How to give the Graviton a Mass?

12^{th} IMPRS seminar

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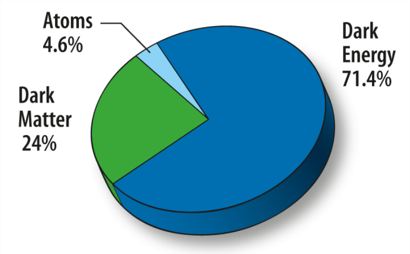
The usual picture...

Our Universe today:



Dark Matter WIMPS, sterile νs, axions, PBH, etc.

$\rightarrow$ Non-particle physics solution?

Dark Energy Cosmological Constant, Quintessence, etc.

$\rightarrow$ Connection to the other dark stuff in the Universe?

Motivation

Dark Matter modified gravity, UV-compete version of MOND

massive Graviton as DM

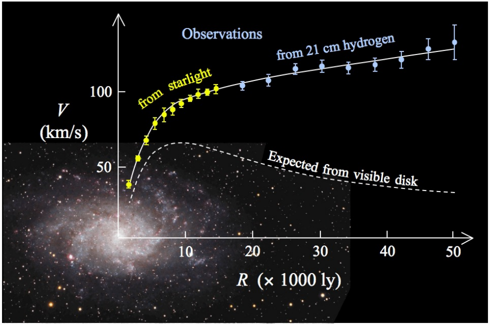
Dark Energy $\Lambda \sim m_g^2$?

Theory perturbation of GR in "theory space"

test stability of GR predictions

WARNING: I'm not an expert!!!

Dark Matter – galaxy rotation curves



Newton:

$$a(r) = v(r)^2/r = \frac{G_N M(r)}{r^2}$$

Model galaxy:

$$M(r) = M_0 \frac{r^3}{(r+r_0)^3}$$

$$v(r) = \sqrt{\frac{G_N M_0 r^2}{(r+r_0)^3}} \sim \begin{cases} r & r \ll r_0 \ \checkmark \\ \frac{1}{\sqrt{r}} & r \gg r_0 \ \text{✗} \end{cases}$$

$$\Rightarrow M(r) \to M(r) + M_{\mathrm{DM}}(r) \text{ or } G_N \to G_N^0 \left[1 + a(r/r_0) + b(r/r_0)^2 \cdots\right]$$

Dark Energy – Degravitation

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

$\Rightarrow$ the CC can be viewed as a vacuum energy $\rho_\Lambda = \frac{\Lambda}{8\pi G_N} \stackrel{?}{=} \left\langle T^0_{\mu\nu} \right\rangle \sim M_{\text{pl}}^4$

BUT: $\rho_\Lambda \sim (10^{-3}\ \text{eV})^4$!

“de-gravitate” the Cosmological Constant:

$$\Lambda g_{\mu\nu} = 8\pi G_N \left\langle T^0_{\mu\nu} \right\rangle \rightarrow 8\pi \left\langle G_N(m^{-2}\Box)\, T^0_{\mu\nu} \right\rangle$$

Degravitation[Dvali et al., 2007]

usually Why is the vacuum energy so much smaller than expected?

Degravitation Why does the vacuum energy density gravitate so little?

$\Rightarrow$ "high-pass filter" that decouples sources with wavelengths $\gtrsim m_g^{-1}$

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Spin-1 analogy – uniform charge distribution $J_\mu = \delta^0_\mu \rho$

$$\partial^\mu F_{\mu\nu} = J_\nu \Leftrightarrow \nabla \cdot \vec{E} = \rho \quad \Rightarrow \quad \vec{E} = \vec{x}\,\rho/3$$

insert "high-pass filter":

$$(1 + m_\gamma^2 \Box^{-1})\partial^\mu \widetilde{F}_{\mu\nu} \overset{\rightarrow}{=} (\Box + m_\gamma^2)\widetilde{A}_\nu = J_\nu \quad (\partial_\alpha \widetilde{A}^\alpha = 0)$$

$$\tilde{\vec{E}} = \frac{\rho}{3}\vec{x}\cos(m_\gamma t)$$

Degravitation – continued

To mimic gravity, one needs to include non-linearities that damp these oscillations: $\sim \lambda(A_\mu A^\mu)^2$.

