is k_{max} beyond which a new interaction modifies the system.
- The problem is, which k_{max} ?. If we assume as limit the Planck energy

$$E_{Planck} = 10^{19} GeV$$

we get

$$\rho_{vacuum} = 10^{92} g/cm^3$$

Now, the experimental limit is

$$\rho = 3H^2/8\pi G \simeq 10^{-29} g/cm^3$$

then, the theoretical estimate is off by 120 orders of magnitude!
- This fundamental theoretical problem is still **open**.

Astronomy of Dark Energy

- We have seen that the Friedmann equation has the form

$$H^2 = \frac{8\pi}{3}(\rho_M + \rho_\Lambda + \rho_k + ...?)$$

- Suppose we know nothing of the matter content of the universe. How to study the structure of the cosmos ?

- *Three levels*:

1. **background** effects (age, distances)
2. **linear** perturbations (large scale clustering, CMB)
3. **non-linear** perturbations (formation of collapsed objects, halo profiles).

Notice that a smooth component, as the Λ, has no **local** observational effects: The Poisson equation remains invaried, because in linear GR the Poisson equation

$$\triangle\Psi = -4\pi G\rho$$

becomes

$$\triangle\Psi = -4\pi G\delta\rho$$

That's why we need cosmology !

From observations to theory

- What we really observe in cosmology is light from **sources** and from **backgrounds**.
- How do we connect these observables to cosmological quantities like $\rho_m, \rho_\gamma, k, a(t), H_0$ etc?
- First, define

$$\Omega_M = \frac{8\pi\rho_0}{3H_0^2}, \quad \Omega_\Lambda = \frac{8\pi\rho_\Lambda}{3H_0^2}, \quad \Omega_k = \frac{8\pi k}{3H_0^2}$$

  and note that

$$1 = \Omega_M + \Omega_\Lambda + \Omega_k$$

  so rewrite Friedman equation as ($a_0 = 1$)

$$H^2 = H_0^2(\Omega_m a^{-3} + \Omega_\Lambda a^0 + \Omega_k a^{-2})$$

- Then, generalize it to several components:

$$\begin{aligned} H^2 &= H_0^2(\Omega_m a^{-3(1+w_m)} + \Omega_\Lambda a^{-3(1+w_\Lambda)} + \ldots) \\ &= H_0^2 \sum_i \Omega_i a^{-3(1+w_i)} = H_0^2 E(a)^2 \end{aligned}$$

- For instance...

name	density Ω_i	state w_i
baryons	0.05	≈ 0
CDM	0.2	0
radiation	0.0001	1/3
massive neutr.	<0.05	≈ 0
cosm. const.	0.75	-1
curvature	<0.03	-1/3
other ?	?	?

- Unknown quantities: H_0, Ω_i, w_i to be determined using:

1. Angular positions of sources, e.g. galaxies: θ_i, φ_i
2. Redshifts: z_i
3. Apparent magnitudes: m_i
4. galaxy ellipticities
5. Age of Universe/age of stars
6. Background radiation/polarization e.g. CMB: $\Delta T / T$

- BASIC RELATION redshift/scale factor:

$$\boxed{a = \frac{1}{1+z}}$$

First basic observable: Age of the Universe

- The age of the universe can be deduced from the Friedmann equation:

$$\left(\frac{da}{dt}\right)^2 = H_0^2 a^2 E(a)^2$$

we get

$$\left(\frac{dz}{H_0 dt}\right) = (1+z)E(z)$$

and finally

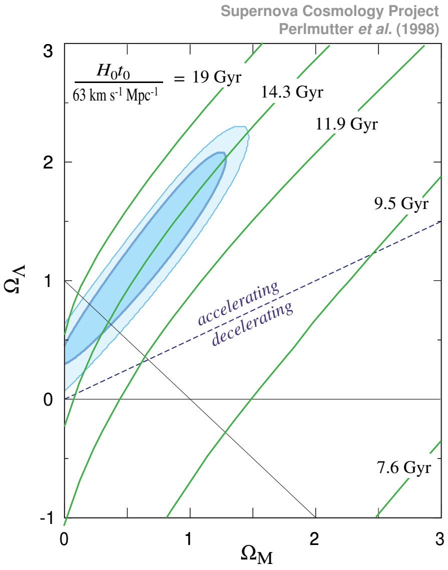
$$t_0 - t_1 = H_0^{-1} \int_0^{z_1} \frac{dz}{(1+z)E(z)}$$

Notice that the Hubble constant is

$$H_0^{-1} = \frac{1}{100hkm/sec/Mpc} = 9.76h^{-1}Gyr$$

For $z_1 \to \infty$ we get then the age of the universe.

