Sensitivity of GERDA

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Reference: GSTR-06-006 A.C., K. Kröninger

1. Analysis strategy
2. Mathematical framework
3. GERDA simulations & sensitivity results
4. Proposed GERDA standard plots

Analysis logic:

1. Decide if you have evidence for a signal
2. Then,
   a) If yes, proceed to determine best value of parameters
   b) If no, set probability limits on possible value of parameters

Note:

• for phase I, we expect 10^{-2}/(kg keV yr), MT=30 kg-yr; I.e.,0.3 events/keV

• for phase II, we expect 10^{-3}/(kg keV yr), MT=100-200 kg-yr; I.e.,0.1-0.2 events/keV

We need to use an analysis technique which is appropriate for small event numbers !

How do we decide whether or not we have evidence for a signal ?

Define the proposition:

H = The observed spectrum is due to background only

We then evaluate $p(H|\text{spectrum})$, the probability (degree-of-belief) assigned to this proposition after observing the spectrum.

If $p(H|\text{spectrum}) < \text{cut}$, claim evidence for something beyond background. If we assume that **what is not background is signal**, then we claim evidence for the signal.

Proposal: $p(H|\text{spectrum}) < 0.01$, evidence for signal
$p(H|\text{spectrum}) < 0.0001$, discovery criterion

Note: intended to be the real ‘degree-of-belief’. **No fudging allowed afterwards. I.e., must believe the numbers**.

What we know how to calculate:
p(spectrum|H) - the probability to observe the spectrum given H
(We assume Poisson statistics are valid)

How do we go from p(spectrum|H) to p(H|spectrum) ?

Certainly p(A|B) $\neq$ p(B|A)
(e.g., 3σ deviation from SM expectations does not mean SM model ruled out with 3σ certainty)

Start with joint probability p(spectrum,H). Then,

$$p(spectrum,H) = p(H \mid spectrum)\, p(spectrum) = p(spectrum \mid H)\, p(H)$$

$$p(H \mid spectrum) = \frac{p(spectrum \mid H)\, p(H)}{p(spectrum)}$$

Bayes' Theorem

p(H) is called the prior belief in H (before we do the experiment). We will write it with a subscript in the following $p_0(H)$. It is a critical part of the Bayesian analysis. Our belief in the truthfulness of H always depends on prior beliefs. In addition, it depends on other information, usually summarized by letter I (see note). E.g.,

The existing limits are $T_{1/2}$>4 10^{25} yr; a positive claim for a signal exists at the level $T_{1/2}$=1.2 10^{25} yr; my favorite theorist believes strongly that neutrinos are Majorana particles, but he wont tell me the neutrino mass; the theorist at a neighboring university says that he believes strongly in Leptogenesis, and in that context the neutrino is a Majorana particle but it must be very light, such that neutrinoless double beta decay is unobservable,...

What is p(spectrum) ? Expand (law of total probability)

$$p(spectrum) = p(spectrum \mid H)\,p(H) + p(spectrum \mid \overline{H})\,p(\overline{H})$$

Now we need also the negation of H. In our case, we assume perfect knowledge of the background, so

$\overline{H}$ **= The spectrum is due to background + signal (neutrinoless double beta decay).**

I.e., we assume all backgrounds are known, and the only possibility other than known background is signal from neutrinoless double beta decay. It is possible to add other possibilities, but the analysis is considerably messier.

$$p(H \mid spectrum) + p(\overline{H} \mid spectrum) = 1$$

so

$$p(H \mid spectrum) = \frac{p(spectrum \mid H)\, p_0(H)}{p(spectrum \mid H)\, p_0(H) + p(spectrum \mid \overline{H})\, p_0(\overline{H})}$$

$$p(\overline{H} \mid spectrum) = \frac{p(spectrum \mid \overline{H})\, p_0(\overline{H})}{p(spectrum \mid H)\, p_0(H) + p(spectrum \mid \overline{H})\, p_0(\overline{H})}$$

Now we know how to perform all calculations:

$$p(spectrum \mid H) = \int p(spectrum \mid B)\, p_0(B)\, dB$$

$$p(spectrum \mid \overline{H}) = \int p(spectrum \mid S, B)\, p_0(S)\, p_0(B)\, dB$$

Where B is the expected number of background events and S is the expected number of signal events. These quantities come with their own priors.

$$n_i = \text{ observed number of events in bin i}$$

$$\lambda_i = \text{ expected number of events in bin i}$$

$$\lambda_i = S \int_{\Delta E_i} f_S(E)\, dE + B \int_{\Delta E_i} f_B(E)\, dE$$

Where f_S and f_B are the normalized signal and background shape functions then

$$p(spectrum \mid B) = \prod_{i=1}^{N} \frac{\lambda_i(0,B)^{n_i}}{n_i!} e^{-\lambda_i(0,B)}$$

$$p(spectrum \mid S,B) = \prod_{i=1}^{N} \frac{\lambda_i(S,B)^{n_i}}{n_i!} e^{-\lambda_i(S,B)}$$

To determine parameter values or set limits, we need

$$p(S,B \mid spectrum) = \frac{p(spectrum \mid S,B)\, p_0(S)\, p_0(B)}{\int p(spectrum \mid S,B)\, p_0(S)\, p_0(B)\, dS\, dB}$$