For $t \to \infty$, the potential will settle in a static solution $A_\mu^\infty = (A_0^\infty, \vec{0})$ with $m^2 A_0^\infty + \lambda(A_0^\infty)^3 = \rho$. Thus, $\vec{E}_\infty = \vec{0}$!

Screening of the homogeneous "vacuum charge density"

Some remarks

- Is this straight-forward generalised to Gravity?
- If yes, how does this mechanism operate?
- Does $m_g \neq 0$ automatically imply Degravitation?
- Ongoing debate!

Could both approaches be related?

$$G_N \to G_N(r)$$

Outline

Consistent Massive Gravity

Phenomenology

Conclusions

Consistent Massive Gravity

What's wrong with $m_g \neq 0$?

Spin-1:

$$\mathcal{L} = a\,\partial_\mu A^\nu \partial_\nu A^\mu + b\,(\partial_\mu A^\mu)^2$$

Gauge-invariance removes 1 dof and requires $b = -a = \frac{1}{2}$.

$$A_\mu = A_\mu^\perp + \partial_\mu \chi \;\Rightarrow\; \mathcal{L} \supset (a+b)(\Box\chi)^2\,,\; \partial^\mu A_\mu^\perp = 0.$$

χ constitutes 2 ghost dof: kinetic energy < 0: **INCONSISTENT**

Conclusion:

gauge-invariance $\Leftrightarrow$ consistency

$$b = -a = \frac{1}{2}$$

Now add a mass term

The mass term in

$$\mathcal{L}_{\text{Proca}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu$$

breaks gauge-invariance, but the kinetic term is unique by consistency!

Abelian Higgs mechanism:

$$\mathcal{L}_{\text{Higgs}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(D_\mu\phi)^\dagger(D^\mu\phi) - V(|\phi|)$$

with $\phi = (v+h)e^{i\chi}$ this gives

$$\mathcal{L}_{\text{Higgs}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}g^2v^2(A_\mu - \partial_\mu\chi)^2 - \frac{1}{2}(\partial_\mu h)^2 - V'(h)$$

<u>gauge-invariant</u> if $A_\mu \to A_\mu + \partial_\mu\alpha$ and $\chi \to \chi + \alpha$.

Spin-2 – What's the difference?

consistency implies gauge invariance $h_{\mu\nu} = h^{\perp}_{\mu\nu} + \partial_\mu \chi_\nu + \partial_\nu \chi_\mu$

There are now 4 possible combinations

no Higgs mechanism known $\rightarrow$ retain only the Stückelberg fields χ_μ

discontinuity if $\mathbf{m_g \rightarrow 0}$: No analogy for spin-1

$$\mathcal{L}_{\text{source}} = A_\mu J^\mu = (A^{\perp}_\mu + \partial_\mu \chi) J^\mu = A^{\perp}_\mu J^\mu \text{ if } \partial_\mu J^\mu = 0$$

i.e. χ doesn't couple to conserved, external sources!

Most importantly:

In Gravity the consistency (absence of ghosts) is spoiled by non-linearity!

Fierz-Pauli mass term

$$\mathcal{L}_{\mathrm{FP}} = -\frac{1}{8} m_g^2 \left[h_{\mu\nu} h^{\mu\nu} - \left(h_\mu{}^\mu \right)^2 \right]$$ [Fierz and Pauli, 1939]

can be made gauge-invariant under $h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$ by Stückelberg fields χ^α:

$$\mathcal{L}'_{\mathrm{FP}} = -\frac{1}{8} m_g^2 \left[\left(h_{\mu\nu} + 2\partial_{(\mu}\chi_{\nu)} \right)^2 - \left(h_\mu{}^\mu + 2\partial_\alpha \chi^\alpha \right)^2 \right],$$

if simultaneously $\chi^\alpha \to \chi^\alpha + \frac{1}{2}\xi^\alpha$.