- The effect of the cosmological constant , when $\Omega_{tot} = \Omega_M + \Omega_\Lambda$ is fixed, is to **increase the cosmological age.**



Best fit age of universe: $t_o = 14.5 \pm 1\ \ (0.63/h)$ Gyr

Second basic observable: Luminosity distance

- From flat Friedmann's metric

$$ds^2 = c^2 dt^2 - a^2 dr^2$$

and integrating along the null geodesics, we get the **proper distance** which is what you would measure with fixed rods

$$r = \int \frac{cdt}{a(t)} = c \int \frac{da}{\dot{a}a} = c \int \frac{dz}{H(z)}$$

$\rightarrow$ **generalized Hubble law:** measuring distances means measuring cosmology**.**

- If we compare the energy L emitted by a source at proper distance r with flux f arriving at the observer, we define the **luminosity distance** $d(z)$ such that

$$f = \frac{L}{4\pi r^2 (1+z)^2} = \frac{L}{4\pi d^2}$$

- The two extra factors of $1+z$ take into account the **loss** of energy due to redshift and the **spread** of energy due to the relative dilatation of the emission time versus observer's time. We get

$$d(z) = r(1+z) = cH_0^{-1}(1+z) \int_0^{z_1} \frac{dz}{E(z)}$$

where $cH_0^{-1} = \frac{300.000 km/sec}{100 h km/sec/Mpc} = 3000 h^{-1} Mpc.$

- Remember our "reference" cosmology

$$E^2(z) = \Omega_M(1+z)^3 + \Omega_\Lambda + \Omega_K(1+z)^2$$

- The luminosity distance therefore depends upon the cosmological constant and, like for the age, increases for Ω_Λ increasing. Therefore, a larger cosm. const. induces a **smaller luminosity of the standard candles**.

- Suppose we have a source of **known** absolute luminosity $M = -2.5 \log L + const$. Then one defines instead of the flux f an **apparent magnitude** $m = -2.5 \log f + const$ as

$$m - M = 25 + 5 \log d(z; \Omega_M, \Omega_\Lambda)$$

If M is the same for every object, then the apparent magnitude gives directly $d(z)$ and is then possible to test for the presence of a cosmological constant.

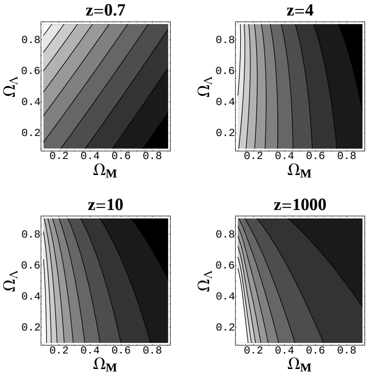
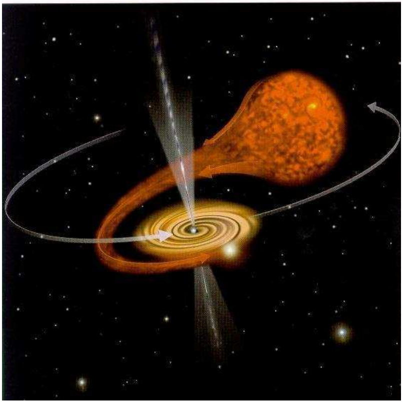


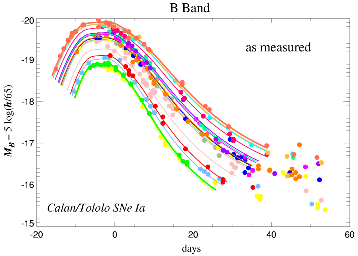
Figure: **Curves of constant $d(z;\Omega_M,\Omega_\Lambda)$.**

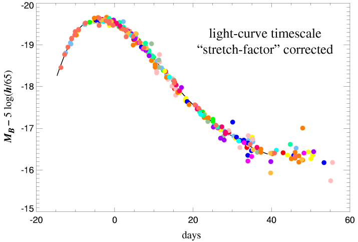
Standard candles

- Are there **standard candles** in nature ?
- The best such thing so far are **supernovae Ia**.