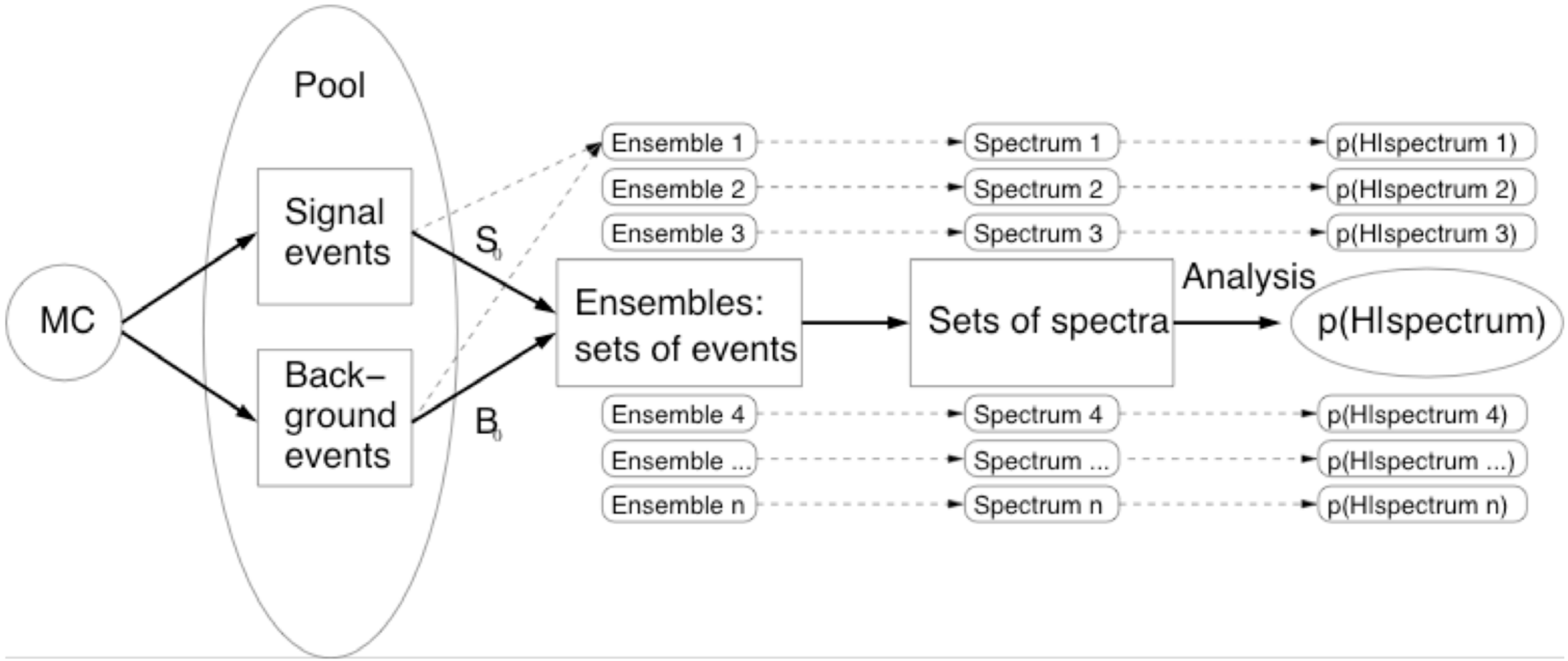
and then marginalize

$$p(S \mid spectrum) = \int p(S,B \mid spectrum)\, dB$$

e.g., 90% probability upper limit, S_{90} from solving

$$\int_0^{S_{90}} p(S \mid spectrum)\, dS = 0.90$$

So we know how to calculate probabilities given an experimental outcome. What do we do to check the sensitivity of the experiment ? We generate ensembles of possible experimental results, which will depend on particular choices of background and signal, B_0 and S_0. Then we can make distributions of the probabilities which could result under these conditions.



Assumptions for GERDA:

$$p_0(H) = p_0(\overline{H}) = 1/2$$

$$p_0(S) = \frac{1}{S_{\max}} \quad 0 \le S \le S_{\max} \quad p_0(S) = 0 \text{ otherwise}$$

$$p_0(B) = \frac{e^{-\frac{(B-\mu_B)^2}{2\sigma_B^2}}}{\int_0^\infty e^{-\frac{(B-\mu_B)^2}{2\sigma_B^2}} dB} \quad B \ge 0; \quad p_0(B) = 0 \; B < 0$$

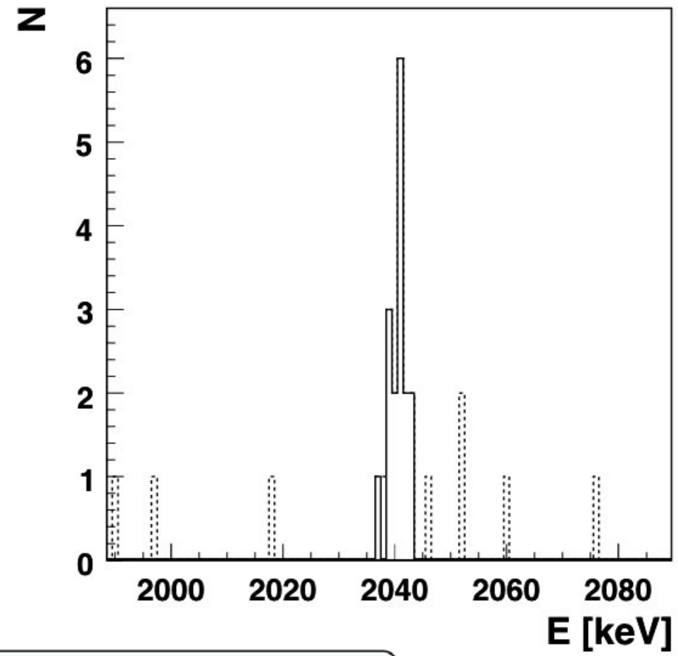
S_{max} was calculated assuming $T_{1/2}$=0.5 10^{25} yr

