Nov. 10, 2008

Pulse Shape Simulation for Germanium Detectors

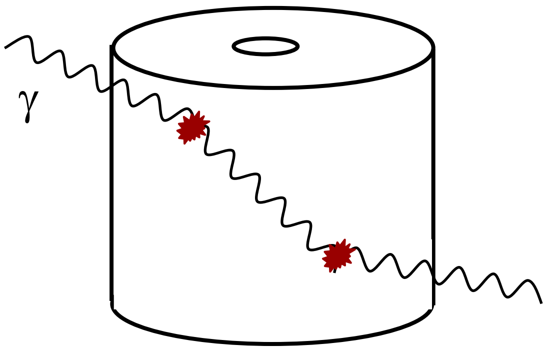
An Overview

MaGe Pulse Shape Simulation Force

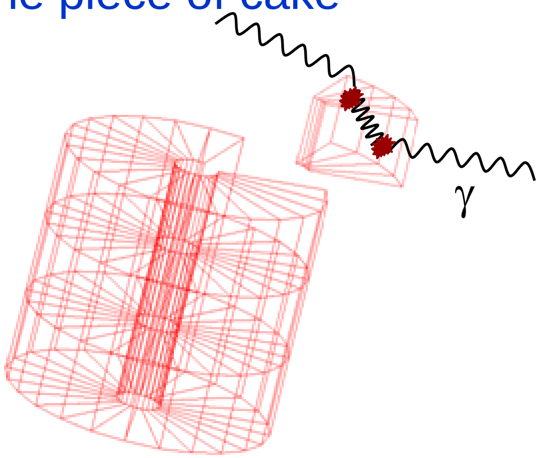
Jing Liu @ GERDA Collab. Meeting, Gran Sasso

Why we need it

Multi-site event in
unsegmented detector

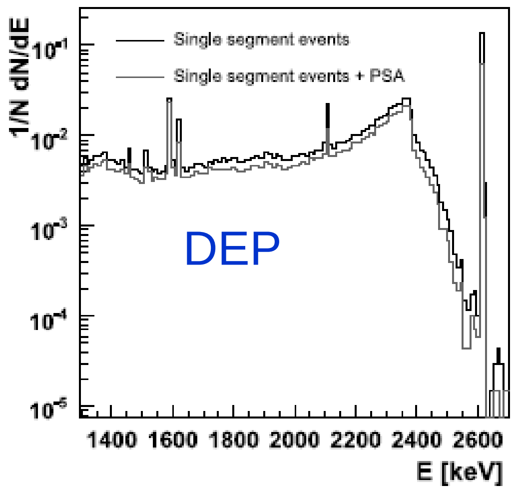


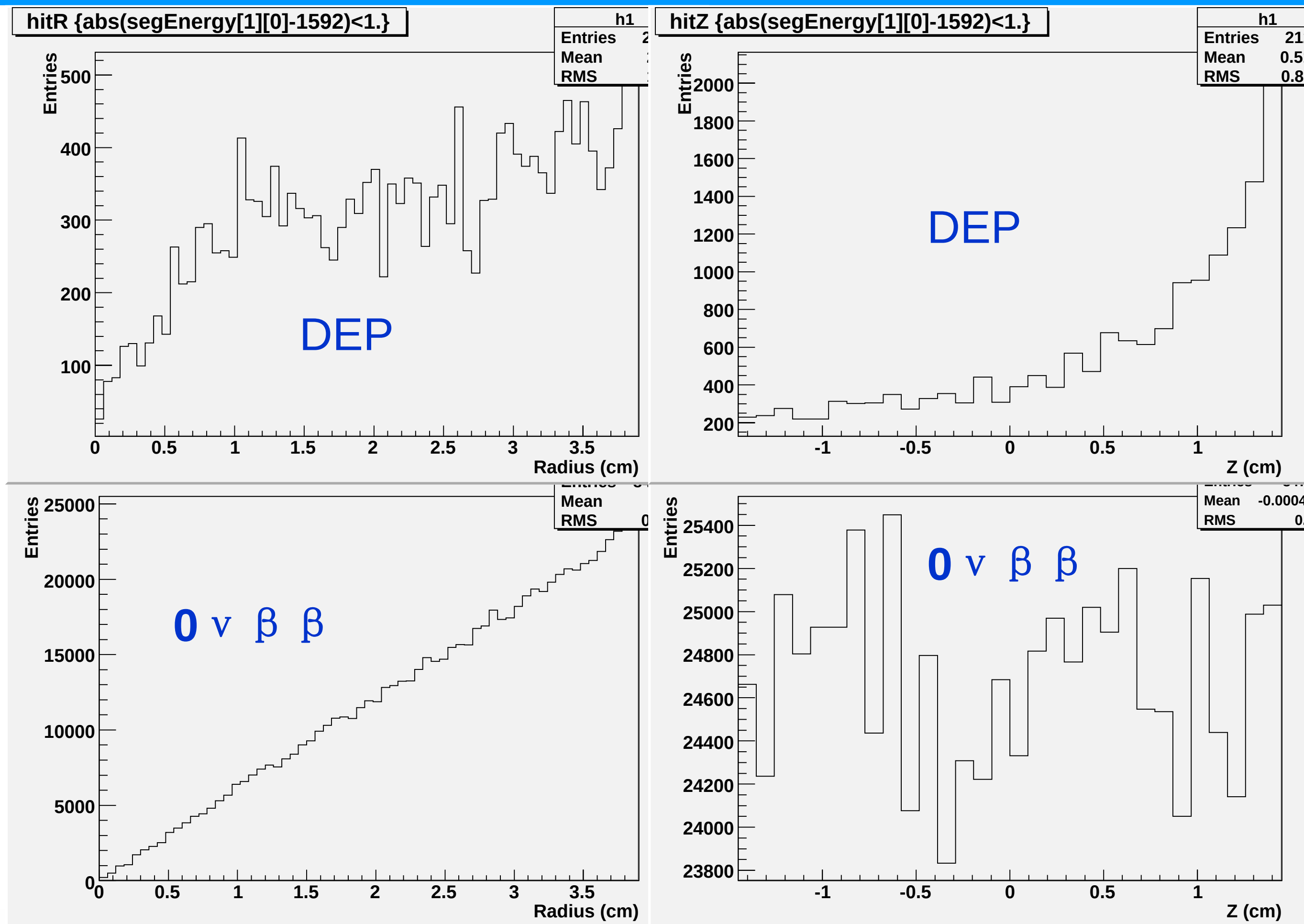
Multi-site event in
one piece of cake



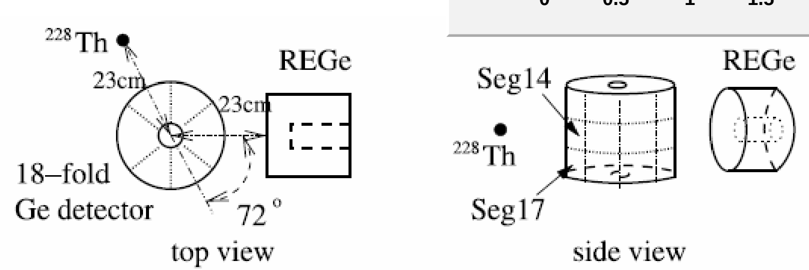
We need pulse shape analysis
AND
Good signal & background samples
to train PSA method
And
verify discrimination efficiency