- This hypothesis can be tested and calibrated through a local sample whose distance we know by other means.





Kim, *et al.* (1997)

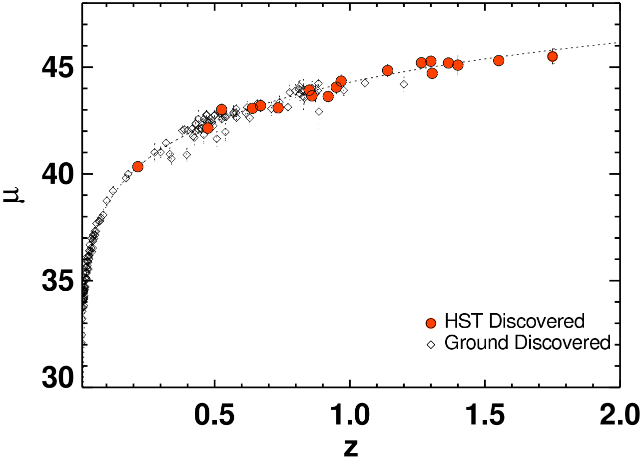
- Then, we compare $m_{obs}(z)$ with

$$m_{theor}(z) = M + 25 + \log d(z; \Omega_M, \Omega_\Lambda, ..)$$

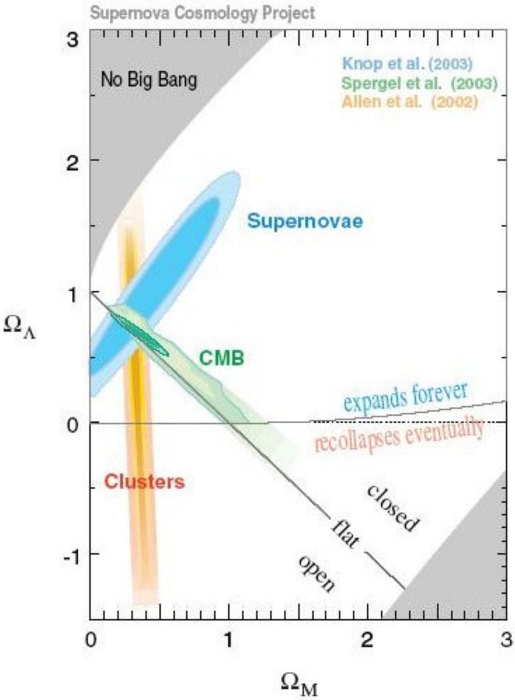
- For instance

$z = 1, \quad M = -19.5$	
$\Omega_M = 0, \quad \Omega_\Lambda = 1$	$\Omega_M = 1, \quad \Omega_\Lambda = 0,$
$m_{theor} = 24.4$	$m_{theor} = 23.2$

More than twice as bright !



45
40
35
30
μ
0.5
1.0
1.5
2.0
z
HST Discovered
Ground Discovered



Supernova Cosmology Project
Knop et al. (2003)
Spergel et al. (2003)
Allen et al. (2002)
No Big Bang
Supernovae
CMB
Clusters
expands forever
recollapses eventually
closed
flat
open
Ω_Λ
Ω_M
3
2
1
0
-1
0
1
2
3

Linear perturbations

- General problem

$$\delta(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) = 8\pi\delta(T_{\mu\nu})$$

- This introduces, at linear level, the following quantities:

$$\delta\rho, \delta p, \delta v, \delta g_{\mu\nu}$$

- The general perturbed metric can be written as

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + g_{\mu\nu}^{(1)}$$

  where in all generality

$$g_{\mu\nu}^{(1)} = a^2 \begin{pmatrix} 2\psi & w_i \\ w_i & 2\phi\delta_{ij} + h_{ij} \end{pmatrix} \tag{5}$$