Great! But Gravity is non-linear. This is why it took 70 years to come up with this...

Consistent massive Gravity

The key observation is that we can introduce the Stückelberg fields by redefining the background metric

$$\eta_{\mu\nu} \to \eta_{\mu\nu} + 2\partial_{(\mu}\chi_{\nu)}$$

in the linear case. Extending to the non-linear case is achieved by

$$f_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b f'_{ab}$$

for a *general* background f and 4 Stückelberg fields ϕ^a.

The FP mass term is then generalised by $h_{\mu\nu} \to (\mathbb{1} - g^{-1}f)_{\mu\nu}$.

NOT UNIQUE and NOT GHOST-FREE

Ghost-free Massive Gravity [de Rham et al., 2011, Hassan and Rosen, 2012]

$$\mathcal{S}_{\mathrm{bi}} = \frac{M_g^2}{2}\int \mathrm{d}^4x\sqrt{-|g|}R_g + \int \mathrm{d}^4x\sqrt{-|g|}\mathcal{L}_{\mathrm{matter}} +$$

$$+\, m^2 M_{\mathrm{eff}}^2 \int \mathrm{d}^4x\sqrt{-|g|}\sum_{n=0}^{4}\beta_n e_n(\mathbb{X})$$

where $\mathbb{X} = \sqrt{g^{-1}f}$, i.e. $\mathbb{X}^{\mu}_{\alpha}\mathbb{X}^{\alpha}_{\nu} = g^{\mu\alpha}f_{\alpha\nu}$, $M_{\mathrm{eff}}^2 = \left(\frac{1}{M_g^2}+\frac{1}{M_f^2}\right)^{-1}$, and

$$e_0 = 1,\ e_1 = \mathrm{tr}\left(\mathbb{X}\right),\ e_2 = \frac{1}{2}\left[\mathrm{tr}\left(\mathbb{X}\right)^2 - \mathrm{tr}\left(\mathbb{X}^2\right)\right],$$

$$e_3 = \frac{1}{6}\left[\mathrm{tr}\left(\mathbb{X}\right)^3 - 3\,\mathrm{tr}\left(\mathbb{X}\right)\mathrm{tr}\left(\mathbb{X}^2\right) + 2\,\mathrm{tr}\left(\mathbb{X}^3\right)\right],\ e_4 = \det(\mathbb{X})$$

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$$+\, m^2 M_{\mathrm{eff}}^2 \int \mathrm{d}^4 x\sqrt{-|g|}\sum_{n=0}^{4}\beta_n e_n(\mathbb{X}) + \frac{M_f^2}{2}\int \mathrm{d}^4 x\sqrt{-|f|}R_f$$

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Einstein equations

$$G(g)_{\mu\nu} + m^2 \cos^2(\theta) \sum_{n=0}^{3} \beta_n V^{(n)}(g)_{\mu\nu} = 8\pi G_N T_{\mu\nu},$$

$$G(f)_{\mu\nu} + m^2 \sin^2(\theta) \sum_{n=1}^{4} \sqrt{|g|/|f|} \beta_n V^{(n)}(f)_{\mu\nu} = 0,$$

where $\cos^2(\theta) = \frac{M_{\text{eff}}^2}{M_g^2}$, the $\sin^2(\theta) = \frac{M_{\text{eff}}^2}{M_f^2}$, and the interaction or mass terms $V(g/f)$ follow from the variation of the e_n, e.g. $V_{\mu\nu}^{(0)}(g) = g_{\mu\nu}$.