Why we need it



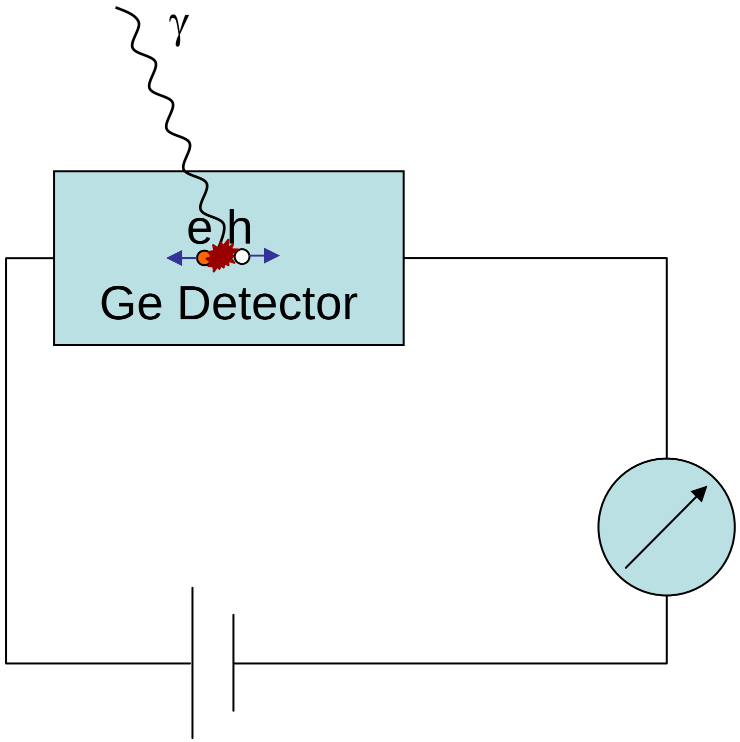


Single Compton
Scattering events

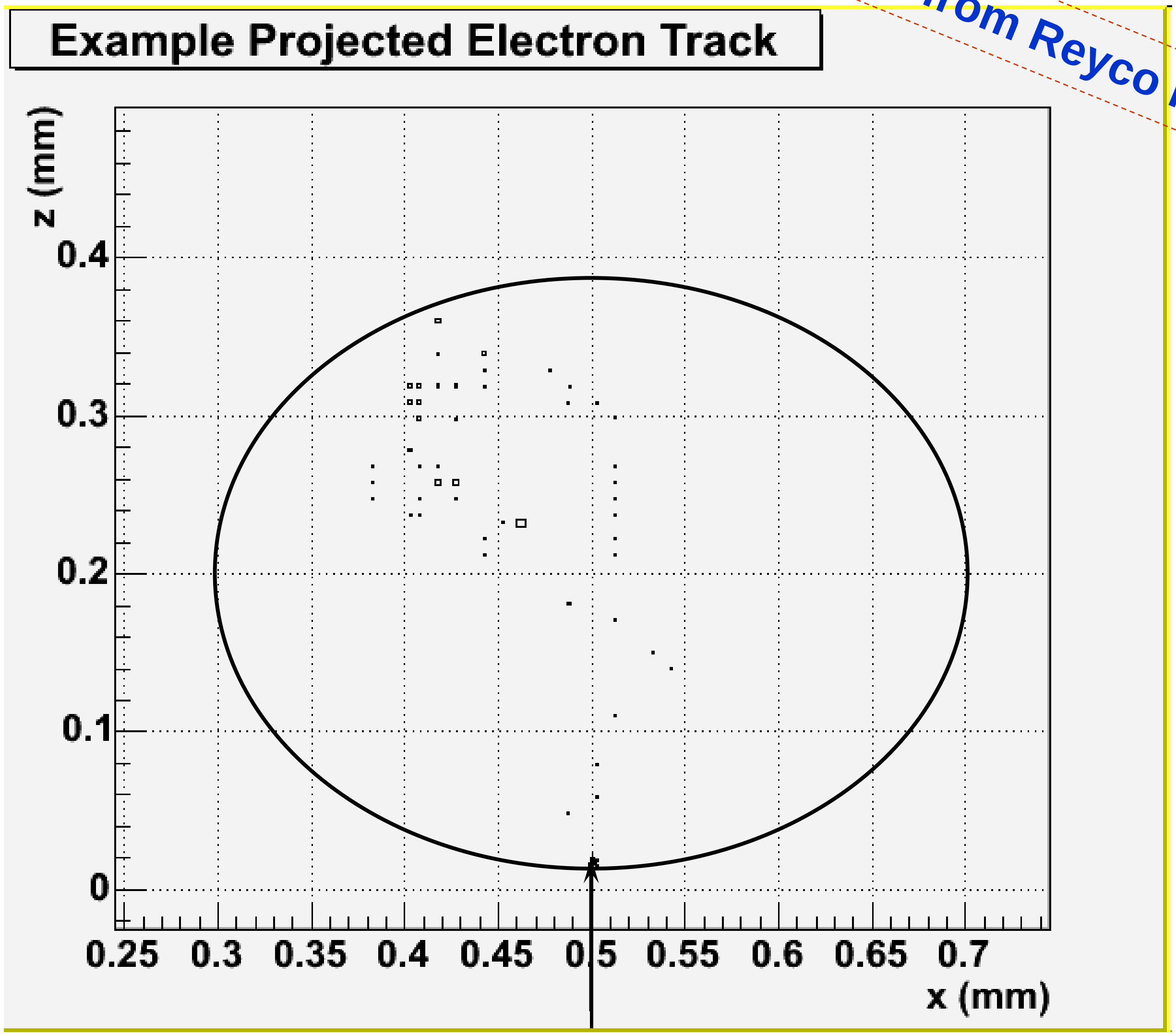


How to do it

1. Simulate E_{dep} using MaGe
2. Group hits according to limits on bandwidth and sampling rate, determine the amount of e-h pairs and their original position
3. Calculate electric field
4. Calculate the drift of charge carriers
5. Calculate induced signals in the electrodes based on the drift trajectory and the weighing potential
6. Fold in effects of electronics, eg. noise, bandwidth, decay time, etc.



Group hits

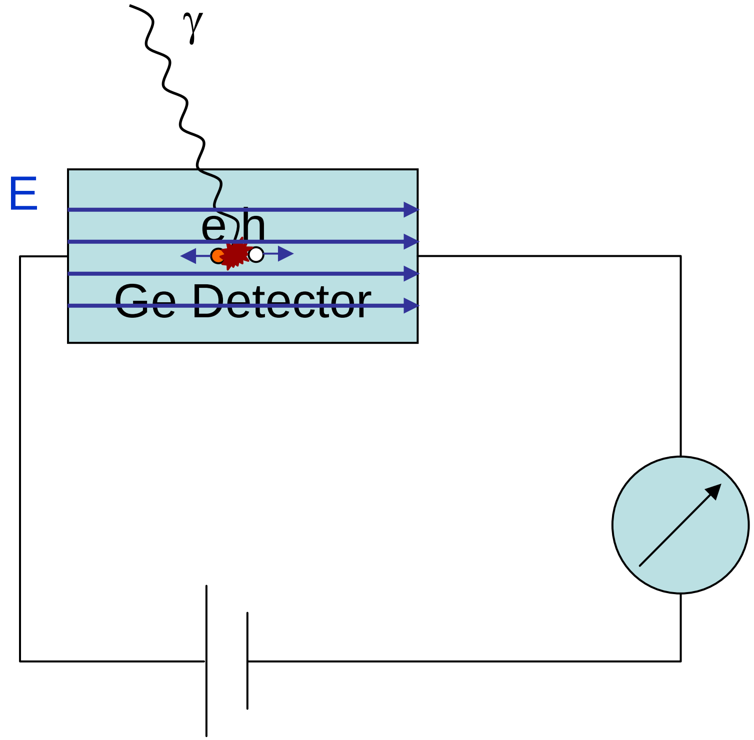


1.02 MeV electron

Electric field – please refer to Daniel's talk for greater detail

Poisson's Equation:

$$\nabla \cdot E = \frac{\rho}{\varepsilon}$$



Calculate the drift :: Mobility

One can define the mobility $\mu_{e,h}$ of the electrons e and holes h as the variable which gives the relation between the electric field $\mathbf{E}(\mathbf{r})$ and the drift velocity

$$\mathbf{v}(\mathbf{r}) = \mu_{e,h}\mathbf{E}(\mathbf{r}), \tag{9}$$

where $\mathbf{r}$ indicates the position. $\mu_{e,h}$ depend on the temperature, the electric field and the structure of the germanium crystal. As long as the electron and hole temperatures do not differ much from the lattice temperature, the drift velocity is proportional to the electrical field and the lattice orientation has no influence. In this case the mobility can be simplified to just a number $\mu_{e,h} = \mu_0$. In germanium detectors cooled at liquid nitrogen temperature the electron/hole pairs are hotter than the lattice. The drift velocity in this condition is influenced by the crystal lattice orientation and not always parallel to the applied electrical field.

Calculate the drift :: Crystal structure

Germanium has the same crystalline structure as silicon and diamond, namely, a face-centered cubic (FCC) structure, in which each atom lies at the center of a regular tetrahedron, and is surrounded at its apices by four atoms as shown in Fig. 1

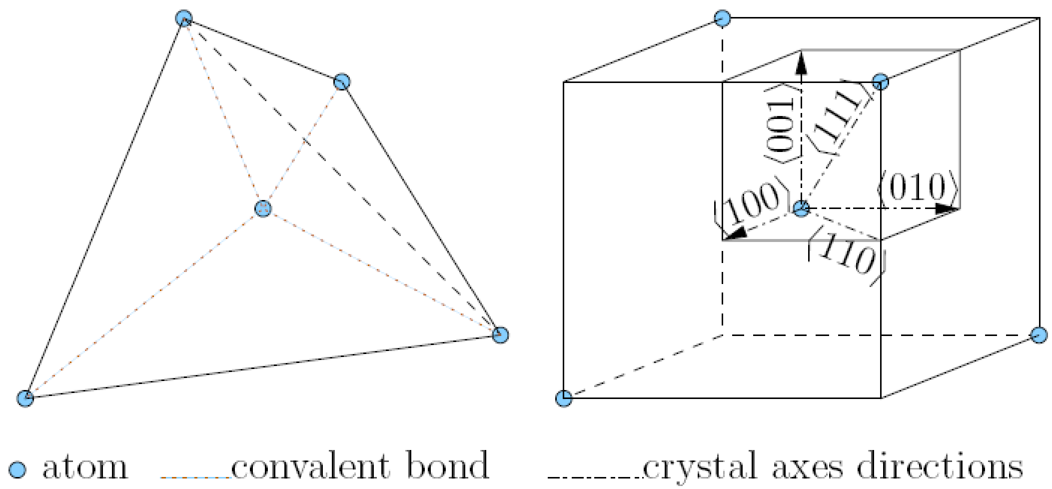


Figure 1: Structure of germanium crystal.

Due to the crystal lattice symmetry in germanium, in three directions, the crystallographic ⟨100⟩, ⟨110⟩ and ⟨111⟩, the mobility is always aligned with the electrical field.

Calculate the drift :: Mobility parameterization

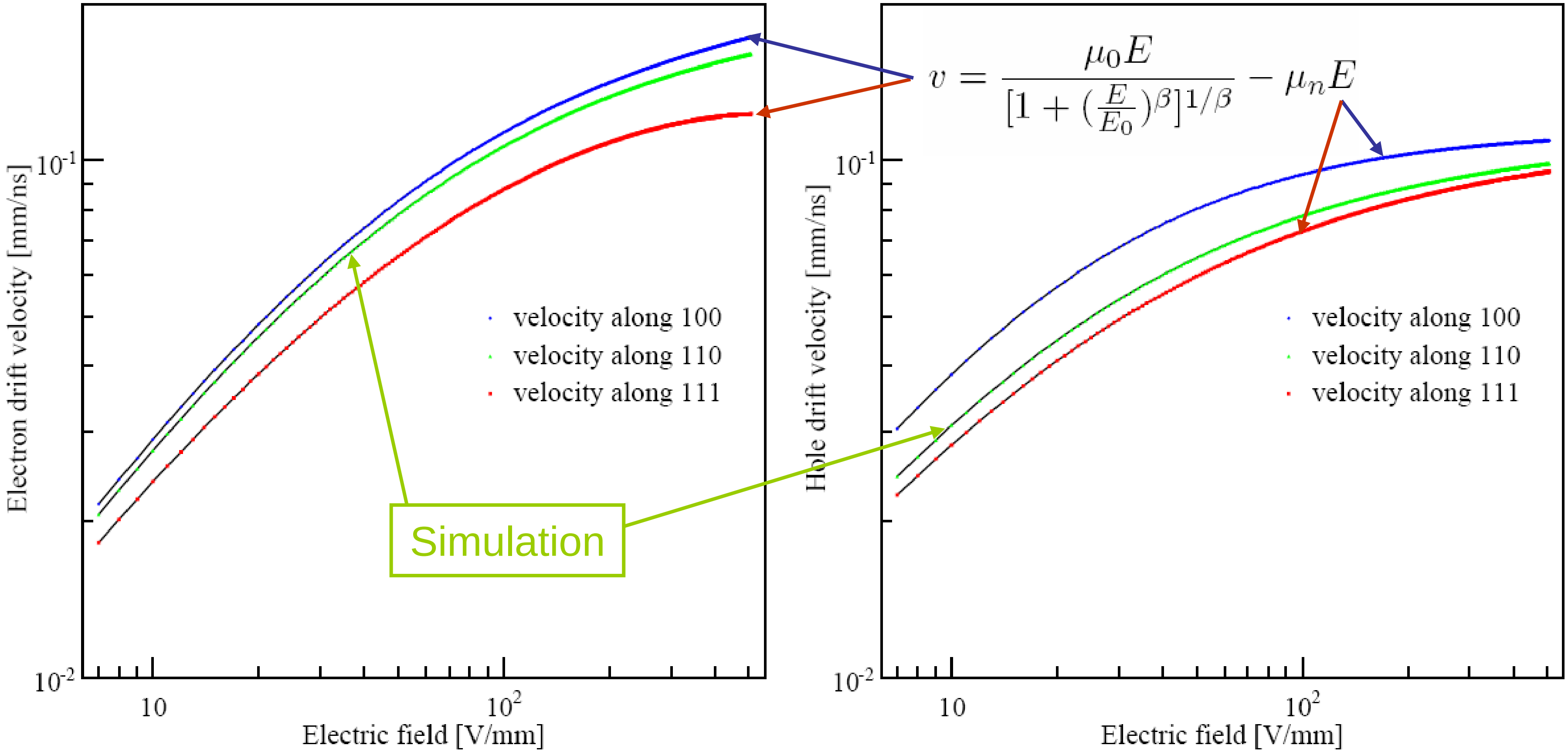
Experimental data on the longitudinal anisotropy in these specific directions can be found in literature. The mobility data can be well fitted in any principal crystallographic direction with the parameterization:

$$v = \frac{\mu_0 E}{[1 + (\frac{E}{E_0})^\beta]^{1/\beta}} - \mu_n E, \tag{10}$$