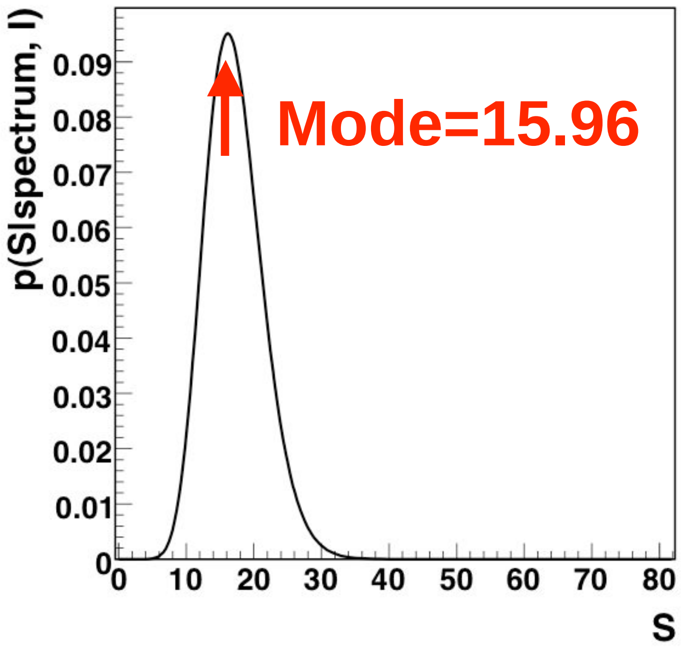
$\mu_B = B_0$, $\sigma_B = B_0/2$

100 keV window analyzed. B_0 total background in this window.

Example:

$S_{true}=16$, $B_{true}=9$





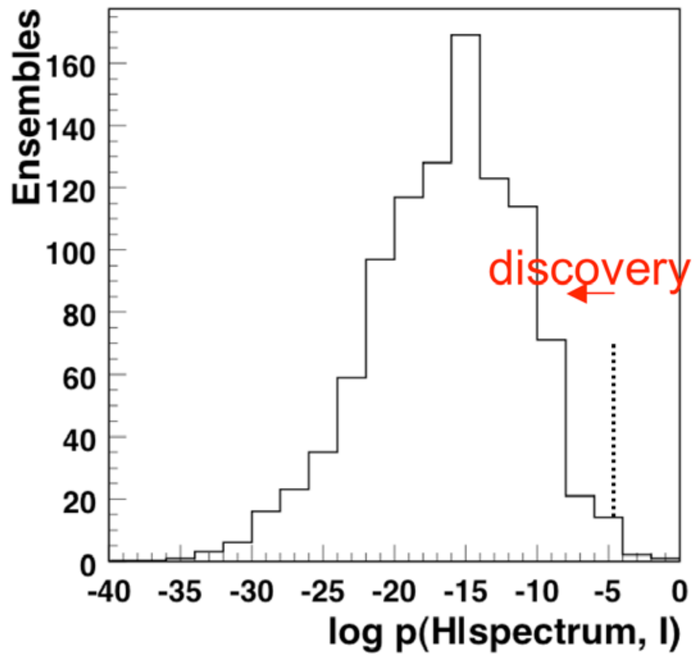
$$p(H \mid spectrum) = 2.2 \cdot 10^{-12}$$

1000 experiments simulated with

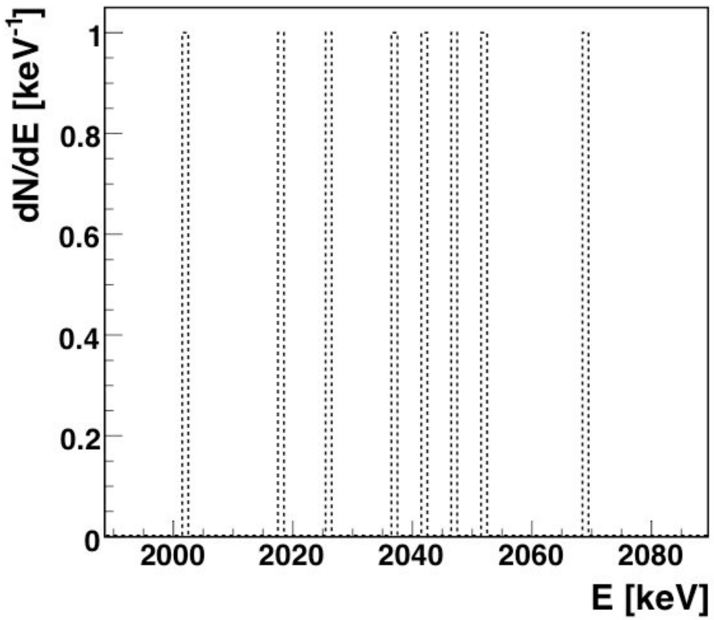
$T_{1/2}=2\ 10^{25}$ yr, 10^{-3}/(kg keV yr)