- However, this is far too general for what we need. First, as any tensor field, the metric tensor can be written as a sum of terms that depend on purely scalar, vector and tensor quantities. For instance

$$w \equiv w^{\parallel} + w^{\perp} = \nabla w_s + w^{\perp} \tag{6}$$

and

$$h_{ij} \equiv h\delta_{ij}/3 + h_{ij}^{\parallel} + h_{ij}^{\perp} + h_{ij}^{T} \tag{7}$$

Only the scalar quantities couple to $\delta\rho$ so we consider only these now.

- Then, we can choose any reference frame, i.e. we can change the coordinates to $y^{\mu} = f(x^{\nu}) \rightarrow 4$ conditions on the metric coefficients. However, we would like to keep the unperturbed part $g_{\mu\nu}^{(0)}$ as it is. Then we can subject the perturbed part to 4 extra conditions: this is called ***gauge choice*****.**
- One of the simplest choice is called *longitudinal* or *Newtonian*: we put $w_i = h = 0$ and obtain finally

$$g_{\mu\nu}^{(1)} = a^2 \begin{pmatrix} 2\Psi & 0 \\ 0 & 2\Phi\delta_{ij} \end{pmatrix} \tag{8}$$

- Then we get the general perturbation equation. In particular, we also obtain $\Psi = \Phi$ if the fluid is a perfect fluid (no anisotropic stress).

- **Newtonian regime**: small scales with respect to horizon H^{-1}, small velocities, for a single component

$$\begin{aligned}\dot{\delta} &= -\nabla v \\ \dot{v}_i &= H v_i - \nabla\Phi \\ \triangle\Phi &= -4\pi a^2 \rho\delta\end{aligned}$$

- Deriving the first we get for the density perturbations $\delta = (\rho - \rho_M)/\rho_M$

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi\rho_M\delta \equiv \frac{3}{2}H^2\delta$$

$\rightarrow$ evolution of a density contrast under a gravitational potential proportional to ρ_M.

- To solve the equation is sufficient to know the behavior of $H(t)$ and $\rho(t)$. If there is only matter, $H^2 = H_0^2 a^{-3}$ and the solutions are

$$\delta \sim a, \quad \delta \sim a^{-3/2}$$

- Dark energychanges **the growth of perturbations.**

Growth of perturbations

- If there is a smooth component as the cosmological constant (for which $\delta_\Lambda = 0$) then all it changes is that $H^2 = H_0^2 \Omega_m a^{-3}$ where $\Omega_m < 1$: weaker gravity forcing. If $\Omega_m \approx const.$ then

$$\begin{aligned} \delta &\sim a^p \\ p &= \frac{1}{4}(-1 \pm \sqrt{1 + 24\Omega_m}) \end{aligned}$$

- If the cosmological constant is the dominating component, the second member vanishes (there is no potential gradient) : $\delta \sim$ const.
- A better way to study perturbation growth is to parametrize the growth in this way

$$\frac{d \log \delta}{d \log a} \approx \Omega_m(a)^\gamma, \quad \gamma \approx 0.55$$

Beyond the cosmological constant

- A cosm. constant, as we have seen, has a negative pressure and does not fluctuate.
- But any fluid with pressure $p = w\rho$ such that $w < -1/3$ has in fact similar properties.
- The simplest case is a scalar field ϕ. Pressure and energy for a potential $V(\phi)$ are

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

  So that the conservation equation

$$\dot{\rho} + 3H(\rho + p) = 0$$

  is in fact the **Klein Gordon** equation

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0$$

The eq. of state is

$$w = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}$$

kin. energy dominates	$w \to 1$	stiff fluid
equipartition	$w \to 0$	dust
pot. energy dominates	$w \to -1$	cosm. const.
negative kin. energy	$w < -1$	phantom

- In general, the equation of state will vary with time.
- As a first approximation

$$\begin{aligned} w &= w_0 + w_1 z \\ w &= w_0 + w_1 (a - 1) \end{aligned}$$

- This matter component, denoted dark energy or quintessence, has properties similar to the cosm. constant but

1. it **fluctuates** at large scale
2. is composed of particles of microscopical **mass**

$$m = V_{,\phi\phi} = H\hbar = 10^{-32} eV$$

   (the energy of a particle with Compton wavelength is equal to the horizon scale 3000 Mpc !)