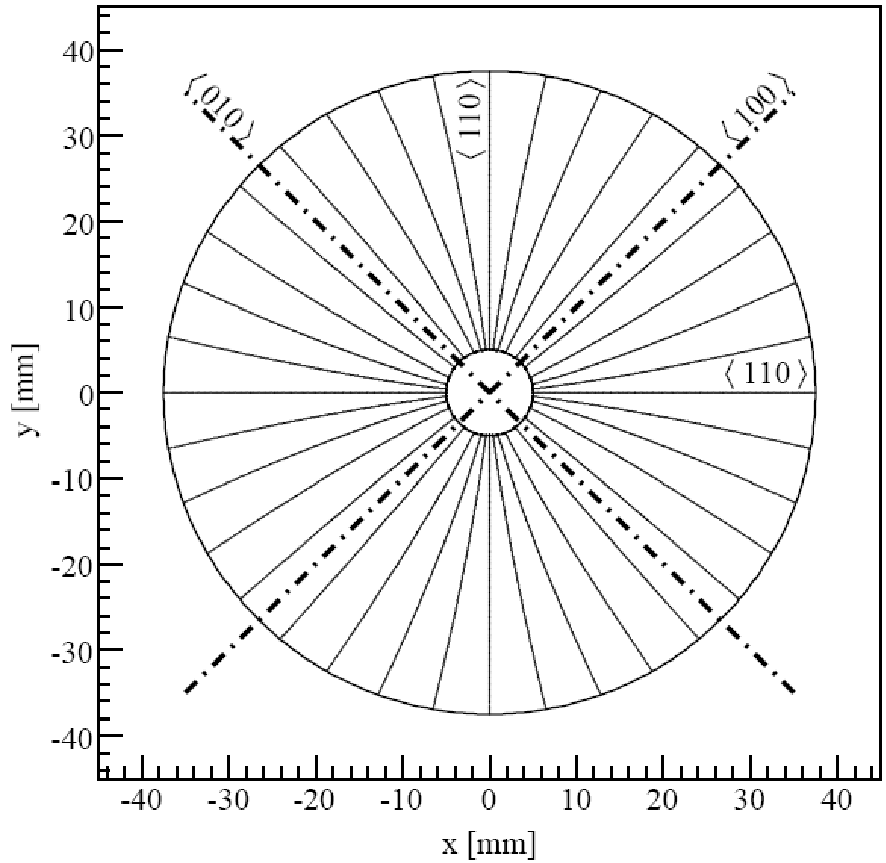
where E, v are the magnitudes of the electric field and drift velocity, respectively, μ_0, μ_n, E_0 and β are the fitting parameters. At low fields, the mobility becomes isotropic and therefore the mobility fit parameter μ_0 is expected to become independent of the crystallographic direction. For hot electrons, the departure from a linear $v \sim E$ relation is modeled through the parameters E_0 and β. At high field, Mihailescu *et al.* [8] have added the term $\mu_n E$ to account for the *Gunn effect* that was observed by Ottaviani *et al.* [9] for field strengths above 3 kV/cm at 80 K. However, this effect is insignificant in our detector operated with field strengths 10-300 V/mm.

Calculate the drift :: Determine anisotropy in any direction

The anisotropy in any direction is related to the longitudinal anisotropy in the $\langle 100 \rangle$ and $\langle 111 \rangle$ directions. The drift velocity in any direction can be calculated accordingly.



Calculate the drift :: Drift trajectories



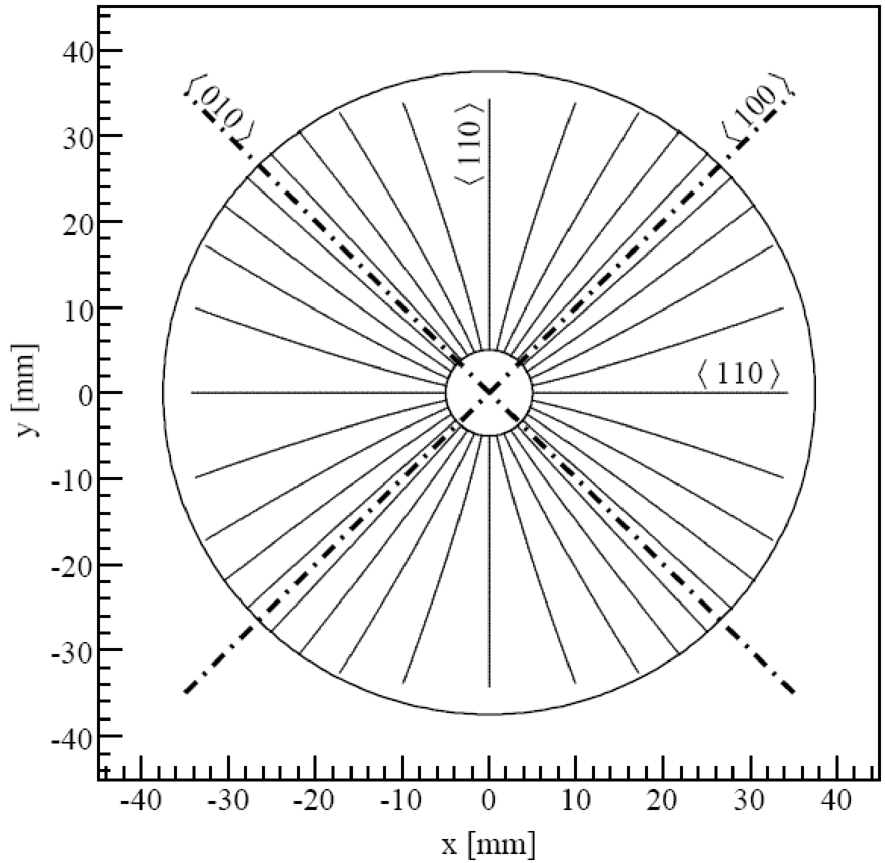
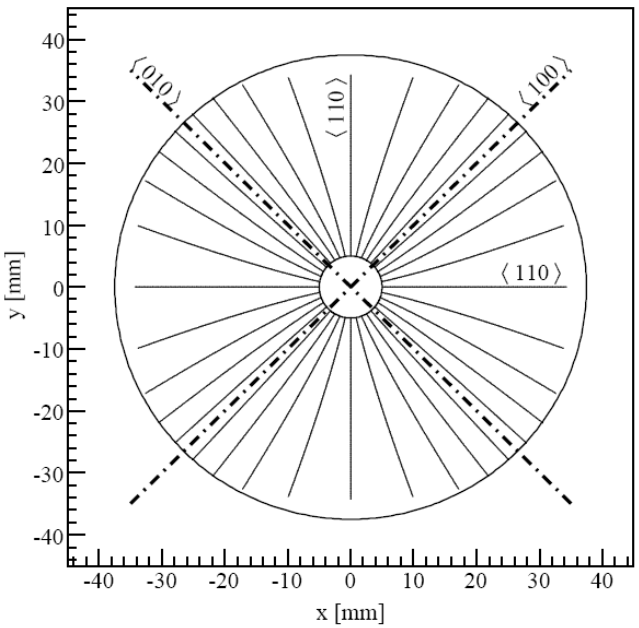
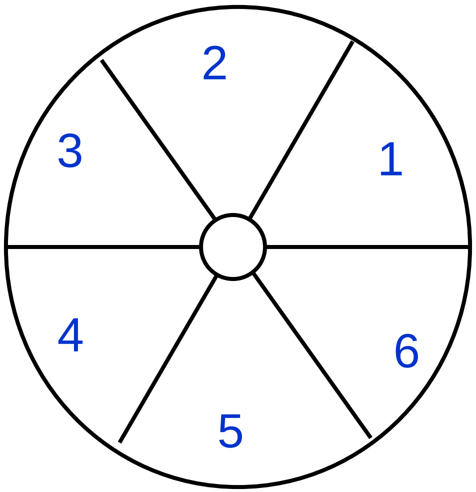
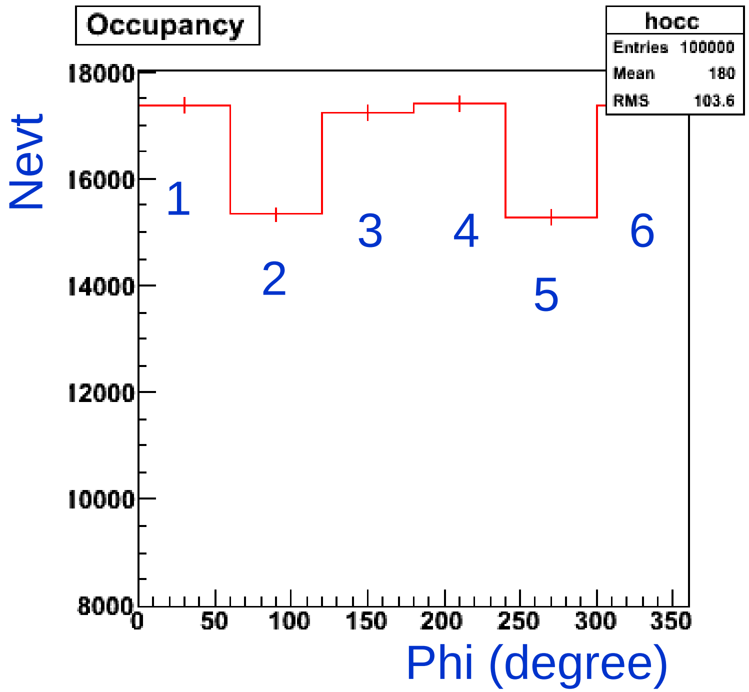


Figure 9: Charge carrier drift trajectories on X-Y plane. The transverse anisotropy causes the bend of the trajectories. Also shown are the cross section of a true coaxial cylindrical germanium detector with inner radius of 5 mm and outer radius of 37.5 mm. The crystal axes are indicated with the signs ⟨100⟩, ⟨110⟩ and ⟨010⟩.