Exposure 100 kg-yr

About 95% chance a discovery could be claimed



Example:

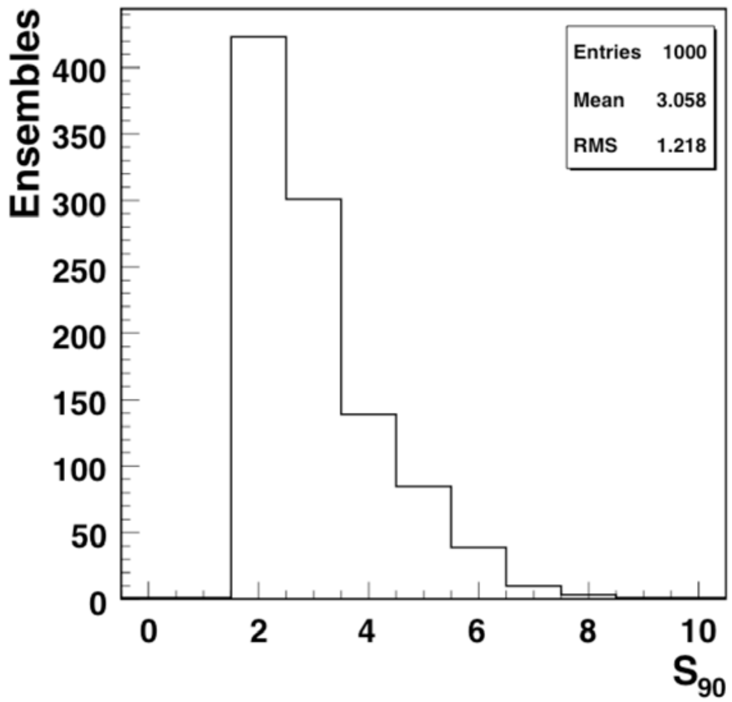


$$p(H \mid spectrum) = 0.93$$

S_{true}=0, B_{true}=8

$$S_{90} = 3.99$$

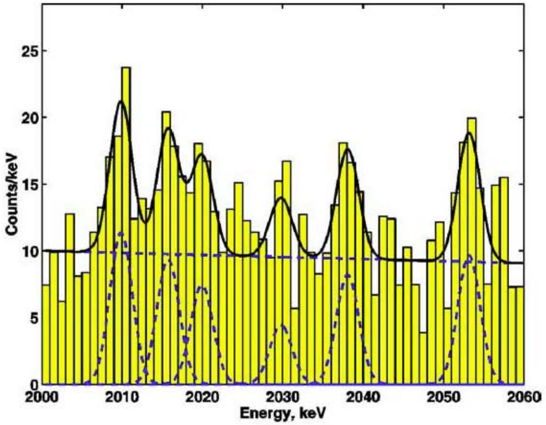
10^{-3}/(kg keV yr)
Exposure 100 kg-yr

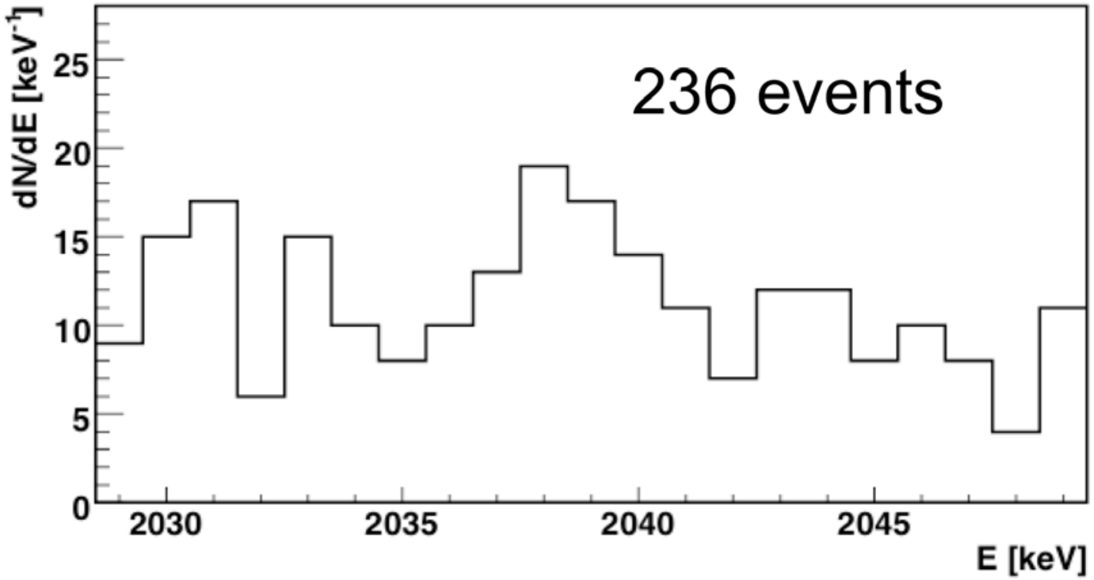


1000 experiments simulated

0 false claims of a discovery

Example: analysis of Klapdor-Kleingrothaus et al. spectrum





Region 2039±10 keV analyzed

Background 9 counts/bin
20% uncertainty

$$p_0(H) = p_0(\overline{H}) = 1/2$$

$$P_0(B) = \frac{e^{-\frac{(B-\mu_B)^2}{2\sigma_B^2}}}{\int_0^\infty e^{-\frac{(B-\mu_B)^2}{2\sigma_B^2}} dB} \qquad \mu_B = 184, \sigma_B = 37$$