Effective CC

$$\Lambda_{\text{eff}} = m^2 \cos^2(\theta) \beta_0$$

Einstein equations – linearised

For a specific choice of $\vec{\beta} = (3, -1, 0, 0, +1)$, one recovers the FP mass term by expanding around a background η

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{\delta g_{\mu\nu}}{M_g}, \;\; f_{\mu\nu} = \eta_{\mu\nu} + \frac{\delta f_{\mu\nu}}{M_f}$$

$$\begin{aligned}\mathcal{S}_{\text{mass}} =& m^2 M_{\text{eff}}^2 \int \mathrm{d}^4 x \sqrt{-|g|} \sum_{n=0}^{4} \beta_n e_n(\mathbb{X}) \\ \simeq & - m^2 M_{\text{eff}}^2 \int \mathrm{d}^4 x \left[\left(\frac{\delta g}{M_g} - \frac{\delta f}{M_f} \right)^{\mu\nu} \left(\frac{\delta g}{M_g} - \frac{\delta f}{M_f} \right)_{\mu\nu} - \right. \\ & \left. - \left(\frac{\delta g^{\mu}{}_{\mu}}{M_g} - \frac{\delta f^{\mu}{}_{\mu}}{M_f} \right)^2 \right]\end{aligned}$$

Einstein equations – linearised 2

Therefore, the mass eigenstates are

$$\begin{pmatrix} u \\ v \end{pmatrix} \equiv \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \delta g \\ \delta f \end{pmatrix}$$

$$\Rightarrow \mathcal{S}_{\mathsf{mass}} = -\frac{m^2}{8} \int \mathrm{d}^4 x \left[u^{\mu\nu} u_{\mu\nu} - (u^{\mu}{}_{\mu})^2 \right],$$

while v remains massless.

two interesting limits

$\theta \to 0$: Only massive mode couples to matter

$\theta \to \frac{\pi}{2}$: GR limit, but without the discontinuity!

Bi-Schwarzschild BH

Starting off with an ansatz

$$g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = -e^{\nu_1(r)}\mathrm{d}t^2 + e^{\lambda_1(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2,$$

$$f_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = -e^{\nu_2(r)}\mathrm{d}t^2 + e^{\lambda_2(r)}(r + r\mu(r))'^2\mathrm{d}r^2 + (r + r\mu(r))^2\mathrm{d}\Omega^2,$$

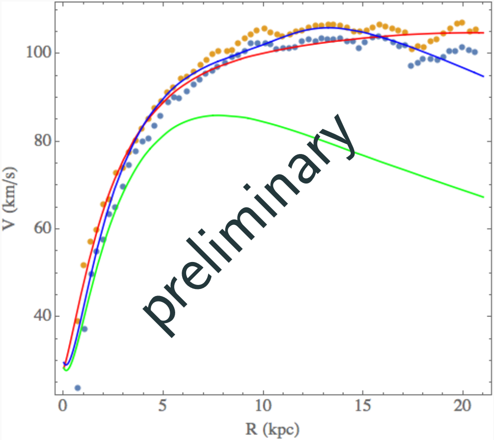
we can calculate the "Newtonian" potential in the weak field regime:

$$\nu_1(r) = \begin{cases} -\frac{r_S}{r} - r^2\,\frac{\Lambda_{\text{eff}}}{3} & r \ll \sqrt[3]{\frac{r_S}{m_g^2}} \\ -\frac{r_S}{r}\left[h(\theta) + 2\cos^2(\theta)e^{-m_g\,r}\left(1 + \frac{\Lambda'_{\text{eff}}}{3m_g^2}\right)\right] - \\ \qquad -r^2\sin^2(\theta)\,\frac{\Lambda_{\text{eff}}}{3} & r \gg \sqrt[3]{\frac{r_S}{m_g^2}} \end{cases}$$

Degravitation for $\sin^2(\theta) \to 0$!