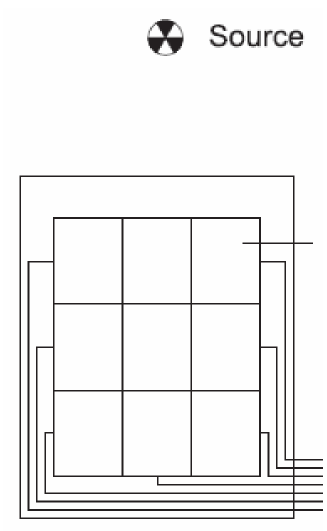
Numbers of signals recorded by different segments



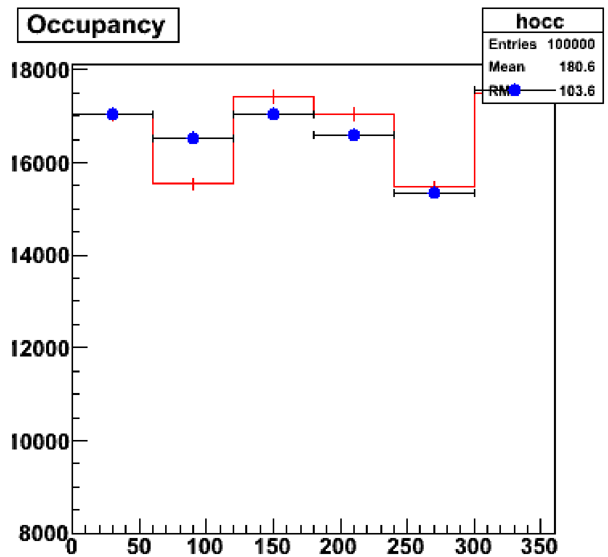




Determine the crystal axis orientation using the occupancy plot

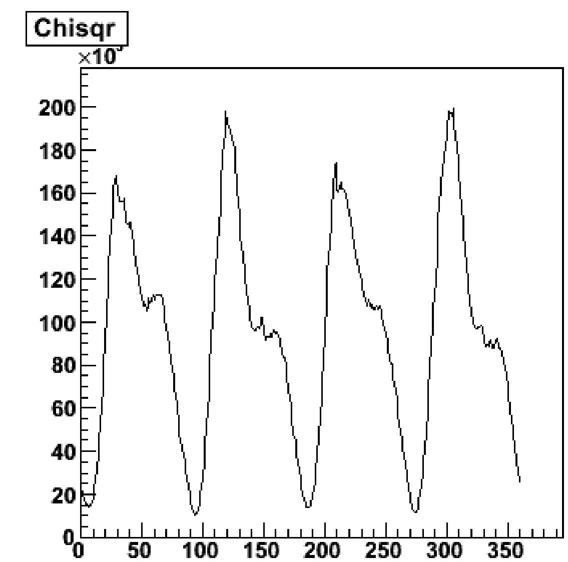


Nevt



Phi (degree)

Difference between MC & data



Angle between axis and boundary

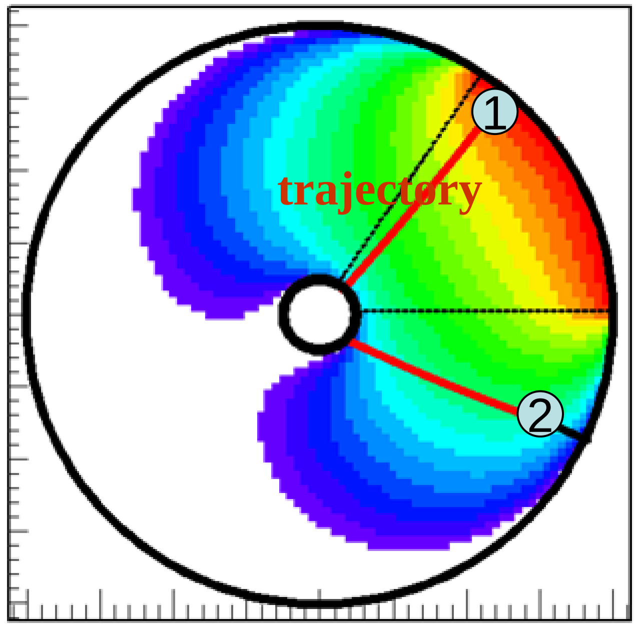
Signals induced in the electrodes – please refer to Daniel's talk for detail

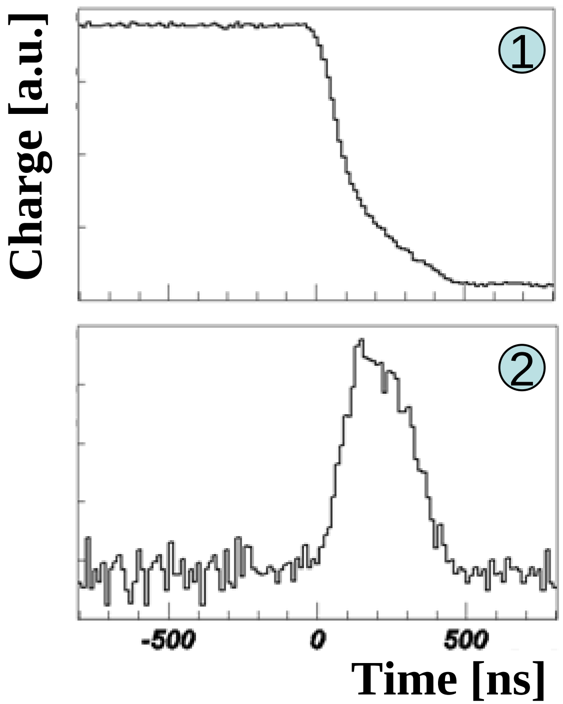
Ramo's Theorem:

$$Q(t) = -N_{e/h}(E_{dep})\varphi_w(r(t))$$

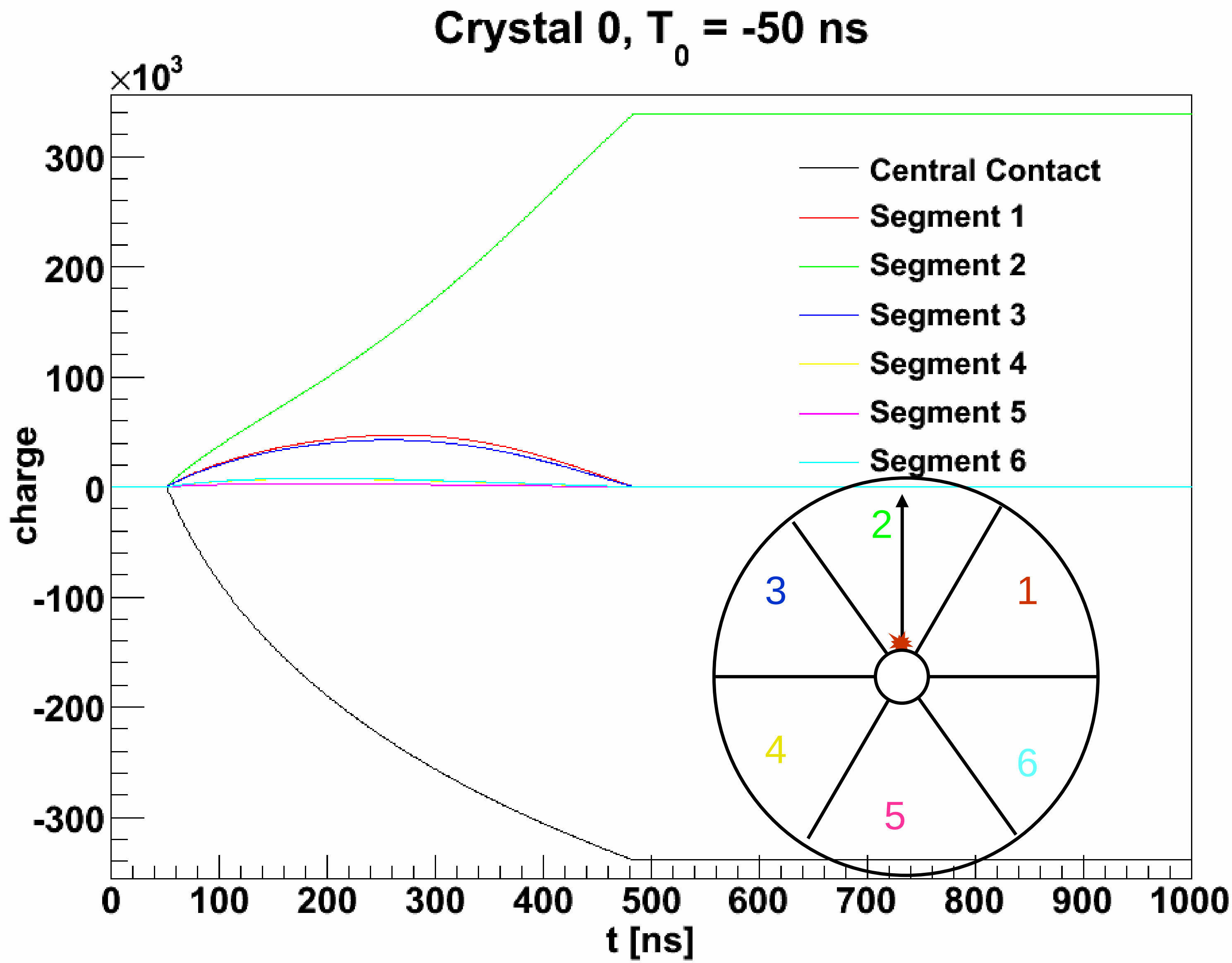
$$I(t) = N_{e/h}(E_{dep})\vec{E}_w(r(t))v(t)$$

- $N_{e/h}(E_{dep})$: Number of e/h created by energy deposition E_{dep}
- $\varphi_w, \vec{E}_w$: Weighting potential, field
- $r(t)$: Drift trajectory
- $v(t)$: Drift velocity