Calculation yields p(H|spectrum)=0.023

To translate the event numbers into lifetimes, we use

$$S = \ln 2 \cdot \kappa \cdot M \cdot \varepsilon_{sig} \cdot \frac{N_A}{M_A} \cdot \frac{T}{T_{1/2}}$$

Where:

N_A is Avogadro’s number

M_A is the atomic mass of ^{enr}Ge

M is the total mass of Germanium

κ is the enrichment factor (by atom, 0.86 used)

ε_{sig} is the signal efficiency (taken to be 87%)

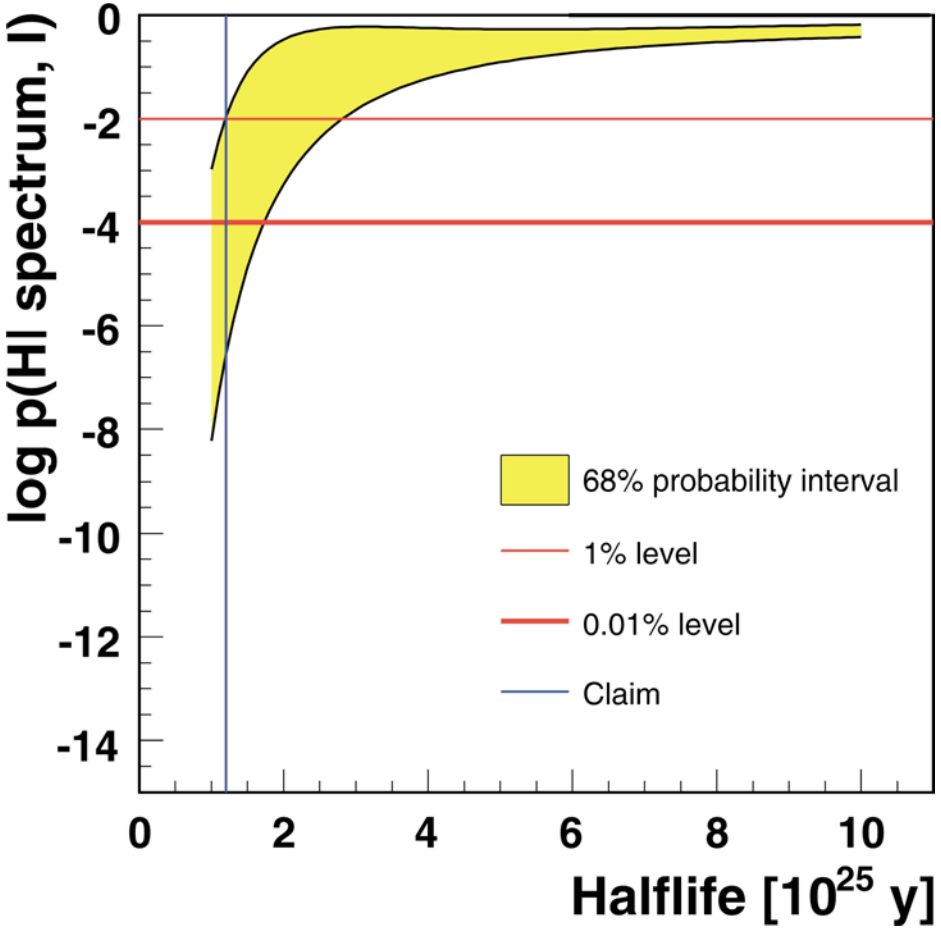
To translate $T_{1/2}$ to a mass $$\langle m_\nu \rangle = \left(T_{1/2} G^{0\nu}\right)^{-1/2} \cdot \frac{1}{M_{0\nu}}$$