3. gives an expansion different from the cosm. constant, and therefore in principle **observable** with the same methods as above

- For instance, if the potential is an exponential

$$V(\phi) = \exp(-\mu\phi)$$

the scale factor grows, when the dark energy is dominating, as a power law

$$a \sim t^{p=\frac{3}{\mu^2}}$$

instead of as an exponential (as for Λ), just as a perfect fluid with **constant** equation of state

$$w = \frac{2}{3p} - 1$$

In general, of course, $w = w(z)$.

- The crucial point is that a scalar field does not cluster because its "**sound speed**"

$$c_s^2 = \frac{\delta p}{\delta \rho}$$

is equal to the speed of light. That is, its own pressure resists gravitational collapse. Perturbing the Klein-Gordon equation in Fourier space:

$$\ddot{\varphi} + 2H\dot{\varphi} + c_s^2 k^2 \varphi + a^2 U'' \varphi = 0$$

At small scales, k dominates and the solution for φ **oscillates acoustically** around zero instead of growing. Therefore, a scalar field is a good candidate for dark energy.

- This is true only for a scalar field with canonical kinetic energy i.e. if the field Lagrangian is

$$L = \frac{1}{2}\phi_{;\mu}\phi^{;\mu} - V(\phi)$$

- while if (k essence)

$$L = P(\frac{1}{2}\phi_{;\mu}\phi^{;\mu}) - V(\phi)$$

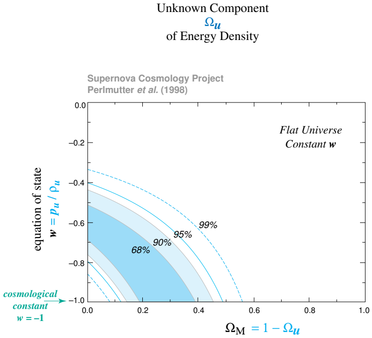
then

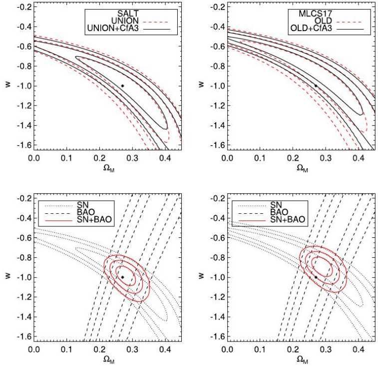
$$c_s^2 = \frac{P_{,X}}{2XP_{,XX} + P_{,X}}$$

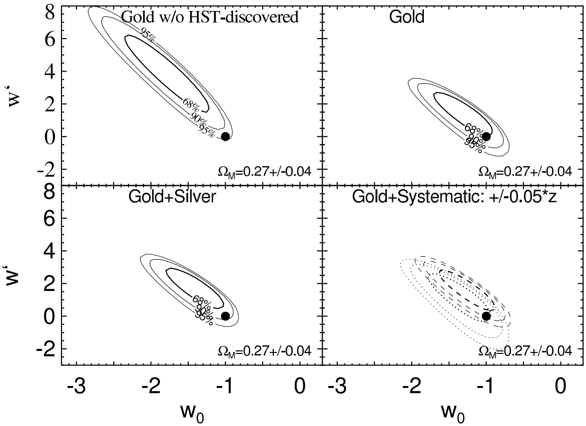
and DE can cluster.