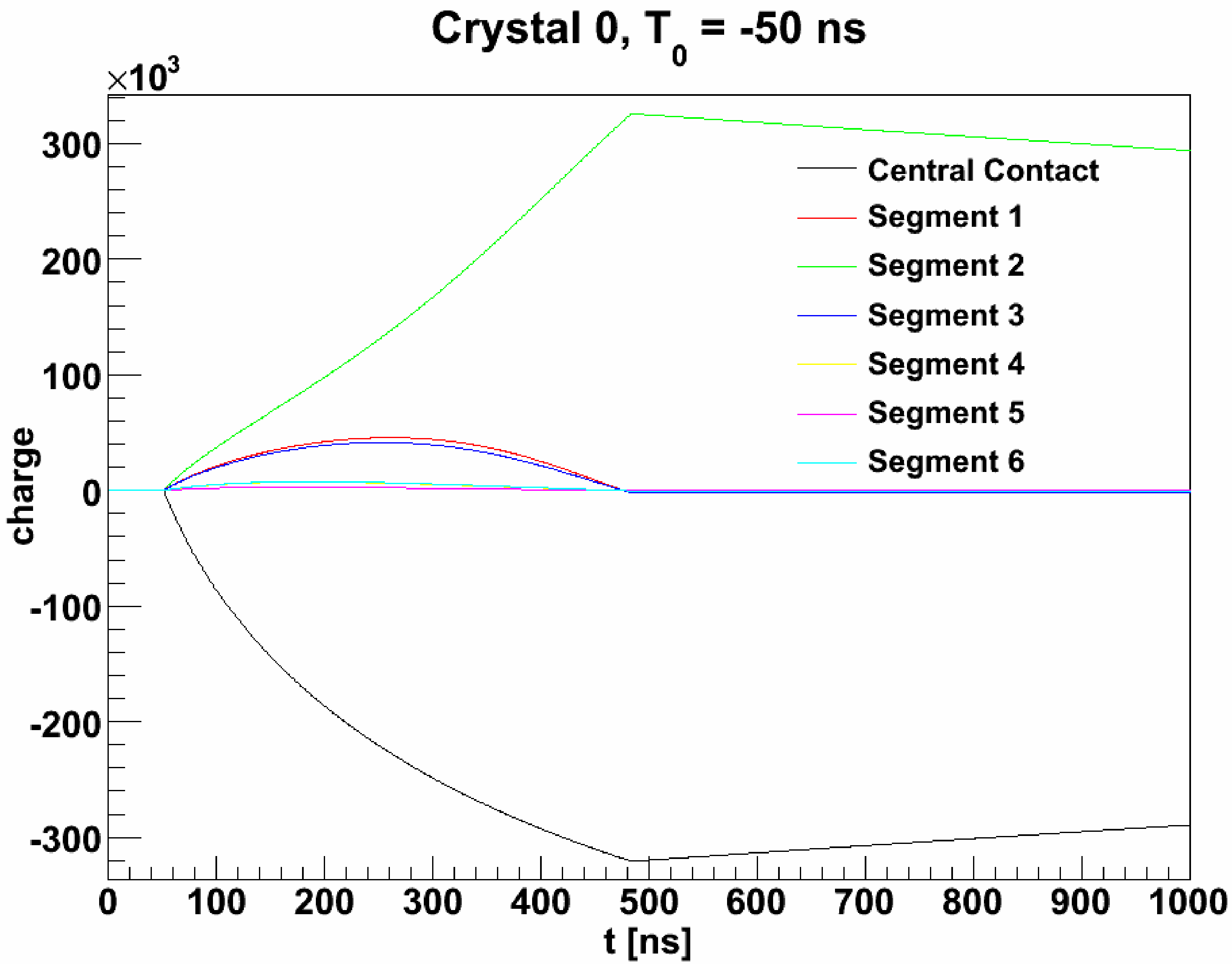




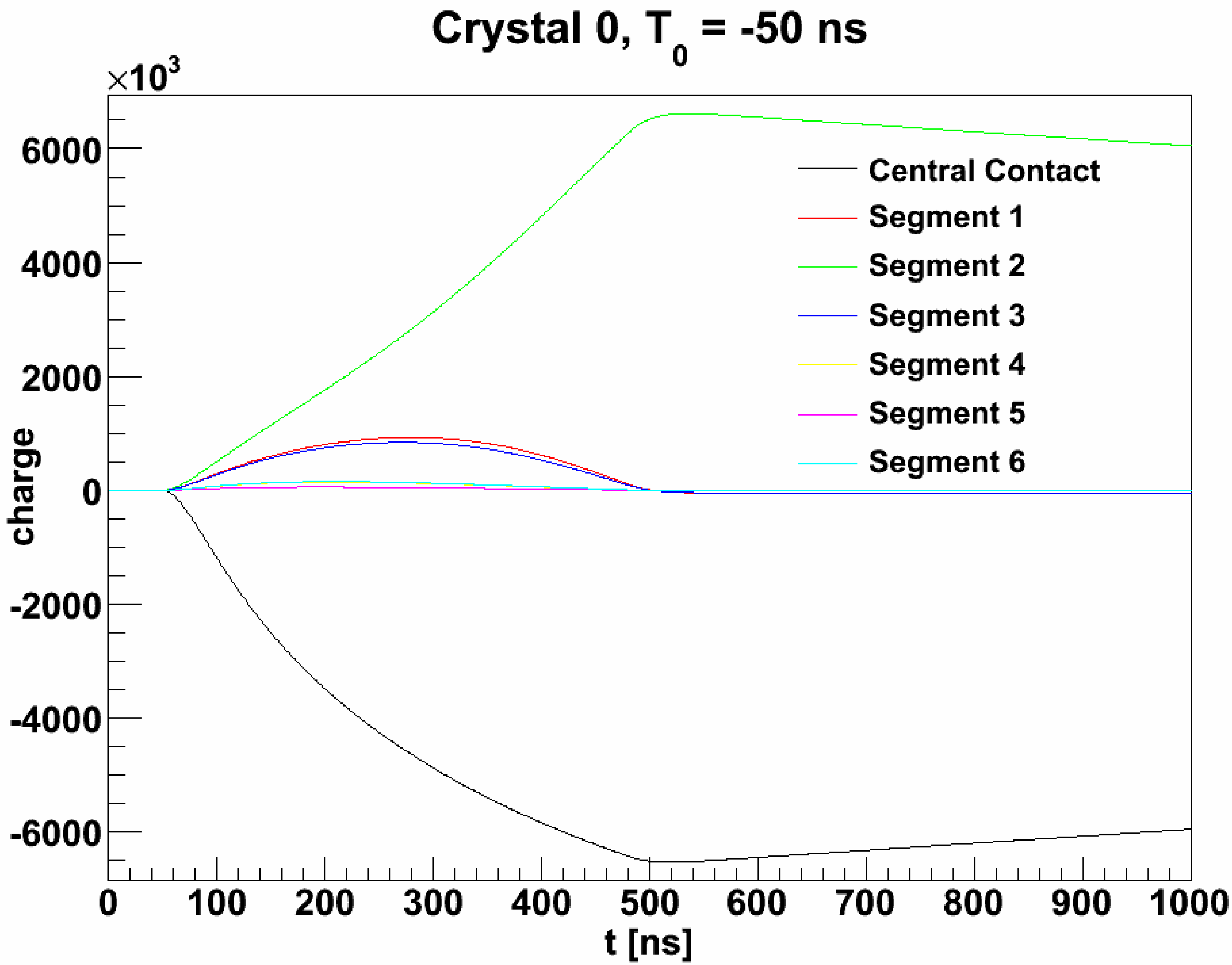
Ideal pulse shape



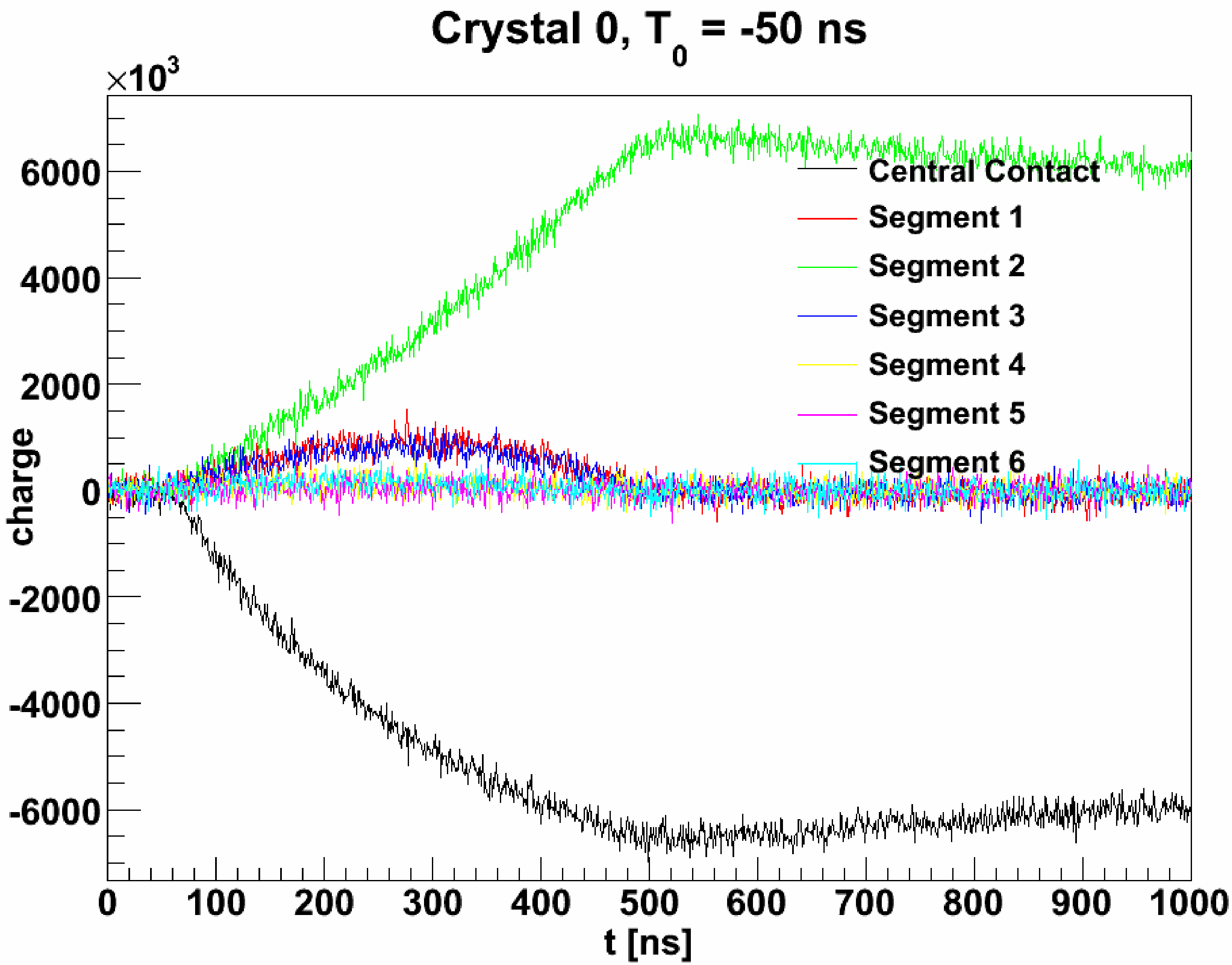
Simulation of decay time



Fold in bandwidth

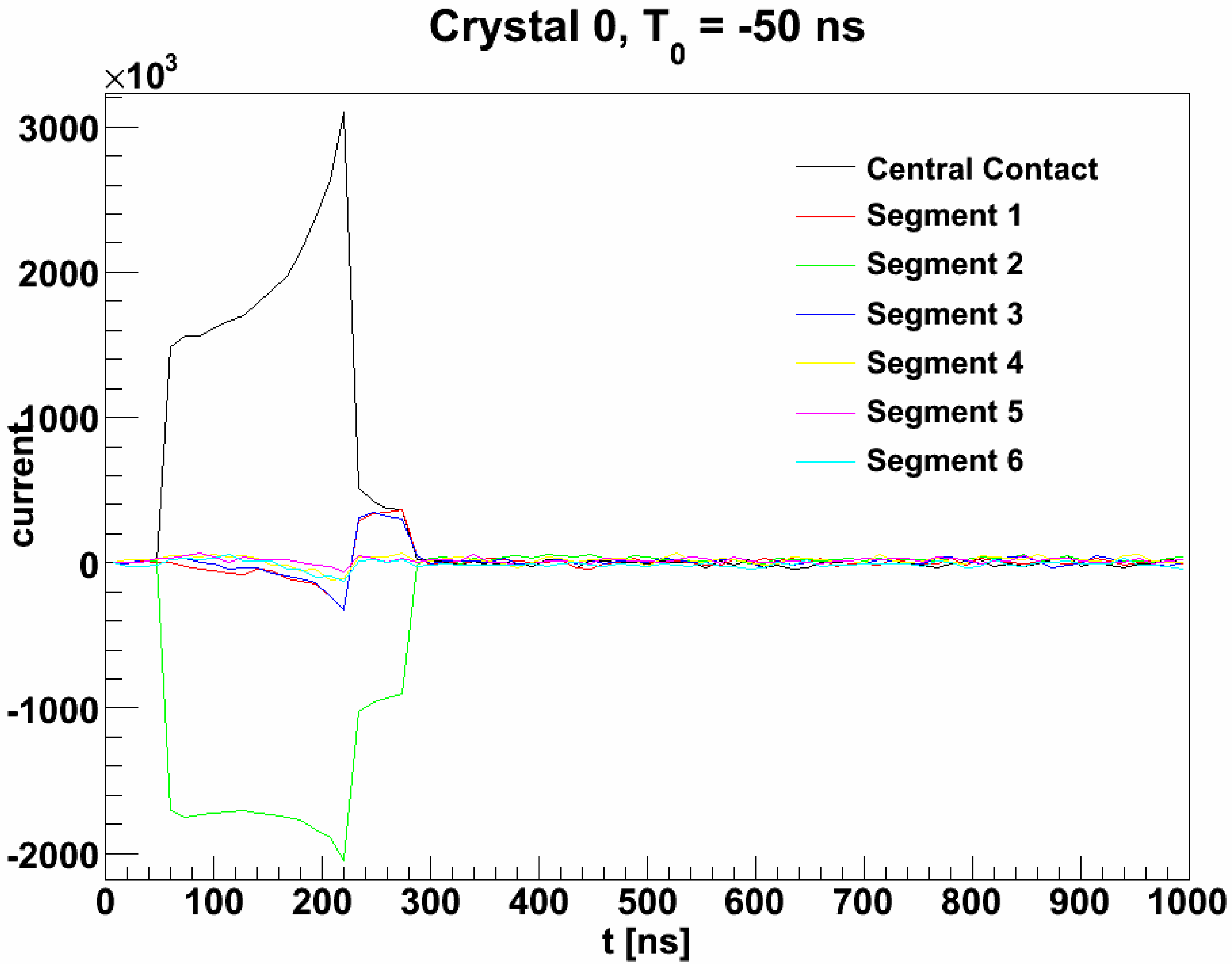


Let's add some noise



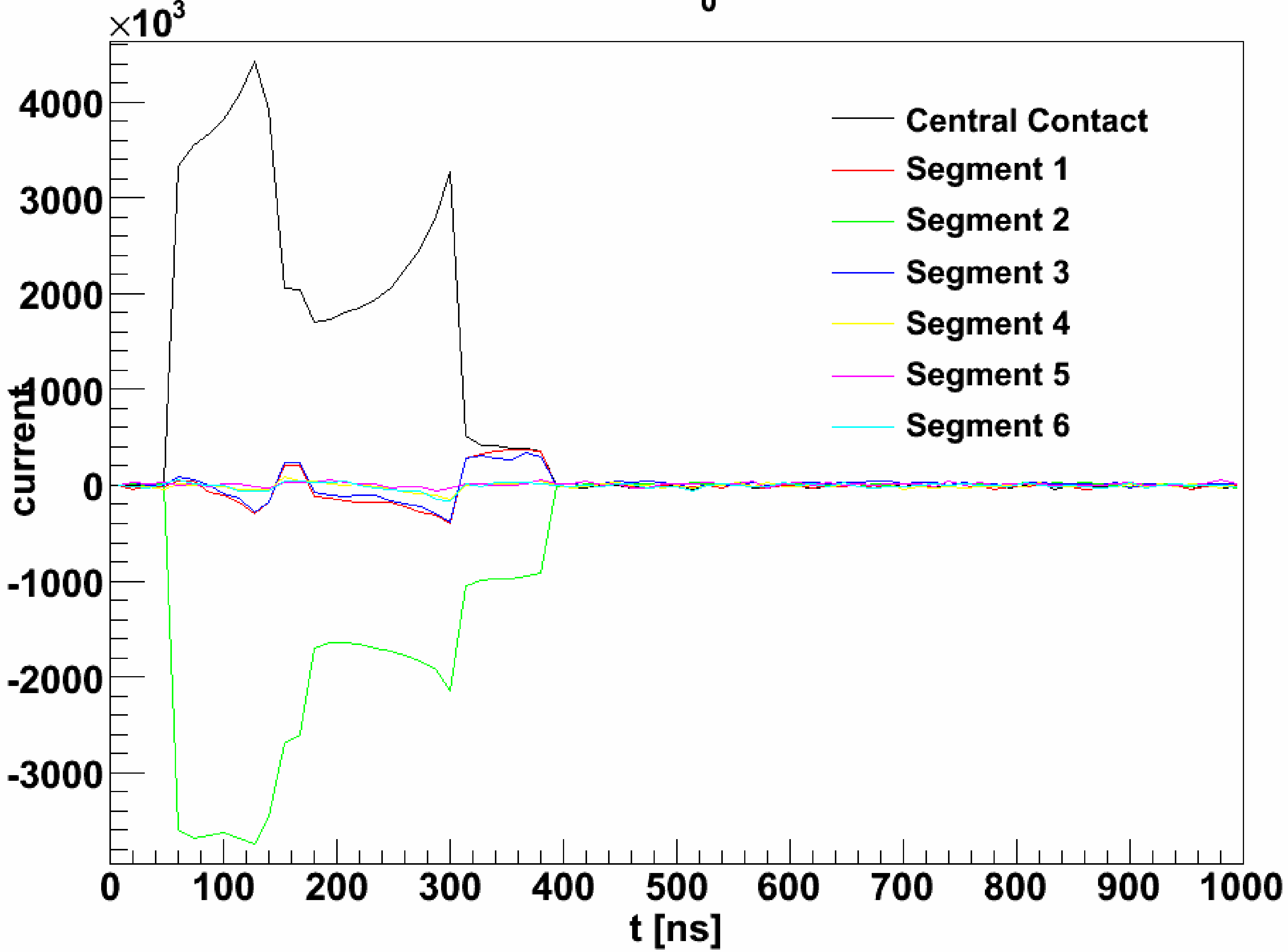
Summary, Acknowledgement & Outlook

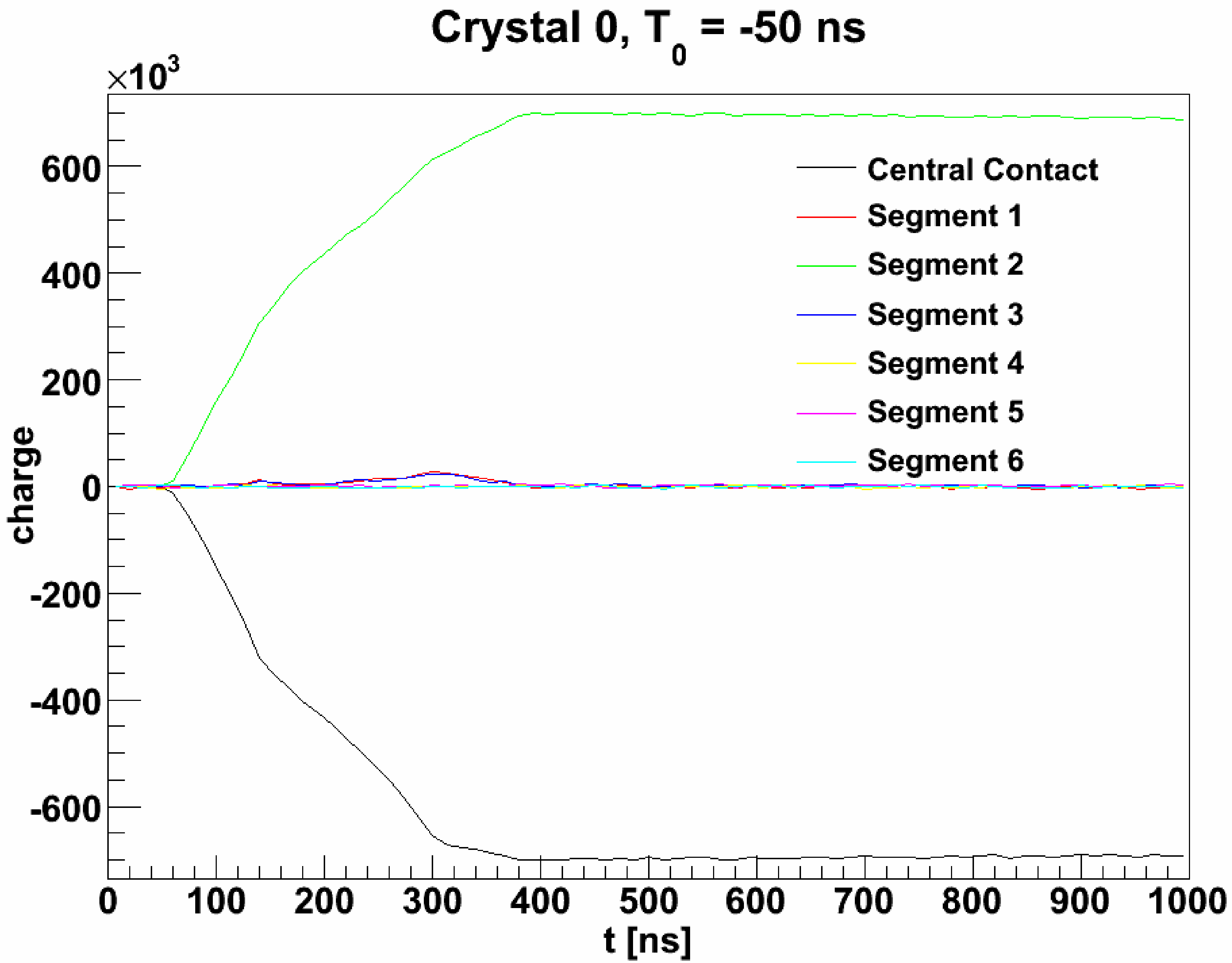