$G^{0\nu}$ and $T_{1/2}$ from Rodin, Faessler, Simkovic, Vogel nucl-th/0503063

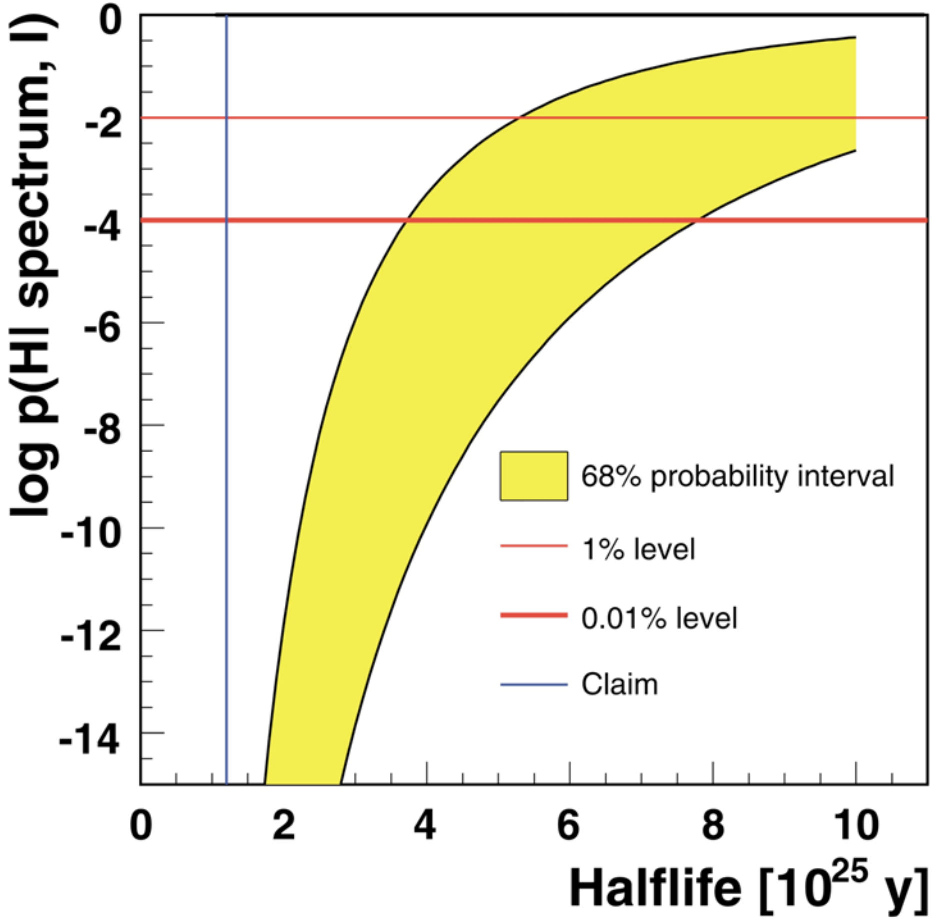
Proposed official GERDA plots

Discovery potential

Phase I: 30 kg-yr, 10^{-2}/(kg keV yr)

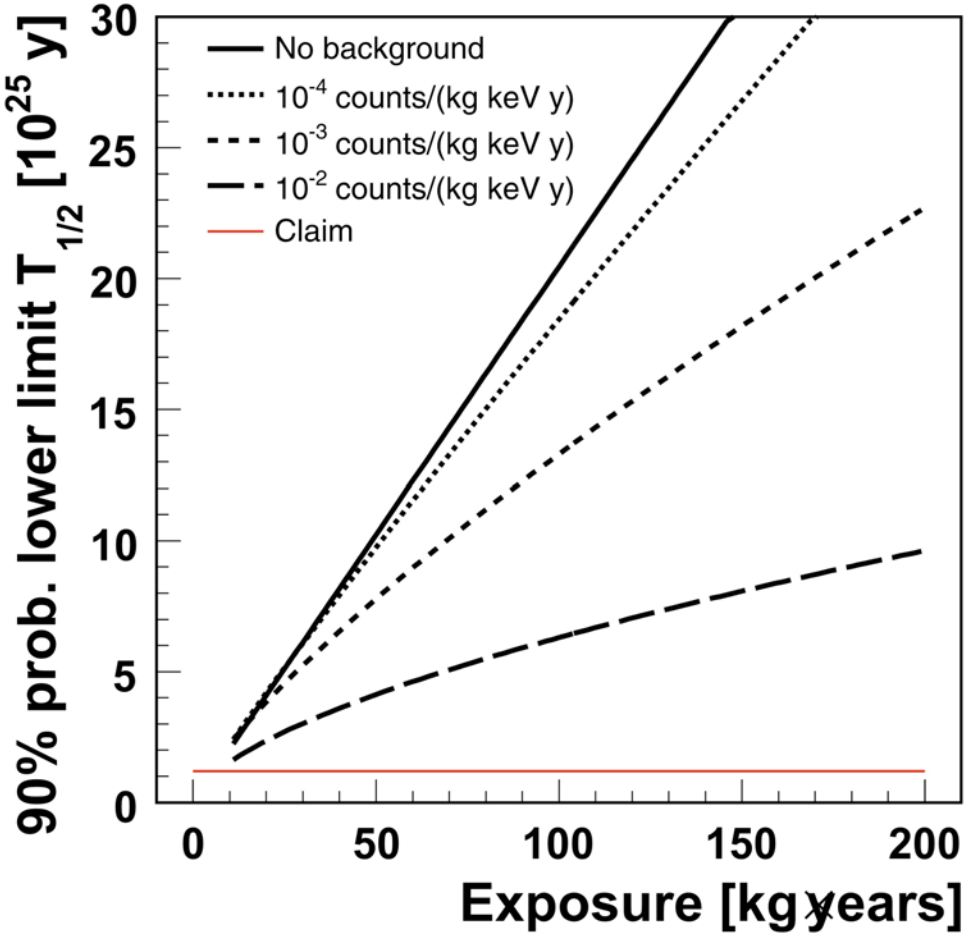


Phase II: 100 kg-yr, 10^{-3}/(kg keV yr)