- Repeat the SNIa fit with

$$E^2(z) = \Omega_M (1+z)^3 + \Omega_{DE} (1+z)^{3+3w_{DE}}$$






Gold w/o HST-discovered
Gold
Gold+Silver
Gold+Systematic: +/-0.05*z
Ω_M=0.27+/-0.04
68%
90%
95%
w'
w_0

Other evidences for dark energy

- A similar analysis can be done for the position of the **acoustic peaks** on the CMB.
- The Cosmic Microwave Radiation is produced when protons and electrons first combine in the cosmic plasma at $T \approx 1$eV (lower than 13 eV due to the high photon-to-baryon ratio) or $z \approx 1100$ or $t \approx 400,000$ years. Photons scattered at this epoch travel imperturbed in a rarefied, neutral medium and can reach our instruments.
- The photons keep trace of the conditions of the universe at their last scatter. Their map reproduces the local inhomogeneities at that time.
- Consider the distance a pressure wave with speed $c_s = adr/dt$ in the primordial photon/baryon fluid can travel:

$$r = \int_0^{t_{dec}} \frac{c_s(t)dt}{a(t)} \tag{9}$$

- We can take $c_s^2 \approx 1/3$ as for a relativistic fluid. Then the pressure wave can travel for roughly $400,000 c_s \approx 300,000$ light-years≈ 0.1 Mpc. More exactly

$$r_s = H_0^{-1} c_s \int_z^{\infty} \frac{dz'}{E(z')}$$

These pressure waves oscillate under the influence of the gravity of the fluid. The CMB photons that reach us show these waves as oscillations of roughly 1 degree; more exactly

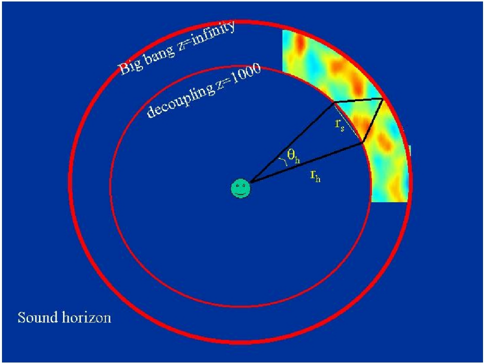
$$\theta_s = \frac{wave}{horizon} = r_s/r_d$$

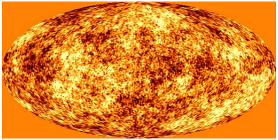
where

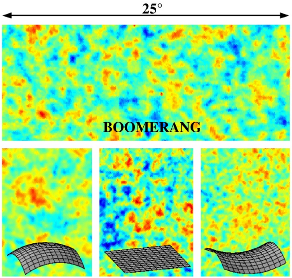
$$r_d = H_0^{-1} S\left[c\int_0^{z_d}\frac{dz'}{E(z')}\right] = H_0^{-1} S\left[c(\tau_0 - \tau_d)\right]$$

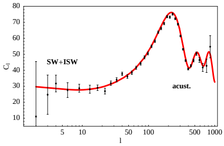
where

$$S(R) = \begin{array}{cc} |\Omega_k|^{-1/2}\sin(|\Omega_k|^{1/2}R) & (k=1) \\ R & (k=0) \\ |\Omega_k|^{-1/2}\sinh(|\Omega_k|^{1/2}R) & (k=-1) \end{array} \tag{10}$$


Big bang z=infinity
decoupling z=1000
r_s
θ_h
r_h
Sound horizon

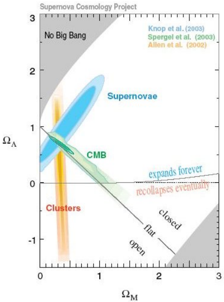


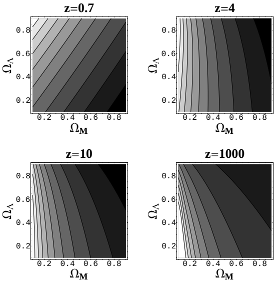

25°
BOOMERANG


SW+ISW
acust.
C_l
l
80
70
60
50
40
30
20
10
5
10
50
100
500
1000

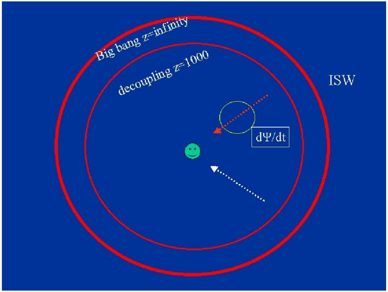
- This is indeed a *standard rod*. A measure of θ_s is, once again, a measure of $E(z)$, this time for $z \approx 1000$.
- The recent data (WMAP experiment) indicate