- Now we can simulate the whole signal formation process in germanium detector system!
- Many thanks to Majorana MC group!
- Validation of the simulation on the way
- Pulse shape analysis follows



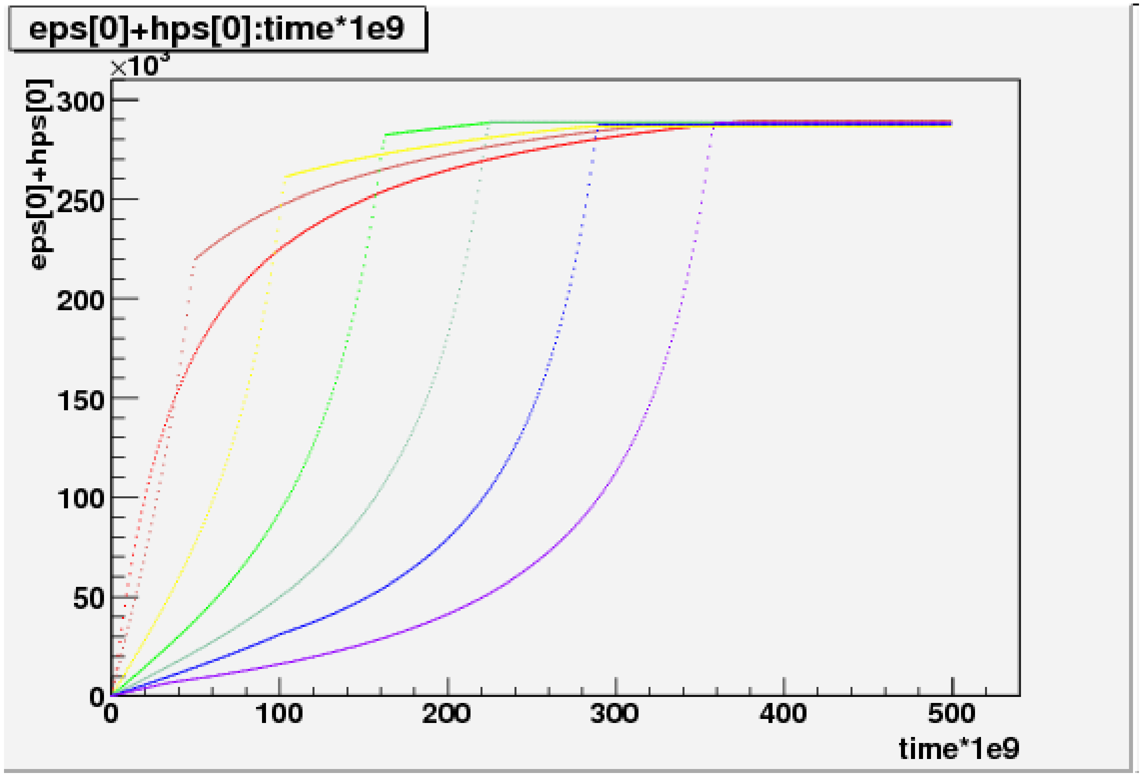
Crystal 0, T0 = -50 ns
×10³
3000
2000
1000
0
-1000
-2000
current
Central Contact
Segment 1
Segment 2
Segment 3
Segment 4
Segment 5
Segment 6
0
100
200
300
400
500
600
700
800
900
1000
t [ns]