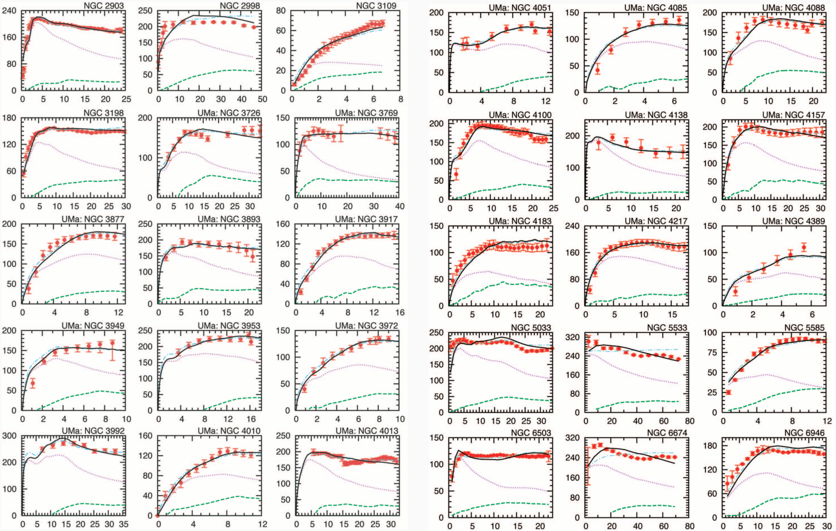
Phenomenology

Spiral galaxy rotation curves



courtesy of Juri Smirnov

Previous analyses[Brownstein and Moffat, 2006]



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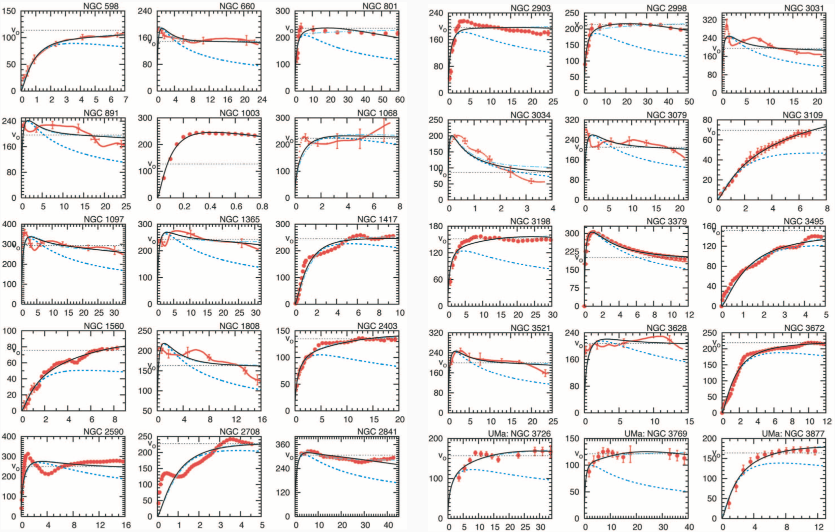
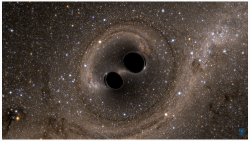


Bounds on the graviton mass

solar system tests $\lambda_g > 2.8 \cdot 10^{12}\,\mathrm{km},\ m_g < 7.2 \cdot 10^{-23}\,\mathrm{eV}$

weak lensing $\lambda_g > 2 \cdot 10^{21}\,\mathrm{km},\ m_g < 6 \cdot 10^{-32}\,\mathrm{eV}$

rely on a Yukawa potential $\propto e^{-m_g r}$

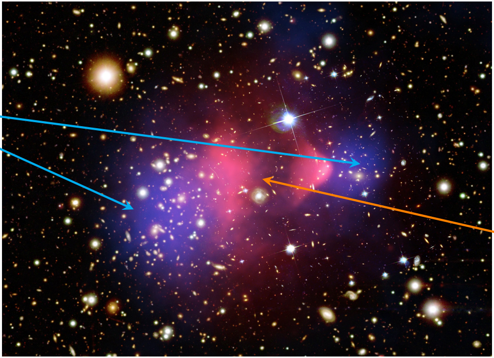


GW150914 $\lambda_g > 4.2 \cdot 10^{11}\,\mathrm{km},\ m_g < 1.2 \cdot 10^{-22}\,\mathrm{eV}$

due to a modified dispersion relation $v_g = \sqrt{1 - \frac{m_g^2}{E^2}}$

But do these bounds apply here?