Proposed official GERDA plots

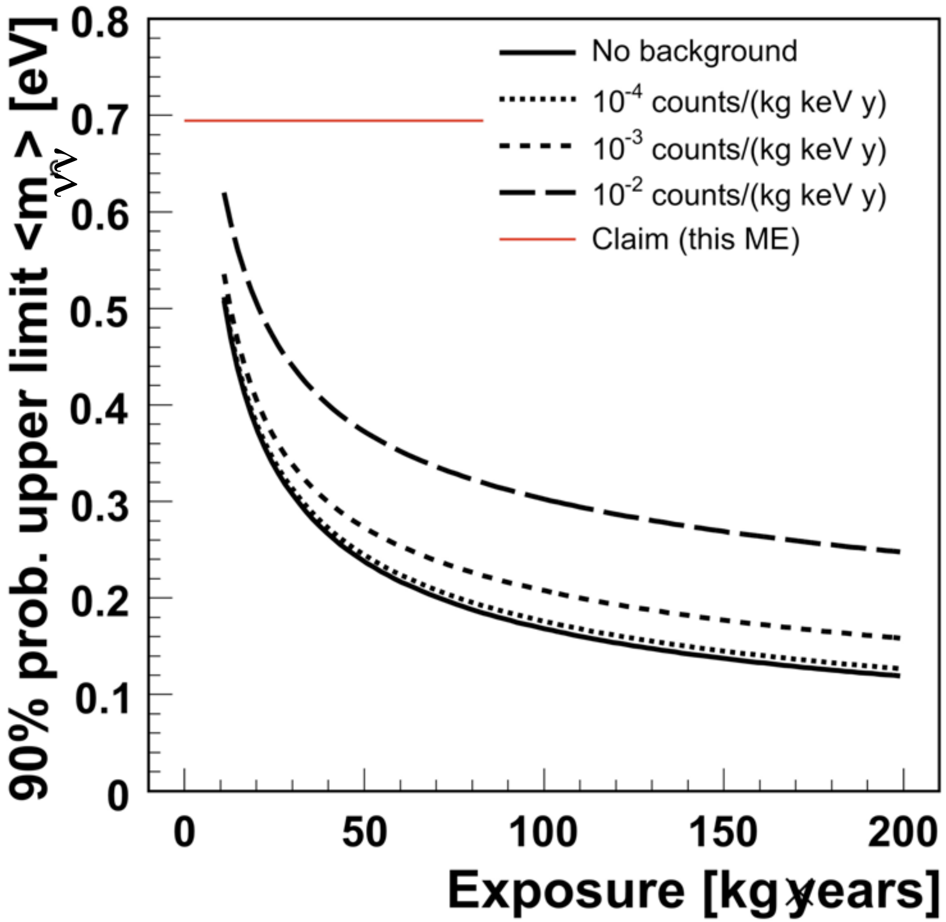
Limit Setting



Note: in this case, we plot the expected limit (otherwise plot would be too messy).