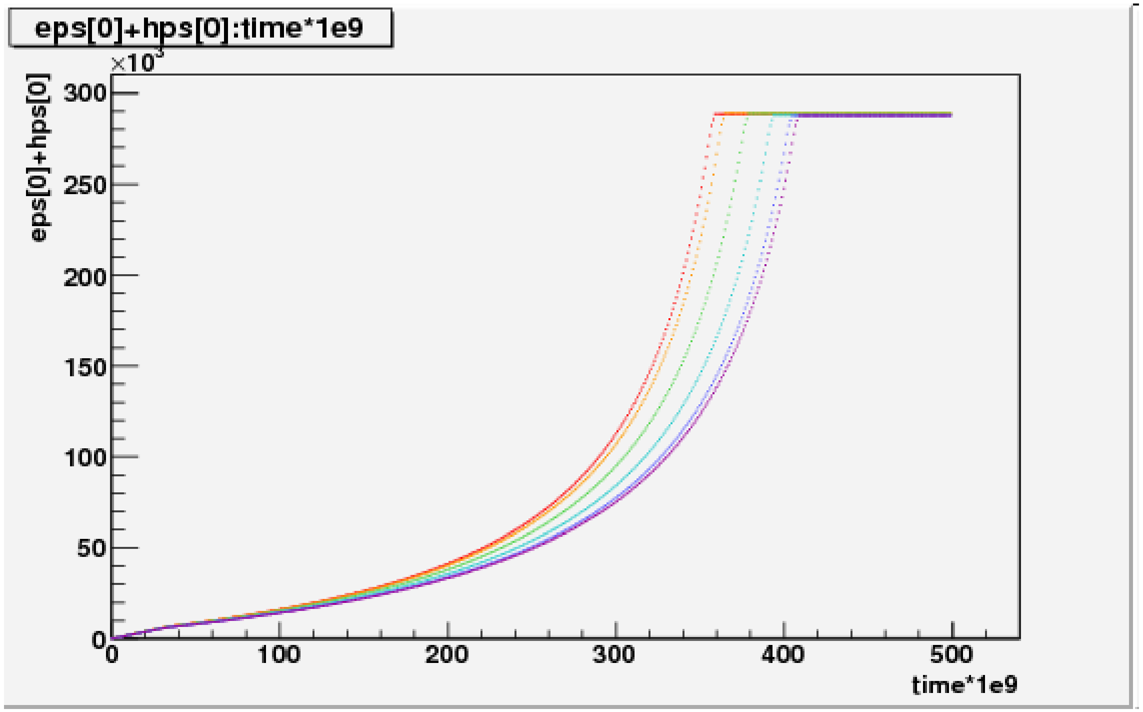
Crystal 0, T_0 = -50 ns

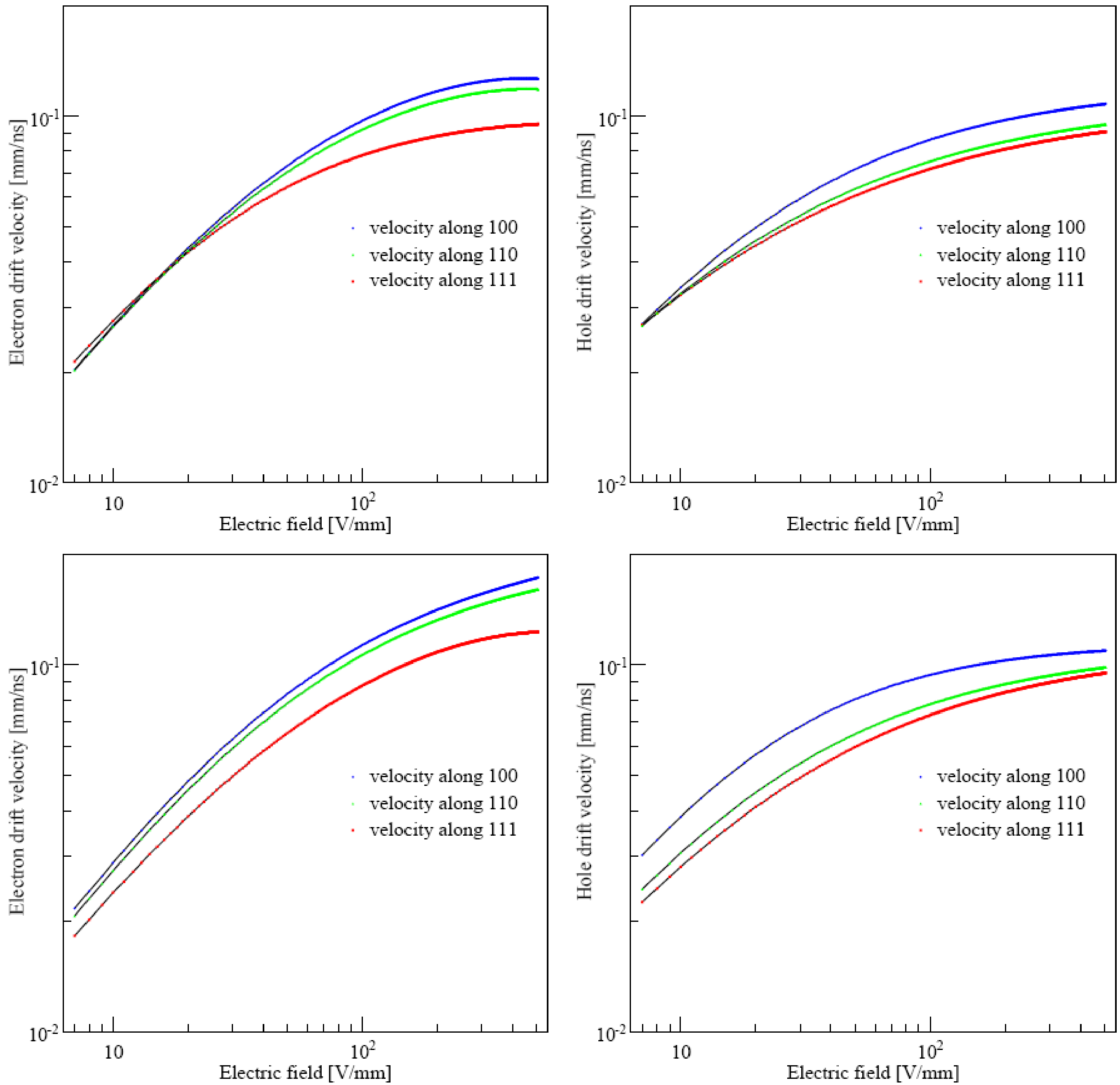




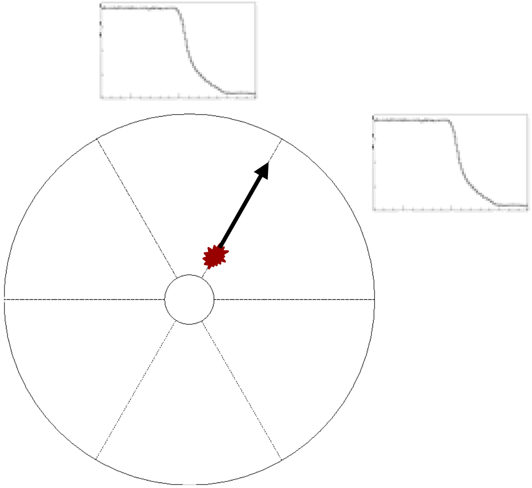
Crystal 0, T_0 = -50 ns
×10[3]
600
400
200
0
-200
-400
-600
charge
0
100
200
300
400
500
600
700
800
900
1000
t [ns]
Central Contact
Segment 1
Segment 2
Segment 3
Segment 4
Segment 5
Segment 6


eps[0]+hps[0]:time*1e9
×10³
300
250
200
150
100
50
0
eps[0]+hps[0]
0
100
200
300
400
500
time*1e9


eps[0]+hps[0]:time*1e9
×10³
eps[0]+hps[0]
300
250
200
150
100
50
0
0
100
200
300
400
500
time*1e9


Electron drift velocity [mm/ns]
10^{-1}
10^{-2}
10
10^2
Electric field [V/mm]
velocity along 100
velocity along 110
velocity along 111
Hole drift velocity [mm/ns]
10^{-1}
10^{-2}
10
10^2
Electric field [V/mm]
velocity along 100
velocity along 110
velocity along 111
Electron drift velocity [mm/ns]
10^{-1}
10^{-2}
10
10^2
Electric field [V/mm]
velocity along 100
velocity along 110
velocity along 111
Hole drift velocity [mm/ns]
10^{-1}
10^{-2}
10
10^2
Electric field [V/mm]
velocity along 100
velocity along 110
velocity along 111

Find back 0νββ events
occur between two segments



Calculate the drift :: Parameters fitted with experimental data

Parameterization values fitted with the experimental data are summarized in Table 1. The values from two different references are quite different from each other. We'd better measure it by ourselves again.

Reference	Carrier	Direction	μ_0 $\left[\frac{cm^2}{V \cdot s}\right]$	E_0 $\left[\frac{V}{cm}\right]$	β	μ_n $\left[\frac{cm^2}{V \cdot s}\right]$
Ref. [8]	Electron	⟨111⟩	40180	493	0.72	589
		⟨100⟩	42420	251	0.87	62
	Hole	⟨111⟩	107270	100	0.58	0
		⟨100⟩	66333	181	0.744	0
Ref. [10]	Electron	⟨111⟩	38536	538	0.641	510
		⟨100⟩	38609	511	0.805	-171
	Hole	⟨111⟩	61215	182	0.662	0
		⟨100⟩	61824	185	0.942	0

Table 1: Fit parameters for the experimental drift velocities in the ⟨111⟩ and ⟨100⟩ directions.