A word of honesty – the bullet cluster



dark matter (lensing)

visible matter (X-rays)

Bullet cluster favours particle DM

Need extra, non-baryonic degrees of freedom!

Conclusions

Summary

- We know how to give the graviton a mass! ✓
- non-linearities make this a difficult, yet interesting problem!
- Bigravity is a consistent framework which allows to study some of the effects
  - ⇒ solve (in part) the DM problem ✓
  - ⇒ Degravitation seems to work ✓
  - ⇒ $\Lambda \sim m_g^2$ ✓
- Many open questions and lots of work to be done!
  - ⇒ CMB, GW, cosmological implications etc.

Thank you!

(and please comment!)

Back-up slides

Back-up – Einstein Eqs. continued

Recall that $\mathbb{X}^{\mu}{}_{\nu} = \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\nu}$:

$$
\begin{aligned}
V^{(0)}(g)^{\mu}{}_{\nu} =& \delta^{\mu}{}_{\nu}, \\
V^{(1)}(g)^{\mu}{}_{\nu} =& \mathrm{tr}\left(\mathbb{X}\right)\delta^{\mu}{}_{\nu} - \mathbb{X}^{\mu}{}_{\nu}, \\
V^{(2)}(g)^{\mu}{}_{\nu} =& \left(\mathbb{X}^2\right)^{\mu}{}_{\nu} - \mathrm{tr}\left(\mathbb{X}\right)\mathbb{X}^{\mu}{}_{\nu} + \frac{\delta^{\mu}{}_{\nu}}{2}\left[\mathrm{tr}\left(\mathbb{X}\right)^2 - \mathrm{tr}\left(\mathbb{X}^2\right)\right], \\
V^{(3)}(g)^{\mu}{}_{\nu} =& -\left(\mathbb{X}^3\right)^{\mu}{}_{\nu} + \mathrm{tr}\left(\mathbb{X}\right)\left(\mathbb{X}^2\right)^{\mu}{}_{\nu} - \frac{1}{2}\left[\mathrm{tr}\left(\mathbb{X}\right)^2 - \mathrm{tr}\left(\mathbb{X}^2\right)\right]\mathbb{X}^{\mu}{}_{\nu} + \\
& + \frac{\delta^{\mu}{}_{\nu}}{6}\left[\mathrm{tr}\left(\mathbb{X}\right)^3 - 3\,\mathrm{tr}\left(\mathbb{X}\right)\mathrm{tr}\left(\mathbb{X}^2\right) + 2\,\mathrm{tr}\left(\mathbb{X}^3\right)\right]
\end{aligned}
$$

Back-up – Einstein Eqs. continued

Recall that $\mathbb{X}^{\mu}{}_{\nu} = \left(\sqrt{g^{-1}f}\right)^{\mu}{}_{\nu}$:

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\begin{aligned}
V^{(1)}(f)^{\mu}{}_{\nu} =& \mathbb{X}^{\mu}{}_{\nu}, \\
V^{(2)}(f)^{\mu}{}_{\nu} =& -\left(\mathbb{X}^{2}\right)^{\mu}{}_{\nu} + \mathrm{tr}\left(\mathbb{X}\right)\mathbb{X}^{\mu}{}_{\nu}, \\
V^{(3)}(f)^{\mu}{}_{\nu} =& \left(\mathbb{X}^{3}\right)^{\mu}{}_{\nu} + \mathrm{tr}\left(\mathbb{X}\right)\left(\mathbb{X}^{2}\right)^{\mu}{}_{\nu} + \frac{1}{2}\left[\mathrm{tr}\left(\mathbb{X}\right)^{2} + \mathrm{tr}\left(\mathbb{X}^{2}\right)\right]\mathbb{X}^{\mu}{}_{\nu}, \\
V^{(4)}(f)^{\mu}{}_{\nu} =& \delta^{\mu}{}_{\nu}
\end{aligned}
$$

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