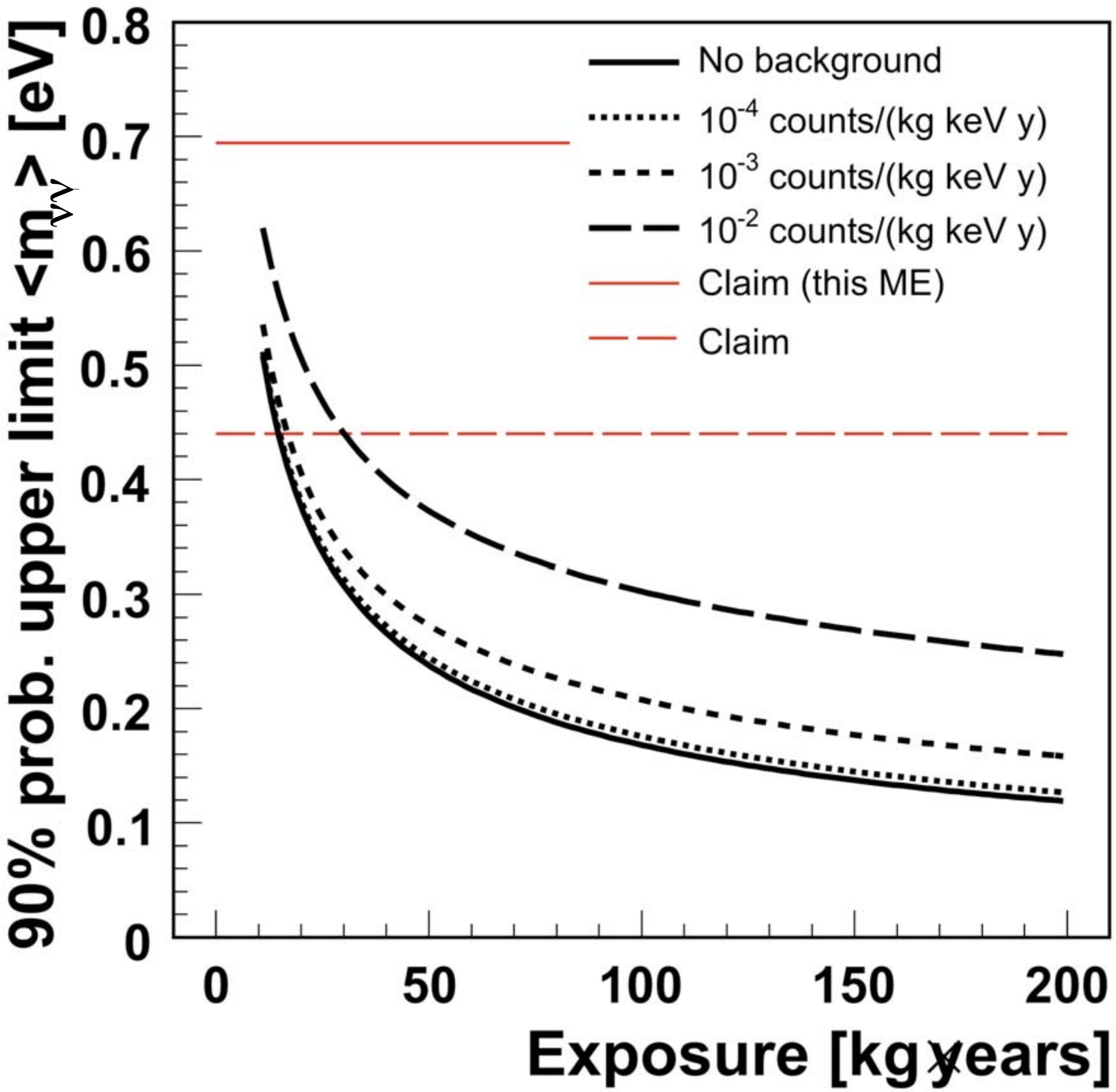
Proposed official GERDA plots

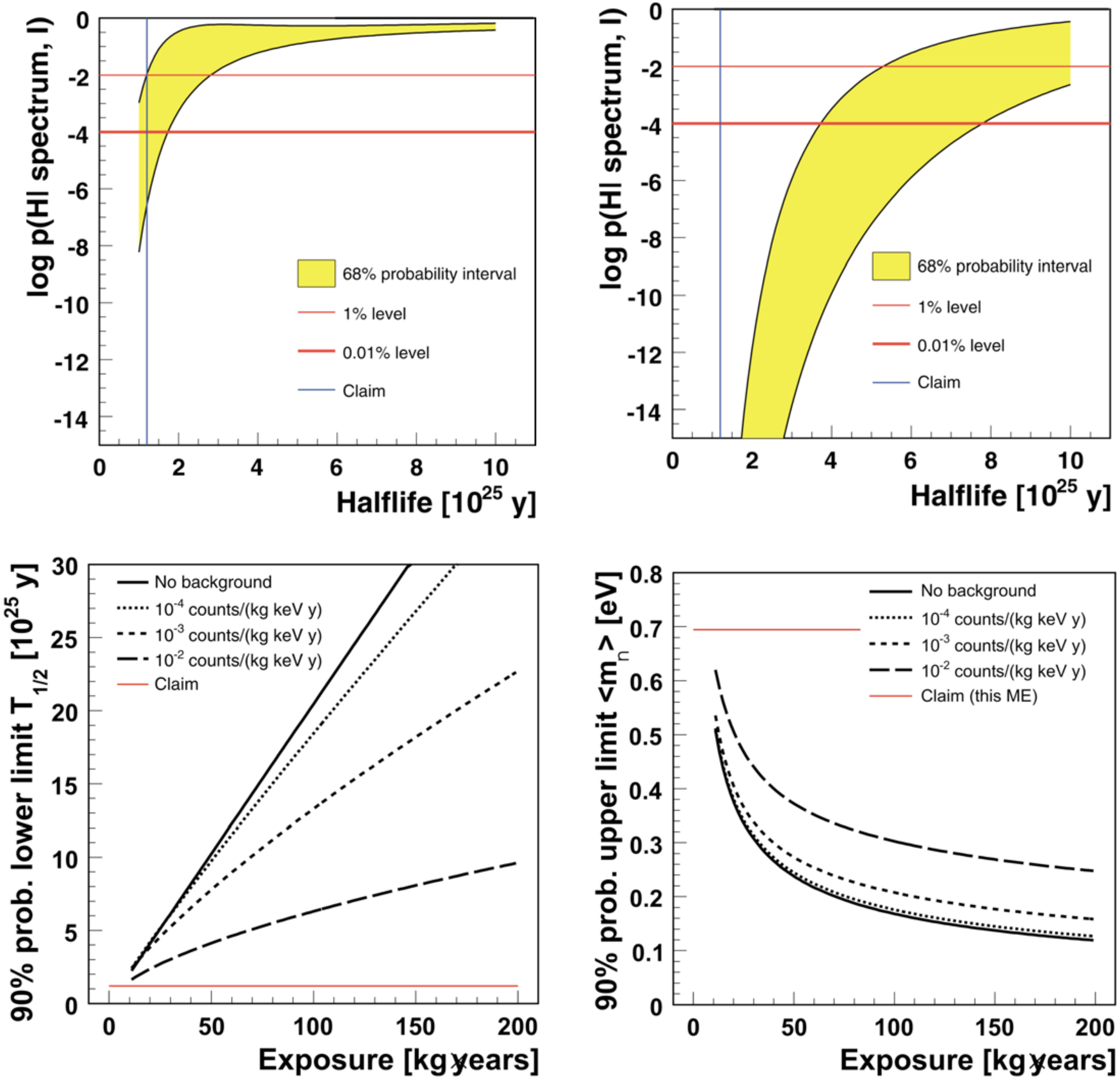
Limit Setting



Note that KK et al. claim corresponds to much larger mass than reported by KK et al. due to different matrix element.

Comparison with KK et al. matrix element





Proposed Official GERDA Plots