Calculate the drift :: Electron drift velocity

$$\gamma_0 \equiv \begin{pmatrix} m_t^{-1} & 0 & 0 \\ 0 & m_l^{-1} & 0 \\ 0 & 0 & m_t^{-1} \end{pmatrix} \quad m_t = 1.64 m_e \quad m_l = 0.0819 m_e$$

Effective electron mass tensor: $\gamma_j = R_j^{-1}\gamma_0 R_j = R_j^T \gamma_0 R_j$

$$\mathbf{v}_e(\mathbf{E}) = \mathcal{A}(E) \sum_j \frac{n_j}{n} \frac{\gamma_j \mathbf{E_0}}{\sqrt{\mathbf{E_0}\gamma_j\mathbf{E_0}}}, \text{ with } j = 1,2,3,4$$

$$\frac{n_j}{n} = \mathcal{R}(E)\left[\frac{\sqrt{\mathbf{E_0}\gamma_j\mathbf{E_0}}}{\sum_i \sqrt{\mathbf{E_0}\gamma_i\mathbf{E_0}}} - \frac{n_e}{n}\right] + \frac{n_e}{n}$$

Fraction of electrons in certain directions

$$v = \frac{\mu_0 E}{[1 + (\frac{E}{E_0})^\beta]^{1/\beta}} - \mu_n E \text{ in the } \langle 111 \rangle \text{ and } \langle 100 \rangle \text{ directions}$$

Calculate the drift :: Hole drift velocity

$$\begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix} = R_z(\phi + \frac{\pi}{4} + \phi_{110}) R_{y'}(\theta) \begin{pmatrix} v_{x'} \\ v_{y'} \\ v_{z'} \end{pmatrix}$$

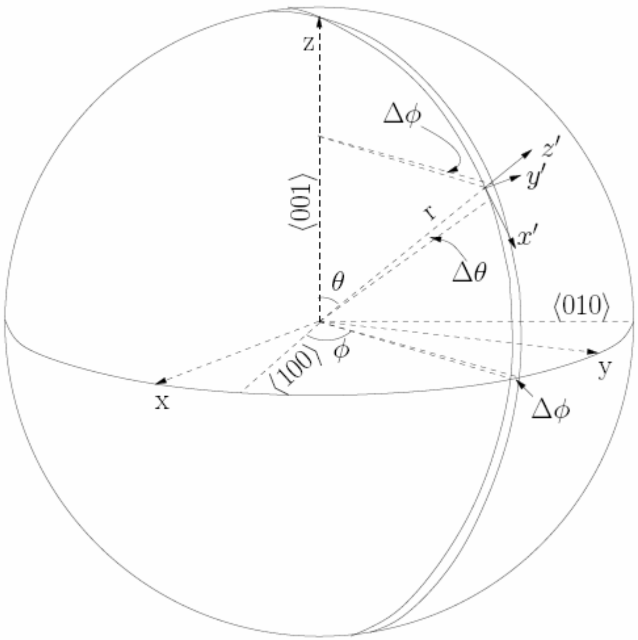
$$\begin{aligned} v_{x'} = v_r &= v_h^{100}(E)[1 - \Lambda(k_0)(\sin(\theta)^4 \sin(2\phi)^2 + \sin(2\theta)^2)], \\ v_{y'} = v_\theta &= v_h^{100}(E)\Omega(k_0)[2\sin(\theta)^3 \cos(\theta)\sin(2\phi)^2 + \sin(4\theta)] \\ v_{z'} = v_\phi &= v_h^{100}(E)\Omega(k_0)\sin(\theta)^3 \sin(4\phi). \end{aligned}$$

$$\Lambda(k_0) = -0.01322k_0 + 0.41145k_0^2 - 0.23657k_0^3 + 0.04077k_0^4$$

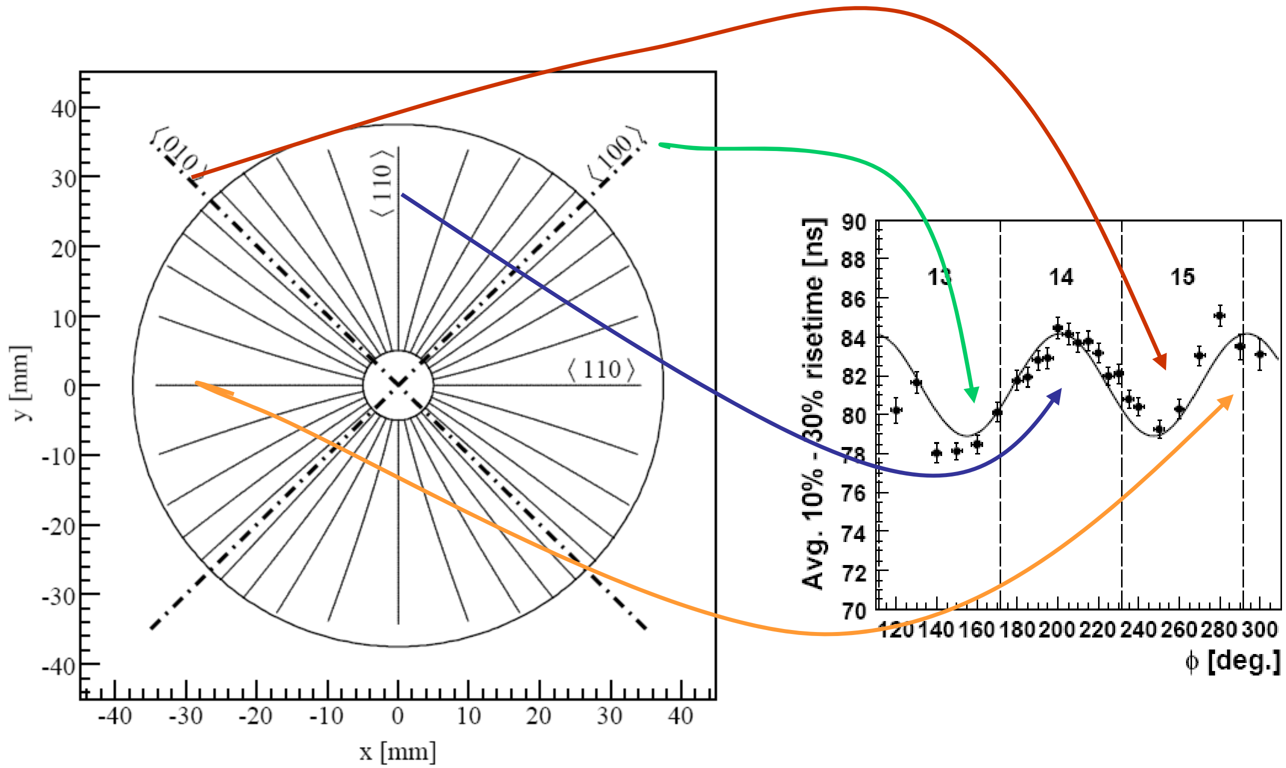
$$\Omega(k_0) = 0.006550k_0 - 0.19946k_0^2 + 0.09859k_0^3 - 0.01559k_0^4$$

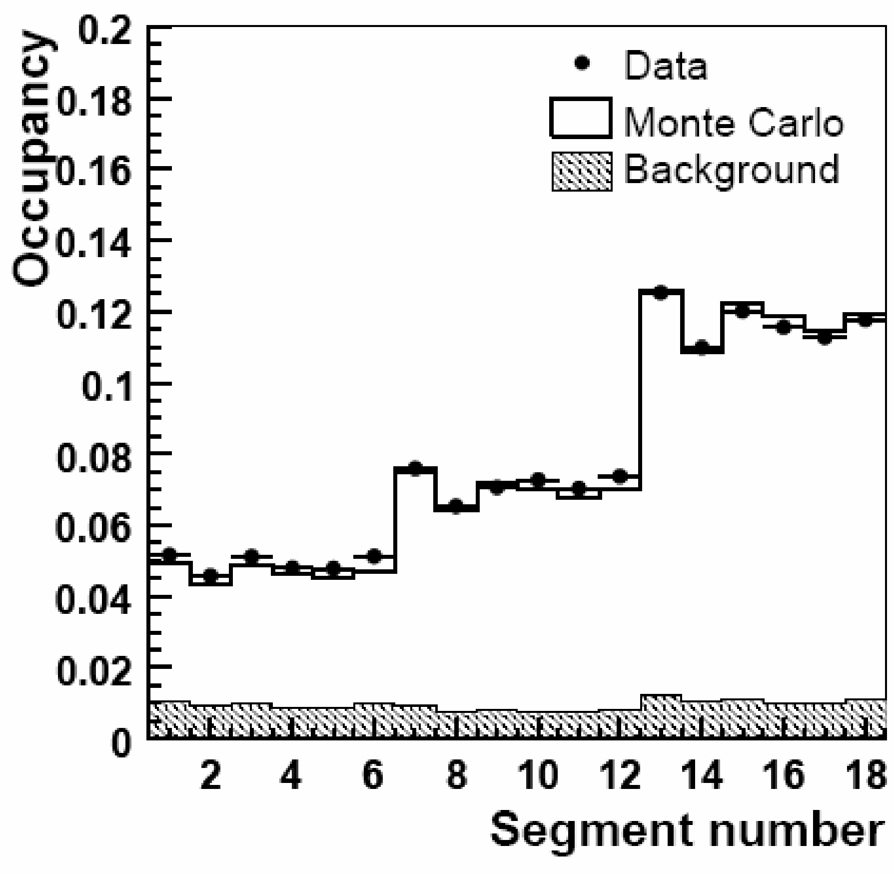
$$k_0(v_{rel}) = 9.2652 - 26.3467v_{rel} + 29.6137v_{rel}^2 - 12.3689v_{rel}^3, \; v_{rel} = v_h^{111}(E)/v_h^{100}(E)$$

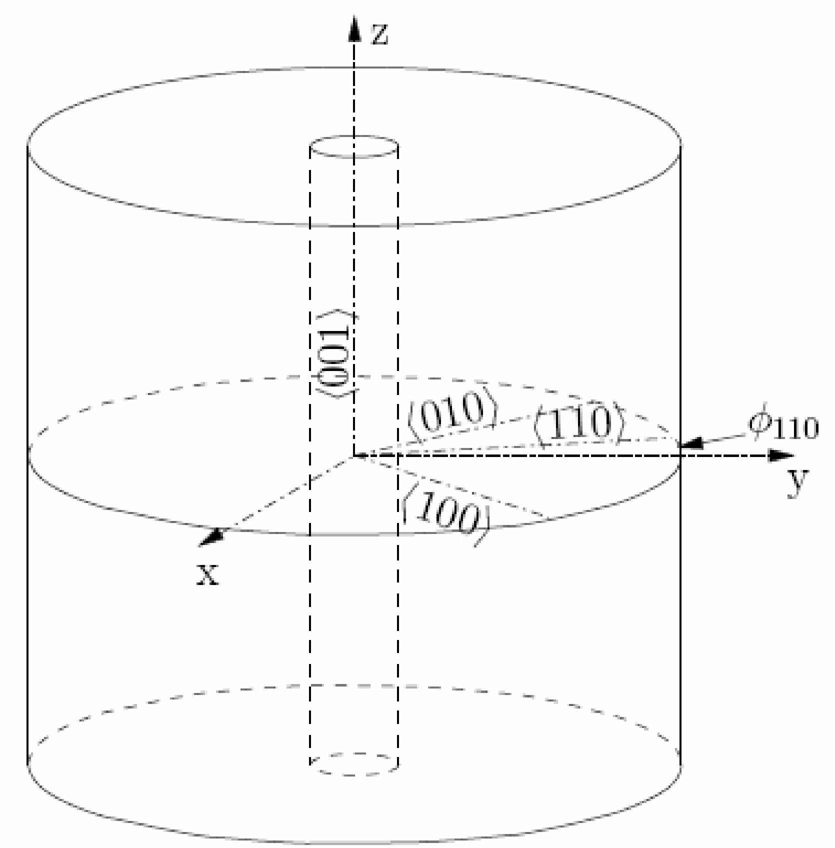
$$v = \frac{\mu_0 E}{[1 + (\frac{E}{E_0})^\beta]^{1/\beta}} - \mu_n E \text{ in the } \langle 111 \rangle \text{ and } \langle 100 \rangle \text{ directions}$$



Pulse rise time




Occupancy
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Data
Monte Carlo
Background
2
4
6
8
10
12
14
16
18
Segment number


z
⟨001⟩
⟨010⟩
⟨110⟩
ϕ_{110}
y
⟨100⟩
x