$$\Omega_{tot} = 1.02 \pm 0.02$$


Supernova Cosmology Project
No Big Bang
Knop et al. (2003)
Spergel et al. (2003)
Allen et al. (2002)
Supernovae
CMB
expands forever
recollapses eventually
Clusters
closed
flat
open
Ω_Λ
Ω_M


z=0.7
z=4
z=10
z=1000
Ω_Λ
Ω_M

Integrated Sachs-Wolfe effect



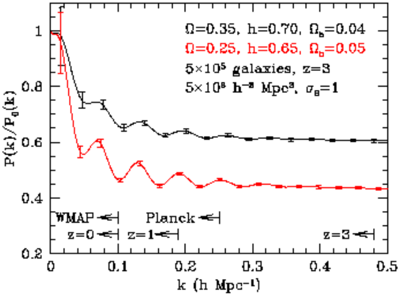
- From the Poisson equation

$$\triangle\Phi = -4\pi a^2 \rho\delta \sim a^{p-3+2}$$

we see that the grav. pot. is constant only when $p = 1$. CMB photons passing through a varying grav. pot. are *red/blue-shifted* and can be seen in CMB maps.

And between z=1 and 1000 ??

- SN give us a glance of the universe up to $z \approx 1$. CMB at $z \approx 1000$. And in between ?
- The same *baryon acoustic oscillations* remain also imprinted on the matter fluctuations after the decoupling.

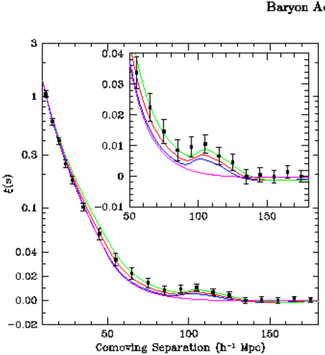


- But now, we can in principle measure the oscillation wavelength (our standard rod) at several z's, looking for the fluctuation in the matter distribution at various distances.

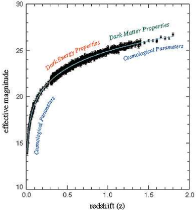
- Estimate the spectrum $P(k;z)$ at several z's, i.e. evaluate for each z's

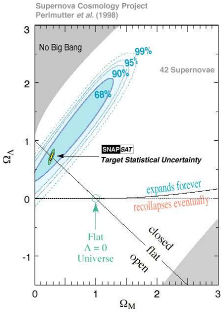
$$\theta_s = \frac{wave}{distance} = r_s/r_d$$

SDSS baryon acoustic oscillations (Eisenstein et al. 2004)



The future of SN



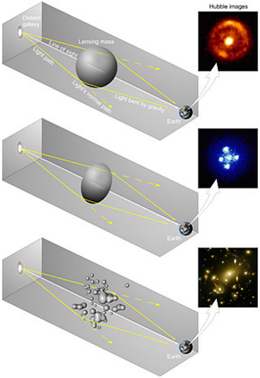


Summary of observational tests

Test	Range	Pro's	Con's
CMB	$\approx 10^3$	known phys.	mostly background integral effect
SN	$0-2$	"easy"	uncertain physics only background limited range
Weak Lens.	$0-3$	known phys. pert.	huge surveys 10^9 galx non-linearities
Baryon oscill.	$0-3$	known phys.	huge surveys bias
ISW corr.	$0-5$	known phys.	cosmic variance

Weak Lensing

- Quasars can be gravitationally lensed into multiple images
- Galaxies can be lensed into distorted shapes (arcs) by clusters
- Galaxies can acquire a non-random correlation of ellipticities due to foreground mass



Weak Lensing

- In the weak-field regime, the correlation of ellipticities is proportional to the gravitational potential correlation, and this is proportional to the matter correlation

$$< \Phi_1 \Phi_2 > \rightarrow < \delta_1 \delta_2 > \rightarrow < \varepsilon_1 \varepsilon_2 >$$

- In terms of power spectra then we have

$$P_{WL} = \int dz K(z) P_m(k)$$

- So finally, measuring the galaxy ellipticity amounts to measuring the dark matter potential